Hyperfine Splitting in the Bottomonium System on the Lattice and in the Continuum

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in collaboration with
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Overview

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$$M(\Upsilon_{1S}) - M(\eta_b) = E_{\text{hfs}}$$

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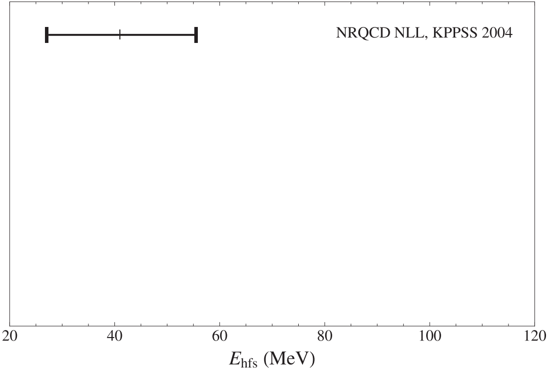

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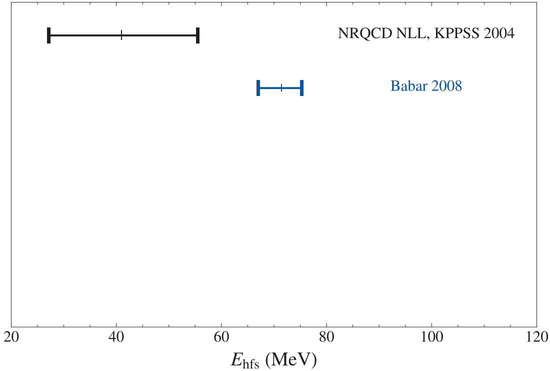

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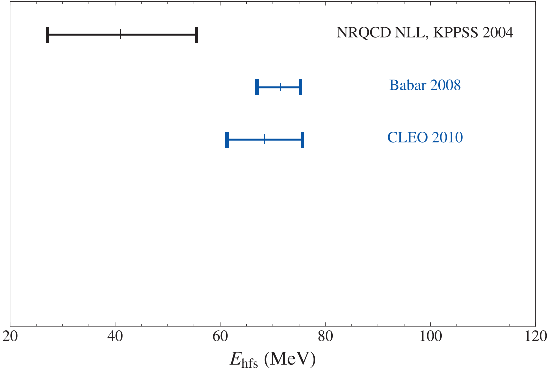

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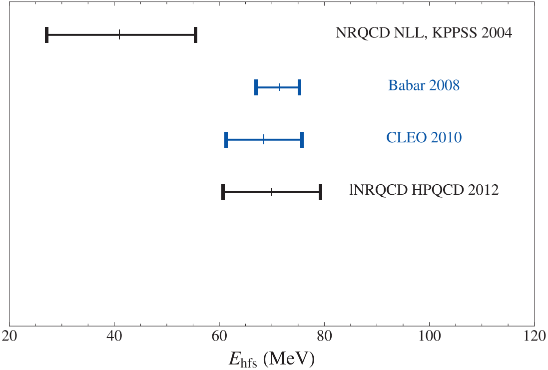

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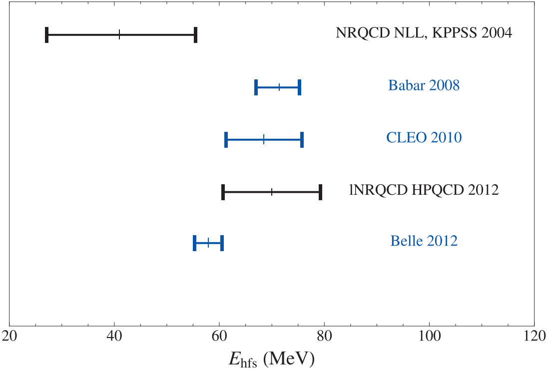

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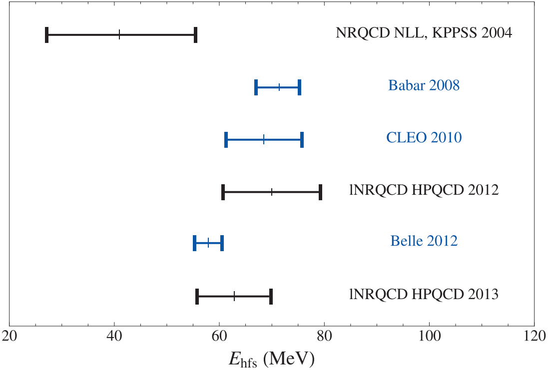

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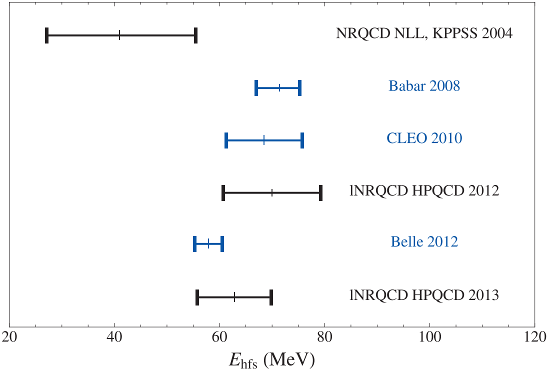

- Is perturbative QCD breaking down?
- Is “new physics” needed? [Domingo *et al.* 2009]

Basics

- Quantum configuration:

	q's	$SU(3)_{\text{Color}}$	$SU(2)_{\text{Spin}}$	n	L	$I^G(J^{PC})$
η_b	$b\bar{b}$	1	1	1	0	$0^+(0^{-+})$
Υ_{1S}	$b\bar{b}$	1	3	1	0	$0^-(1^{--})$
h_b	$b\bar{b}$	1	1	1	1	$0^-(1^{+-})$
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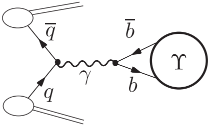
- Υ_{1S} production:



$$\sqrt{s} = M(\Upsilon_{1S}) = 9460.30 \pm 0.26\ \text{MeV}$$

[E288 collaboration 1977]

[PDG 2014]

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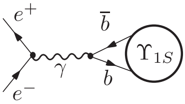

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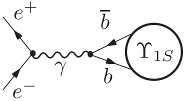

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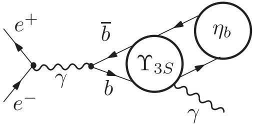
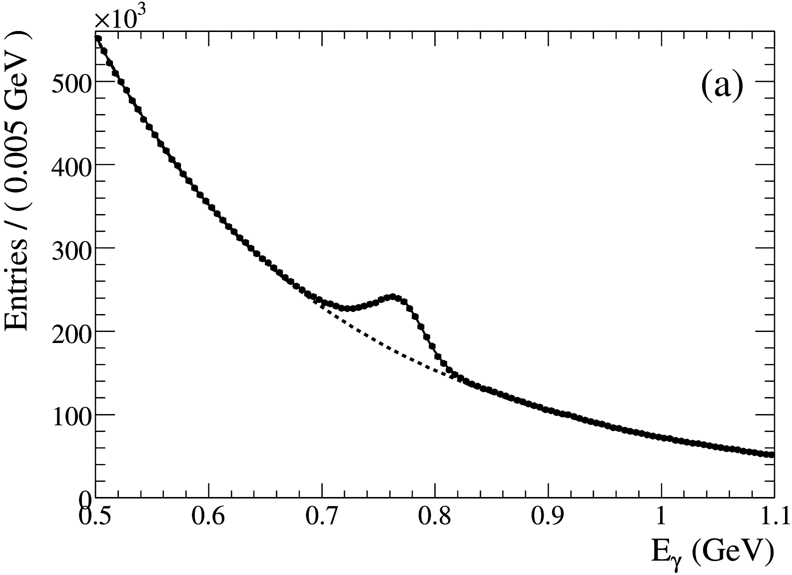


[BaBar 2008]

Experimental Results

- BaBar 2008: Inklusive photon spectrum



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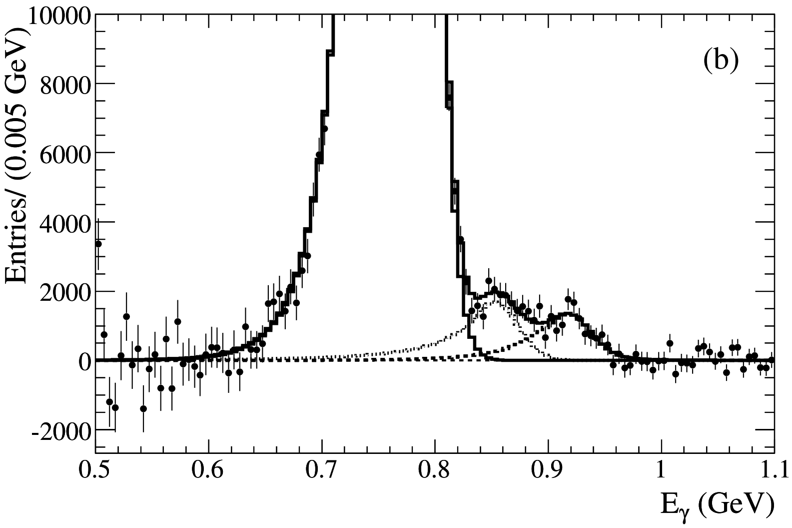

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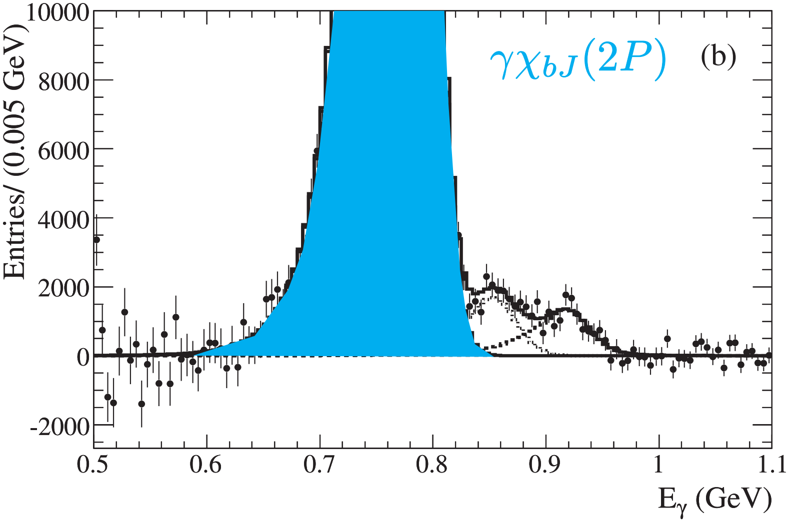

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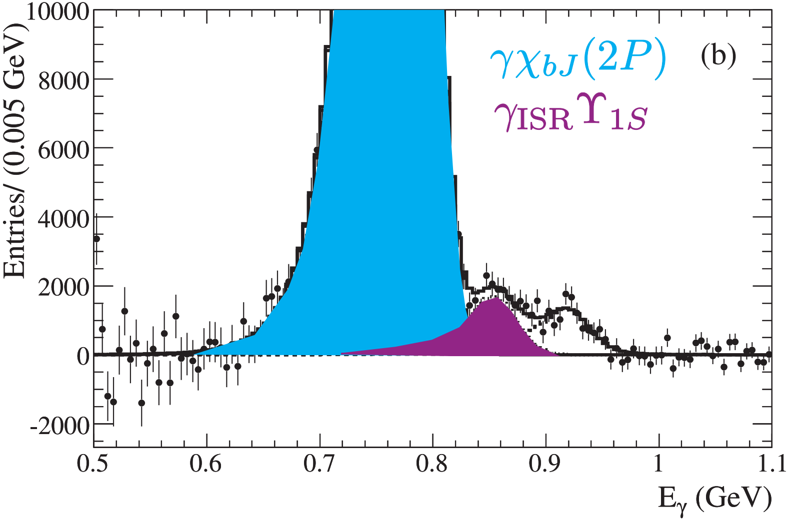

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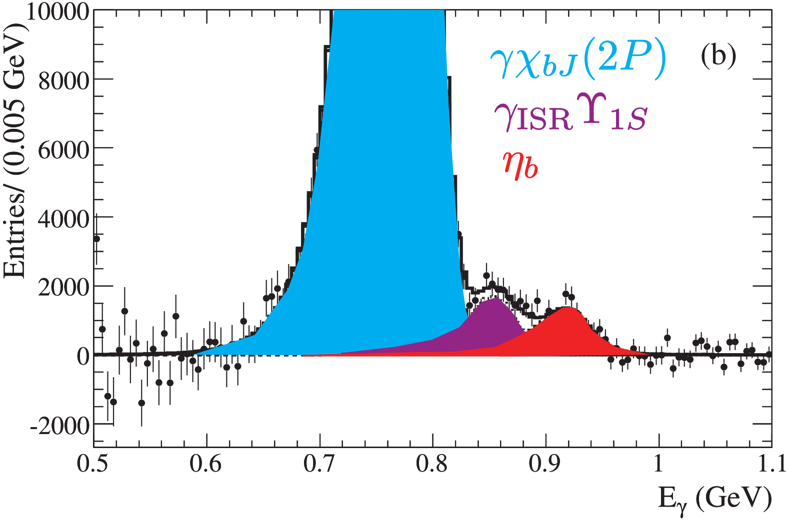

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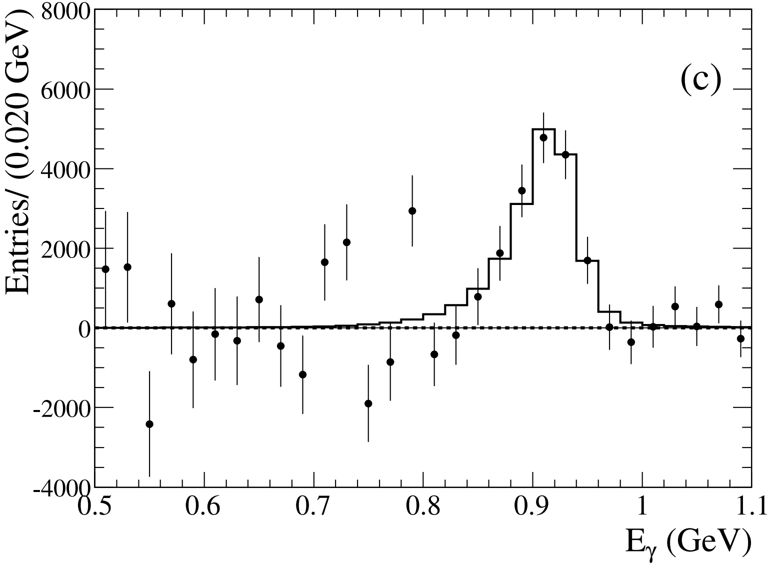
- BaBar 2008: (η_b signal)



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$$E_{\text{hfs}} = 71.4^{+2.3}_{-3.1} \pm 2.7\ \text{MeV}\,.$$

Experimental Results

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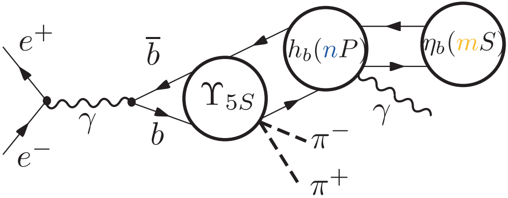

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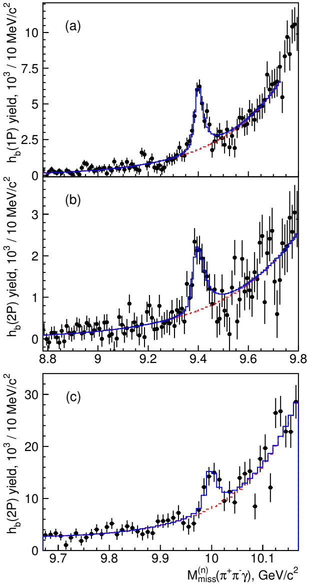

$$h_b(1P) \to \gamma\eta_b(1S)$$

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$$E_{\text{hfs}} = 71.4^{+2.3}_{-3.1} \pm 2.7\ \text{MeV}$$

- Belle 2012:

$$E_{\text{hfs}} = 57.9 \pm 2.3\ \text{MeV}$$

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- Lattice NRQCD simulation
  - ⊕ non-perturbative effects
  - ⊕ less demanding for hardware (scales separation)
  - ⊕ systematic ordering of radiative corrections in α_s, v

Perturbative pNRQCD calculation

- EFT-Tower:

QCD $\qquad \overline{\Psi}(i\not{D} - m)\Psi$

$\downarrow$ integrating out m

$NRQCD$ $\qquad \psi^\dagger(iD_0 + \frac{\mathbf{D}^2}{2m} - g_s \frac{c_F}{2m}\sigma \cdot \mathbf{B})\psi + ...$

$\downarrow$ integrating out vm ($v =$ velocity)

$pNRQCD$ $\qquad \hat{H}|\phi\rangle = E|\phi\rangle\,, \qquad \hat{H} = -\frac{\nabla^2}{m} - \frac{C_F\alpha_s}{r}(1 + \dots) + V_{\mathbf{S}} + \dots$

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- Spin dependence:

$NRQCD$ $\qquad \mathcal{L}_\sigma = g_s \frac{c_F}{2m}\psi^\dagger\sigma \cdot \mathbf{B}\psi + (\psi \to \chi_c) + d_\sigma \frac{C_F\alpha_s}{m^2}\psi^\dagger\sigma\psi\chi_c^\dagger\sigma\chi_c$

$pNRQCD$ $\qquad V_{\mathbf{S}}(\mathbf{q}^2) = (c_F^2 - \frac{3}{2}d_\sigma)\frac{4\pi C_F\alpha_s(\mathbf{q}^2)\mathbf{S}^2}{3m^2} + \ldots$

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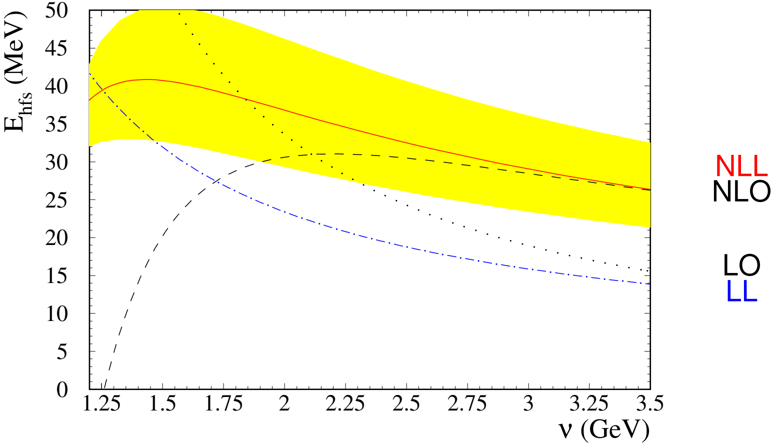
- E_{hfs} calculated within pNRQCD using TIPT

Perturbative pNRQCD calculation

- $E_{\rm hfs}$ result: [Kniehl, Penin, Pineda, Soto, Steinhauser, Smirnov 2004]



$$E_{\rm hfs} = 41 \pm 11({\rm th})^{+9}_{-8}(\delta\alpha_s)$$

Lattice NRQCD Simulation

Basic idea

- take $vm < 1/a < m$ as *NRQCD* factorization scale
- use *NRQCD* on the lattice (*lNRQCD*) to simulate E_{hfs} at finite a
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Matching *QCD* to *lNRQCD*

QCD $\quad \int d^4x\, \overline{\Psi}(i\not{D} - m)\Psi$

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NRQCD $\quad \int d^4x\, \psi^\dagger(iD_0 + \frac{\mathbf{D}^2}{2m} - g_s \frac{c_F}{2m} \sigma \cdot \mathbf{B})\psi + \ldots$

$\downarrow$ descretizing/introducing lattice spacing a

lNRQCD $\quad a^3 \sum_x \psi^\dagger(x)(1 - U_{-0} + \cdots - a g_s \frac{c_F}{2m} \sigma \cdot \mathbf{B})\psi(.) + \ldots$

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Matching QCD to $lNRQCD$

QCD $$\int d^4x\, \overline{\Psi}(i\not{D} - m)\Psi$$

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Determination of MCs

- Lattice Perturbation Theory

Spin Dependent Matching Status

$$\mathcal{L}_\sigma = g_s \frac{c_F}{2m}\psi^\dagger \sigma \cdot \mathbf{B}\psi + (\psi \to \chi_c) + d_\sigma \frac{C_F \alpha_s}{m^2}\psi^\dagger \sigma \psi \chi_c^\dagger \sigma \chi_c$$

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- d_σ (local four-quark interaction)
  - result available at $\mathcal{O}(\alpha_s)$
  - currently sign of $\log$ depends on specific publication (typo)
  - we do not agree with constant of QCD amplitude ✖
  - finite quark mass are kept $\rightarrow$ includes uncontrolled HO terms in $1/m$
  - Coulomb singularity numerically subtracted

Naive Lattice Action

- Wilson's glue action

$$\mathcal{S}_g = -\sum_x \sum_{\mu,\nu} \frac{\beta}{2} \left(\frac{1}{2N_F} \text{tr} \left[\Box_{\mu\nu} + \Box^\dagger_{\mu\nu} \right] - 1 \right) , \qquad \beta = 2N_F/g_s^2 ,$$

$$\Box_{\mu\nu} = U_{+\mu} U_{+\nu} U_{-\mu} U_{-\nu} , \qquad U_{\pm\mu}(x) = \exp\{iag_s T^a A^a_\mu(x \pm \tfrac{1}{2} a\hat{e}_\mu)\} .$$

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$$\mathcal{S}_\psi = \sum_x a^4 \psi^\dagger D_0^- \psi .$$

$$D^+_\mu = \frac{1}{a}(U_{+\mu} - 1) , \qquad D^-_\mu = \frac{1}{a}(1 - U_{-\mu}) .$$

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- Spin flip action

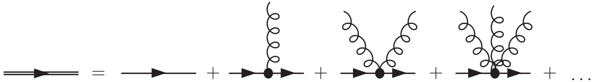
$$\mathcal{S}_{\psi\sigma} = \sum_x a^4 \frac{i}{2m} \epsilon^{ijk} \psi^\dagger \sigma_k \underbrace{\frac{1}{2} (D_i^- D_j^+ + D_i^+ D_j^-)}_{G_{ij}} \psi$$

$$D_\mu^+ = \frac{1}{a}(U_{+\mu} - 1) , \qquad D_\mu^- = \frac{1}{a}(1 - U_{-\mu}) .$$

Lattice Action $\overset{g_s\to 0}{\to}$ Feynman-rules

- Expansion of a single gauge link for small g_s

$$x_\pm = x \pm \tfrac{1}{2} a\hat{e}_\mu \,,$$
$$U_{\pm\mu}(x) = 1 + iag_s T^a A^a_\mu(x_\pm) - \frac{a^2 g_s^2}{2} T^a T^b A^a_\mu(x_\pm) A^b_\mu(x_\pm) + \ldots$$



- Switching to momentum space

$$\phi(x) = \frac{1}{V}\sum_x e^{ip\cdot x}\tilde{\phi}(p)\,, \qquad \phi \in \{A, \psi, \chi_c\}\,,$$
$$1.BZ: \qquad -\tfrac{\pi}{a} < p_i \leq \tfrac{\pi}{a}\,.$$

- Feynman-rules
  - propagators: Inverse of 2-field Fourier coefficient
  - n-vertex: n-field Fourier coefficient (symmetrized)

Analytic Integral Treatment

Evaluation in the limit $a\lambda \to 0$ [Becher, Melnikov 2002]

- Full integral

$$I = \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{\left[\sum_\mu \left(\frac{2}{a}\sin(a k_\mu/2)\right)^2 + \lambda^2\right]^2} \,.$$

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- Soft region $k \sim \lambda \Rightarrow$ expand in ak

$$I_{\text{soft}} = \frac{1}{8\pi^4}\left[\frac{1}{2\alpha} - \log\lambda + \ldots\right] + \mathcal{O}(a^2\lambda^2)\,.$$

- Hard region $k \sim 1/a \Rightarrow$ expand in $a\lambda$

$$I_{\text{hard}} = \frac{1}{8\pi^4}\left[-\frac{1}{2\alpha} - \log a + \ldots\right] + \mathcal{O}(a^2\lambda^2)\,.$$

Analytic Integral Treatment

Evaluation in the limit $a\lambda \to 0$ [Becher, Melnikov 2002]

- Full integral

$$I = I_{\text{soft}} + I_{\text{hard}} = -\frac{1}{8\pi^4}\left[\log(a\lambda) + \dots\right] + \mathcal{O}(a^2\lambda^2)\,.$$

- Soft region $k \sim \lambda \Rightarrow$ expand in ak

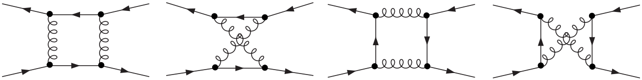
$$I_{\text{soft}} = \frac{1}{8\pi^4}\left[\frac{1}{2\alpha} - \log\lambda + \dots\right] + \mathcal{O}(a^2\lambda^2)\,.$$

- Hard region $k \sim 1/a \Rightarrow$ expand in $a\lambda$

$$I_{\text{hard}} = \frac{1}{8\pi^4}\left[-\frac{1}{2\alpha} - \log a + \dots\right] + \mathcal{O}(a^2\lambda^2)\,.$$

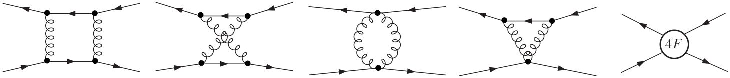
4-Fermion Operator Matching

QCD



$\parallel\, q^2 = 0$

lNRQCD



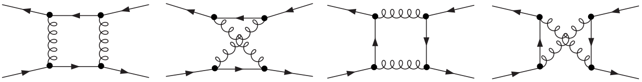
$+\ldots$

$$M_{1\mathrm{PI}}^{QCD} = \frac{C_F\alpha_s^2}{m_q^2}\left[\frac{C_A}{2}\log\left(\frac{m_q}{\lambda}\right) + (\ln 2 - 1)\,T_F + \left(1 - \frac{2\pi m_q}{3\lambda}\right)C_F\right]\psi^\dagger\boldsymbol{\sigma}\psi\chi_c^\dagger\boldsymbol{\sigma}\chi_c,$$

$$M_{1\mathrm{PI}}^{lNRQCD} = \frac{C_F\alpha_s^2}{m_q^2}\left[\quad ? \quad + 0\times T_F - \frac{2\pi m_q}{3\lambda}C_F\right]\psi^\dagger\boldsymbol{\sigma}\psi\chi_c^\dagger\boldsymbol{\sigma}\chi_c + \mathcal{O}\left(a^2\right),$$

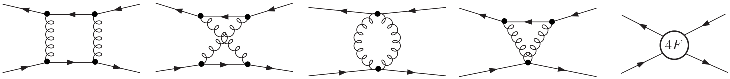
4-Fermion Operator Matching

QCD



$\parallel \; q^2 = 0$

lNRQCD



$+ \ldots$

$$M^{QCD}_{1PI} = \frac{C_F\alpha_s^2}{m_q^2}\left[\frac{C_A}{2}\log\left(\frac{m_q}{\lambda}\right) + (\ln 2 - 1)\,T_F + \left(1 - \frac{2\pi m_q}{3\lambda}\right)C_F\right]\psi^\dagger\boldsymbol{\sigma}\psi\chi_c^\dagger\boldsymbol{\sigma}\chi_c,$$

$$M^{lNRQCD}_{1PI} = \frac{C_F\alpha_s^2}{m_q^2}\left[-\left(\delta + \frac{1}{2}\ln(\lambda a)\right)C_A - \frac{2\pi m_q}{3\lambda}C_F\right]\psi^\dagger\boldsymbol{\sigma}\psi\chi_c^\dagger\boldsymbol{\sigma}\chi_c + \mathcal{O}\left(a^2\right),$$

$$d_\sigma = \alpha_s\left[\left(\delta + \frac{1}{2}L\right)C_A + (\ln 2 - 1)\,T_F + C_F\right].$$

4-Fermion Operator Matching

- Naive action

$$\delta^{\text{naive}} = -\frac{7}{3} + 28\pi^2 b_2 - 256\pi^2 b_3 = 0.288972\ldots$$

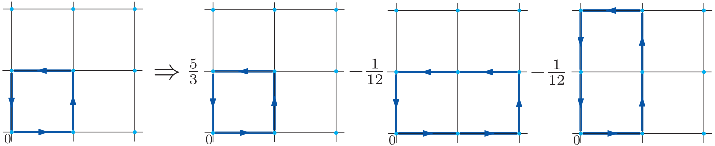
  - ⊕ calculated analytically
  - ⊖ naive action simulation has large discretization error

- Improved lattice action are in use for improved $a \to 0$ limit

  - ⊖ no analytic calculation
  - ➡ numeric calculation

Action Improvements

- Glue action Wilson ⇒ Lüscher & Weisz



Action Improvements

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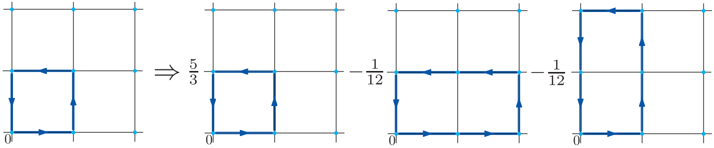
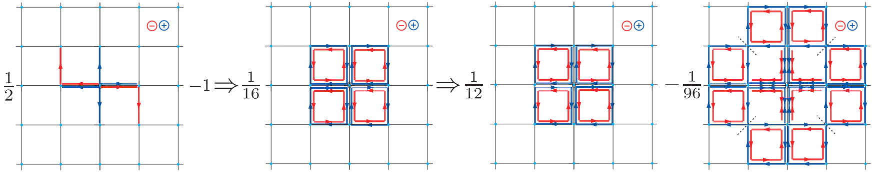
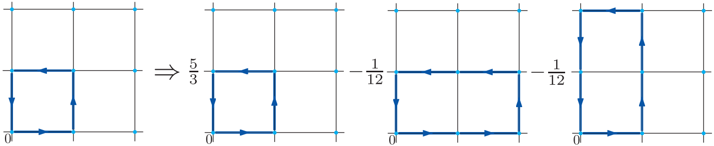


- Naive G_{ij} ⇒ cloverleaf G_{ij} ⇒ improved cloverleaf G_{ij}

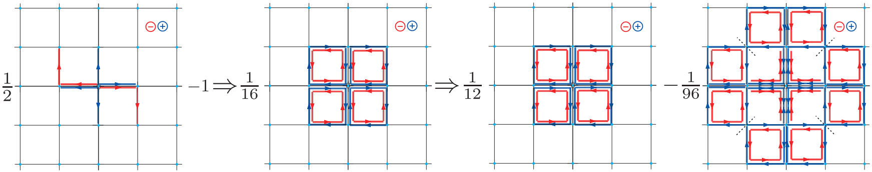


Action Improvements

- Glue action Wilson ⇒ Lüscher & Weisz



- Naive G_{ij} ⇒ cloverleaf G_{ij} ⇒ improved cloverleaf G_{ij}



- Additional smoothing time slice in quark-/anti-quark action

$$\sum_x a^4 \frac{i}{2m} \epsilon^{ijk} \psi^\dagger \sigma_k G_{ij} \psi \quad \Rightarrow \quad \sum_x a^4 \frac{i}{2m} \epsilon^{ijk} \psi^\dagger \sigma_k (U_{-t} G_{ij} + G_{ij} U_{-t}) \psi$$

Numeric Setup

fastcol.m [NZ]
Full automatic lattice amplitude generator (Mathematica)

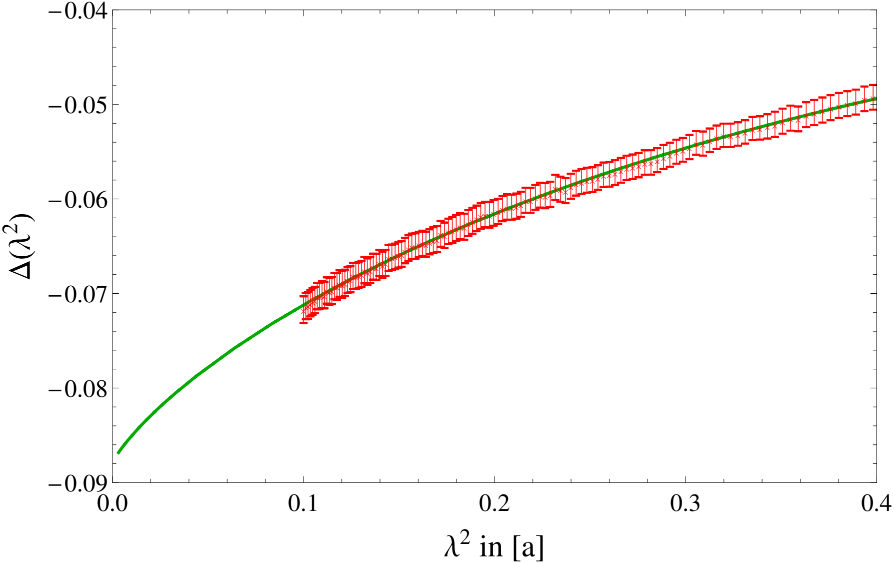
- QGRAF [Nogueira 91]
  diagram generator (FORTRAN)
- COLOR [Ritbergen,Schellekens,Vermaseren 99]
  precalculating color amplitudes (FORM)
- HiPPy [Hart,Hippel,Horgan,Muller 09]
  numeric expansion coefficients for gauge link configurations
  "numeric Feynman-rules" (Python)
- HPsrc [Hart,Hippel,Horgan,Muller 09]
  numeric vertex & propagator functions (FORTRAN)
- CUBA [Hahn 05]
  integrator package with multicore support (FORTRAN)

δ for Improved Action

Numeric evaluation of the difference

$$\Delta\delta = \delta^{\text{improved}} - \delta^{\text{naive}}$$

δ for Improved Action



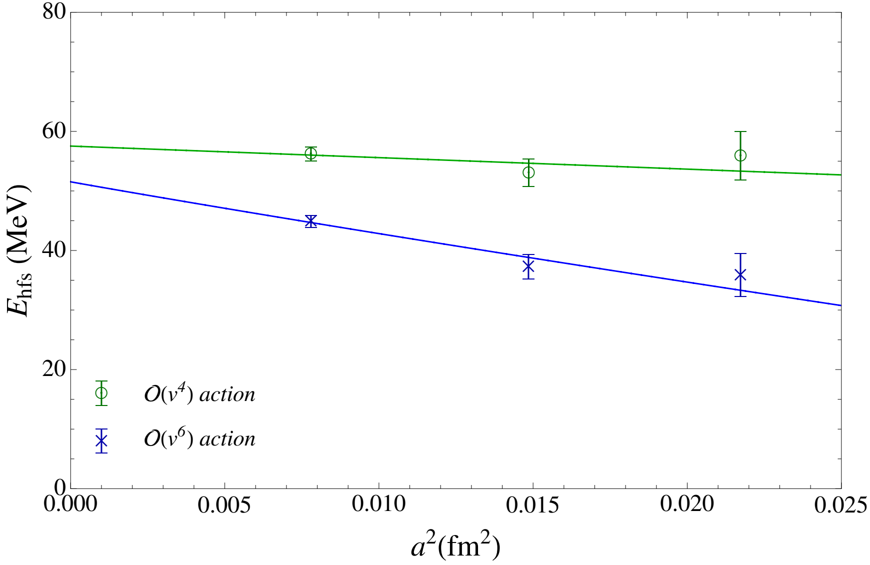
$\Delta(0) = \frac{6}{\pi^2}\Delta\delta$

$$\Delta(\lambda^2) = \Delta(0) + c_1\lambda^2 + c_2\lambda^2 \log(\lambda^2)$$

Determination of E_{hfs}

- Lattice simulations with $d_\sigma = 0$
  - $\mathcal{O}(v^4)$ action [HPQCD 12]
  - $\mathcal{O}(v^6)$ action [HPQCD 14]
- Four quark operator contribution
  - $$\Delta E_{\text{hfs}} = -d_\sigma \frac{4C_F\alpha_s}{m_q^2}|\psi(0)|^2$$
  - $m_b, \alpha_s, |\psi(0)|^2$ from the lattice

Determination of E_{hfs}



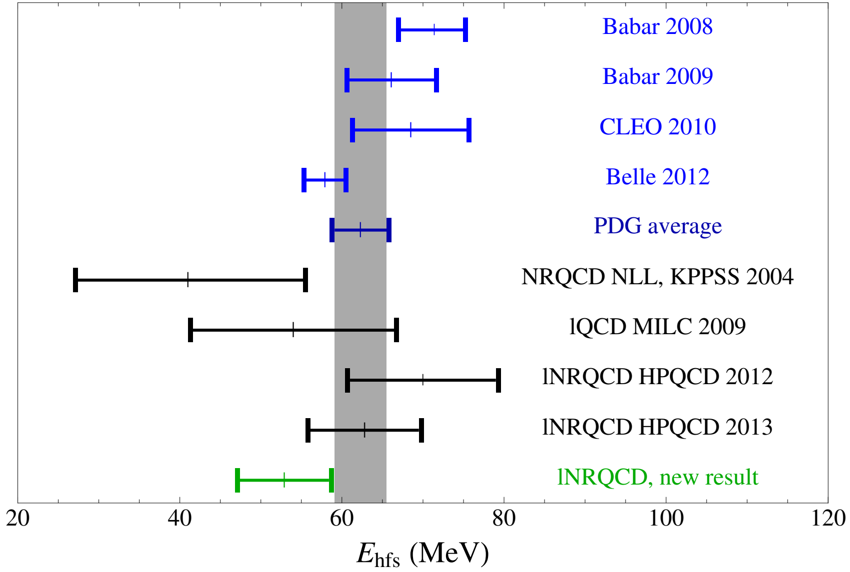
New value: $E^{\text{th}}_{\text{hfs}} = 52.9 \pm 5.5$ MeV

Summary of $E_{\rm hfs}$ results

Experiment	
$\Upsilon(3S)$ decays (Babar, 2008)	$71.4^{+2.3}_{-3.1}(\text{stat}) \pm 2.7(\text{syst})$
$\Upsilon(2S)$ decays (Babar, 2009)	$66.1^{+4.8}_{-4.9}(\text{stat}) \pm 2.0(\text{syst})$
$\Upsilon(3S)$ decays (CLEO, 2010)	$68.5 \pm 6.6(\text{stat}) \pm 2.0(\text{syst})$
$h_b(1P)$ decays (Belle, 2012)	$57.9 \pm 1.5(\text{stat}) \pm 1.8(\text{syst})$
PDG average	62.3 ± 3.2

Theory	
NRQCD, NLL (2004)	$41 \pm 11(\text{th})^{+9}_{-8}(\delta\alpha_s)$
Lattice QCD (MILC, 2009)	$54.0 \pm 12.4^{+1.2}_{-0.0}$
Lattice NRQCD $\mathcal{O}(v^4)$ (HPQCD, 2012)	70 ± 9
Lattice NRQCD $\mathcal{O}(v^6)$ (HPQCD, 2013)	62.8 ± 6.7
Lattice NRQCD, new result	52.9 ± 5.5

Summary of E_{hfs} results



Summary

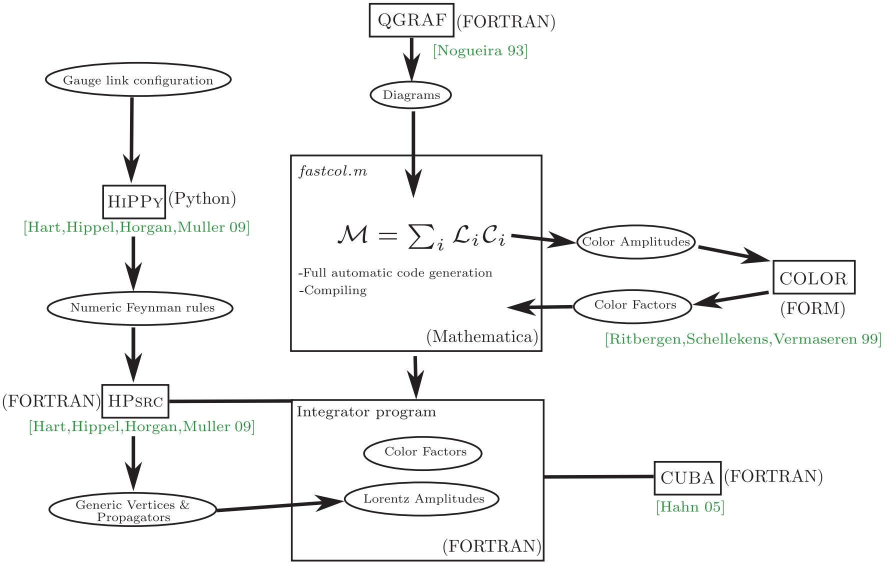
1. employed perturbative *lNRQCD* to establish a radiative UV improvement (4FO) on top of *lNRQCD* simulations
2. $E_{\text{hfs}}^{\text{th}} = 52.9 \pm 5.5\ \text{MeV}$
3. QCD works / no evidence for "new physics"
4. lattice amplitude generator

M. Buffer

Checks

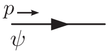
- Naive action
  - analytic & numeric setup yield corrections to the static potential ✓
  - analytic result for d_σ is gauge independent ✓
  - numeric calculation $\overset{\lambda\to 0}{=}$ analytic calculation ✓
  - analytic Feynman rules checked independently (MATHEMATICA)
  - integrands of amplitudes checked against MATHEMATICA implementation ✓
- Improved action
  - numeric $\log$ coefficient $=$ analytic $\log$ coefficient ✓

Numeric Setup

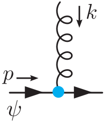


Feynman-rules for naive action

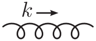
$a = 1$



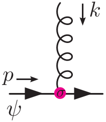
$$\sim \frac{\delta^i_j \delta^{s_2}_{s_1}}{1-e^{-ip_0}}$$



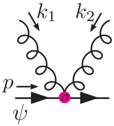
$$\sim -g_s T^a \delta_{\mu 0} \delta^{s_2}_{s_1} e^{-i(p_0+\frac{1}{2}k_0)}$$



$$\sim \frac{\delta_{ab}\delta_{\mu\nu}}{\hat{k}^2+\lambda^2}$$



$$\sim \frac{g_s}{2m} T^a \delta_{\mu i} \epsilon^{ilm} (\sigma^m)^{s_2}_{s_1} \widehat{k}_l \widetilde{\rho}_{k\,il}$$



$$\sim \frac{g_s^2}{2m} \delta_{\mu_1 i} \delta_{\mu_2 j} (\sigma^m)^{s_2}_{s_1} \Big[-f^{a_1 a_2 c} C^c_A \left(\epsilon^{ijm} \widetilde{k_{1+2}}_{ji} \widetilde{\zeta}_{ij} \right) + \delta_{a_1 a_2} C_S \left(\epsilon^{ijm} \widehat{k_{1+2}}_{ji} \widehat{\zeta}_{ij} - \delta_{ij} \epsilon^{ilm} \widehat{k_{1+2}}_{ll} \widehat{\zeta}_{jl} \right) \Big]$$

Shorthands

$$
\begin{aligned}
\widehat{k}_\mu &= 2\sin\left(\tfrac{1}{2}k_\mu\right), \\
\widetilde{\rho}_{kil} &= \cos\{\tfrac{1}{2}(k_i - k_l + 2p_i - 2p_l)\}, \\
\widehat{\zeta}_{ij} &= \sin\{(\tfrac{1}{2}(k_{1i} + k_{2i} - k_{1j} - k_{2j} + 2p_i - 2p_j)\}, \\
\widetilde{\zeta}_{ij} &= \cos\{(\tfrac{1}{2}(k_{1i} + k_{2i} - k_{1j} - k_{2j} + 2p_i - 2p_j)\}, \\
\widehat{k_{1+2}}_{ij} &= 2\sin\{(\tfrac{1}{2}(k_{1i} + k_{2j})\}, \\
\widetilde{k_{1+2}}_{ij} &= 2\cos\{(\tfrac{1}{2}(k_{1i} + k_{2j})\}, \\
\mathcal{C}_A^c &= \frac{i}{2}T^c, \\
\mathcal{C}_S &= \frac{I_{2,F}}{N_F}\mathbb{I}_{N_F\times N_F} + \frac{1}{2}\delta_{ab}^{-1}d_{abc}T^c .
\end{aligned}
$$

Numeric Data L&W 4ICL action