N-jettiness as a subtraction scheme for NNLO

Xiaohui Liu

based on:

arXiv:1504.02131, Boughezal, Focke, XL and Petriello
arXiv:1505.03893, Boughezal, Focke, Giele, XL and Petriello
arXiv:1504.02540, Boughezal, XL and Petriello

Subtraction scheme

- Local subtraction schemes at NNLO
  - sectors, STRIPPER, Antenna subtraction and Colorful NNLO   Anastasiou, Melnikov, Petriello '03; M. Czakon '10; Boughezal, Melnikov, Petriello'11; Gehrmann-De Ridder, Gehrmann, Glover'05
    Aglietti, Bolzoni, Del Duca, Duhr, Moch, Somogyi, Tramontano, Trosanyi' 05-13
  - Successfully applied to LHC physics
    - Higgs+1jet   Boughezal, Caola, Melnikov, Petriello, Schulze '13, '15; Chen, Gehrmann, Glover, Jaquier '14
    - Top pair production   Czakon, Fiedler, Mitov '13, Abelof, Gehrmann-De Ridder, Majer' 15
    - Partial results for di-jet production
      Gehrmann-De Ridder, Gehrmann, Glover, Pires '13

Subtraction scheme

- Local subtraction schemes at NNLO
  - Construct IR subtraction point by point in phase space. Relatively smooth integrand

$$\mathrm{d}\sigma \sim \mathrm{d}\Phi_{\mathrm{Res.}} \int \mathrm{d}\Phi_{\mathrm{UnRes.}} \left\{ F[\Phi_{\mathrm{Res.}};\Phi_{\mathrm{UnRes.}}] - R[\Phi_{\mathrm{UnRes.}}] \otimes H[\Phi_{\mathrm{Res.}}] \right\}$$
$$+ \, \mathrm{d}\Phi_{\mathrm{Res.}} \, H[\Phi_{\mathrm{Res.}}] \otimes \int \mathrm{d}\Phi_{\mathrm{UnRes}} \, R[\Phi_{\mathrm{UnRes.}}]$$

  - Build up code from the scratch and it is difficult to directly recycle known NLO results (MCFM …)

Subtraction scheme

- Non-local subtraction schemes at NNLO
  - Pick some physical observable ν, with nice properties: ν ->0 limit catches all IR behavior which can be predicted using simplified formalisms (CSS or SCET). Non-local due to integrating over part of the phase space to get the observable ν

$$v \to 0\,, \qquad \mathrm{d}\sigma \to \mathrm{d}\sigma_{\mathrm{sing.}}$$

$$\mathrm{d}\sigma_{\mathrm{sing.}} \sim \mathrm{d}\sigma_0 \sum \alpha_s^n \left(C_{n,m} \left[\frac{\log^{2n-m}(v)}{v} \right]_+ + C_{n,0}\, \delta(v) \right) + v\,[\ldots]$$

Subtraction scheme

- Non-local subtraction schemes at NNLO
  - Small parameter v_{res}
    - $v > v_{res}$, at least 1 additional radiation can be resolved, NO singularities related to NNLO in the problem: NLO problem for Born+1j
    - $v < v_{res}$, NO additional radiations are resolved, true NNLO: use $\mathrm{d}\sigma_{\mathrm{sing.}}$
    - Combine to achieve NNLO, v_{res} independent

Subtraction scheme

- Non-local subtraction schemes at NNLO
  - Maximally recycle the NLO tools, easy to implement
    - qT-subtraction Catani and Grazzini ’07; Catani, Cieri, Ferrera, de Florian, Grazzini ’09 …
      - color neutral final state (ggH, Drell-Yan, Di-boson …)
    - top quark decay and top pair production in lepton annihilation Gao, Li and Zhu ’13; Gao and Zhu ‘14

Subtraction scheme

- Non-local subtraction schemes at NNLO
  - Maximally recycle the NLO tools, easy to implement
  - A more generic observable
    - catches all IR behaviors?
    - universal
    - YES, jettiness

Outline

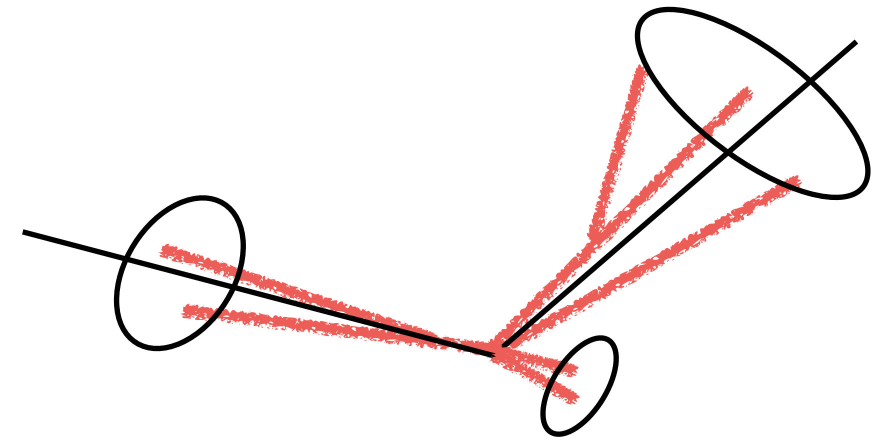
- N-Jettiness
- Jettiness-subtraction scheme for NNLO
- NNLO W+1j and Higgs+1j
- Conclusion

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
    - A global inclusive event-shape to distinguish between N-jet events and more-than-N-jet events

$$\mathcal{T}_N = \sum_k \min\{w_a\, n_a \cdot q_k\,, w_b\, n_b \cdot q_k\,, w_i\, n_i \cdot q_k\,, \ldots\,, w_N\, n_N \cdot q_k\}$$

Jettiness

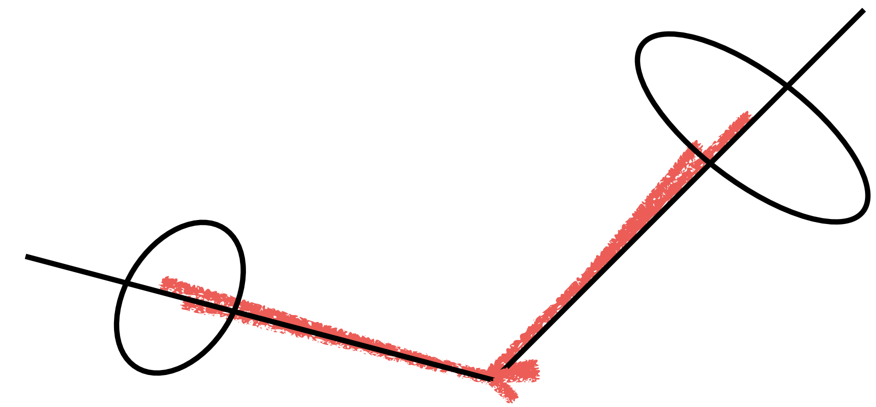


- N-Jettiness   Stewart, Tackmann, Waalewijn'10
  - A global inclusive event-shape to distinguish between N-jet events and more-than-N-jet events

$$\mathcal{T}_N = \sum_k \min\{w_a\, n_a \cdot q_k\,, w_b\, n_b \cdot q_k\,, w_i\, n_i \cdot q_k\,, \ldots\,, w_N\, n_N \cdot q_k\}$$

  - n_i's are arbitrary unit light-like reference vectors, but in practice they are determined most efficiently by a jet algorithm. w_i's are arbitrary positive weights. k runs over all colored partons in the final state. q_k is the four momentum for k. Does not include color neutral particles like leptons, W/Z/H…

Jettiness



- N-Jettiness   Stewart, Tackmann, Waalewijn'10
  - A global inclusive event-shape to distinguish between N-jet events and more-than-N-jet events

$$\mathcal{T}_N = \sum_k \min\{w_a\, n_a \cdot q_k\,, w_b\, n_b \cdot q_k\,, w_i\, n_i \cdot q_k\,, \ldots\,, w_N\, n_N \cdot q_k\}$$

- $\tau_N = 0$ forces an N-jet final state, i.e. q_k's must be soft or collinear to one of p_i, hence τ_N will control all the IR behaviors for N-jet. In the IR limit, n_i's do not depend on any IR safe jet algorithm and are solely determined by the Born topology. $\tau_N = 0$ behavior is universal for any physical IR safe measurement on N jets.

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Universal IR behavior for N-jet in the small jettiness region

$$\mathrm{d}\sigma_{\mathrm{sing.}}(\mathcal{T}_N) \sim \mathrm{Tr}\left[\underline{H} \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_{i}^{N} J_i$$

    - Hard function
      - $\mathcal{T}_N$-independent; process dependent, virtual corrections (1-loop,2-loop …)
      - known for some processes up to 2-loop

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Universal IR behavior for N-jet in the small jettiness region

$$\mathrm{d}\sigma_{\mathrm{sing.}}(\mathcal{T}_N) \sim \mathrm{Tr}\left[H \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i$$

  - Jet function
    - depends on the jet species, quark jet or gluon jet
    - both known up to 2-loop for a long time

Becher and Neubert' 06, Becher and Bell '10

Jettiness

- N-Jettiness   Stewart, Tackmann, Waalewijn'10
  - Universal IR behavior for N-jet in the small jettiness region

$$d\sigma_{\text{sing.}}(\mathcal{T}_N) \sim \text{Tr}\left[H \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i$$

  - Beam function

$$B_i(x) = \int_x^1 \frac{dz}{z} I_{ij}\left(\frac{x}{z}\right) f_j(z)$$

    - quark or gluon beam function

Fleming, Leibovich, Mehen '06, Stewart, Tackmann, Waalewijn '09

    - further matching onto pdfs, and known up to 2-loop

Gaunt, Stahlhofen, Tackmann '14

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Universal IR behavior for N-jet in the small jettiness region

$$d\sigma_{\text{sing.}}(\mathcal{T}_N) \sim \text{Tr}\left[H \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i$$

- Soft function
  - Color source, depends on the Number of the jets

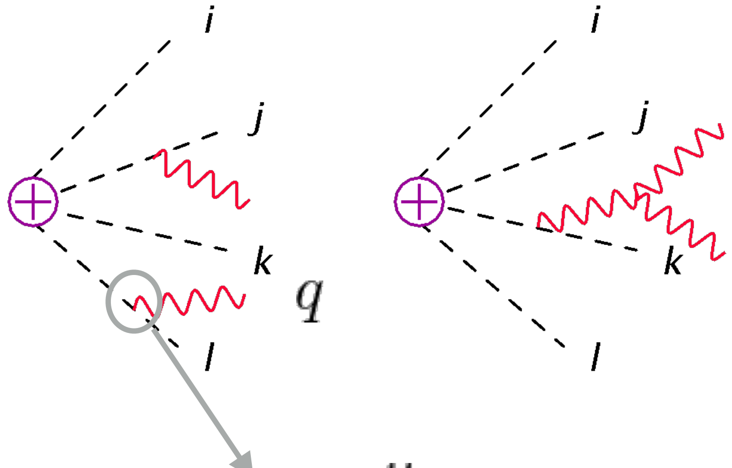
Stewart, Tackmann, Waalewijn '09, Jouttenus, Stewart, Tackmann, Waalewijn '11

  - 2-loop numerical result using a general framework, anomalous dimension for NNLL as a by product

Boughezal, XL and Petriello '15

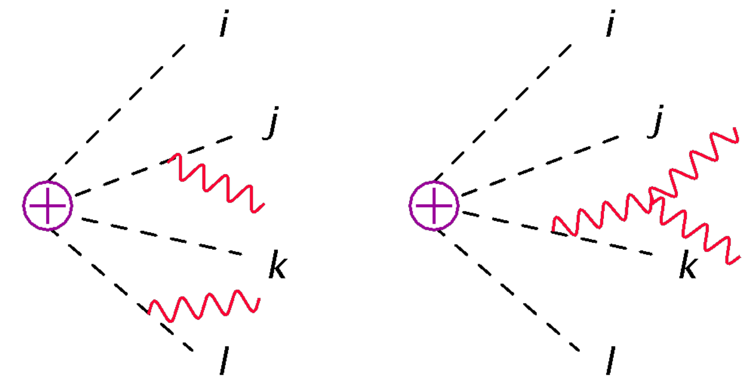
Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15
    - Radiations from eikonal lines.



$$\frac{n_l^\mu}{n_l \cdot q}$$

Jettiness



- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$$\frac{n_l^\mu}{n_l \cdot q}$$

    - Dipole structure Catani, Grazzini '98
      - only need to calculate the building blocks subject to the jettiness cut (4 reference vectors)

$$\mathcal{J}_{ij} = \mathcal{J}_{ij}^I + \mathcal{J}_{ij}^{II}$$

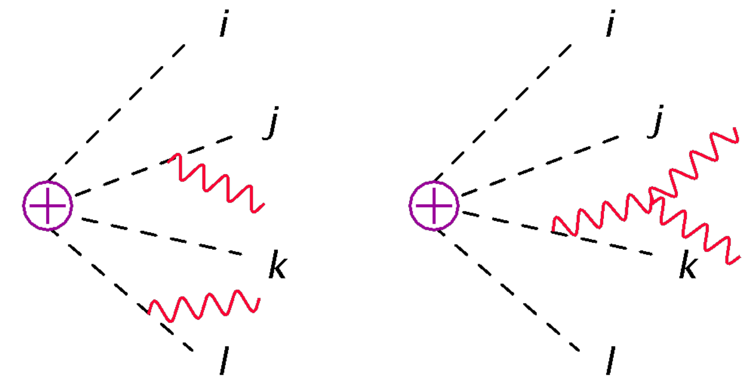
$$\mathcal{J}_{ij}^I = -2\frac{[p_i \cdot q_1 p_j \cdot q_2 + p_j \cdot q_1 p.q_2]^2}{(q_1 \cdot q_2)^2 [p_i \cdot (q_1+q_2)]^2 [p_j \cdot (q_1+q_2)]^2},$$

$$\mathcal{J}_{ij}^{II} = 2\frac{p_i \cdot p_j}{(q_1 \cdot q_2)[p_i \cdot (q_1+q_2)][p_j \cdot (q_1+q_2)]}.$$

$$\mathcal{T}_{ij} = (1-\epsilon)\mathcal{J}_{ij}^I + 2\mathcal{J}_{ij}^{II} + \left(\frac{p_i \cdot q_1 p_j \cdot q_2 + p_j \cdot q_1 p_i \cdot q_2}{[p_i \cdot (q_1+q_2)][p_j \cdot (q_1+q_2)]} - 2\right) S_{ij}^{(s.o.)},$$

$$S_{ij}^{(s.o.)} = \frac{p_i \cdot p_j}{q_1 \cdot q_2}\left(\frac{1}{p_i \cdot q_1 p_j \cdot q_2} + \frac{1}{p_j \cdot q_1 p_i \cdot q_2}\right) - \frac{(p_i \cdot p_j)^2}{p_i \cdot q_1 p_i \cdot q_2 p_j \cdot q_1 p_j \cdot q_2}.$$

Jettiness



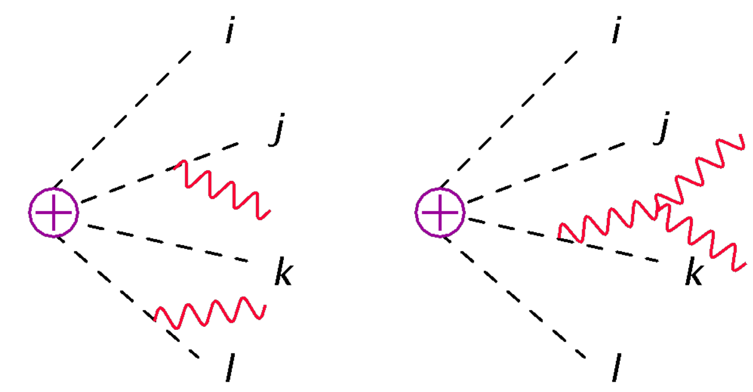
$$\frac{n_l^\mu}{n_l \cdot q}$$

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15
    - Dipole structure Catani, Grazzini '98
      - only need to calculate the building blocks subject to the jettiness cut (4 reference vectors)

V/H + 1j

$$S_{q\bar{q}} = T_R \left[C_a \mathcal{J}_{12} + C_{a3} \left(-\frac{1}{2}\mathcal{J}_{12} + \frac{1}{2}\mathcal{J}_{13} + \frac{1}{2}\mathcal{J}_{23} \right) \right]$$

Jettiness



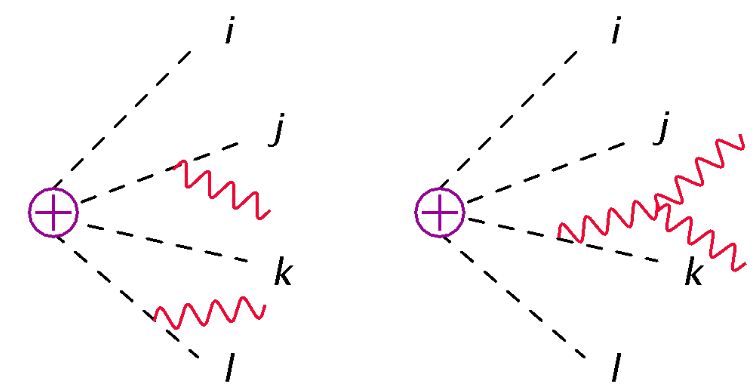
- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$$\frac{n_l^\mu}{n_l \cdot q}$$

    - Has both UV and IR divergence
      - No hard scales ~ Q >> τ_N in the soft function, relax cuts
      - Allows the soft radiation energy E can go to infinity, regularized using dim-reg.

$$\int^{\sim Q} \mathrm{d}E \to \int^{\infty} \mathrm{d}E$$

Jettiness



- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$$\frac{n_l^\mu}{n_l \cdot q}$$

    - Finite $\mathcal{T}_N$ forces ni.q = 0, when E-> infinity. Map UV divergence to collinear divergence: variable change

$$\mathcal{T}_N = \sum_k \min\left\{w_a\, n_a \cdot q_k\, , w_b\, n_b \cdot q_k\, , w_i\, n_i \cdot q_k\, , \ldots\, , w_N\, n_N \cdot q_k\right\}$$

    - Use standard techniques to calculate, e.g. sector decomposition. Framework valid for a large class of observables!

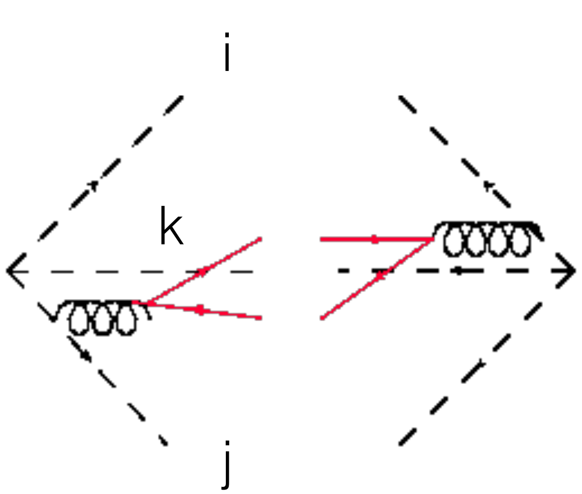
Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$$2\, n_{i,j}\, \mathrm{d}PS_1\, \mathrm{d}PS_2\, \frac{1}{(q_1 \cdot q_2)\, n_i \cdot (q_1 + q_2)\, n_j \cdot (q_1 + q_2)}$$
$$\times \quad \delta(\tau - n_k \cdot q_1 - n_k \cdot q_2)\Theta^M_{k,1;k,2} \,.$$

$$q_1^+ = n_k \cdot q_1 \,, \qquad q_2^+ = n_k \cdot q_2 \,, \qquad q_1^- = n_i \cdot q_1 \,, \qquad q_2^- = n_i \cdot q_2$$

$$\Theta^M_{k,1;k,2} = \Theta(q_1^- - q_1^+)\Theta(q_2^- - q_2^+) \prod_{j \neq i,k} \Theta(n_j \cdot q_1 - q_1^+)\, \Theta(n_j \cdot q_2 - q_2^+)$$



$$q_1^+ = \mathcal{T}_N \xi, \quad q_1^- = \frac{\mathcal{T}_N \xi}{s}, \quad q_2^+ = \mathcal{T}_N(1-\xi), \quad q_2^- = \frac{\mathcal{T}_N(1-\xi)}{t},$$
$$\phi_1 = 2\pi x_4, \quad \lambda = \sin^2(\pi x_5/2), \quad \cos(\alpha) = 1 - 2x_6.$$

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$$q_1^+ = \mathcal{T}_N\xi, \quad q_1^- = \frac{\mathcal{T}_N\xi}{s}, \quad q_2^+ = \mathcal{T}_N(1-\xi), \quad q_2^- = \frac{\mathcal{T}_N(1-\xi)}{t},$$
$$\phi_1 = 2\pi x_4, \quad \lambda = \sin^2(\pi x_5/2), \quad \cos(\alpha) = 1 - 2x_6.$$

$$\begin{aligned} I_{q\bar{q},kk}^{(2),ij,II} &= 2^{1-8\epsilon} B_{RR} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{T}_N^{-1-4\epsilon} n_i \cdot n_j [n_i \cdot n_k]^{-1+2\epsilon} \int_0^1 d\xi\, ds\, dt\, dx_4\, dx_5\, dx_6\, d\Omega^{(\epsilon)} \\ &\times [\xi(1-\xi)]^{-2\epsilon} [\lambda(1-\lambda)]^{-\epsilon} \sin^{-2\epsilon}(\phi_1) [-\epsilon x_6^{-1-\epsilon}] (1-x_6)^{-\epsilon} [s\,t]^{1+\epsilon} |s-t|^{-1-2\epsilon} \\ &\times \frac{\{(\sqrt{s}-\sqrt{t})^2 + 4\lambda\sqrt{st}\}^{2\epsilon}}{\xi\, t A_{ki,j}(s,\phi_1) + (1-\xi) s A_{ki,j}(t,\phi_{2k})} \frac{1}{\xi t + (1-\xi) s} \\ &\times \prod_{l\neq i,k} \theta\left[A_{ki,l}(s,\phi_{1l}) - s\right] \theta\left[A_{ki,l}(t,\phi_{2l}) - t\right], \end{aligned}$$

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$$1 = \theta(s-t) + \theta(t-s) \qquad s = x_1\,, \quad t = (1-x_2)x_1 \quad \text{or} \quad s = (1-x_2)x_1\,, \quad t = x_1$$

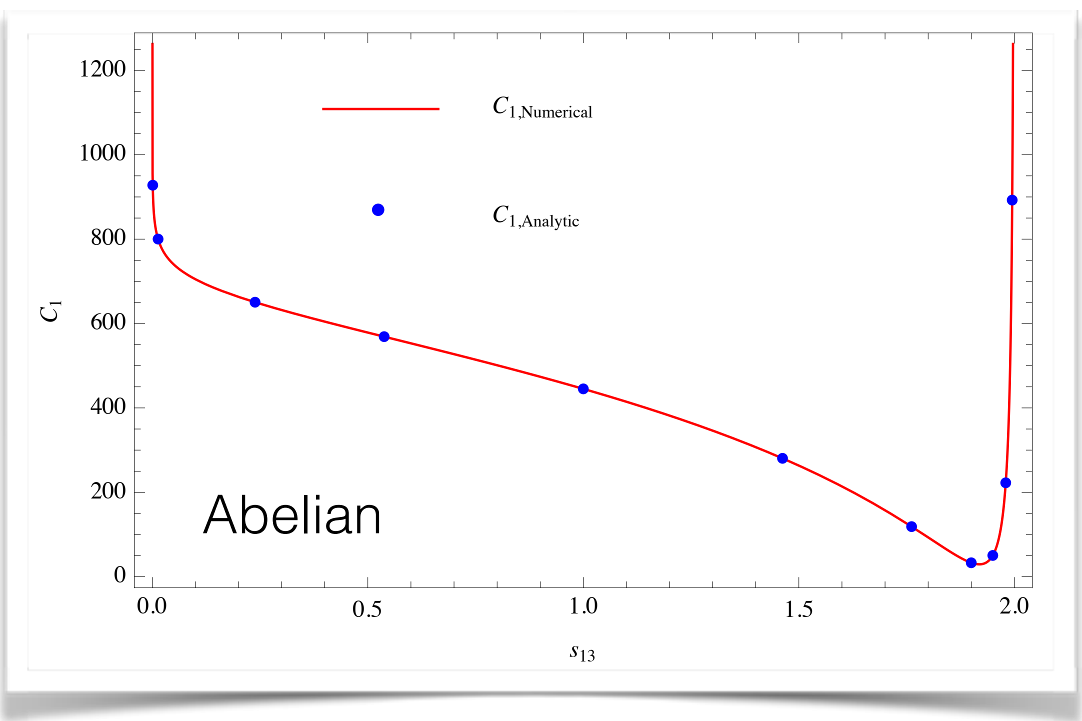
$$\begin{aligned} I^{(2),ij,II}_{q\bar{q},kk,s>t} &= 2^{1-8\epsilon} B_{RR} \left(\frac{\alpha_s}{2\pi}\right)^2 \mathcal{T}_N^{-1-4\epsilon} n_i \cdot n_j [n_i \cdot n_k]^{-1+2\epsilon} \int_0^1 dx_1\, dx_2\, dx_3\, dx_4\, dx_5\, dx_6\, d\Omega^{(\epsilon)} \\ &\times [x_1(1-x_1)]^{-2\epsilon} [\lambda(1-\lambda)]^{-\epsilon} \sin^{-2\epsilon}(\phi_1) [-\epsilon x_6^{-1-\epsilon}] (1-x_6)^{-\epsilon} x_2^{2\epsilon} (1-x_3)^{1+\epsilon} x_3^{-1-2\epsilon} \\ &\times \frac{\left\{(1-\sqrt{1-x_3})^2 + 4\lambda\sqrt{1-x_3}\right\}^{2\epsilon}}{x_1(1-x_3)\, t A_{ki,j}(x_2,\phi_1) + (1-x_1) A_{ki,j}(x2(1-x_3),\phi_{2k})} \frac{1}{1-x_1 x_3} \\ &\times \prod_{l \neq i,k} \theta\left[A_{ki,l}(x_2,\phi_{1l}) - x_2\right] \theta\left[A_{ki,l}(x_2(1-x_3),\phi_{2l}) - x_2(1-x_3)\right]. \end{aligned}$$

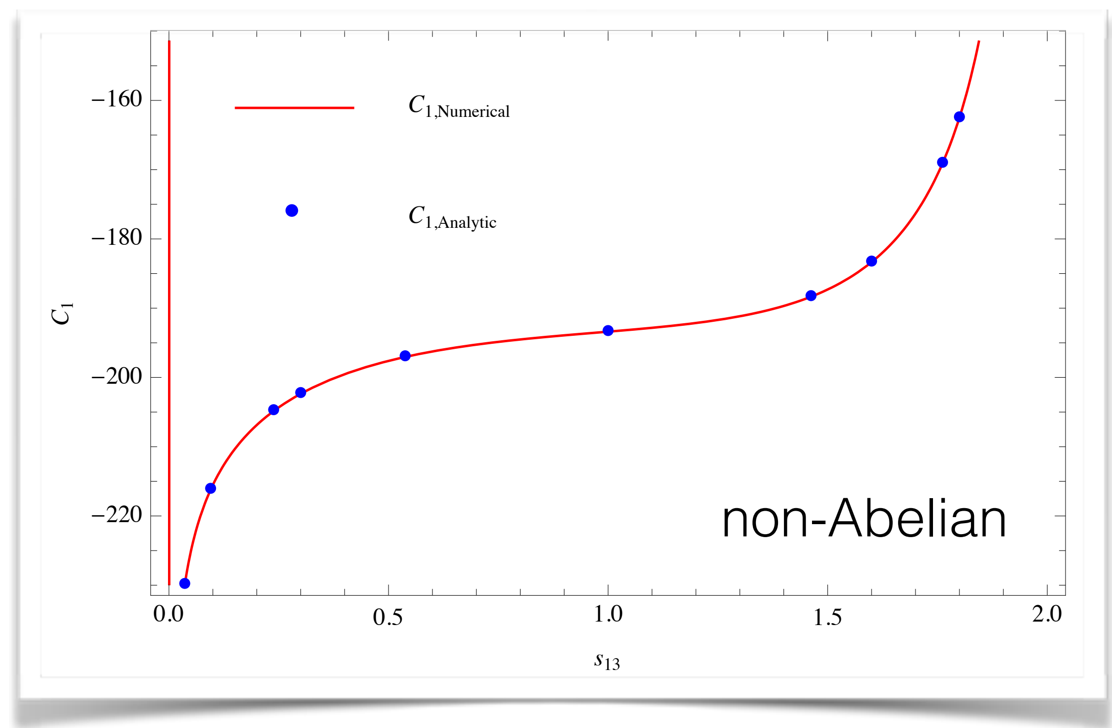
suitable for numerical calculation after Laurent expansion

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$\mathcal{T}_1$ for pp collision



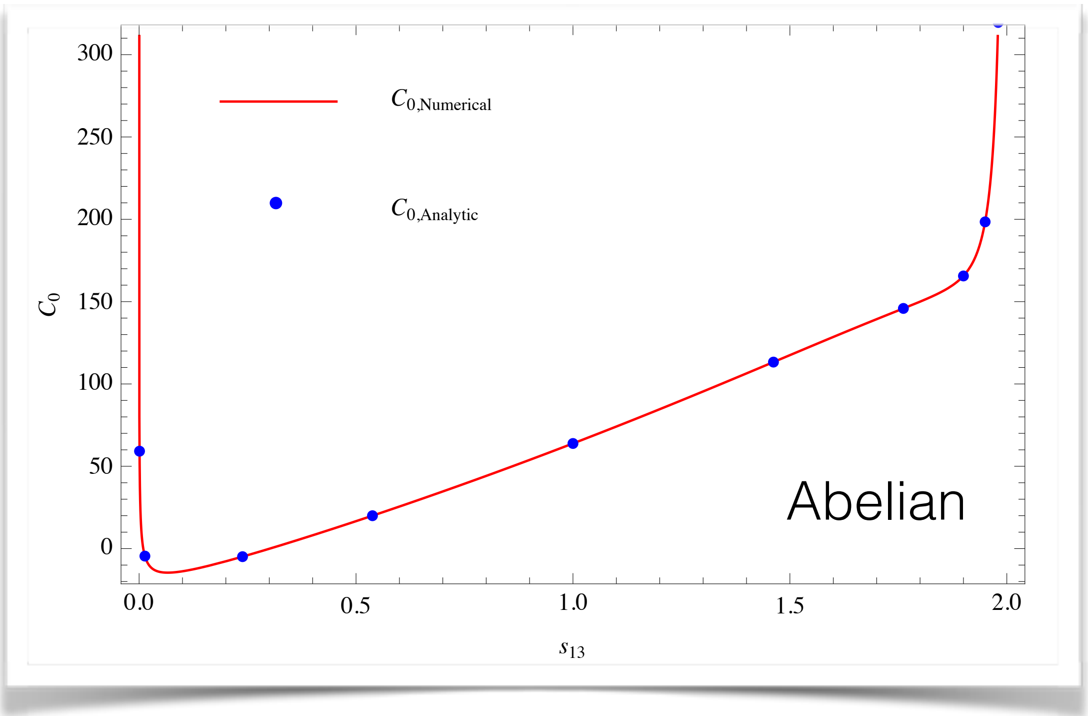


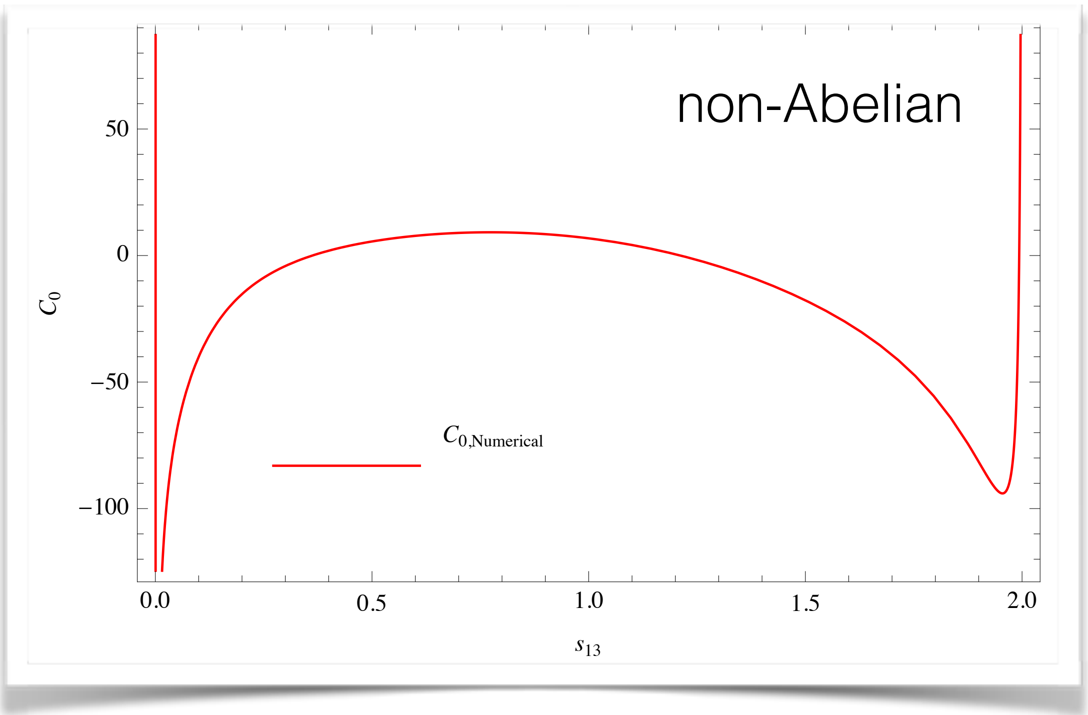
$$\Sigma^{(2)}_{soft} = \int_0^{\mathcal{T}_N^{cut}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N} = \left(\frac{\alpha_s}{2\pi}\right)^2 \left(C_4 L^4 + C_3 L^3 + C_2 L^2 + C_1 L + C_0\right) \qquad L = \log \frac{\mathcal{T}_N^{cut}}{\mu}$$

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Calculate the 2-loop soft function Boughezal, XL and Petriello '15

$\mathcal{T}_1$ for pp collision





$$\Sigma^{(2)}_{soft} = \int_0^{\mathcal{T}_N^{cut}} d\mathcal{T}_N \frac{d\sigma}{d\mathcal{T}_N} = \left(\frac{\alpha_s}{2\pi}\right)^2 (C_4L^4 + C_3L^3 + C_2L^2 + C_1L + C_0) \qquad L = \log\frac{\mathcal{T}_N^{cut}}{\mu}$$

Jettiness

- N-Jettiness Stewart, Tackmann, Waalewijn'10
  - Universal IR behavior for N-jet in the small jettiness region

$$\mathrm{d}\sigma_{\mathrm{sing.}}(\mathcal{T}_N) \sim \mathrm{Tr}\left[H \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i$$

  - Each building block is know up to NNLO

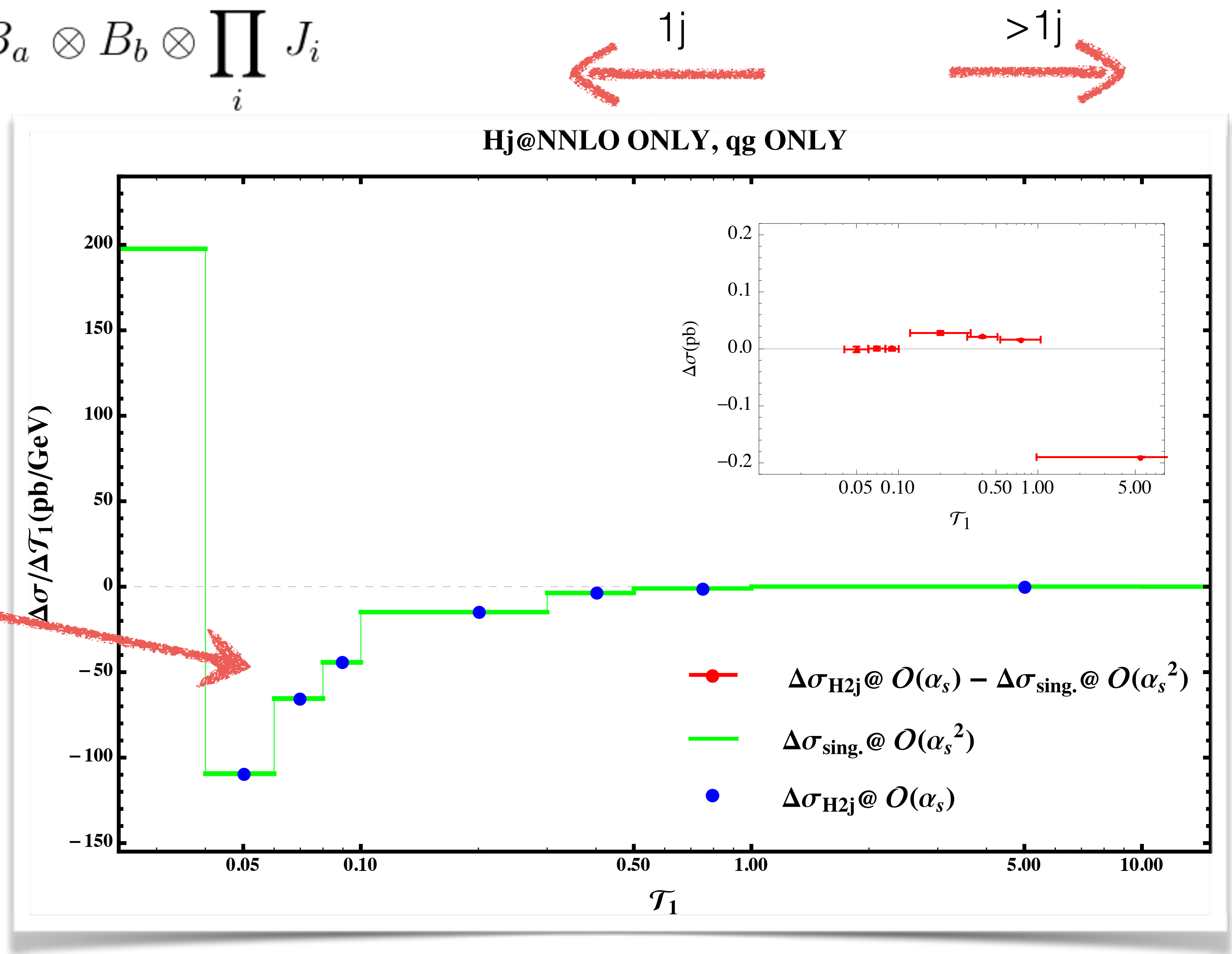
Becher and Neubert' 06, Becher and Bell '10, Gaunt, Stahlhofen, Tackmann '14, Boughezal, XL and Petriello '15

Jettiness

- N-Jettiness   Stewart, Tackmann, Waalewijn'10

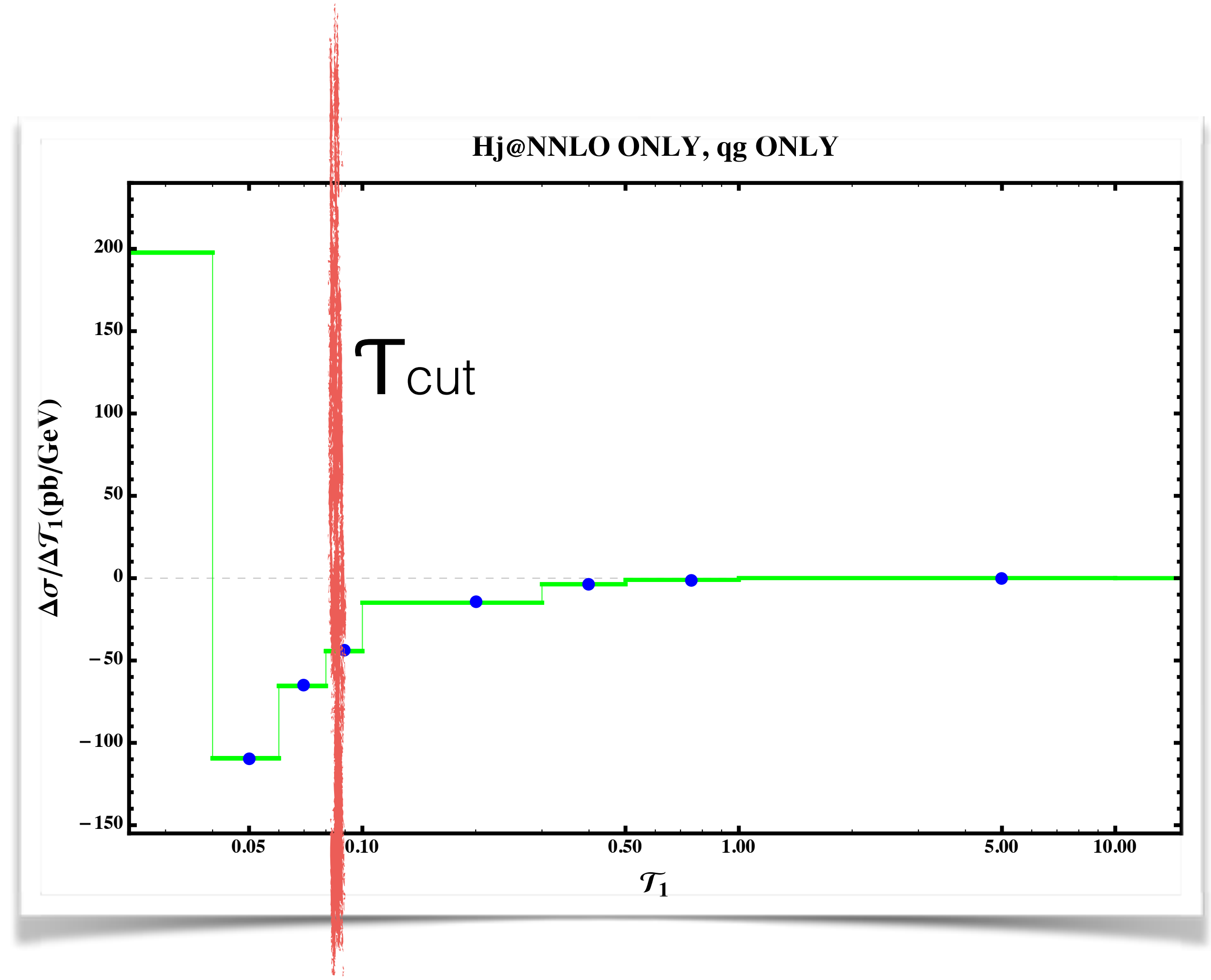
$$\mathrm{d}\sigma_{\text{sing.}}(\mathcal{T}_N) \sim \mathrm{Tr}\left[H \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i$$

Does catch the singular behavior!



Jettiness-subtraction scheme

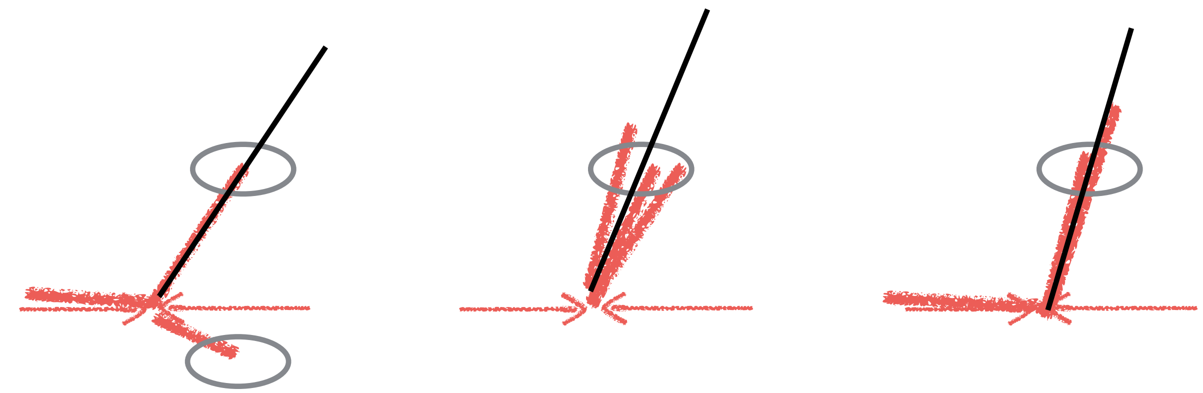
- Jettiness-subtraction Boughezal, Focke, XL and Petriello '15, see also Gaunt, Stahlhofen, Tackmann, Walsh '15
  - introduce a small τ_{cut}

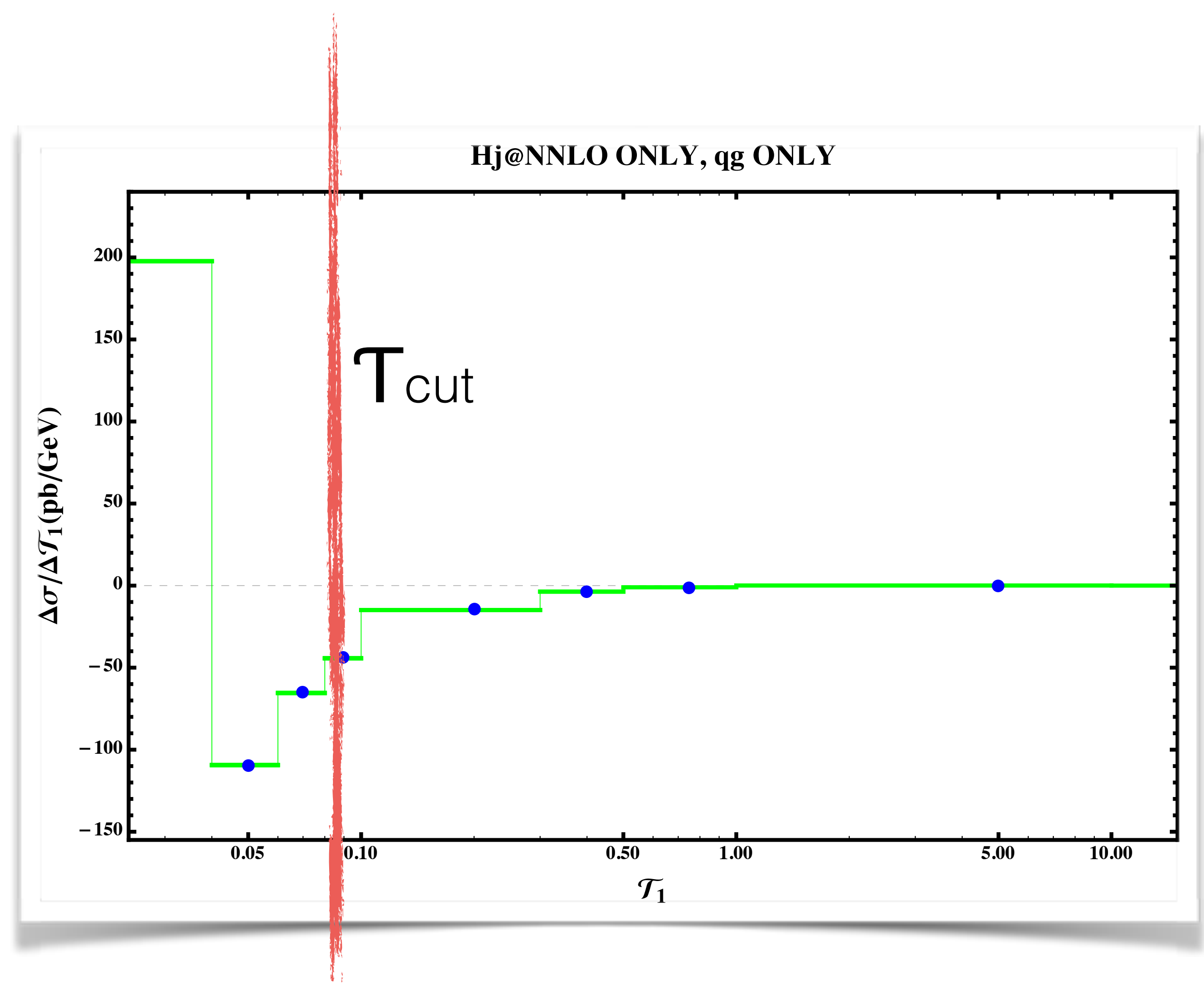


Jettiness-subtraction scheme

- Jettiness-subtraction
  - introduce a small $\mathcal{T}_{cut}$
  - at least N jets, calculate $\mathcal{T}_N$

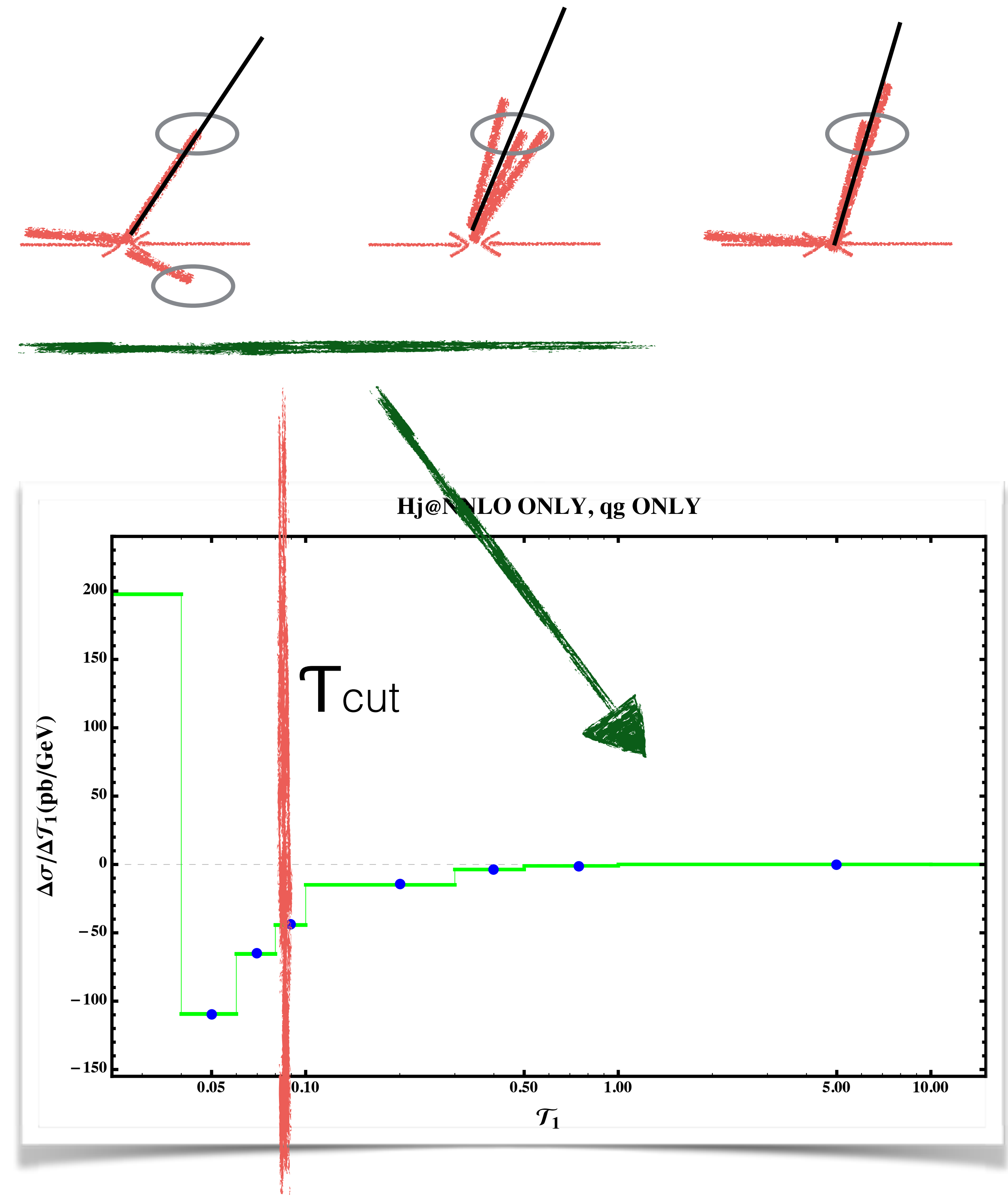
$$\mathcal{T}_N = \sum_k \min\left\{w_a\, n_a \cdot q_k\,, w_b\, n_b \cdot q_k\,, w_i\, n_i \cdot q_k\,, \ldots\,, w_N\, n_N \cdot q_k\right\}$$





Jettiness-subtraction scheme

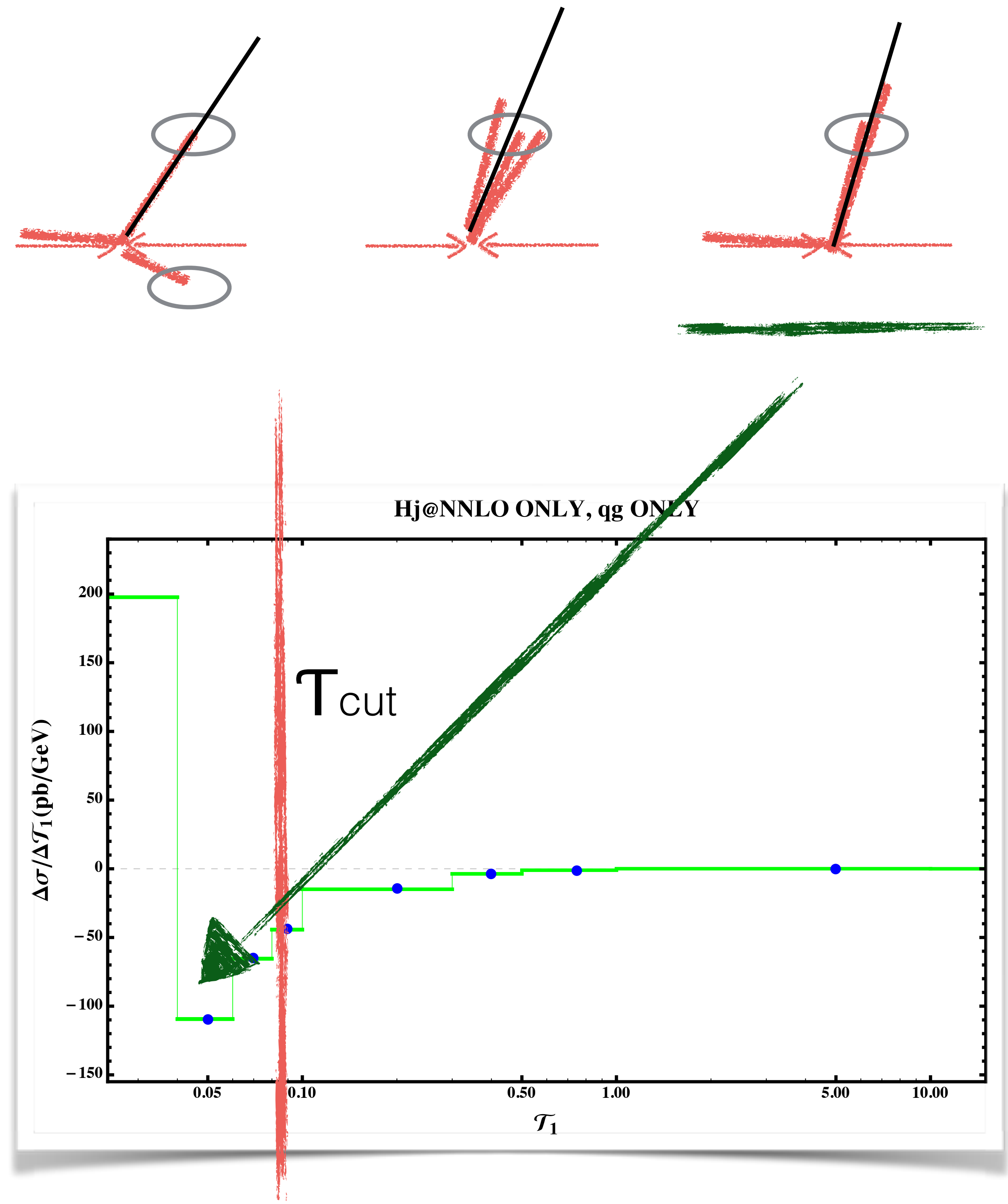
- Jettiness-subtraction
  - introduce a small τ_{cut}
  - at least N jets, calculate τ_N
  - if $\tau_N > \tau_{cut}$, use NLO N+1j



Jettiness-subtraction scheme

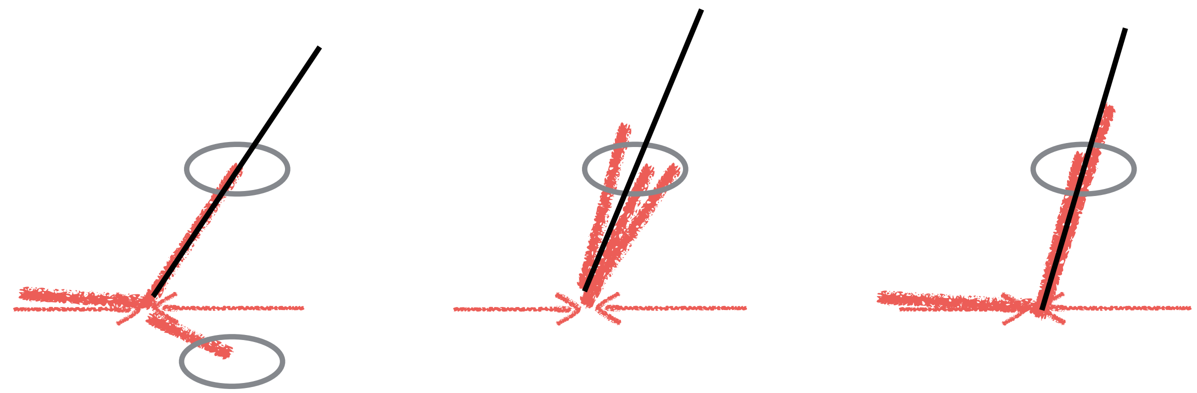
- Jettiness-subtraction
  - introduce a small $\mathcal{T}_{cut}$
  - at least N jets, calculate $\mathcal{T}_N$
  - if $\mathcal{T}_N > \mathcal{T}_{cut}$, use NLO N+1j
  - if $\mathcal{T}_N < \mathcal{T}_{cut}$, use $\mathrm{d}\sigma_{\mathrm{sing.}}$ to include the true NNLO

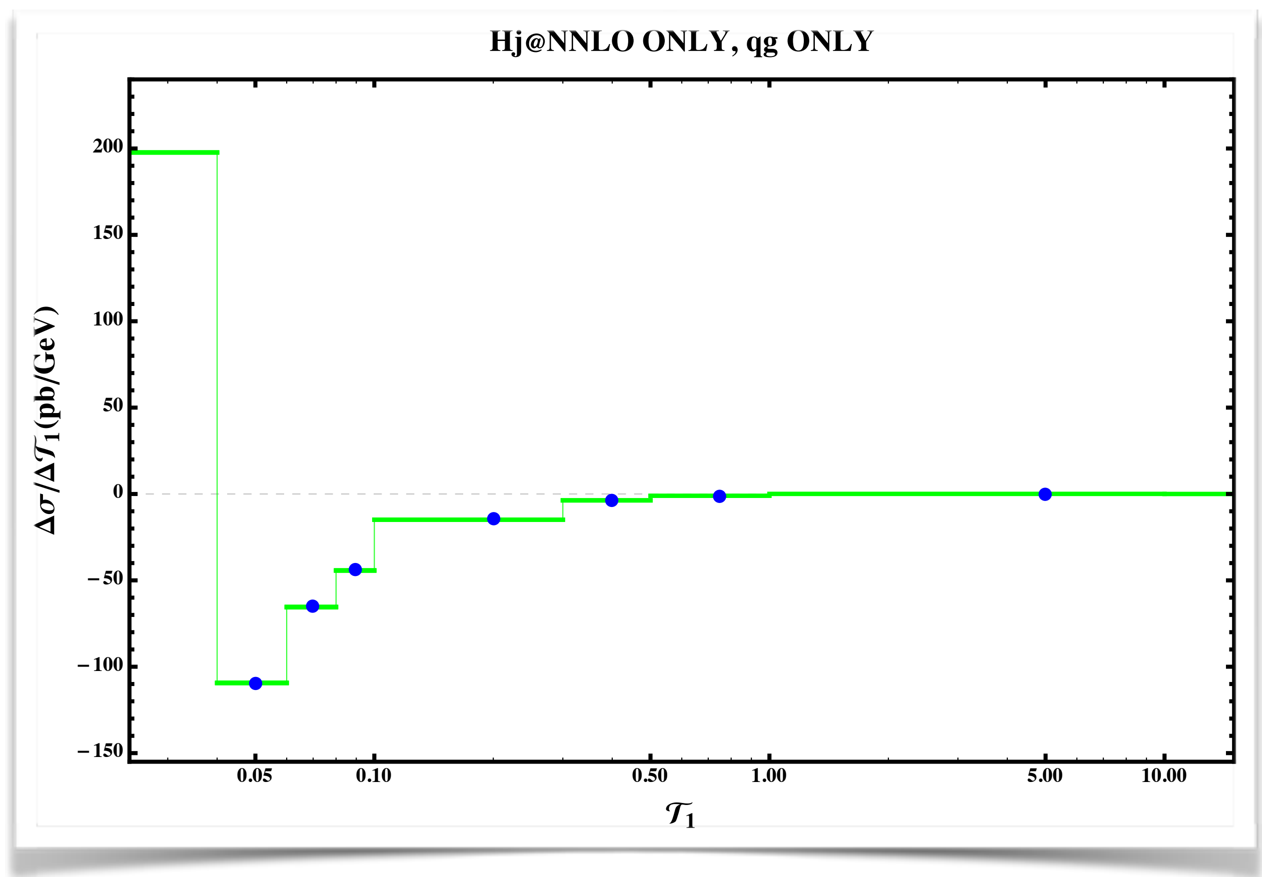
$$\mathrm{d}\sigma_{\mathrm{sing.}}(\mathcal{T}_N) \sim \mathrm{Tr}\left[H \cdot S_N\right] \otimes B_a \otimes B_b \otimes \prod_i^N J_i$$



Jettiness-subtraction scheme

- Jettiness-subtraction
  - introduce a small $\mathcal{T}_{cut}$
  - at least N jets, calculate $\mathcal{T}_N$
  - if $\mathcal{T}_N > \mathcal{T}_{cut}$, use NLO N+1j
  - if $\mathcal{T}_N < \mathcal{T}_{cut}$, use $\mathrm{d}\sigma_{\mathrm{sing.}}$ to include the true NNLO
  - result is $\mathcal{T}_{cut}$-independent





Jettiness-subtraction scheme

- Jettiness-subtraction
  - Final massive colored state?
    - less final collinear singularities, less N.
    - For example, need only τ_0 for pp -> ttbar

      similar ideas using transverse momentum see also: Zhu, Li, Li, Shao and Yang'12; Catani, Grazzini and Torre '15
    - Same beam/jet functions. Soft function can be obtained using the same framework in 1504.02540 Boughezal, XL and Petriello '15

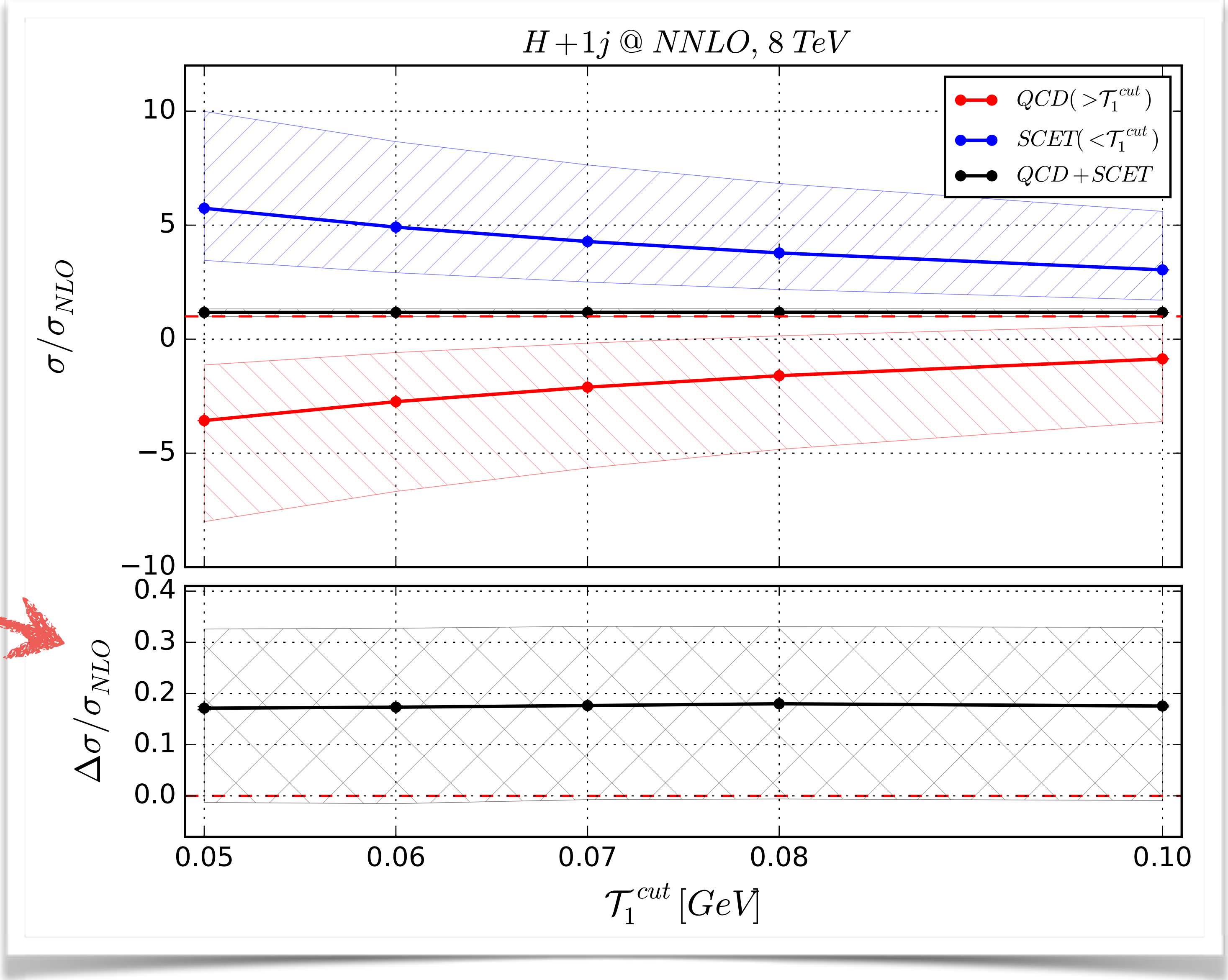
$$\frac{n_i^\mu}{n_i \cdot q} \to \frac{v_i^\mu}{v_i \cdot q}$$

Application

- Fully differential W/H + 1 jet at the LHC

Boughezal, Focke, XL and Petriello ‘15, Boughezal, Focke, Giele, XL and Petriello ‘15

$\mathcal{T}_{cut}$ independent !

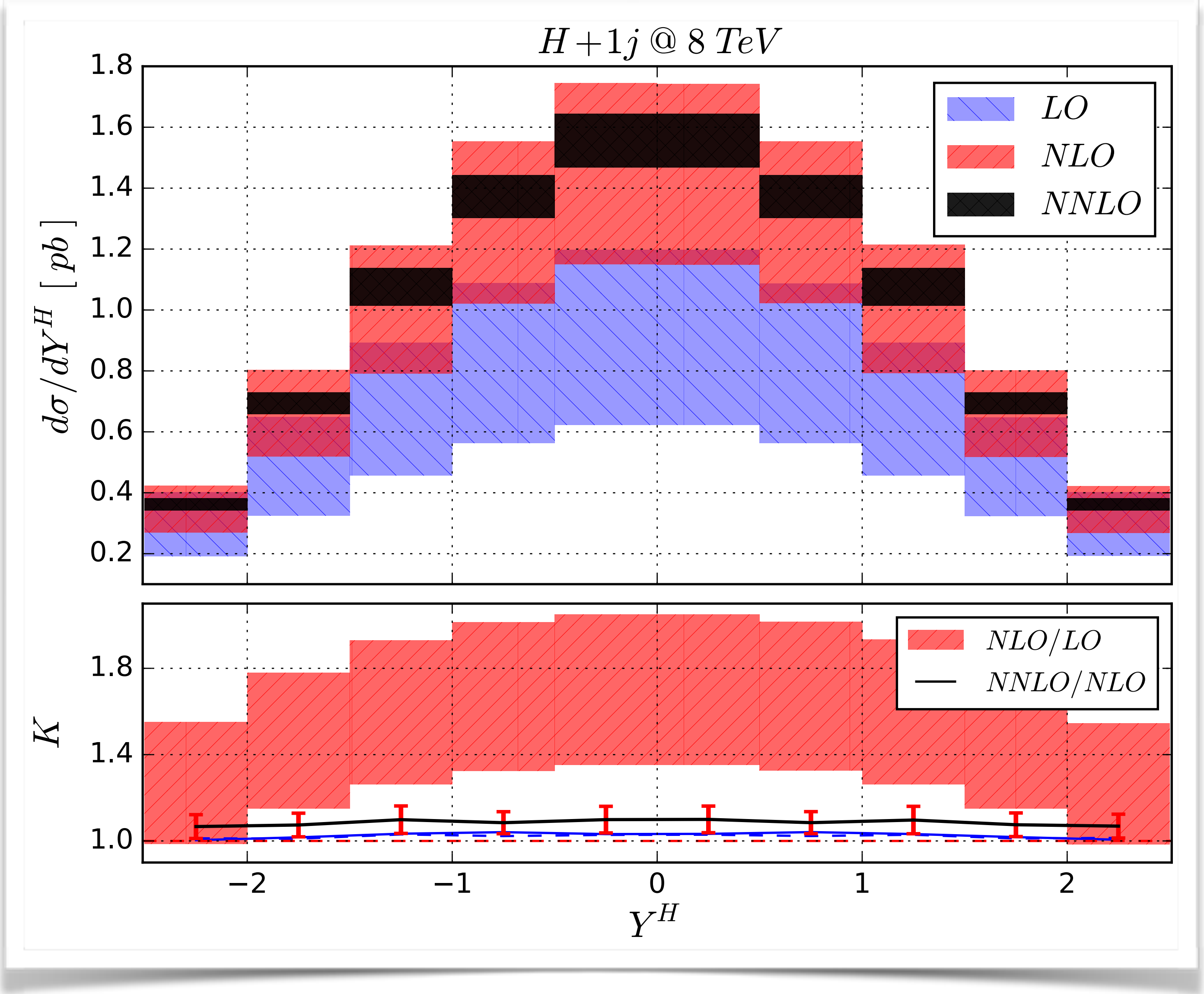


Application

- Fully differential W/H + 1 jet at the LHC

Boughezal, Focke, XL and Petriello '15, Boughezal, Focke, Giele, XL and Petriello '15

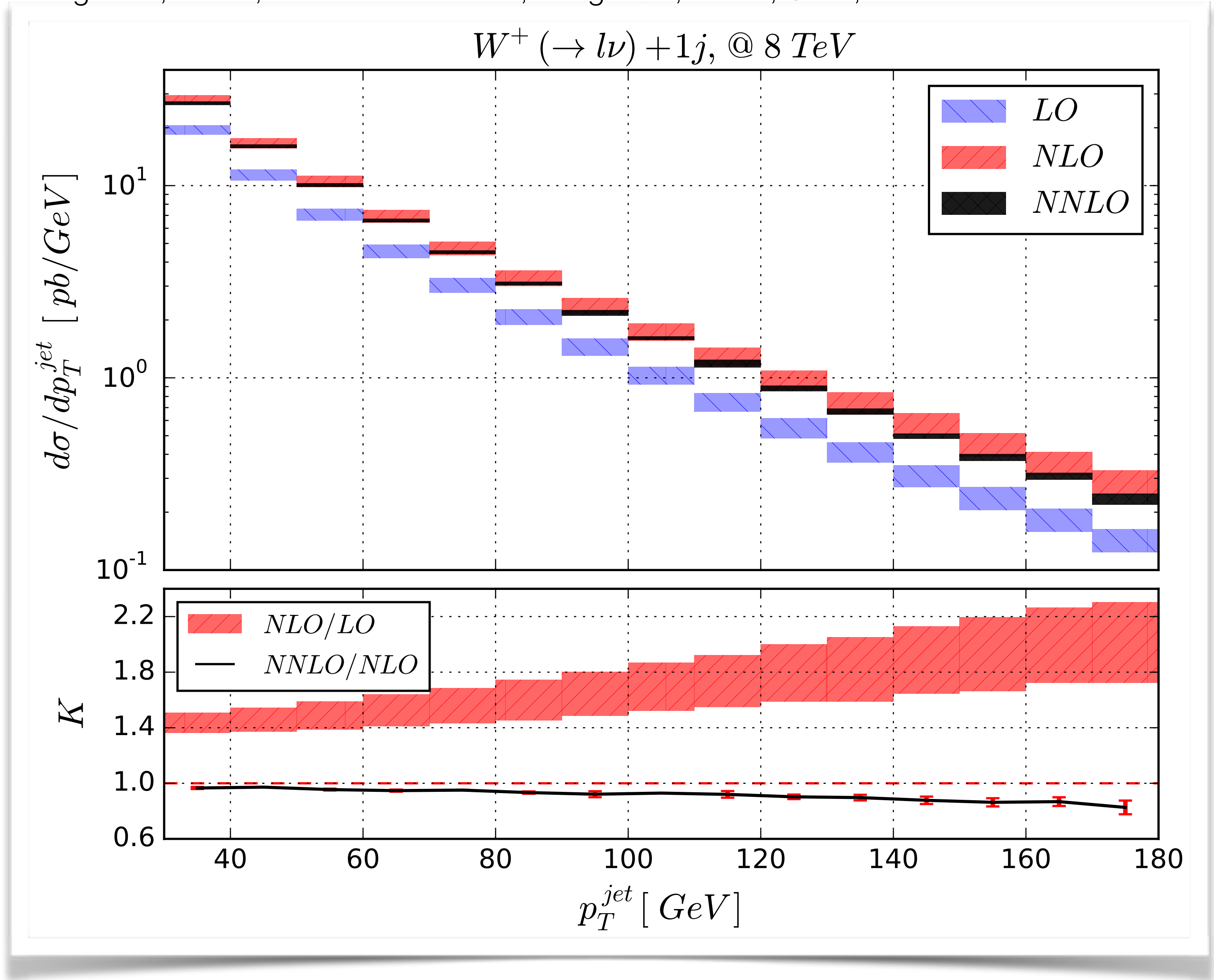
  - all channels
  - convergent series
  - reduced uncertainty



Application

- Fully differential W/H + 1 jet at the LHC
  - all channels
  - convergent series
  - reduced uncertainty

Boughezal, Focke, XL and Petriello ‘15, Boughezal, Focke, Giele, XL and Petriello ‘15



Conclusions

- Jettiness-subtraction for generic N-jet production at NNLO
  - applicable to both massless and massive cases
- All building blocks are known to NNLO
  - soft function calculated recently using a general framework suitable for a large class of observables
- Applied to W+1j and Higgs+1j at the LHC
  - fully differential
  - all channels included
- Looking forward to more applications in FO.

thanks

Backup Slides

- Power corrections Gaunt, Stahlhofen, Tackmann, Walsh ’15, Lee, Stewart ’04, Freedman, Goerke ‘13
  - find to be under control
  - can be systematically predicted using SCET

Lee, Stewart ’04, Freedman, Goerke ‘13

$$\alpha_s^2\, \mathcal{T}_{N,cut}\Big[C_3 \log^3(\mathcal{T}_{N,cut}) \,+\, C_2\, \log^2(\mathcal{T}_{N,cut}) + \ldots\Big]$$