Matching the Nagy-Soper parton shower at NLO

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Motivation

- **Monte Carlo Event Generators**
  - Provide simulations of LHC collisions
  - Present in all experimental analyses
  - Widely used to make predictions
  - Need to be improved as data become more precise

- **Improving theoretical prediction to reach better accuracy**
  - Fixed order: (NLO, NNLO, $N^3LO$) QCD, NLO EW
  - Matching parton shower to fixed order calculation (LO, NLO, NNLO)
  - Merging matched calculation for different jet multiplicities (LO & NLO)
  - Current parton showers with LC/LL accuracy
  - Improvement on the PS front by inclusion of subleading effects
Subleading Effects

- Subleading $N_C$ contributions visible for tailored observables
  - Sensitive to soft (wide angle) splitting

\[ e^+ e^- \rightarrow \text{jets} \]

\[ C_F = 4/3 \quad C_F = 3/2 \]

[Plätzer, Sjödahl '12]
Outline of the Talk

- **Nagy-Soper PS concept allows for parton state evolution to include both spin and color correlations**

- Nagy-Soper PS in a nutshell
  - **DEDUCTOR** (LC+ & Spin averaged)
  
  *Nagy, Soper '07 '08 '12 '14*

- Matching Nagy-Soper parton shower and NLO calculation (MC@NLO)
  - **HELAC-NLO+DEDUCTOR**
  
  *Czakon, Hartanto, Kraus, MW '15*

- First study for $pp \rightarrow t\bar{t}j$ production at the LHC (LC & Spin averaged)

- Comparison with other frameworks
Nagy-Soper Parton Shower

- Cross section for an inclusive observable $F$

$$\sigma[F] = \sum_m \frac{1}{m!} \int [d\{p, f\}_m] \langle \mathcal{M}(\{p, f\}_m) | F(\{p, f\}_m) | \mathcal{M}(\{p, f\}_m) \rangle \frac{f_a(\eta_a, \mu_F^2) f_b(\eta_b, \mu_F^2)}{4 n_c(a) n_c(b) \times \text{flux}}$$

- Quantum density matrix

$$\rho(\{p, f\}_m) \sim |\mathcal{M}(\{p, f\}_m)\rangle \langle \mathcal{M}(\{p, f\}_m)|$$

- $|\mathcal{M}(\{p, f\}_m)\rangle$ is a vector in color $\otimes$ spin space

- Perturbative evolution is described by a unitary operator $U(t_F, t_0)$ obeying

$$\frac{dU(t, t_0)}{dt} = [\mathcal{H}_I(t) - \mathcal{V}(t)] U(t, t_0)$$

- $\mathcal{H}_I(t)$: resolved emission; $\mathcal{V}(t)$: unresolved/virtual emission
- Can be decomposed into color diagonal and off-diagonal parts

$$\mathcal{V}(t) = \mathcal{V}_E(t) + \mathcal{V}_S(t)$$
Nagy-Soper Parton Shower

- Solution of evolution equation

\[ U(t, t_0) = N(t, t_0) + \int_{t_0}^{t} d\tau \ U(t, \tau) [H_I(\tau) - V_S(\tau)] \ N(\tau, t_0) \]

- Sudakov form factor

\[ N(t, t_0) = \exp \left( -\int_{t_0}^{t} d\tau \ V_E(\tau) \right), \quad V(t) = V_E(t) + V_S(t) \]

- Exponentiation of \( V(t) \) can be difficult in the case of non-trivial color evolution

  - Only the color diagonal part \( V_E(t) \) is exponentiated
  - Color off-diagonal part \( V_S(t) \) is treated perturbatively

- Expectation value of observable F including shower effects

\[ \sigma[F] = \langle F | \rho(t_F) | \rangle = \langle F | U(t_F, t_0) | \rho(t_0) \rangle \]

- \( t_F \rightarrow \) scale at which parton emission can not be described perturbatively
Features of Nagy-Soper PS

- Splitting functions are different from Altarelli-Parisi
- Massive initial state charm and bottom quarks
- Constructed to include full spin evolution and full color evolution
- Ordering variable $\Lambda_l$ [Nagy, Soper ‘08 ’12]
  
  $$\Lambda_l^2 = \frac{[(\hat{p}_l \pm \hat{p}_{m+1})^2 - m_i^2]}{2 p_l \cdot Q_0} Q_0^2,$$
  
  $$e^{-t} = \frac{\Lambda_l^2}{Q_0^2}$$

- PDFs are evolved according to shower splitting functions [Nagy, Soper ‘14]

- Public code: DEDUCTOR [Nagy, Soper ‘14]
  
  - LC+ approximation [Nagy, Soper ‘12]
    - Full color for collinear and soft-collinear limits
    - LC for pure soft limits
  - Spin averaged evolution
Matching Inclusive Processes

- NLO density matrix
  \[ |\rho\rangle = |\rho_m^{(0)}\rangle + |\rho_m^{(1)}\rangle + |\rho_{m+1}^{(0)}\rangle + \mathcal{O}(\alpha_s^2) \]
  - Born, \(\mathcal{O}(1)\)
  - Virtual, \(\mathcal{O}(\alpha_s)\)
  - Real, \(\mathcal{O}(\alpha_s)\)

- Shower evolution on the NLO density matrix expanded to \(\mathcal{O}(\alpha_s)\)
  \[ |\rho(t_F)\rangle = U(t_F, t_0) |\rho\rangle \approx |\rho\rangle + \int_{t_0}^{t_F} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}(\tau) \right] |\rho_m^{(0)}\rangle + \mathcal{O}(\alpha_s^2) \]

- Modify density matrix to remove double counting (MC@NLO approach)
  \[ |\tilde{\rho}\rangle \equiv |\rho\rangle - \int_{t_0}^{t_F} d\tau \left[ \mathcal{H}_I(\tau) - \mathcal{V}(\tau) \right] |\rho_m^{(0)}\rangle + \mathcal{O}(\alpha_s^2) \]

- For an infrared safe observable \(F\) we have
  \[ \tilde{\sigma}[F] = \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)|\Phi_m) \left[ (\Phi_m|\rho_m^{(0)}) + (\Phi_m|\rho_m^{(1)}) + \int_{t_0}^{t_F} d\tau (\Phi_m|\mathcal{V}(\tau)|\rho_m^{(0)}) \right] \]
  \[ + \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)|\Phi_{m+1}) \left[ (\Phi_{m+1}|\rho_{m+1}^{(0)}) - \int_{t_0}^{t_F} d\tau (\Phi_{m+1}|\mathcal{H}_I(\tau)|\rho_m^{(0)}) \right] \]
Matching Inclusive Processes

- Shower kernels are used to define subtraction terms of IR singularities, $t_F \rightarrow \infty$

$$\int_{t_0}^{\infty} d\tau \mathcal{H}_l(\tau) = \sum_l S_l \int_{t_0}^{\infty} d\tau \delta(\tau - t_l)\Theta(\tau - t_0) = \sum_l S_l \Theta(t_l - t_0)$$

$$\int_{t_0}^{\infty} d\tau \mathcal{V}(\tau) = \sum_l \int d\Gamma_l S_l \Theta(t_l - t_0) \equiv I(t_0) + K(t_0)$$

- Matched cross section including shower evolution

$$\bar{\sigma}[F]^{PS} = \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)|\Phi_m)(\Phi_m|S)$$

$$+ \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)|\Phi_{m+1})(\Phi_{m+1}|H)$$

$$(\Phi_m|S) \equiv (\Phi_m|\rho_m^{(0)}) + (\Phi_m|\rho_m^{(1)}) + (\Phi_m|[I(t_0) + K(t_0) + P]|\rho_m^{(0)})$$

$$(\Phi_{m+1}|H) \equiv (\Phi_{m+1}|\rho_{m+1}^{(0)}) - \sum_l (\Phi_{m+1}|S_l|\rho_m^{(0)})\Theta(t_l - t_0)$$

- Matching in two steps: generation of $(\Phi_m|S)$ and $(\Phi_{m+1}|H)$
- Application of $U(t_F, t_0)$
Matching Exclusive Processes

- Inclusion of generation cuts

\[
\bar{\sigma}[F]^{PS} = \frac{1}{m!} \int [d\Phi_m](F|U(t_F, t_0)\Phi_m)(\Phi_m|S)F_I(\{\hat{\rho}, \hat{\phi}\}_m) \\
+ \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|U(t_F, t_0)\Phi_{m+1})(\Phi_{m+1}|H)F_I(\{p, f\}_{m+1})
\]

- Expanding the evolution operator

\[
\bar{\sigma}[F]^{PS} \approx \frac{1}{m!} \int [d\Phi_m](F|\Phi_m)(\Phi_m| \left[ |\rho_m^{(0)}\rangle + |\rho_m^{(1)}\rangle + P|\rho_m^{(0)}\rangle \right] F_I(\{\hat{\rho}, \hat{\phi}\}_m) \\
+ \frac{1}{(m+1)!} \int [d\Phi_{m+1}](F|\Phi_{m+1})(\Phi_{m+1}|\rho_{m+1}^{(0)})F_I(\{p, f\}_{m+1}) \\
+ \int \frac{[d\Phi_m]}{m!} \frac{[d\Phi_{m+1}]}{(m+1)!} \int_{t_0}^{t_F} d\tau (F|\Phi_{m+1})(\Phi_{m+1}|\mathcal{H}_I(\tau)|\Phi_m) \\
\times (\Phi_m|\rho_m^{(0)}) \left[ F_I(\{\hat{\rho}, \hat{\phi}\}_m) - F_I(\{p, f\}_{m+1}) \right] + \mathcal{O}(\alpha_s^2)
\]
Matching Exclusive Processes

- Mismatch is cured by enforcing the subtraction terms to fulfill $F_I(\{\hat{p}, \hat{f}\}_m)$

\[
(\Phi_{m+1}|H) \rightarrow (\Phi_{m+1}|\tilde{H}) \equiv (\Phi_{m+1}|\rho_{m+1}^{(0)}) - \sum_l (\Phi_{m+1}|S_l|\rho_{m}^{(0)}) \Theta(t_l - t_0) F_I(Q_I(\{p, f\}_{m+1}))
\]

- $F_I(Q_I(\{p, f\}_{m+1})) = F_I(\{\hat{p}, \hat{f}\}_m)$ and $Q_I$ is inverse momentum mapping

- After modification we have

\[
\bar{\sigma}[F]^{PS} \approx \sigma^{NLO} + \int \frac{[d\Phi_m]}{m!} \frac{[d\Phi_{m+1}]}{(m+1)!} \int_{t_0}^{t_f} d\tau F(\Phi_{m+1})(\Phi_{m+1}|H_I(\tau)|\Phi_m)
\]

\[
\times (\Phi_m|\rho_m^{(0)}) \left[1 - F_I(\{p, f\}_{m+1})\right] F_I(\{\hat{p}, \hat{f}\}_m) + O(\alpha_s^2)
\]

- The double counting is removed if

\[
\left[1 - F_I(\{p, f\}_{m+1})\right] F(\{p, f\}_{m+1}) = 0
\]

\[
F_I(\{p, f\}_{m+1}) = 1 \text{ for } F(\{p, f\}_{m+1}) \neq 0
\]
Summary of Ambiguities

- **Parton masses**
  - Nagy-Soper parton shower treats bottom and charm quarks as massive
  - NLO calculation treats them as massless (bottom in 5FS)
  - Masses for the relevant quarks introduced by the on-shell projection

- **Parton distribution functions**
  - PDFs are evolved differently in the NLO calculation and in the shower
    - NLO calculation: NLO PDFs are used
    - Parton Shower: PDFs are evolved using Nagy-Soper splitting kernels
  - The presence of quark masses
  - The evolution is of higher order $\Rightarrow$ NLO accuracy is maintained if the evolutions share a common point e.g. at the low scale

- **Initial shower time**
  - The choice of $t_0$ in the parton evolution is arbitrary
  - Requirement: NLO prediction recovered for hard emissions
  - Different choices of $t_0$ can achieve this
  - We pick one possible choice of $t_0$ but others are possible
  - Vary $t_0$ to study the uncertainty of NLO+PS matching systematic
Implementation

- Nagy-Soper subtraction scheme in HELAC-DIPOLES

**Catani-Seymour**
- Easier dipole integration
- $n^3$ growth of subtraction terms

**Nagy-Soper**
- More complex dipole integration
- $n^2$ growth of subtraction terms

- Due to differences in
  - splitting functions, momentum mappings, dipole phase space factorization
- Extensively tested for various processes, e.g. study of $pp \rightarrow bbbb$

[Bevilacqua, Czakon, Krämer, Kubocz, MW ‘13]
Modifications in HELAC-DIPOLES

- Momentum mapping for initial state splitting
  - Implementation of subtraction scheme in HELAC-DIPOLES based on the first Nagy-Soper parton shower paper [Nagy, Soper ’07]
  - DEDUCTOR uses revised momentum mapping for initial state splitting
  - Improves log resummation for certain observables [Nagy, Soper ’10]
  - Now implemented in HELAC-DIPOLES

- Event samples are generated using HELAC-1LOOP and HELAC-DIPOLES

- Supply leading color and unpolarized events to DEDUCTOR
Interface to DEDUCTOR

- Use reweighting for m-parton samples
  - Generate unweighted LO events, then reweight according to

\[
\omega_i(\{p, f\}_m) = 1 + \frac{(\{p, f\}_m|\rho^{(1)}_m)}{(\{p, f\}_m|\rho^{(0)}_m)} + \frac{(\{p, f\}_m|I(t_0) + K(t_0) + P|\rho^{(0)}_m)}{(\{p, f\}_m|\rho^{(0)}_m)}
\]

- Use unweighting for (m + 1)-parton samples
  - Pick the most probable diagonal color flow for each event
  - Store the generated events in the LHE file format

- Interface to DEDUCTOR: implementation of LHE file reader
- On-shell projection for charm and bottom quarks
- Translate color flow in the LHE file to internal representation of DEDUCTOR in terms of color strings
ttj production at LHC

- NLO calculations available
  - NLO+PS using **POWHEG** method
    - [Dittmaier, Uwer, Weinzierl ’07; Melnikov, Schulze ’10]
    - [Kardos, Papadopoulos, Trocsanyi ’11]
    - [Alioli, Moch, Uwer ’11]

- $\sqrt{s} = 8$ TeV, $m_t = 173.5$ GeV, $m_b = 4.75$ GeV, $m_c = 1.4$ GeV
- MSTW2008NLO PDF sets, provided in PS at $\mu_F = 1$ GeV
  - $p_t^{\text{gen}} > 30$ GeV, $p_T > 50$ GeV, $|y_j| < 5$ GeV, $\mu_R = \mu_F = m_t$
- anti-$k_T$ jet algorithm with R=1

- LC and **spin averaged shower evolution**, full correlation in the subtraction
- Top decays, hadronization and multiple interactions are not included

  - **HELAC-NLO + DEDUCTOR** v1.0.0 is compared to
  - NLO calculation (from **HELAC-1LOOP** and **HELAC-DIPOLES**)
  - aMC@NLO + (**Pythia8** and **Pythia6Q**) (from **MadGraph5_aMC@NLO**)
  - **POWHEG + Pythia8** (from **POWHEG-BOX**)

  - [Czakon, Hartanto, Kraus, MW ’15]
## Generation Cut

### HELAC-NLO+DEDUCTOR

<table>
<thead>
<tr>
<th>$p_T^{cut}$ [GeV]</th>
<th>$\sigma_{pp\rightarrow \bar{t}tj+X}^{NLO+PS}$ [pb]</th>
<th>$\epsilon$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>86.51 ± 0.21</td>
<td>2.4</td>
</tr>
<tr>
<td>10</td>
<td>86.26 ± 0.17</td>
<td>2.0</td>
</tr>
<tr>
<td>15</td>
<td>86.22 ± 0.14</td>
<td>1.6</td>
</tr>
<tr>
<td>30</td>
<td>86.11 ± 0.13</td>
<td>1.5</td>
</tr>
<tr>
<td>40</td>
<td>86.01 ± 0.08</td>
<td>0.9</td>
</tr>
<tr>
<td>50</td>
<td>84.58 ± 0.07</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Total cross section together with statistical and relative errors for different values of the generation cut

[Czakon, Hartanto, Kraus, MW '15]
Initial Shower Time

- Virtuality rescaling parameter $\mu_T$

$e^{-t_0} = \min_{i \neq j} \left\{ \frac{2p_i \cdot p_j}{\mu_T^2 Q_0^2} \right\}$

- Higher $\mu_T$ value → larger correction in the high-$p_T$ tail
- To recover the NLO prediction, we set $\mu_{T_0} = 1$

[Czakon, Hartanto, Kraus, MW '15]
Uncertainties

Scale uncertainties: $m_t/2 < \mu_{R,F} < 2m_t$
PS initial conditions: $1/2 < \mu_T < 2$
Uncertainties

Scale uncertainties: $m_t/2 < \mu_{R,F} < 2m_t$
PS initial conditions: $1/2 < \mu_T < 2$

[Czakon, Hartanto, Kraus, MW ’15]
Comparison

- **HELAC-NLO**
  \[ \sigma_{pp \rightarrow t\bar{t}j + X}^{NLO} = 86.04^{+5.10}_{-11.41} (\pm 6\%) \text{ pb} \]

- **HELAC-NLO+DEDUCTOR**
  \[ \sigma_{pp \rightarrow t\bar{t}j + X}^{NLO+PS} = 86.11^{+4.38}_{-10.88} (\pm 5\%) \text{ [scales]} +0.80 (\pm 1\%) \text{ [PS time]} \text{ pb} \]

- **Others**
  \[ \sigma_{pp \rightarrow t\bar{t}j + X}^{NLO+PS} (aMC@NLO+PYTHIA6Q) = 84.85^{+8.95}_{-13.75} (\pm 11\%) \text{ [scales]} \text{ pb} \]
  \[ \sigma_{pp \rightarrow t\bar{t}j + X}^{NLO+PS} (aMC@NLO+PYTHIA8) = 89.55^{+8.44}_{-15.41} (\pm 9\%) \text{ [scales]} \text{ pb} \]
  \[ \sigma_{pp \rightarrow t\bar{t}j + X}^{NLO+PS} (POWHEG+PYTHIA8) = 89.12^{+26.22}_{-8.96} (\pm 29\%) \text{ [scales]} \text{ pb} \]

[Czakon, Hartanto, Kraus, MW ’15]
Comparison

- Differences between: matching procedures and showers

- Agreement between different predictions for inclusive distributions

[Czakon, Hartanto, Kraus, MW '15]
Comparison

- Shower sensitive observables

- Helac-NLO+Deductor preserves NLO spectrum
- aMC@NLO+Pythia6Q recovers NLO results, produces softer emission
- Pythia8 (with MC@NLO and POWHEG matching) overshoots NLO at high-$p_T$

[Czakon, Hartanto, Kraus, MW '15]
Summary

 Already done
  - NLO matching scheme for the Nagy-Soper parton shower (MC@NLO approach)
  - Implementation in HELAC-NLO framework
  - LC and spin averaged
  - ttj production at the LHC studied using HELAC-NLO+DEDUCTOR
  - Comparison to other generators performed

 Need to be added
  - In DEDUCTOR
    - Resonance decays
    - Non-perturbative effects
    - Go beyond LC+
    - Spin correlation
  - In HELAC-NLO
    - Full treatments of color and spin correlation in the matching implementation