EW and QCD corrections at NLO with RECOLA

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In collaboration with

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After the discovery of the Higgs boson:

- Precise investigation of the Standard Model and beyond
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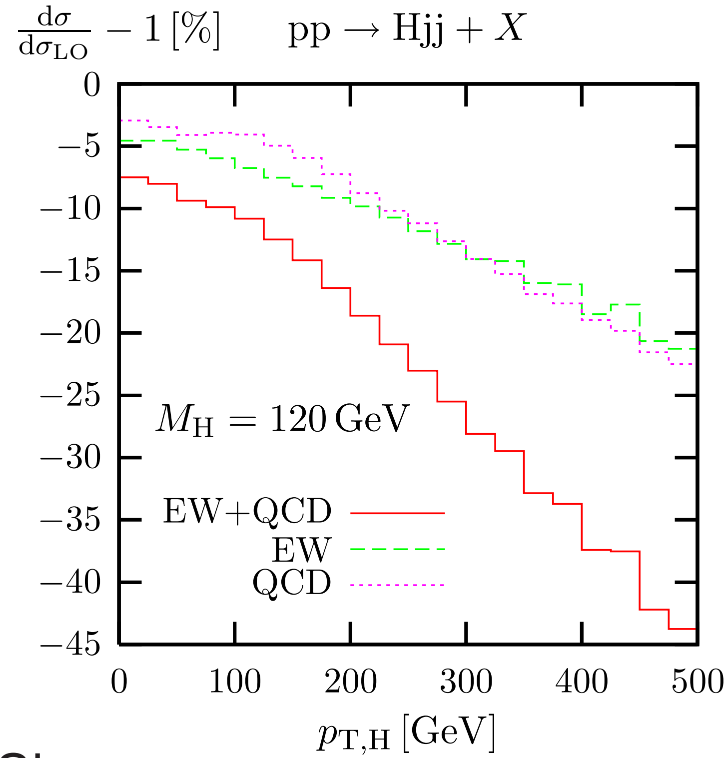
After the discovery of the Higgs boson:

- Precise investigation of the Standard Model and beyond
- Need to have under control potential large corrections for several processes
- QCD corrections are known to be large
- EW corrections can be enhanced:
  - in high energy regions (Sudakov log's)
  - in Higgs physics
  - by photon emission (mass-singular log's)



Many issues at hadronic level:

Multi-channel MCs, Real emission, PDFs, Parton Shower, ...

At least the partonic processes should be **automatized**

Many codes for NLO computations:

MCFM	Campbell, Ellis
FormCalc	Agrawal, Hahn, Mirabella
BlackHat	Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître
VBFNLO	Arnold, Bähr, Bozzi, Campanario, Englert, Figy, Greiner, Hackstein, Hankele, Jäger, Klämke, Kubocz, Oleari, Plätzer, Prestel, Worek, Zeppenfeld
HELAC-NLO	Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
GoSam	Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano
SANC	Sadykov, Arbuzov, Bardin, Bondarenko, Christova, Kalinovskaya, Kolesnikov, Sapronov, Uglov
NJet	Badger, Biedermann, Uwer, Yundin
AMC@NLO	Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
OpenLoops	Cascioli, Maierhöfer, Pozzorini

R E C O L A REcursive Computation of One Loop Amplitudes

⇝ in the **full Standard Model**

Based on **recursive relations** for **off-shell currents**

Structure of the code (fortran 95)

- Definition of the processes

```
call define_process_rcl(1,'u g -> u g e+ e-','NLO')
call define_process_rcl(2,'u g -> u g e+[+] e-[-]','NLO')
call define_process_rcl(3,'u g -> u g Z(e+ e-)','NLO')
```

- Generation phase

```
call generate_processes_rcl
```

- Computation of the amplitudes

```
call compute_process_rcl(1,p)
call compute_process_rcl(2,p)
call compute_process_rcl(3,p)
```

(the momenta `p(1:legs,0:3)` come from MC)

Simple calling of RECOLA

```
program main_rcl

use recola

implicit none
double precision :: p(1:6,0:3)

call define_process_rcl(1,'u u~ -> g g e+ e-','NLO')

call generate_processes_rcl

p(1,:) = [4000.000000d0,    0.000000d0,    0.000000d0, 4000.000000d0]
p(2,:) = [4000.000000d0,    0.000000d0,    0.000000d0,-4000.000000d0]
p(3,:) = [2387.444557d0,-2131.721982d0,  677.671238d0, -834.514588d0]
p(4,:) = [2084.010821d0, 1206.027476d0, 1266.044963d0,-1133.899901d0]
p(5,:) = [1954.132674d0, -173.344284d0, -836.261762d0, 1757.626961d0]
p(6,:) = [1574.411948d0, 1099.038791d0,-1107.454439d0,  210.787528d0]

call compute_process_rcl(1,p)

call reset_recola_rcl

end program main_rcl
```

which produces the output:

```
---------------------------------------------------------------------------
 Pole masses and widths:
 M_Z    =   91.154892493050                    Width_Z    =   2.4421237356891
 M_W    =   80.371978311279                    Width_W    =   2.0842992422669
 M_H    =   120.00000000000                    Width_H    =   0.0000000000000
 m_e    =   0.0000000000000
 m_mu   =   0.0000000000000                    Width_mu   =   0.0000000000000
 m_tau  =   0.0000000000000                    Width_tau  =   0.0000000000000
 m_u    =   0.0000000000000
 m_d    =   0.0000000000000
 m_c    =   0.0000000000000                    Width_c    =   0.0000000000000
 m_s    =   0.0000000000000
 m_t    =   172.60000000000                    Width_t    =   0.0000000000000
 m_b    =   0.0000000000000                    Width_b    =   0.0000000000000
---------------------------------------------------------------------------
 Renormalization done in the complex mass scheme
---------------------------------------------------------------------------
 EW Renormalization Scheme: gfermi             Gf = 0.11663700000000E-04
---------------------------------------------------------------------------
 alpha_s(Q) = 0.11800000000000                 Q =   91.187600000000
---------------------------------------------------------------------------
 Delta_UV    =   0.0000000000000               mu_UV =   91.187600000000
 Delta_IR^2 =   0.0000000000000
 Delta_IR    =   0.0000000000000               mu_IR =   91.187600000000
---------------------------------------------------------------------------
 Dimensional regolarization for soft singularities
---------------------------------------------------------------------------
```

```
u u~ -> g g e+ e-

p1 = (4000.0000000000,    0.0000000000,    0.0000000000, 4000.0000000000)
p2 = (4000.0000000000,    0.0000000000,    0.0000000000,-4000.0000000000)
p3 = (2387.4445571379,-2131.7219821216,  677.6712380335, -834.5145879427)
p4 = (2084.0108209587, 1206.0274745508, 1266.0449626178,-1133.8999008430)
p5 = (1954.1326742459, -173.3442838631, -836.2617619034, 1757.6269608155)
p6 = (1574.4119476575, 1099.0387914340,-1107.4544387478,  210.7875279701)

-----------------------------------------------------------------------------

UNPOLARIZED SQUARED AMPLITUDE

 als |        | A0 |^2                    als |   2*Re{ A1 * A0^* }
-----------------------------            -----------------------------
   0 |  0.00000000000000E+00               0 |  0.00000000000000E+00
   1 |  0.00000000000000E+00               1 |  0.00000000000000E+00
   2 |  0.56539984102420E-14               2 | -0.51565460580846E-15
   3 |  0.00000000000000E+00               3 | -0.18749213659000E-13
   4 |  0.00000000000000E+00               4 |  0.00000000000000E+00
     |                                     5 |  0.00000000000000E+00
-----------------------------            -----------------------------
 SUM |  0.56539984102420E-14             SUM | -0.19264868264808E-13
```

Features of RECOLA

- Full Standard Model:
  - Complex mass scheme
  - Feynman rules for rational parts and on-shell Counterterms
  - Select/unselect powers of α_s in the amplitude
  - Selection of resonant contributions

Features of RECOLA

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- Computation of Colour- and Spin-correlations
- Need external libraries for Tensor Integrals $\rightsquigarrow$ link to the COLLIER library

see talk of L. Hofer

Performances

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- **CPU time** (processor Intel(R) Core(TM) i5-2450M CPU @ 2.50GHz):

⇝ QCD corrections ($W^+ \to l^+\nu_l, W^- \to l^-\bar{\nu}_l$, colour and helicity summed)

$u\,\bar{d} \to W^+\,g\,g$	t_{gen}: 2.4 s	t_{TIs}: 4.0 ms	t_{TCs}: 1.1 ms
$u\,\bar{d} \to W^+\,g\,g\,g$	t_{gen}: 15 s	t_{TIs}: 67 ms	t_{TCs}: 45 ms
$u\,\bar{u} \to W^+\,W^-\,g\,g$	t_{gen}: 76 s	t_{TIs}: 83 ms	t_{TCs}: 16 ms

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$u\,\bar{u} \to W^+\,W^-\,g\,g$	$t_{\rm gen}$: 76 s	$t_{\rm TIs}$: 83 ms	$t_{\rm TCs}$: 16 ms

⇝ EW + QCD corrections (colour and helicity summed)

$u\,\bar{u} \to l^+l^-\,g\,g$	$t_{\rm gen}$: 3.2 s	$t_{\rm TIs}$: 27 ms	$t_{\rm TCs}$: 25 ms
$u\,\bar{u} \to l^+l^-\,u\,\bar{u}$	$t_{\rm gen}$: 5 s	$t_{\rm TIs}$: 68 ms	$t_{\rm TCs}$: 35 ms
$u\,\bar{u} \to l^+l^-\,g\,g\,g$	$t_{\rm gen}$: 44 s	$t_{\rm TIs}$: 331 ms	$t_{\rm TCs}$: 684 ms
$u\,\bar{u} \to l^+l^-\,u\,\bar{u}\,g$	$t_{\rm gen}$: 50 s	$t_{\rm TIs}$: 835 ms	$t_{\rm TCs}$: 632 ms

The process $p\,p \to Z + 2\,\text{jets}$

$p\,p \to V + \text{jets}$:

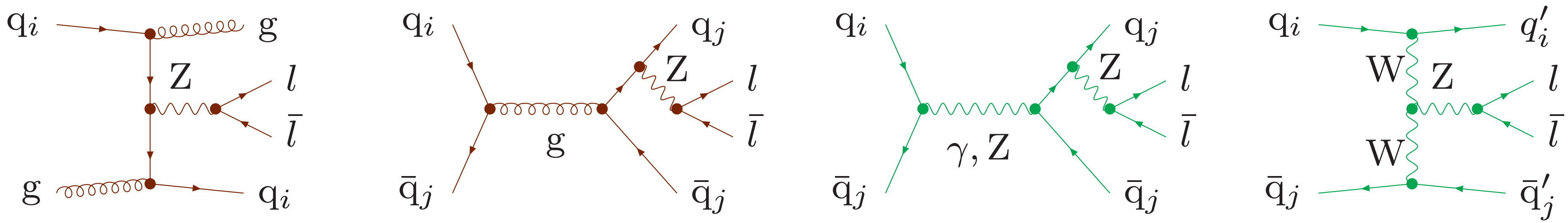
- Tests of QCD and EW Standard Model
- Backgrounds for Higgs studies and new physics searches
- Testing ground for perturbative calculations and event generators
- QCD corrections for $\mathrm{Z} + \leq 4\mathrm{j}$, $\mathrm{W} + \leq 5\mathrm{j}$ [Blackhat collaboration]
- EW corrections for $\mathrm{Z/W} + \mathrm{j}$ [Denner, Dittmaier, Kasprzik, Mück '09, '11, '12]

$p\,p \to Z + 2$ jets:

- Background to Higgs production in Vector Boson Fusion (VBF)
- Background for new physics with large missing transverse momentum

$p\,p \to Z + 2$ jets at LO:

gluon: $q\,g \to q\,g\,\bar{l}l \quad \mathcal{O}(g_s^2 e^2)$ four-quark: $q\,\bar{q} \to q\,\bar{q}\,\bar{l}l \quad \mathcal{O}(e^4)$



$p\,p \to Z + 2$ jets at NLO:

$\mathcal{O}(\alpha_s^3\alpha^2)$: QCD corrections to QCD diagrams [Campbell, Ellis, Rainwater '02, '03]

$\mathcal{O}(\alpha_s^2\alpha^3)$: EW corrections to QCD diagrams

QCD corrections to EW-QCD interferences

$\mathcal{O}(\alpha_s\alpha^4)$: QCD corrections to EW diagrams [Oleari, Zeppenfeld '04]

EW corrections to EW-QCD interferences

$\mathcal{O}(\alpha^5)$: EW corrections to EW diagrams

- Electroweak corrections for $\nu\,\bar{\nu} + 2$ jets in Sudakov limit [Chiesa et al. '13]

General setup:

- G_μ scheme for electromagnetic coupling:

$$\alpha_{G_\mu} = \frac{\sqrt{2} G_\mu M_W^2}{\pi} \left(1 - \frac{M_W^2}{M_Z^2} \right)$$

  Absorbs running of α to EW scale and some universal corrections $\propto m_t^2$

- Complex-mass scheme [Denner, Dittmaier, Roth, Wackeroth, Wieders '99, '05]

  $\mu_\mathrm{Z}^2 = M_Z^2 - iM_Z\Gamma_Z, \quad \mu_\mathrm{W}^2 = M_W^2 - iM_W\Gamma_W, \quad \cos\theta_W = \mu_W/\mu_Z$

- Massless light fermions
- 't Hooft–Feynman gauge
- PDFs: MSTW2008LO [Martin et al. '09]
- Scales: $\mu_R = \mu_F = M_Z$
- Jet clustering: anti-k_T algorithm with $\Delta R = 0.4$ also for photons [Cacciari, Salam, Soyez '08]

Results for $p\,p \to l^+\,l^-\,j\,j$

Basic cuts: motivated by [ATLAS '13]

$$p_{\mathrm{T,j}} > 30\ \mathrm{GeV} \qquad |\eta_j| < 4.5 \qquad p_{\mathrm{T,l}} > 20\ \mathrm{GeV} \qquad |\eta_l| < 2.5$$

$$\Delta R_{jl} > 0.5 \qquad \Delta R_{l^+l^-} > 0.2 \qquad 66\ \mathrm{GeV} < M_{l^+l^-} < 116\ \mathrm{GeV}$$

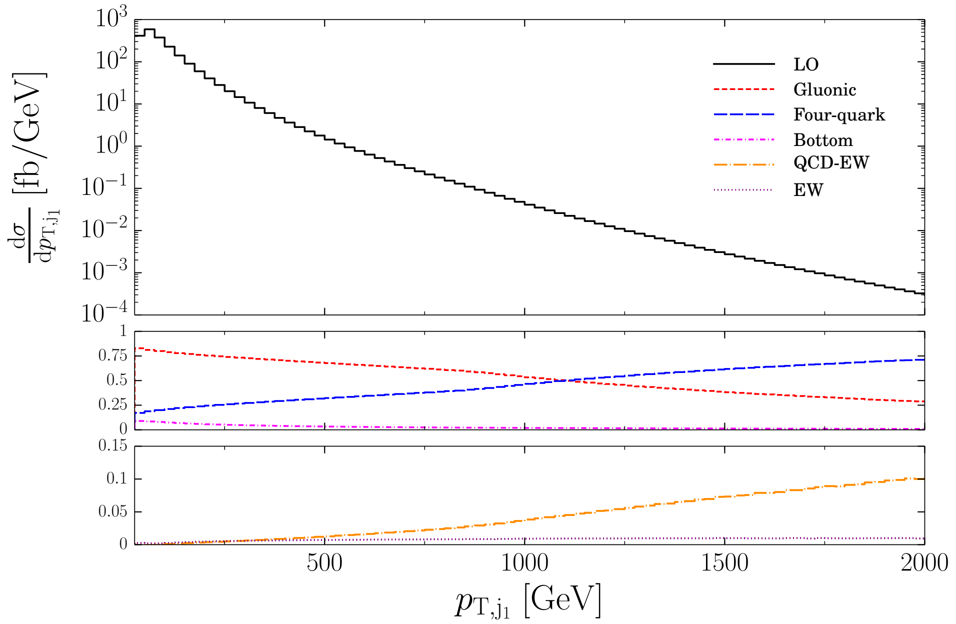
photon energy fraction in jet $z_\gamma < 0.7$

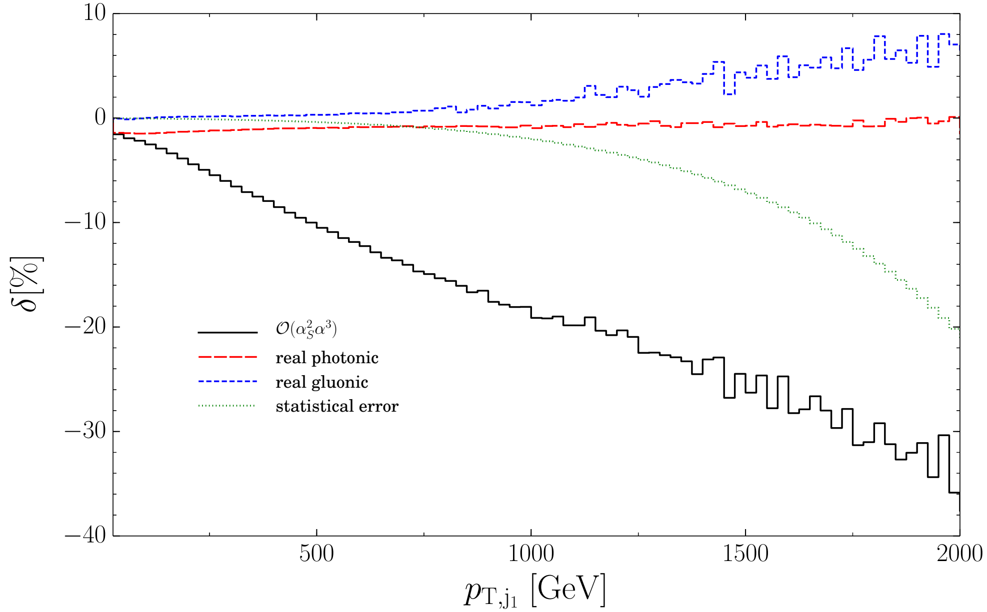
LHC - 13 TeV - Basic cuts - Total cross section

process class	σ^{LO} [fb]	$\sigma^{\mathrm{LO}}/\sigma^{\mathrm{LO}}_{\mathrm{tot}}$ [%]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [fb]	$\frac{\sigma^{\mathrm{NLO}}_{\mathrm{EW}}}{\sigma^{\mathrm{LO}}} - 1$ [%]
gluonic	40910(8)	79.9	39932(9)	- 2.39
four-quark	10299(1)	20.1	10033(1)	- 2.58
sum	51209(8)	100	49965(9)	- 2.43
bottom quarks	4376(3)	8.54		

- $q\, g \to q\, g\, l^- l^+$ channels dominate
- Small EW corrections for total cross section

LHC - 13 TeV - Basic cuts - Distribution in p_{T,j_1} (j_1 = hardest jet)





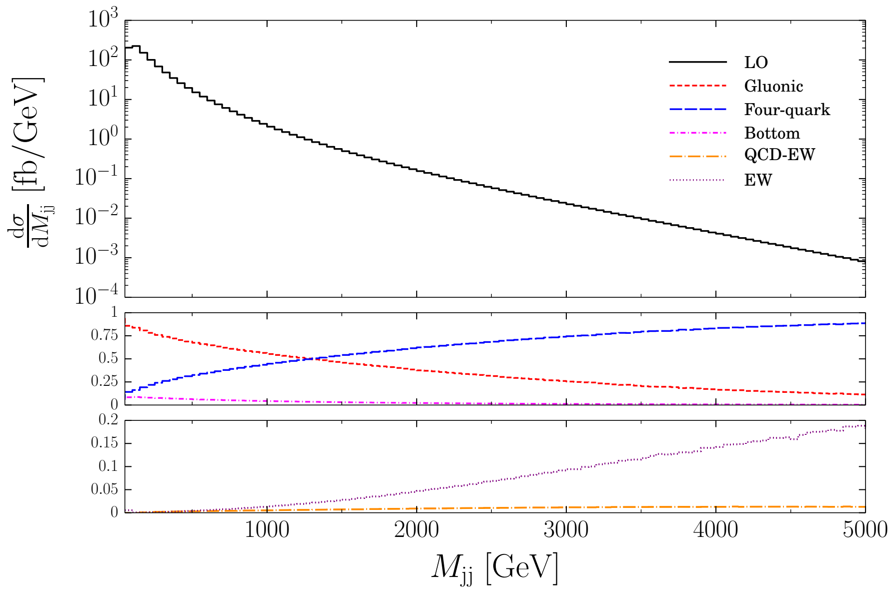
LO

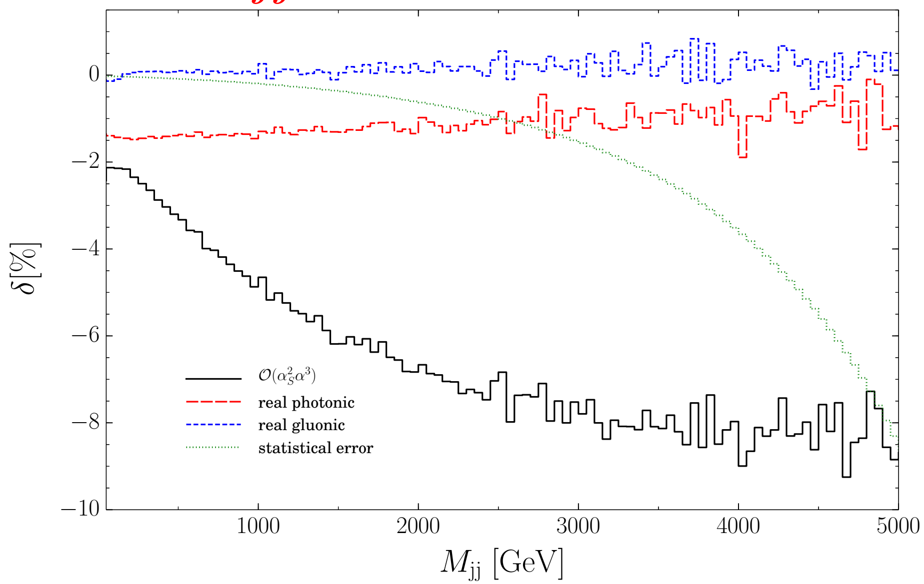
- 4-quark channels dominate at high p_T
- Small bottom contributions

EW NLO

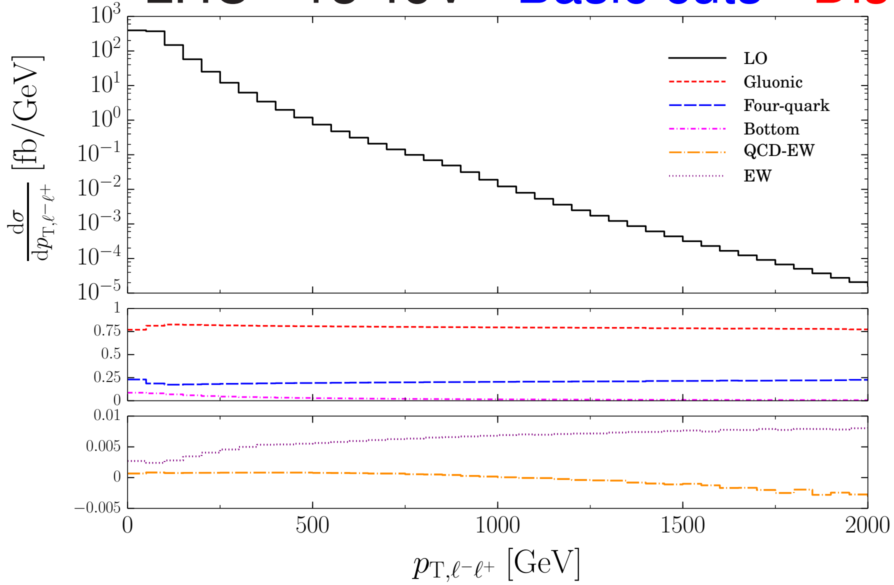
- EW corrections sizeable for large p_T (Sudakov logarithms)
- Small corrections from real + dipoles

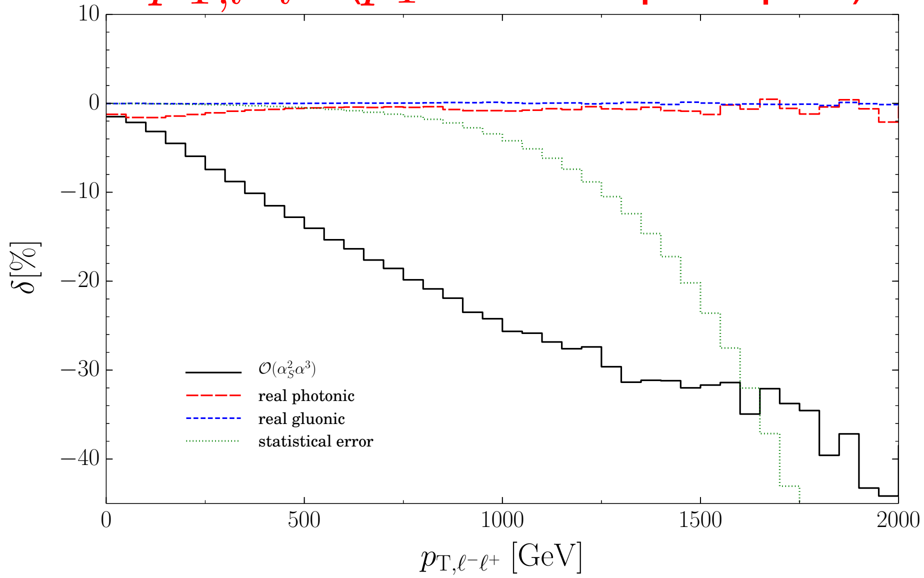
LHC - 13 TeV - Basic cuts - Distribution in M_{jj} (di-jet invariant mass)



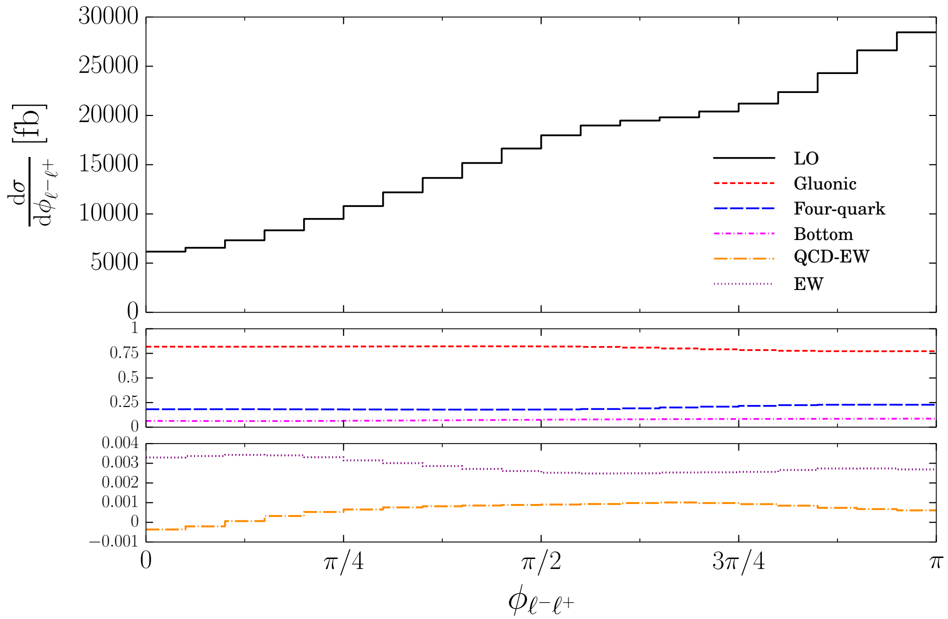


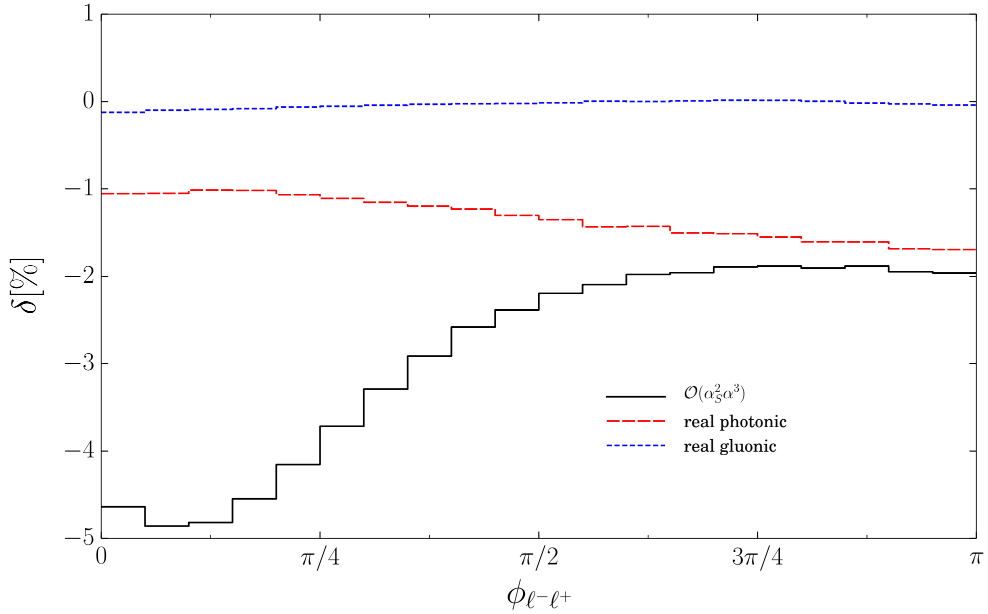
LHC - 13 TeV - Basic cuts - Distribution in p_{T,l^-l^+} (p_T of the lepton pair)





LHC - 13 TeV - Basic cuts - Distribution in $\phi_{l^-l^+}$ (azimutal angle between $l^- l^+$)





LO

- Distribution peaked in backward direction
- Small bottom contributions

EW NLO

- Effect of virtual EW corrections around 3%

Vector boson fusion (VBF) cuts:

$$p_{\mathrm{T,j}} > 30\ \mathrm{GeV} \qquad |\eta_j| < 4.5 \qquad p_{\mathrm{T,l}} > 20\ \mathrm{GeV} \qquad |\eta_l| < 2.5$$

$$\Delta R_{jl^-} > 0.5 \qquad \Delta R_{jl^+} > 0.5 \qquad \Delta R_{l^+l^-} > 0.2$$

$$|y_{\mathrm{j}_1} - y_{\mathrm{j}_2}| > 4 \qquad y_{\mathrm{j}_1} \cdot y_{\mathrm{j}_2} < 0 \qquad \min(y_{\mathrm{j}_1}, y_{\mathrm{j}_2}) < y_l < \max(y_{\mathrm{j}_1}, y_{\mathrm{j}_2})$$

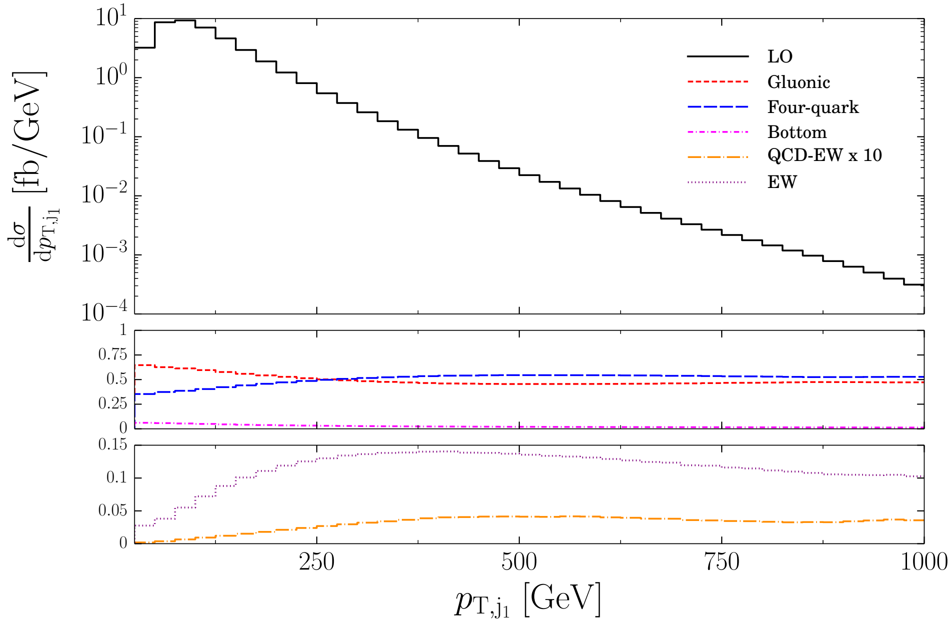
$M_{\mathrm{jj}} > 600\ \mathrm{GeV}$ photon energy fraction in jet $z_\gamma < 0.7$

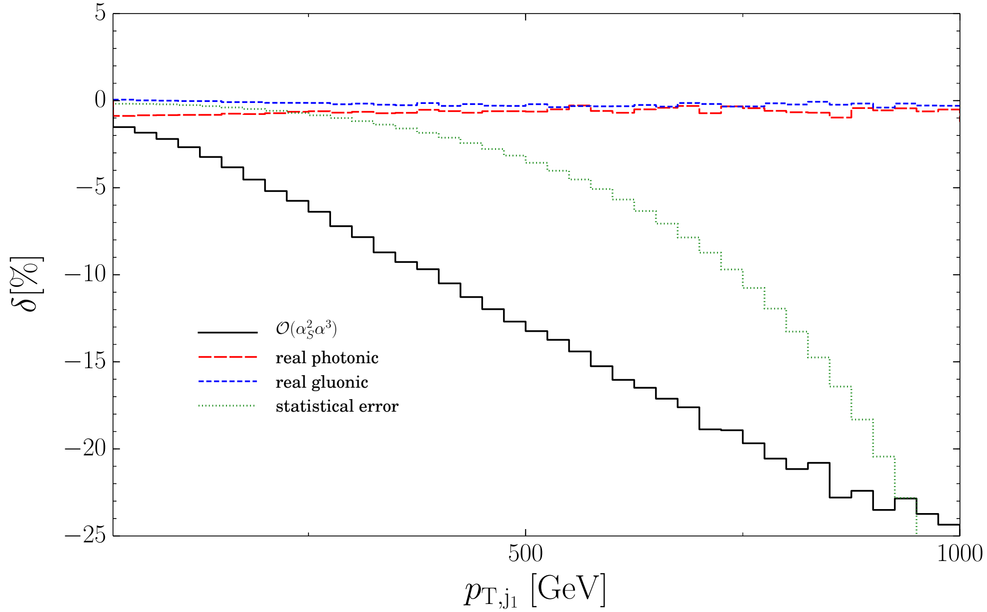
LHC - 13 TeV - VBF cuts - Total cross section

process class	σ^{LO} [fb]	$\sigma^{\mathrm{LO}}/\sigma^{\mathrm{LO}}_{\mathrm{tot}}$ [%]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [fb]	$\frac{\sigma^{\mathrm{NLO}}_{\mathrm{EW}}}{\sigma^{\mathrm{LO}}} - 1$ [%]
gluonic	617.8(4)	59.4	599.2(3)	- 3.01
four-quark	421.7(1)	40.6	410.2(1)	- 2.73
sum	1039.6(4)	100	1009.3(3)	- 2.91

- Cross section reduced by factor 50 w.r.t. basic cuts
- $q\,g \to q\,g\,l^-\,l^+$ channels still dominate, but four-quark channel enhanced
- Still small EW corrections for total cross section

LHC - 13 TeV - VBF cuts - Distribution in p_{T,j_1} (j_1 = hardest jet)





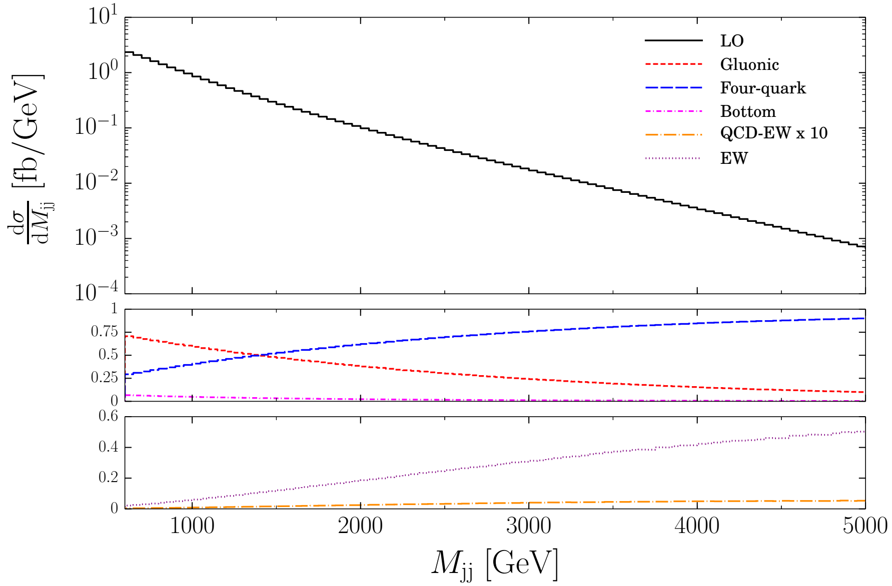
LO

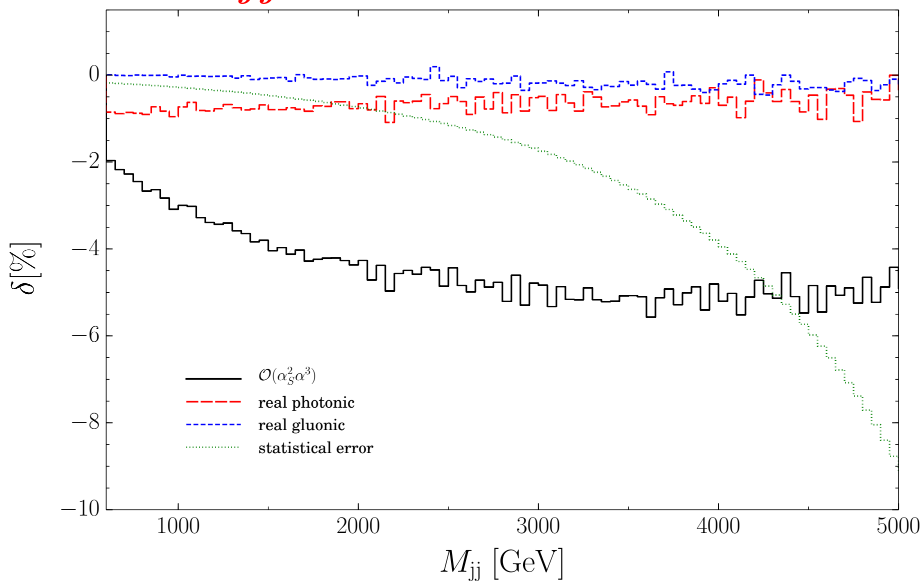
- 4-quark and gluon channels comparable at high p_T
- Small bottom contributions

EW NLO

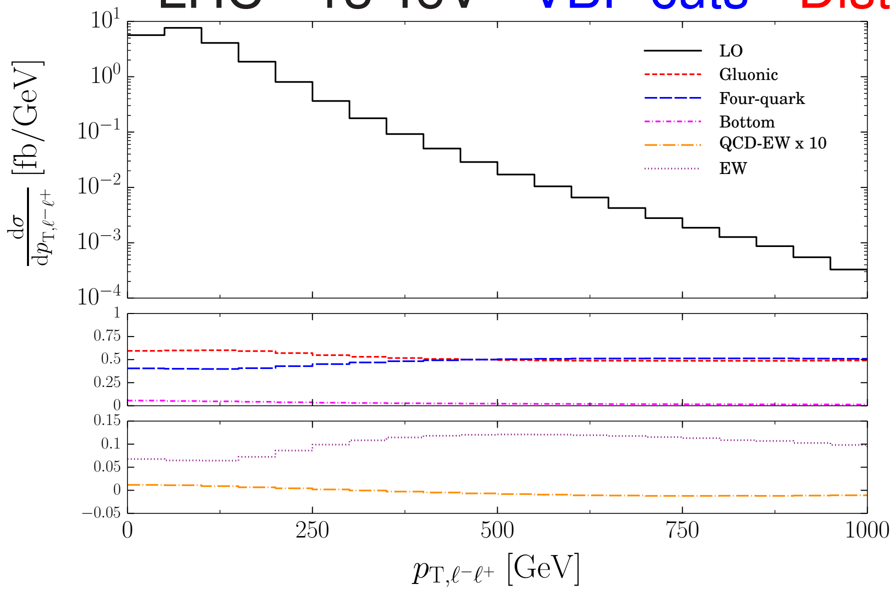
- EW correction more sizeable for large p_T than for basic cuts
- Small corrections from real + dipoles

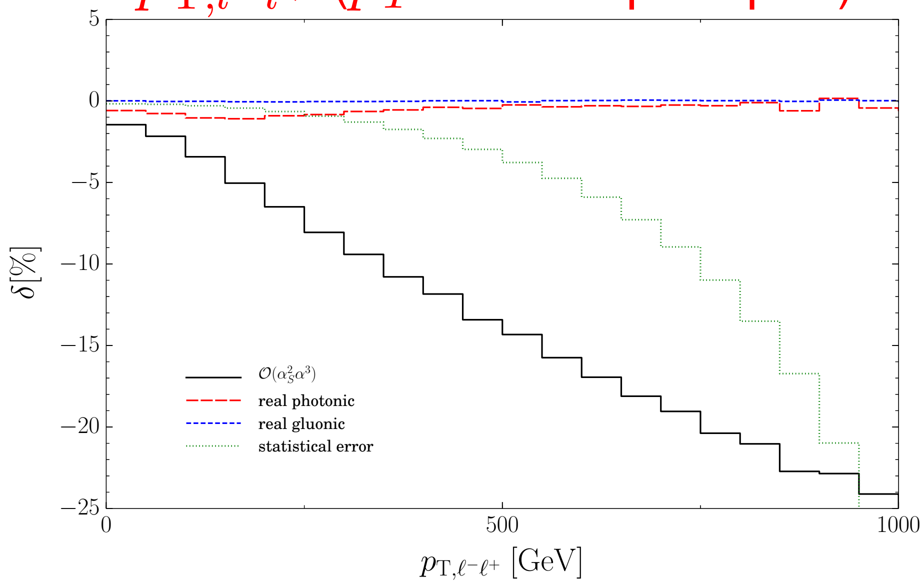
LHC - 13 TeV - VBF cuts - Distribution in M_{jj} (di-jet invariant mass)





LHC - 13 TeV - VBF cuts - Distribution in p_{T,l^-l^+} (p_T of the lepton pair)





Results for $p\,p \to \nu\,\bar{\nu}\,j\,j$

Basic cuts:

$$p_{\mathrm{T,j}} > 30\ \mathrm{GeV} \qquad |\eta_j| < 4.5$$

$$\not{E}_{\mathrm{T}} > 25\ \mathrm{GeV}$$

photon energy fraction in jet $z_\gamma < 0.7$

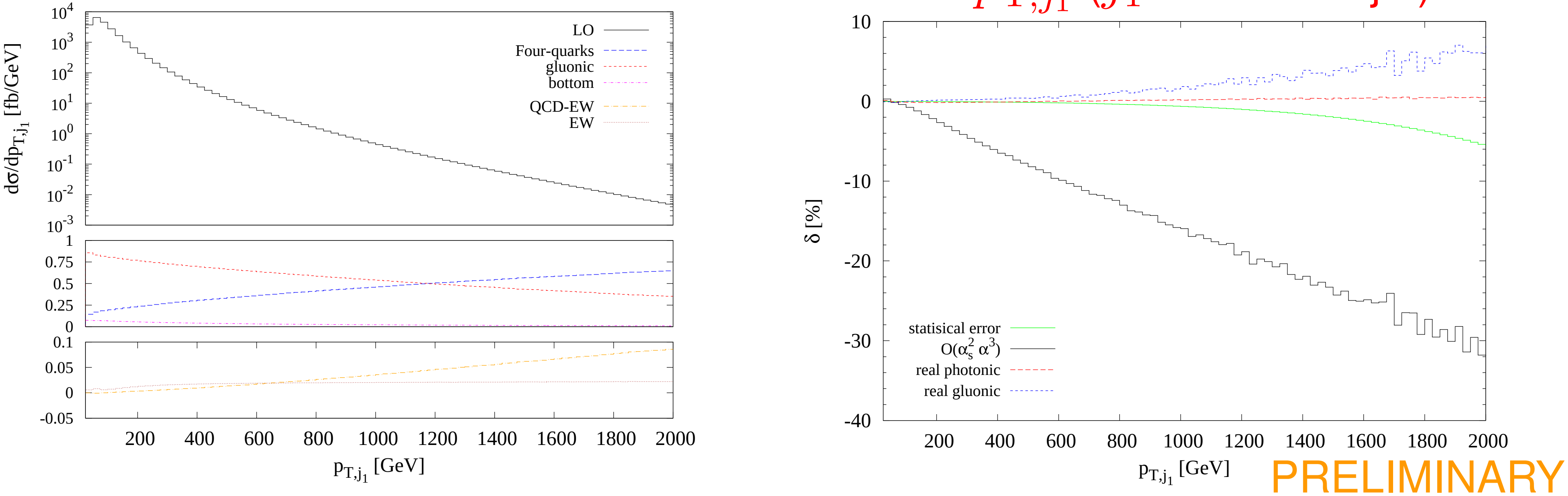
LHC - 13 TeV - Basic cuts - Total cross section

process class	σ^{LO} [pb]	$\sigma^{\mathrm{LO}}/\sigma^{\mathrm{LO}}_{\mathrm{tot}}$ [%]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [pb]	$\frac{\sigma^{\mathrm{NLO}}_{\mathrm{EW}}}{\sigma^{\mathrm{LO}}} - 1$ [%]
gluonic	456.94(11)	81.5	454.34(11)	- 0.57
four-quark	103.66(01)	18.5	102.40(02)	- 1.22
sum	560.60(11)	100	556.74(11)	- 0.69
bottom quarks	37.699(6)	6.72		

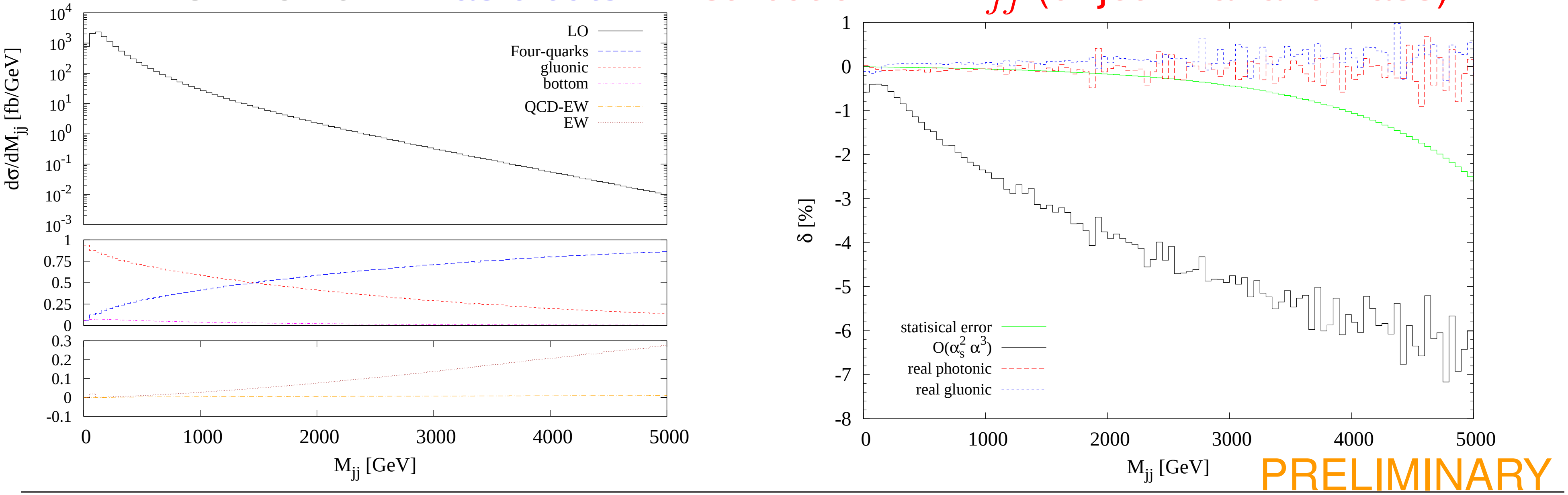
PRELIMINARY

- One order of magnitude larger than for $p\,p \to l^+\,l^-\,j\,j$
- $q\,g \to q\,g\,\nu\,\bar{\nu}$ channels dominate
- Negligible EW corrections for total cross section

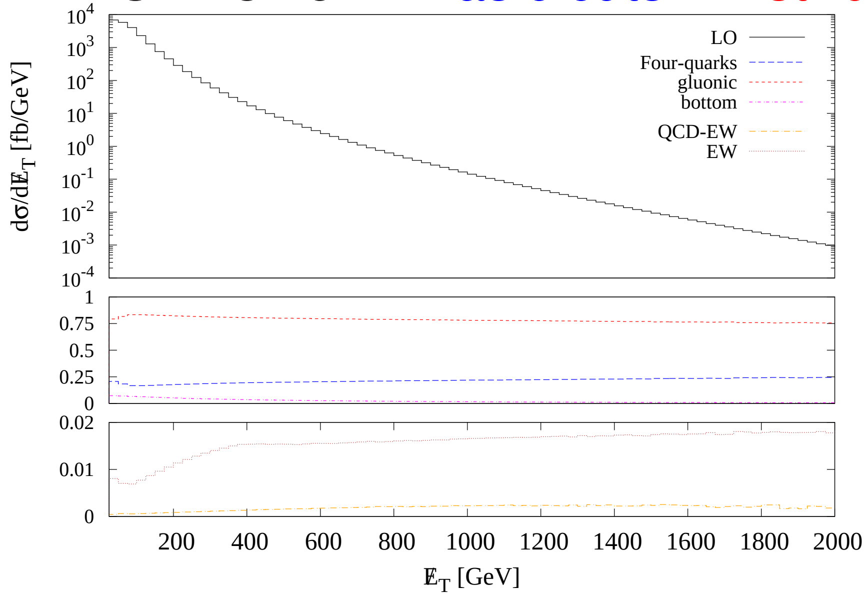
LHC - 13 TeV - Basic cuts - Distribution in p_{T,j_1} (j_1 = hardest jet)

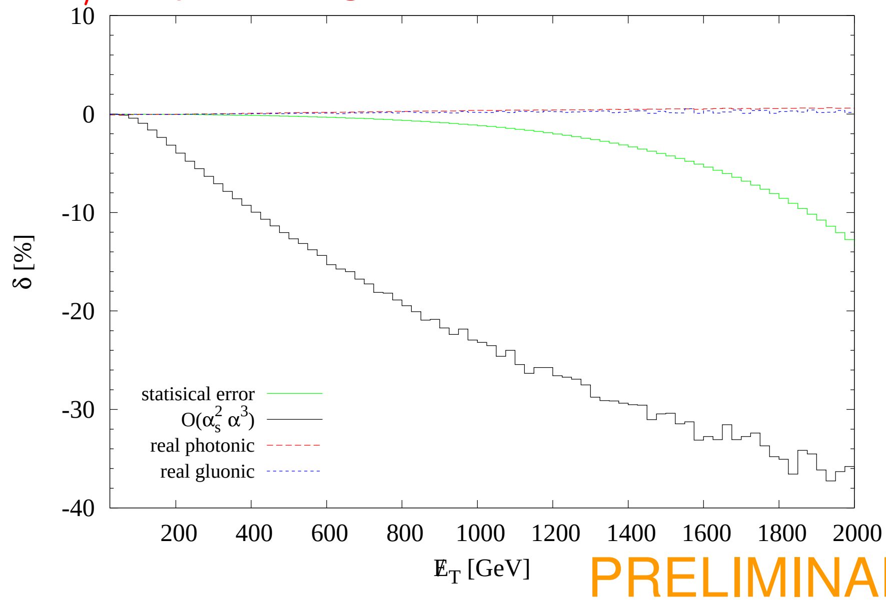


LHC - 13 TeV - Basic cuts - Distribution in M_{jj} (di-jet invariant mass)



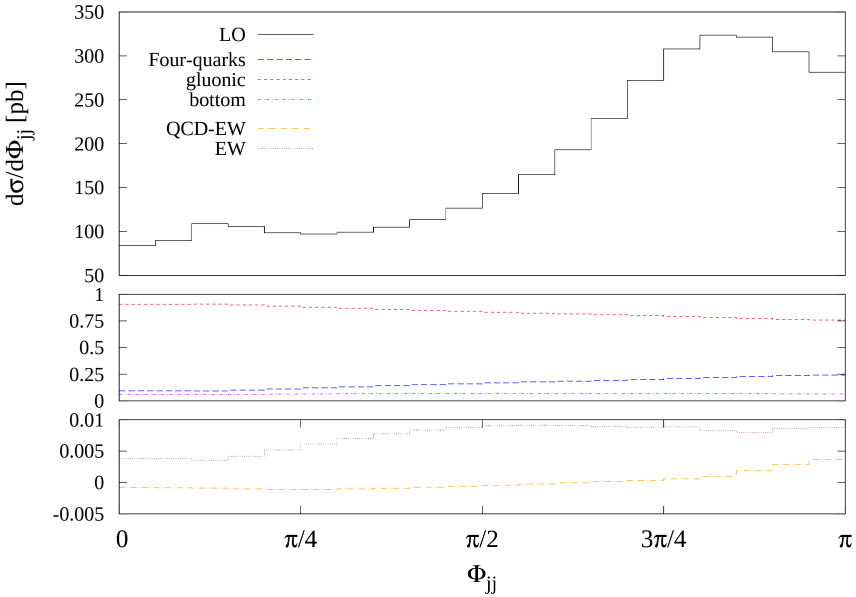
LHC - 13 TeV - Basic cuts - Distribution in $\not{E}_T$ (missing transverse momentum)

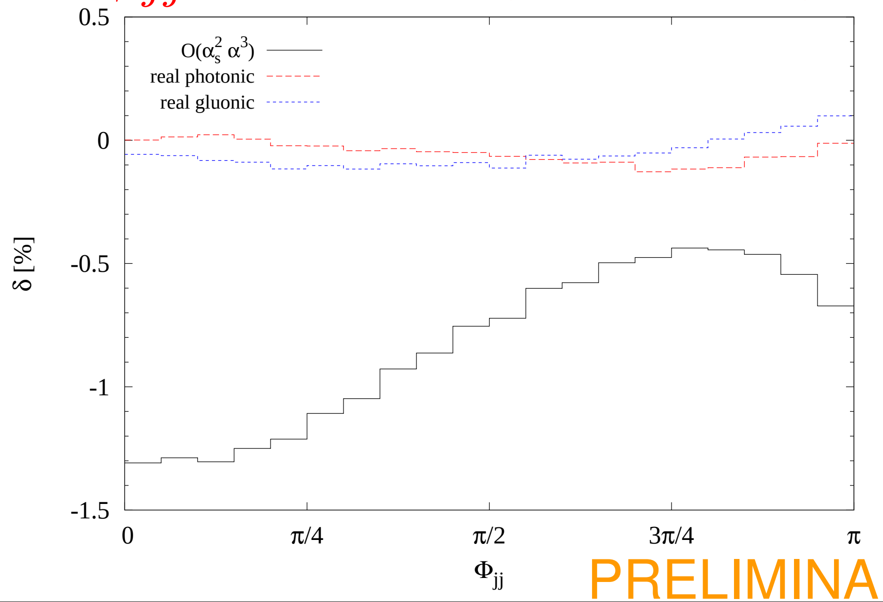




PRELIMINARY

LHC - 13 TeV - Basic cuts - Distribution in ϕ_{jj} (azimutal angle between jets)





PRELIMINARY

ATLAS cuts ([ATLAS Collaboration, JHEP 1409 (2014) 176]):

$$p_{\mathrm{T,j_1}} > 130\ \mathrm{GeV} \qquad p_{\mathrm{T,j_2}} > 60\ \mathrm{GeV} \qquad |\eta_j| < 4.5$$

$$H_{\mathrm{T}} > 800\ \mathrm{GeV} \qquad \not{E}_{\mathrm{T}}/\sqrt{H_{\mathrm{T}}} > 8\,\sqrt{\mathrm{GeV}}$$

$$\Delta\phi_{\not{E}_{\mathrm{T}}j} > 0.4$$ photon energy fraction in jet $z_\gamma < 0.7$

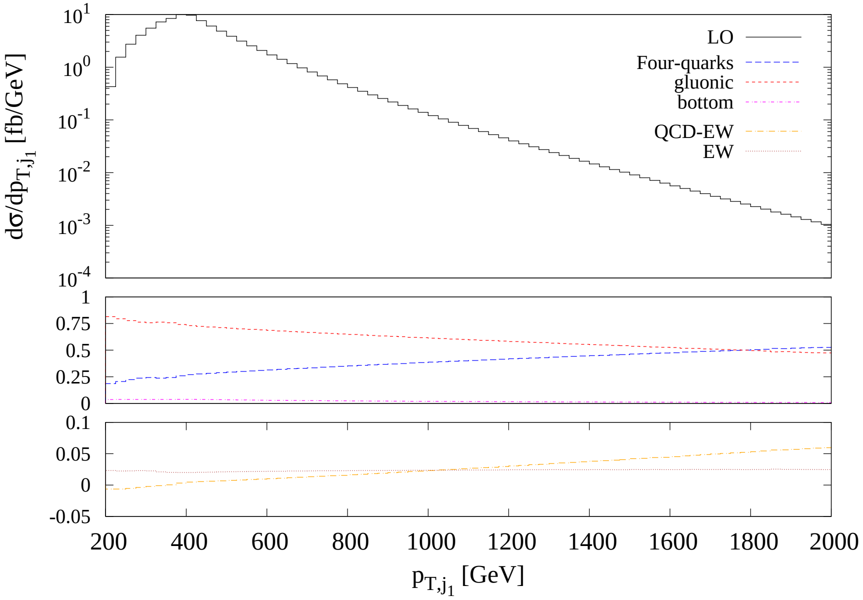
LHC - 13 TeV - ATLAS cuts - Total cross section

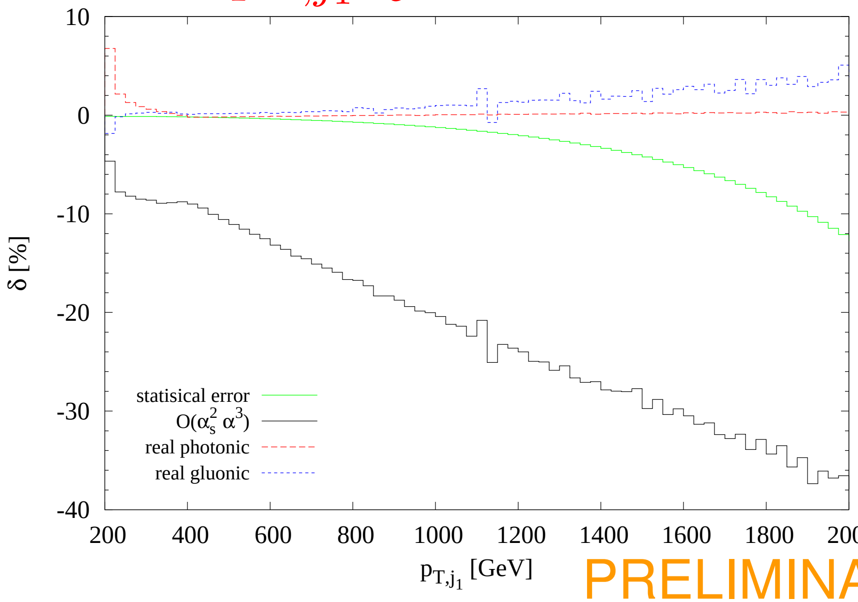
process class	σ^{LO} [fb]	$\sigma^{\mathrm{LO}}/\sigma^{\mathrm{LO}}_{\mathrm{tot}}$ [%]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [fb]	$\frac{\sigma^{\mathrm{NLO}}_{\mathrm{EW}}}{\sigma^{\mathrm{LO}}} - 1$ [%]
gluonic	1649.47(39)	72.7	1479.47(42)	- 10.31
four-quark	618.50(07)	27.3	557.83(16)	- 9.81
sum	2267.97(39)	100	2037.29(45)	- 10.17
bottom quarks	79.69(01)	3.51		

PRELIMINARY

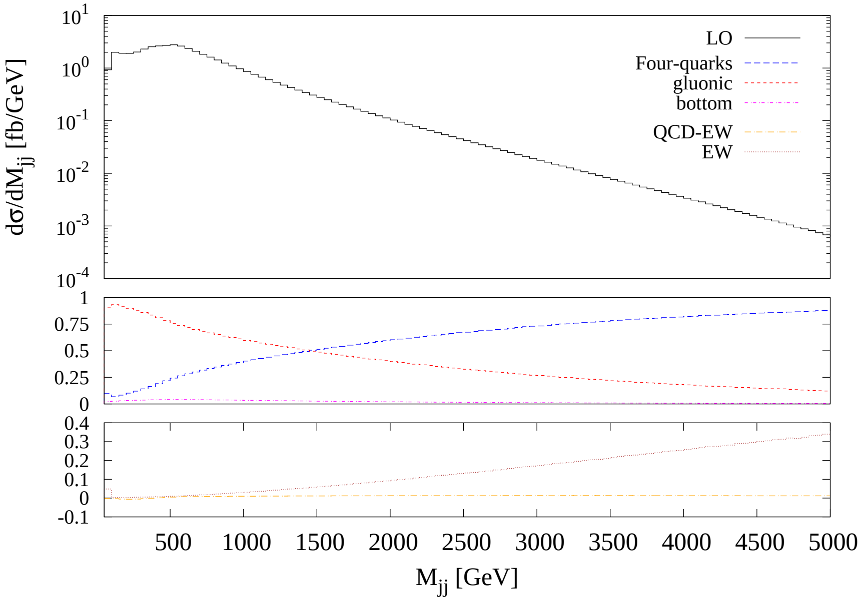
- Cross section reduced by factor 250 w.r.t. basic cuts
- $q\,g \to q\,g\,\nu\,\bar{\nu}$ channels still dominate
- EW corrections for total cross section around -10%

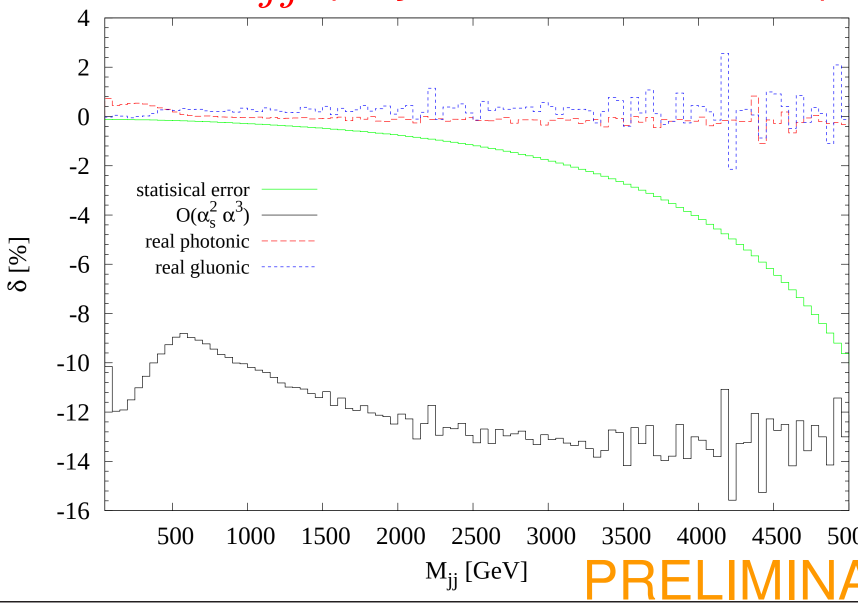
LHC - 13 TeV - ATLAS cuts - Distribution in p_{T,j_1} (j_1 = hardest jet)



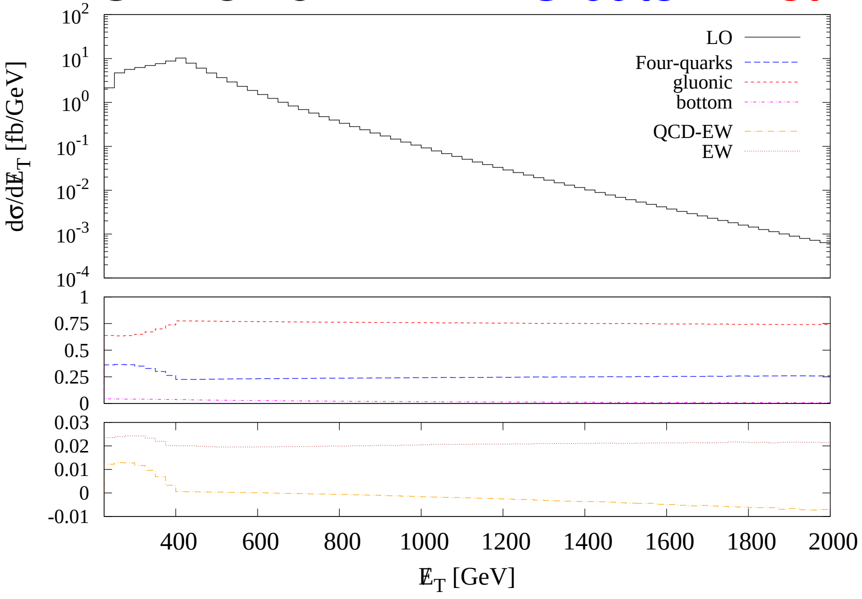


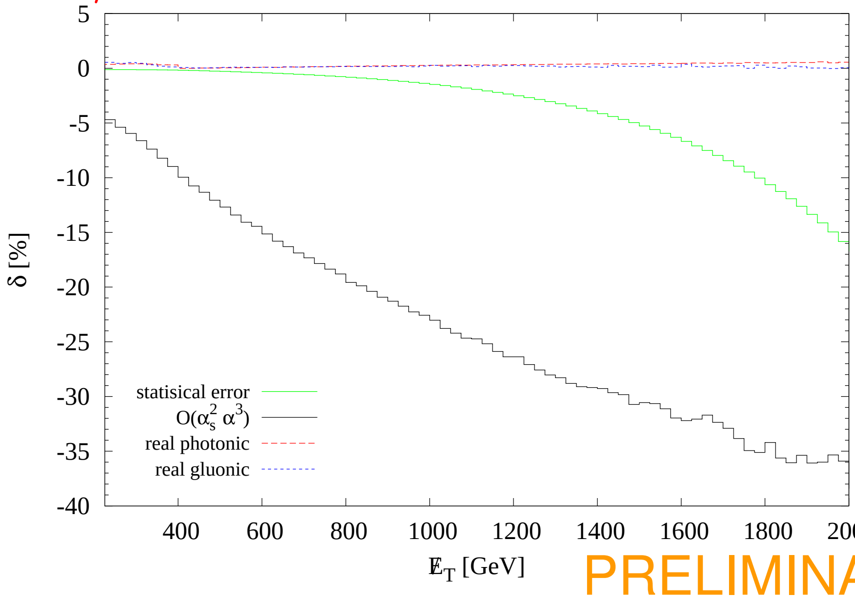
LHC - 13 TeV - ATLAS cuts - Distribution in M_{jj} (di-jet invariant mass)





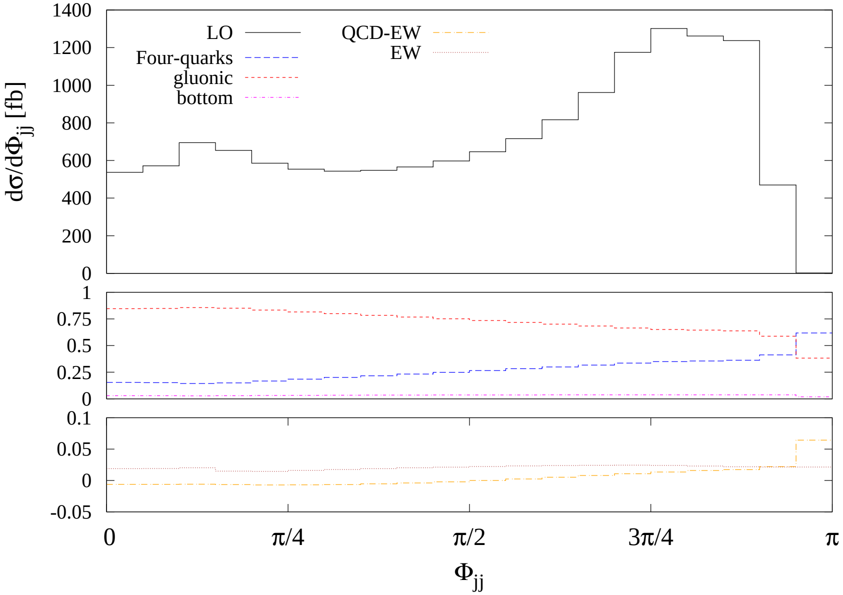
LHC - 13 TeV - ATLAS cuts - Distribution in $\not{E}_T$ (missing transverse momentum)

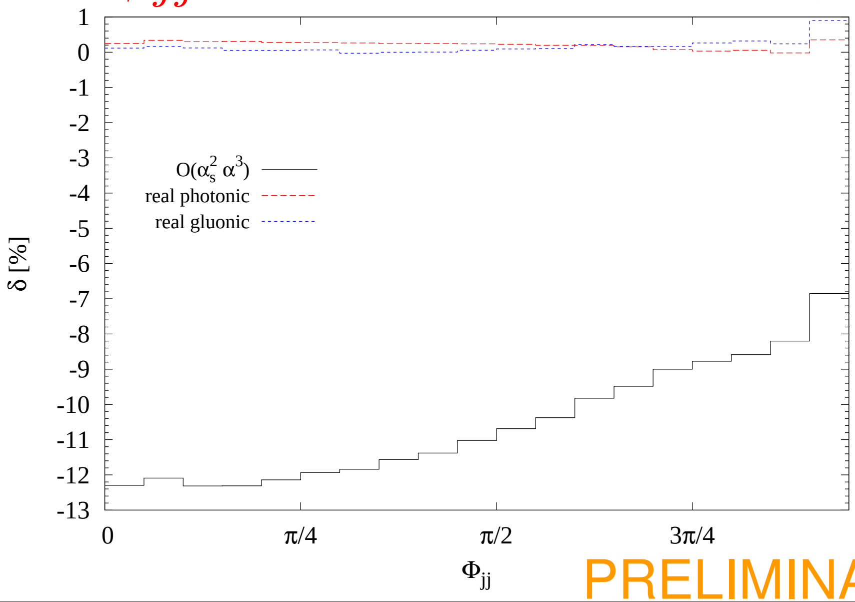




PRELIMINARY

LHC - 13 TeV - ATLAS cuts - Distribution in ϕ_{jj} (azimutal angle between jets)





PRELIMINARY

Summary

- Efficient automatization for elementary EW and QCD processes at NLO
- Recursion relations $\longrightarrow$ good tool also in the EW sector
- **used for EW corrections to** $p\,p \to l^+\,l^-\,j\,j$ **and** $p\,p \to \nu\,\bar{\nu}\,j\,j$

Summary

- Efficient automatization for elementary EW and QCD processes at NLO
- Recursion relations $\rightarrow$ good tool also in the EW sector
- **used for EW corrections to** $p\,p \rightarrow l^+\,l^-\,j\,j$ **and** $p\,p \rightarrow \nu\,\bar{\nu}\,j\,j$

Outlook

- Publication of the code
- Allow extensions to other Models
- Let’s compute other LHC processes

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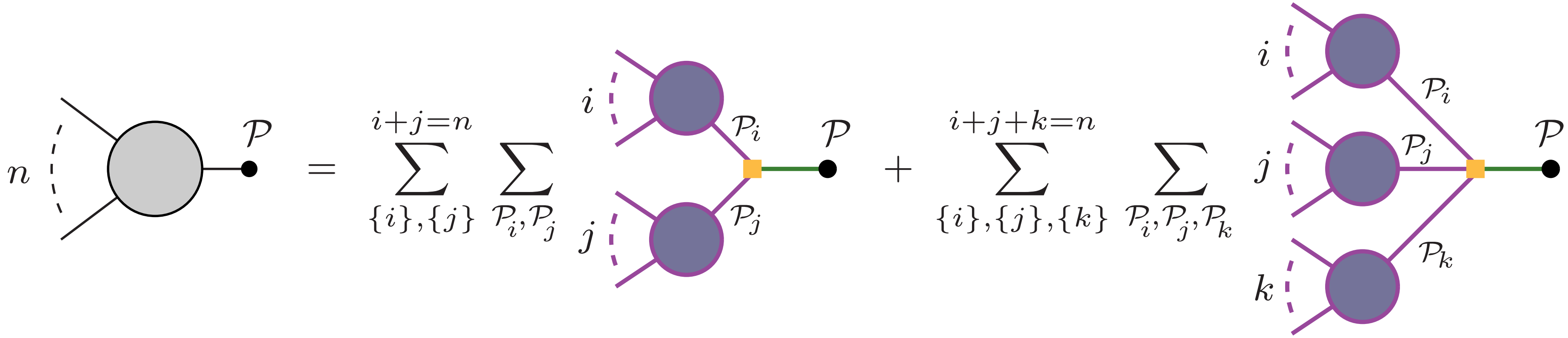
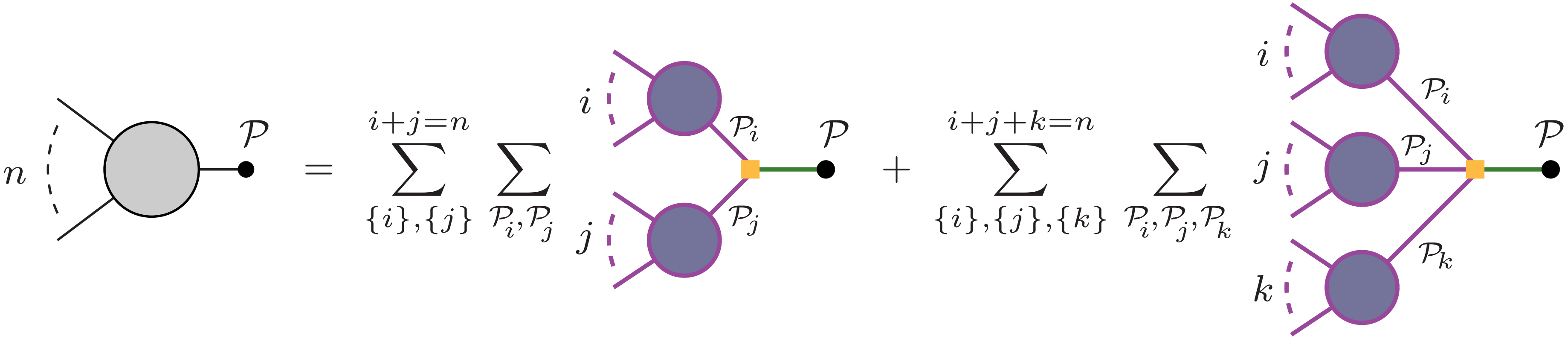
Back-up slides

Recursion relation for tree amplitudes



(**incoming currents**) × (**coupling**) × (**propagator**)

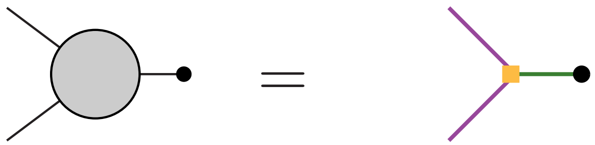
Recursion relation for tree amplitudes



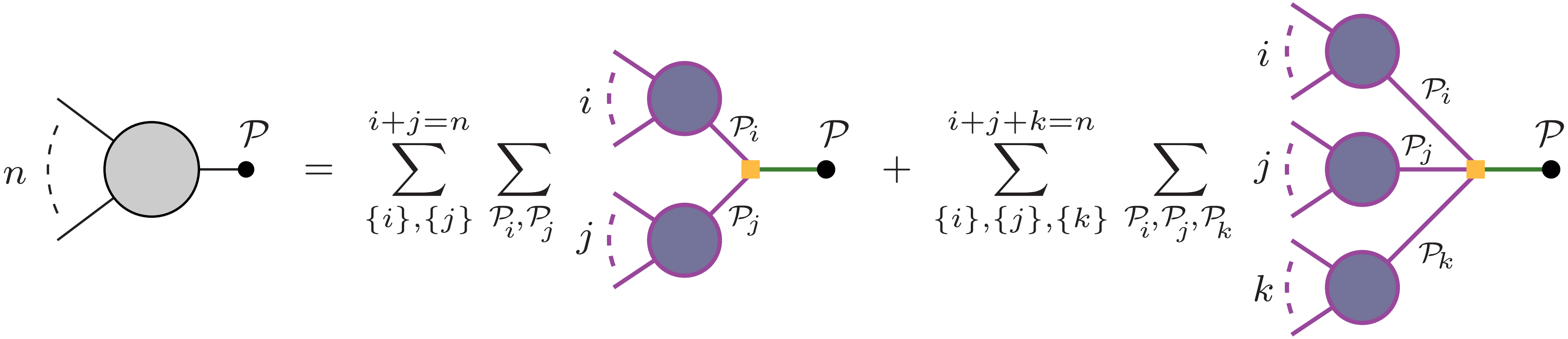
(**incoming currents**) × (**coupling**) × (**propagator**)

- Recursive procedure:

2-leg currents:



Recursion relation for tree amplitudes



(**incoming currents**) $\times$ (**coupling**) $\times$ (**propagator**)

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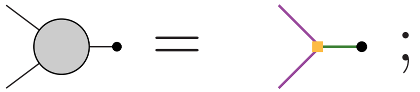
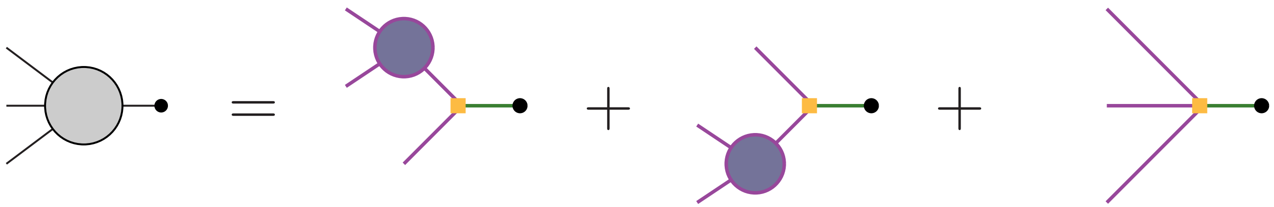
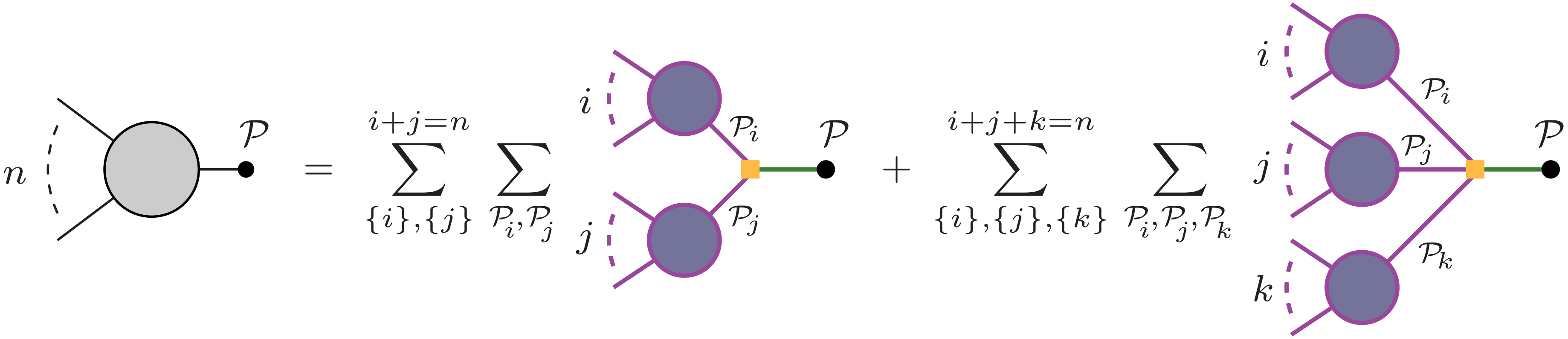


3-leg currents:



Recursion relation for tree amplitudes



(**incoming currents**) × (**coupling**) × (**propagator**)

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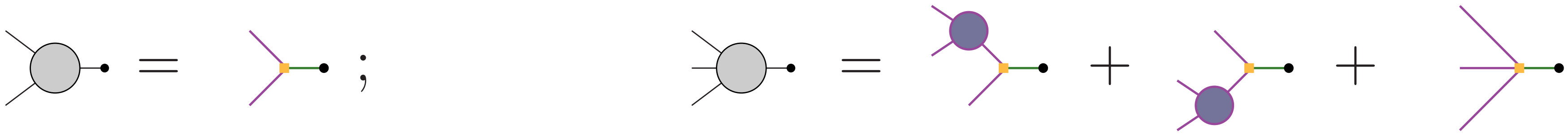
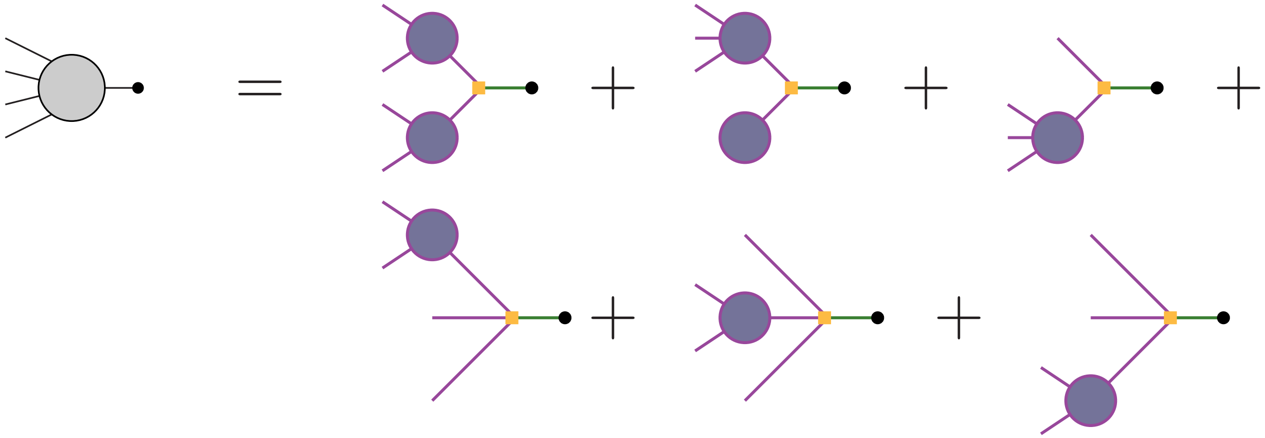
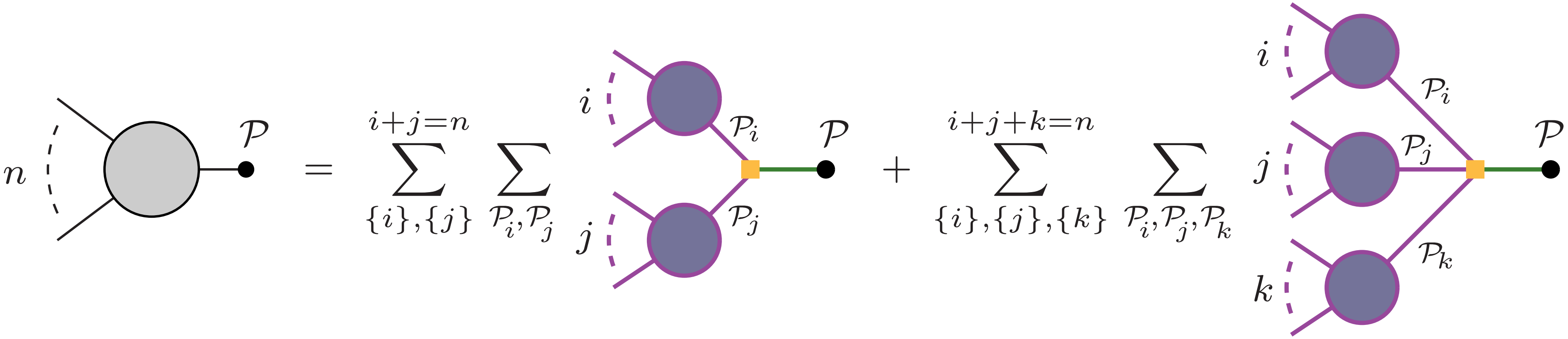


4-leg currents:



Recursion relation for tree amplitudes



(**incoming currents**) × (**coupling**) × (**propagator**)

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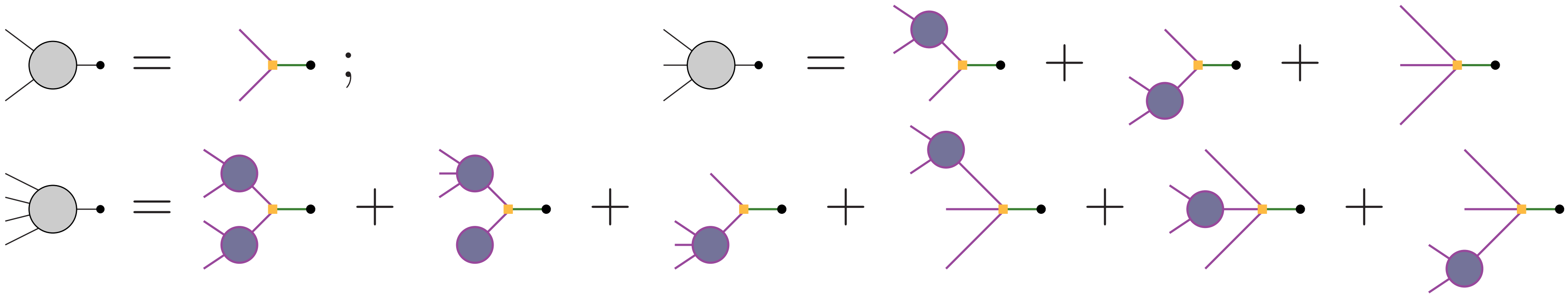
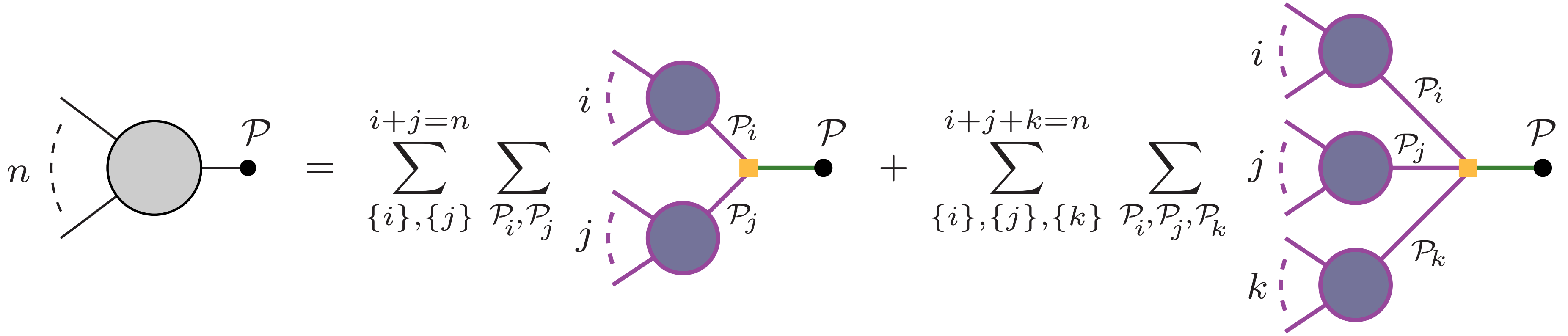


etc. . . .

Recursion relation for tree amplitudes



(**incoming currents**) × (**coupling**) × (**propagator**)

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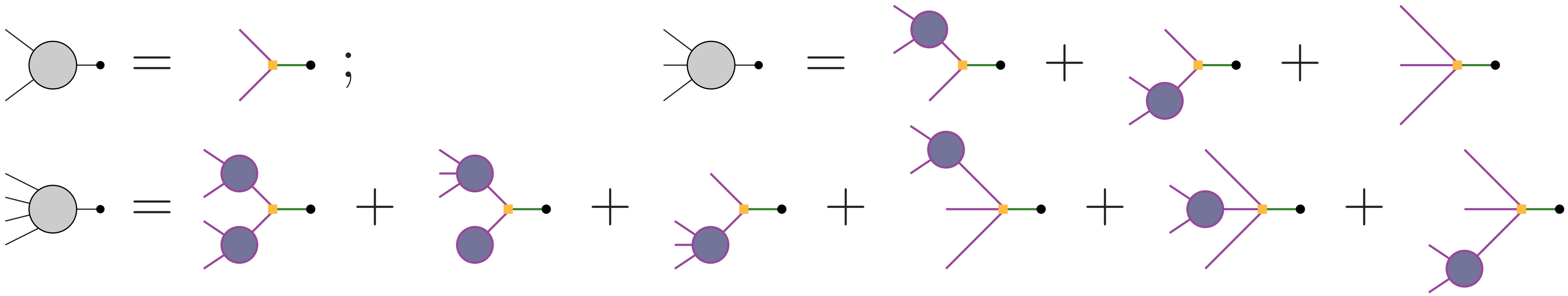

etc. . . .

- Amplitude: $\mathcal{A} = w(\overline{\mathcal{P}}_L, 2^{L-1} - 1) \times (\textbf{propagator})^{-1} \times w(\mathcal{P}_L, 2^{L-1})$

Recursion relation for loop amplitudes

General form of the amplitude:

Tensor Coefficients (TCs)

$$\mathcal{A} = \sum_t c^{(t)}_{\mu_1 \dots \mu_{r_t}} \, T^{\mu_1 \dots \mu_{r_t}}_{(t)}$$

Tensor Integrals (TIs)

$$T^{\mu_1 \cdots \mu_{r_t}}_{(t)} = \int \frac{d^n q \, q^{\mu_1} \cdots q^{\mu_{r_t}}}{D^{(t)}_0 \cdots D^{(t)}_{k_t}} \qquad D^{(t)}_{k_t} = (q + p^{(t)}_{k_t})^2 - (m^{(t)}_{k_t})^2$$

Indices $\mu_1, \dots, \mu_{r_t}$ are computed numerically in D=4 dimensions.

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⇝ Add the rational part $\mathcal{A}_{\mathrm{R2}}$

- Effective Feynman rules
  [Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09-'10]

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Recursion relation for loop amplitudes

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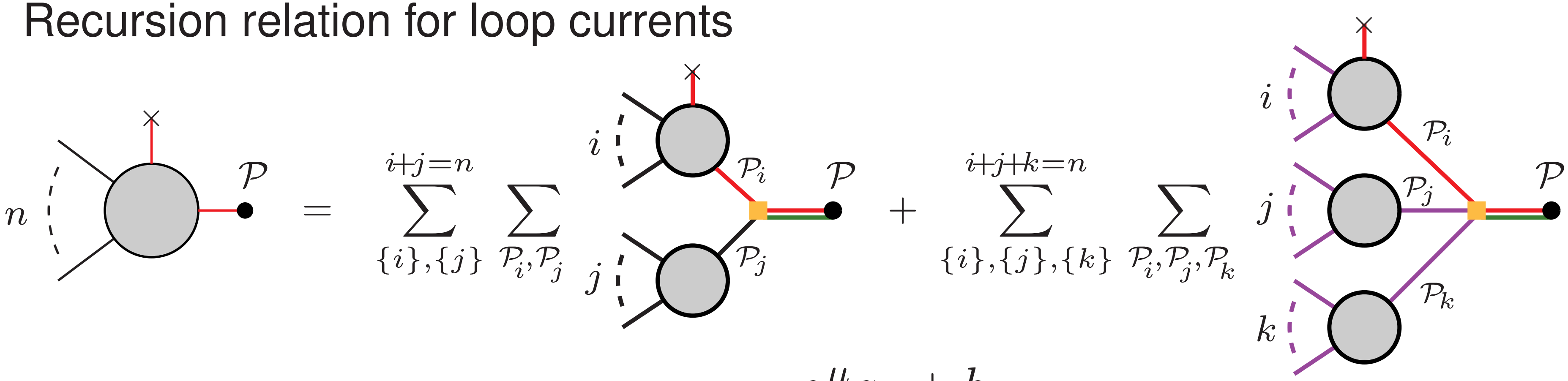
Indices $\mu_1, \dots, \mu_{r_t}$ are computed numerically in D=4 dimensions.

⇝ Add the rational part $\mathcal{A}_{\mathrm{R2}}$ → **tree-like amplitudes**

- Effective Feynman rules
  [Draggiotis, Garzelli, Malamos, Papadopoulos, Pittau '09-'10]

⇝ Add the counterterms contribution $\mathcal{A}_{\mathrm{CT}}$

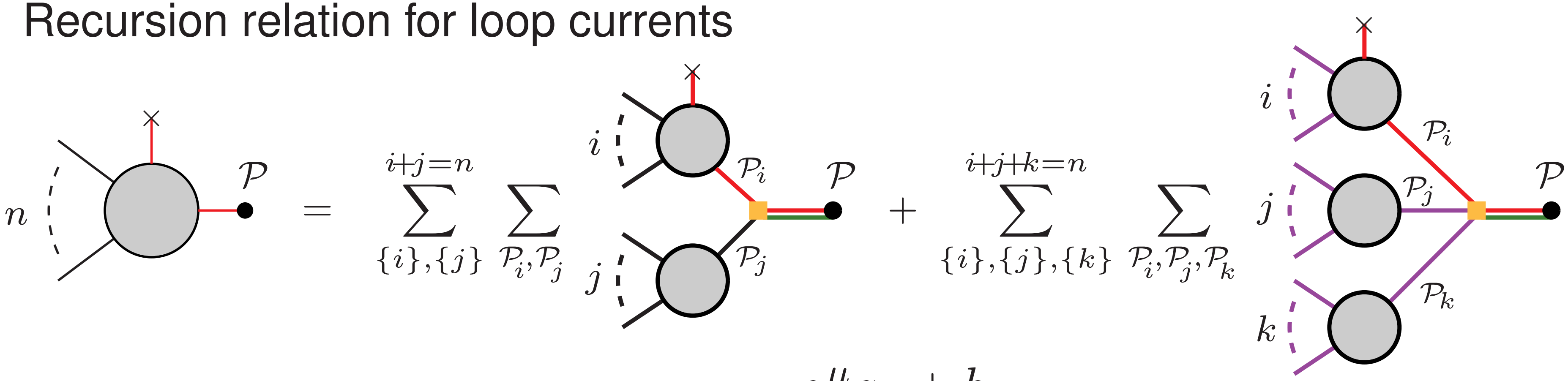
- Recursion relation for loop currents



$$(\textbf{coupling}) \times (\textbf{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2}$$

$q =$ loop momentum

- Recursion relation for loop currents



$$(\text{coupling}) \times (\text{propagator}) = \frac{a^\mu q_\mu + b}{(q+p)^2 - m^2}$$

$q =$ loop momentum

$$\text{loop current } (q) = \sum_{r=0}^{k} a_{k,r}^{\mu_1 \cdots \mu_r} \frac{q_{\mu_1} \cdots q_{\mu_r}}{\prod_{h=0}^{k} [(q+p_h)^2 - m_h^2]}$$

k: number of propagators

r: rank

$a_{k,r}^{\mu_1 \cdots \mu_r}$: computed in the recursion relation

$\frac{q_{\mu_1} \cdots q_{\mu_r}}{\prod_{h=0}^{k} [(q+p_h)^2 - m_h^2]}$: goes in the TIs

Remark: Indices $\mu_1, \ldots, \mu_r$ are symmetrized at each step

- The coefficients $a_{k,r}^{\mu_1 \cdots \mu_r}$ of the last current give the TCs $c^{(t)}_{\mu_1 \ldots \mu_{r_t}}$

Off-shell tree currents

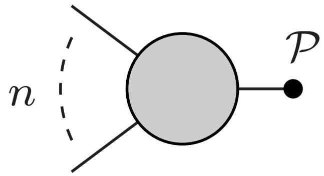
Given a process with L external legs:

$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_{L-1}}_{\text{primary}} + \underbrace{\mathcal{P}_L}_{\text{last}} \to 0$$

Off-shell current of a particle $\mathcal{P}$ with n primary legs:

Def: Amplitude made of n primary on-sheel particles and the off-sheel particle $\mathcal{P}$

$$w(\mathcal{P}, \mathcal{C}, \{l_1, \ldots, l_n\}) =$$



colour

particle: e^-, e^+, γ, ...

List of primary legs

- w is a scalar, spinor or vector
- The off-shell currents for external legs are the wave functions:

 $= u_\lambda(p)$  $= \bar{u}_\lambda(p)$  $= \epsilon_\lambda(p)$  $= 1$

- Binary notation for $\{l_1, \ldots, l_n\}$ (HELAC):

  Binary numbers $1, 2, 4, 8, ..., 2^{L-1}$ for the primary legs

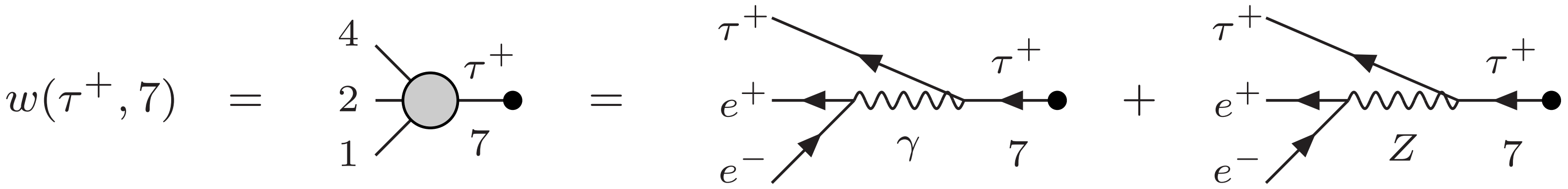
  $\{l_1, \ldots, l_n\}$ can be expressed by $\mathcal{B}_n =$ sum of the n binaries

Example: $\{1, 2, 8\} \quad \to \quad \mathcal{B}_3 = 1 + 2 + 8 = 11$

Note: The off-shell currents just keep trace of the primary legs used to build them, not the way it has been done.

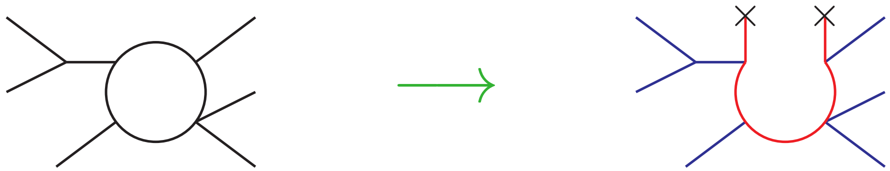
Example: Process $e^- + e^+ + \tau^+ + \tau^- \to 0$

1 2 4



Basic idea: Cut the loop line and consider tree diagrams with two more legs.

[A. van Hameren, JHEP 0907 (2009) 088]



Given the loop process

$$\mathcal{P}_1 + \ldots + \mathcal{P}_L \to 0$$

we consider the tree processes

$$\underbrace{\mathcal{P}_1 + \ldots + \mathcal{P}_L + \mathcal{P}}_{\text{primary}} + \underbrace{\overline{\mathcal{P}}}_{\text{last}} \to 0 \qquad \forall\, \mathcal{P} \in \{\text{Particle of the SM}\}$$

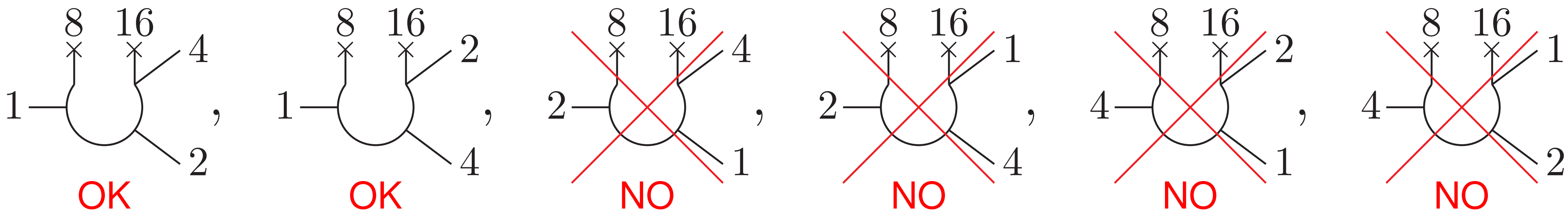
Problem: Associated tree diagrams are more than the original loop diagrams

$\rightsquigarrow$ **Selection rules to avoid double counting of the associated trees**

Rules to avoid double counting of the associated trees:

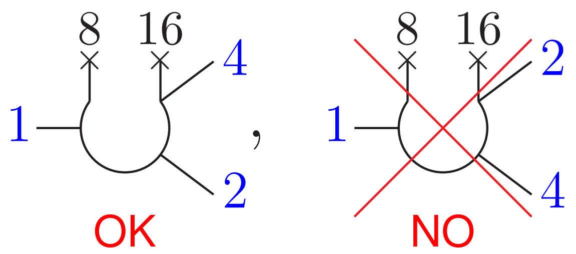
Rule 1: $\rightarrow$ Fix starting point of loop flow

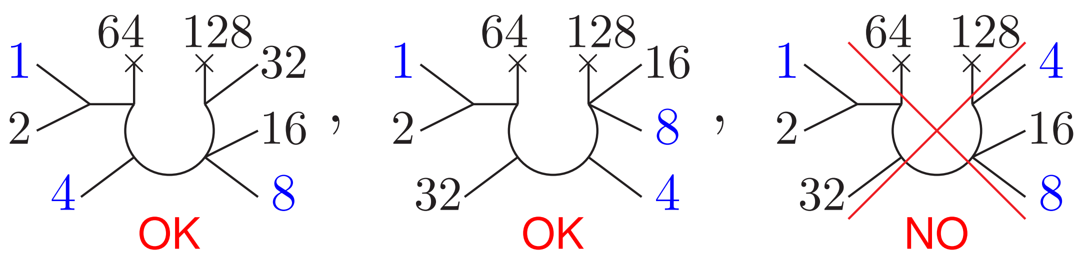
The current containing the first external line enters the loop flow first



Rule 2: $\rightarrow$ Fix direction of loop flow

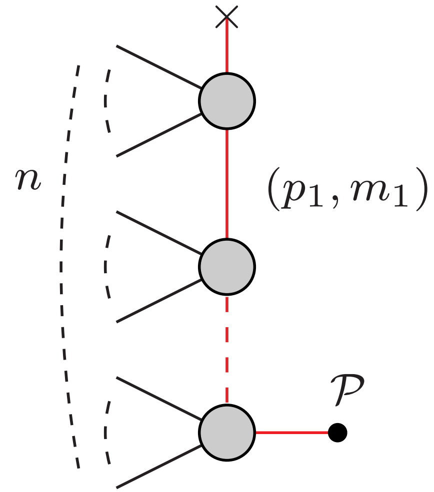
The 3 currents with the 3 smallest binaries enter the loop flow in fixed order





Loop off-shell currents

$$w_{i_k}(\,\mathcal{P},\,\mathcal{C},\,\mathcal{B}_n,\,\{p_1,\dots,p_k\},\,\{m_1,\dots,m_k\}) =$$



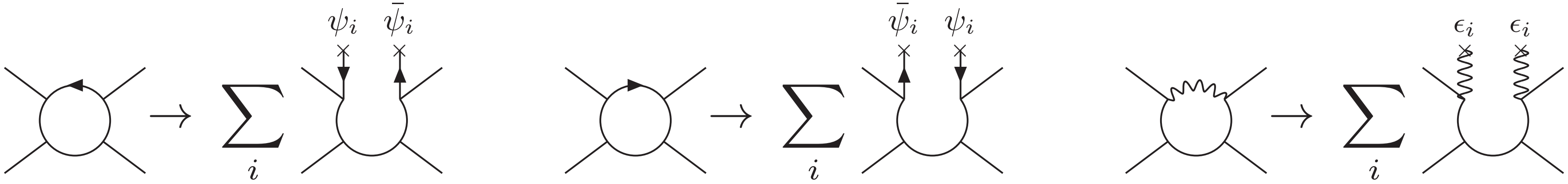
Sequence of momenta in loop propagators

Sequence of masses in loop propagators

- i_k is the tensorial index:

$$\begin{aligned} i_k &= 0 && \rightarrow && w_{i_k} = a_{k,0} \\ i_k &= 1,\dots,4 && \rightarrow && w_{i_k} = a_{k,1}^{\mu_1} \\ i_k &= 5,\dots,14 && \rightarrow && w_{i_k} = a_{k,2}^{\mu_1\mu_2} \\ &\dots \end{aligned}$$

- Special wave functions for the cutted line:



where the components are $(\psi_i)_j = (\bar{\psi}_i)_j = \delta_{ij}$, $\epsilon_i^\mu = \delta_{i\mu}$.

Treatment of the colour

Colour-flow representation [Kanaki, Papadopoulos 2000; Maltoni, Paul, Stelzer, Willenbrock 2002]

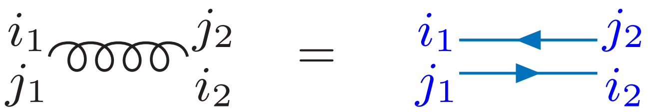
$$\text{Gluon field} : \quad \sqrt{2}\, A^a_\mu\, (\lambda^a)^i_j = (\mathcal{A}_\mu)^i_j$$

A^a_μ: “usual” gluon with colour index $a = 1, \ldots, 8$

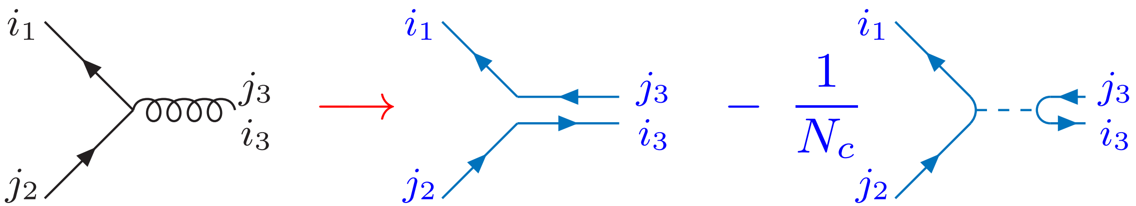
$(\mathcal{A}_\mu)^i_j$: gluon with colour-flow ^i_j
$i, j = 1, 2, 3$
$\sum_i (A_\mu)^i_i = 0$

Feynman rules:

- Multiply gluon fields A^a_μ by $(\lambda^a)^i_j/\sqrt{2}$ and use properties of $(\lambda^a)^i_j$
- The colour part of the Feynman rules is just product of deltas:



$$\times \frac{-i\, g_{\mu\nu}}{p^2} = \delta^{i_1}_{j_2}\delta^{i_2}_{j_1} \times \frac{-i\, g_{\mu\nu}}{p^2}$$



$$= \delta^{i_1}_{j_3}\delta^{i_3}_{j_2} - \frac{1}{N_c}\delta^{i_1}_{j_2}\delta^{i_3}_{j_3}$$

Colour-flow representation

[Kanaki, Papadopoulos ’00; Maltoni, Paul, Stelzer, Willenbrock ’02]

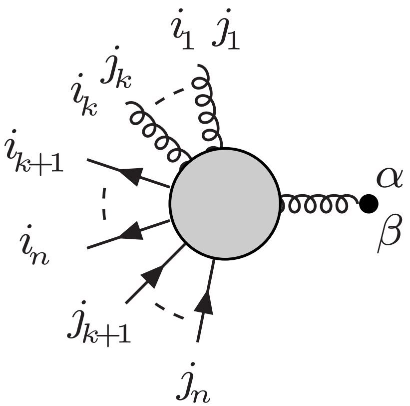
Structure of amplitude: $$\mathcal{A}^{i_1\cdots i_n}_{j_1\cdots j_n} = \sum_{P(j_1,\ldots,j_n)} \delta^{i_1}_{j_1}\cdots\delta^{i_n}_{j_n}\,\mathcal{A}_P$$

Structure-dressed (or colour-ordered) amplitudes:

$\rightarrow$ Compute $\mathcal{A}_P$ for all possible P $(n!)$

Squared amplitude: $$\overline{\mathcal{M}^2} = \sum_{P,P'} \mathcal{A}^*_P\, C_{PP'}\, \mathcal{A}_{P'}$$

It requires structure-dressed currents:



$\rightarrow$ saturated part open part

$$\delta^{i_1}_{j_1}\cdots\delta^{i_{n-1}}_{j_{n-1}}\ \delta^{\alpha}_{j_n}\ \delta^{i_n}_{\beta}$$

$$w(\,\mathcal{P},\ \mathcal{C},\ \mathcal{B}_n\,)$$

with all possible permutations of $j_1,\ldots j_n,\beta$

Optimization for colour

- U(1)-gluons are unphysical:

$$\text{Gluon field} \ : \quad \sqrt{2}\, A_\mu^a \, (\lambda^a)^i_j = (\mathcal{A}_\mu)^i_j$$

"usual" gluon with colour index $a = 1, \ldots, 8$

gluon with colour-flow ${}^i_j$
$i, j = 1, 2, 3$
$\sum_i (A_\mu)^i_i = 0$

Each external gluon with indices i, j has to be contracted with:

$$P^{ii'}_{jj'} = \delta^i_{j'} \delta^{i'}_j - \frac{1}{N_C}\, \delta^i_j \delta^{i'}_{j'}$$

$\rightsquigarrow$ All colour structures with $\ldots \delta^i_j \ldots$ does not contribute

Compute just colour structures without δ^i_j for each ${}^i_j$-gluon

Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

  $\rightsquigarrow$ $g + g + g \to 0$:

$$\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}\,,\quad \delta^{i_1}_{j_1}\delta^{i_2}_{j_3}\delta^{i_3}_{j_2}\,,\quad \delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}\,,\quad \delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}\,,\quad \delta^{i_1}_{j_3}\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\,,\quad \delta^{i_1}_{j_3}\delta^{i_2}_{j_2}\delta^{i_3}_{j_1}$$

Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

  $\rightsquigarrow$ $g + g + g \to 0$: 2 colour structures instead of 6

$$\cancel{\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}}\,,\quad \cancel{\delta^{i_1}_{j_1}\delta^{i_2}_{j_3}\delta^{i_3}_{j_2}}\,,\quad \cancel{\delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}}\,,\quad \delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}\,,\quad \delta^{i_1}_{j_3}\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\,,\quad \cancel{\delta^{i_1}_{j_3}\delta^{i_2}_{j_2}\delta^{i_3}_{j_1}}$$

Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

  - $g+g+g \to 0$: 2 colour structures instead of 6

    ~~$\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}$~~ , ~~$\delta^{i_1}_{j_1}\delta^{i_2}_{j_3}\delta^{i_3}_{j_2}$~~ , ~~$\delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}$~~ , $\delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}$, $\delta^{i_1}_{j_3}\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}$, ~~$\delta^{i_1}_{j_3}\delta^{i_2}_{j_2}\delta^{i_3}_{j_1}$~~

  - $g+g+g+g \to 0$: 9 colour structures instead of 24
  - $g+g+g+g+g \to 0$: 44 colour structures instead of 120
  - $g+g+g+q+\bar{q} \to 0$: 11 colour structures instead of 24
  - $g+g+g+g+q+\bar{q} \to 0$: 53 colour structures instead of 120

Optimization for colour

- **Compute just colour structures without δ^i_j for each ^i_j-gluon**

  $\rightsquigarrow$ $g+g+g \to 0$: 2 colour structures instead of 6

  ~~$\delta^{i_1}_{j_1}\delta^{i_2}_{j_2}\delta^{i_3}_{j_3}$~~ , ~~$\delta^{i_1}_{j_1}\delta^{i_2}_{j_3}\delta^{i_3}_{j_2}$~~ , ~~$\delta^{i_1}_{j_2}\delta^{i_2}_{j_1}\delta^{i_3}_{j_3}$~~ , $\delta^{i_1}_{j_2}\delta^{i_2}_{j_3}\delta^{i_3}_{j_1}$, $\delta^{i_1}_{j_3}\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}$, ~~$\delta^{i_1}_{j_3}\delta^{i_2}_{j_2}\delta^{i_3}_{j_1}$~~

  $\rightsquigarrow$ $g+g+g+g \to 0$: 9 colour structures instead of 24

  $\rightsquigarrow$ $g+g+g+g+g \to 0$: 44 colour structures instead of 120

  $\rightsquigarrow$ $g+g+g+q+\bar{q} \to 0$: 11 colour structures instead of 24

  $\rightsquigarrow$ $g+g+g+g+q+\bar{q} \to 0$: 53 colour structures instead of 120

- **Compute once currents differing just by the colour structure**

Example: $\bar{u} + u + g + g + g \rightarrow 0$

$\bar{u}$	+	u	+	g	+	g	+	g	$\rightarrow$	0
1		2		4		8		16		

Example: $\bar{u} + u + g + g + g \to 0$

$\bar{u}$: 1, u: 2, g: 4, g: 8, g: 16

$$2 \longrightarrow\bullet\, \beta = w(u, \delta^{i_2}_{\beta}, 2)$$

$$4 \;\text{(gluon)}\bullet^{\alpha}_{\beta} = w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$$

$$8 \;\text{(gluon)}\bullet^{\alpha}_{\beta} = w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$$

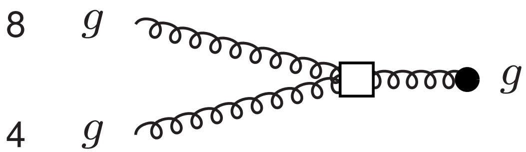
Example: $\bar{u} + u + g + g + g \rightarrow 0$

$\bar{u}$: 1, u: 2, g: 4, g: 8, g: 16

2  $\beta = w(u, \delta_\beta^{i_2}, 2)$

4  ${}^{\alpha}_{\beta} = w(g, \delta_\beta^{i_4}\delta_{j_4}^{\alpha}, 4)$

8  ${}^{\alpha}_{\beta} = w(g, \delta_\beta^{i_8}\delta_{j_8}^{\alpha}, 8)$



$\rightarrow \quad w(g, \delta_{j_4}^{i_8}\delta_\beta^{i_4}\delta_{j_8}^{\alpha}, 12) \qquad w(g, \delta_{j_8}^{i_4}\delta_\beta^{i_8}\delta_{j_4}^{\alpha}, 12)$

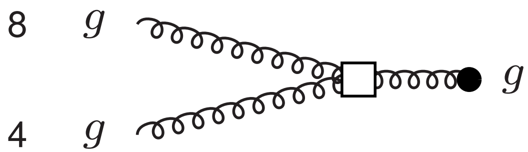
Example: $\bar{u} + u + g + g + g \to 0$

$\bar{u}$	u	g	g	g
1	2	4	8	16

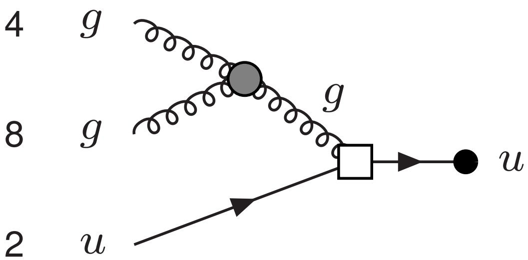
2  $\beta = w(u, \delta_\beta^{i_2}, 2)$

4  $^{\alpha}_{\beta} = w(g, \delta_\beta^{i_4}\delta_{j_4}^{\alpha}, 4)$

8  $^{\alpha}_{\beta} = w(g, \delta_\beta^{i_8}\delta_{j_8}^{\alpha}, 8)$

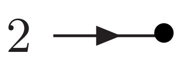


$\to \quad w(g, \delta_{j_4}^{i_8}\delta_\beta^{i_4}\delta_{j_8}^{\alpha}, 12) \qquad w(g, \delta_{j_8}^{i_4}\delta_\beta^{i_8}\delta_{j_4}^{\alpha}, 12)$

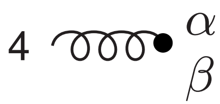


Example: $\bar{u} + u + g + g + g \to 0$

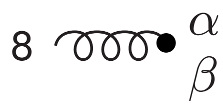
$\bar{u}$	u	g	g	g
1	2	4	8	16



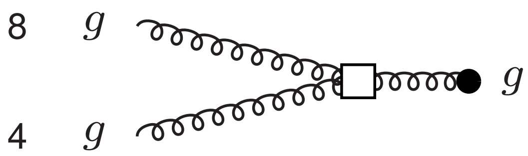
$= w(u, \delta^{i_2}_{\beta}, 2)$



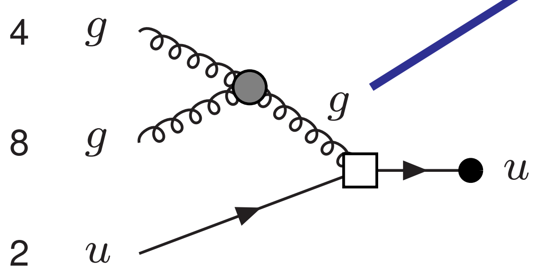
$= w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$



$= w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$



$\to \quad w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12) \qquad w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)$



$\to$

$$w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14)$$
$$w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)$$

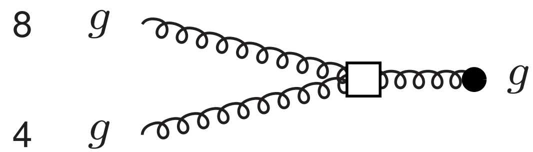
Example: $\bar{u} + u + g + g + g \rightarrow 0$

$$\begin{array}{ccccc} \bar{u} & u & g & g & g \\ 1 & 2 & 4 & 8 & 16 \end{array}$$

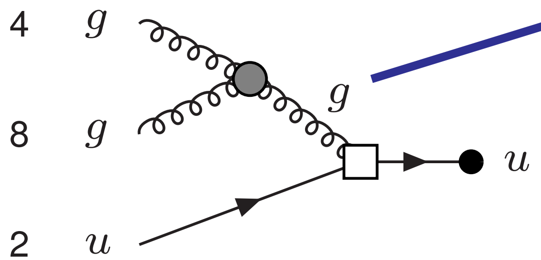
2  $\beta = w(u, \delta^{i_2}_{\beta}, 2)$

4  $^{\alpha}_{\beta} = w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$

8  $^{\alpha}_{\beta} = w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$



$\rightarrow \quad w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12) \qquad w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)$



$\rightarrow$

$w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14)$

$w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)$

$w(u, \delta^{i_2}_{j_4}\delta^{i_4}_{j_8}\delta^{i_8}_{\beta}, 14)$

$w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)$

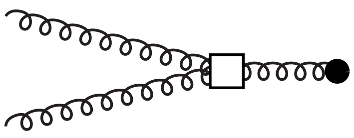
Example: $\bar{u} + u + g + g + g \to 0$

1 2 4 8 16

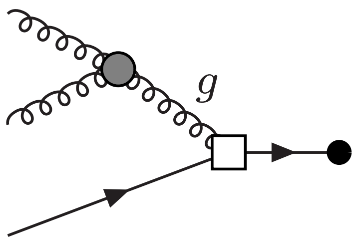
2  $\beta = w(u, \delta^{i_2}_{\beta}, 2)$

4  $^{\alpha}_{\beta} = w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$

8  $^{\alpha}_{\beta} = w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$



$\to \underbrace{w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12)}_{A} \quad \underbrace{w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)}_{-A}$



$\to$

$w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14) \quad w(u, \delta^{i_2}_{j_4}\delta^{i_4}_{j_8}\delta^{i_8}_{\beta}, 14)$

$w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14) \quad w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)$

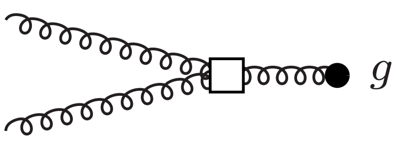
Example: $\bar{u} + u + g + g + g \to 0$

$\bar{u}$	u	g	g	g
1	2	4	8	16

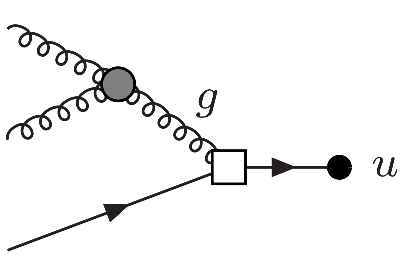
2  $\beta = w(u, \delta^{i_2}_{\beta}, 2)$

4  $^{\alpha}_{\beta} = w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$

8  $^{\alpha}_{\beta} = w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$



$$\to \underbrace{w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12)}_{A} \quad \underbrace{w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)}_{-A}$$

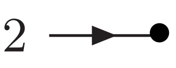


$$\to \overbrace{w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14)}^{B} \quad \overbrace{w(u, \delta^{i_2}_{j_4}\delta^{i_4}_{j_8}\delta^{i_8}_{\beta}, 14)}^{-B}$$

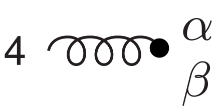
$$\underbrace{w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)}_{-\frac{B}{N_c}} \quad \underbrace{w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)}_{\frac{B}{N_c}}$$

Example: $\bar{u} + u + g + g + g \to 0$

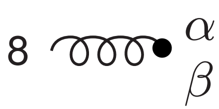
$\bar{u}$	u	g	g	g
1	2	4	8	16



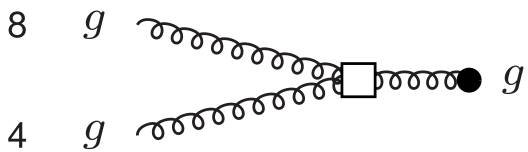
$= w(u, \delta^{i_2}_{\beta}, 2)$



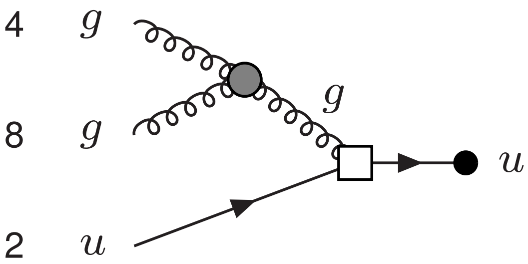
$= w(g, \delta^{i_4}_{\beta}\delta^{\alpha}_{j_4}, 4)$



$= w(g, \delta^{i_8}_{\beta}\delta^{\alpha}_{j_8}, 8)$



$\to$ $\underbrace{w(g, \delta^{i_8}_{j_4}\delta^{i_4}_{\beta}\delta^{\alpha}_{j_8}, 12)}_{A}$ $\underbrace{w(g, \delta^{i_4}_{j_8}\delta^{i_8}_{\beta}\delta^{\alpha}_{j_4}, 12)}_{-A}$



$\to$

$\overbrace{w(u, \delta^{i_2}_{j_8}\delta^{i_8}_{j_4}\delta^{i_4}_{\beta}, 14)}^{B}$ $\overbrace{w(u, \delta^{i_2}_{j_4}\delta^{i_4}_{j_8}\delta^{i_8}_{\beta}, 14)}^{-B}$

$\underbrace{w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)}_{-\frac{B}{N_c}}$ $\underbrace{w(u, \delta^{i_4}_{j_8}\delta^{i_8}_{j_4}\delta^{i_2}_{\beta}, 14)}_{\frac{B}{N_c}}$

COLLIER

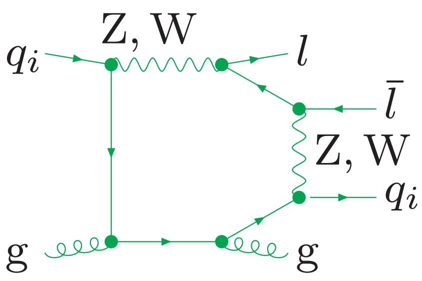
Complex One Loop Library In Extended Regularizations

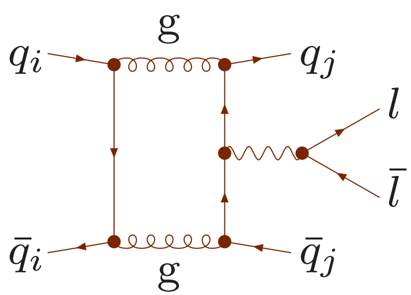
- Compute tensor integrals for:
  - arbitrary number of external momenta N
  - arbitrary rank
- Expansion methods for exceptional phase-space points (e.g. small Gram determinant) to arbitrary order
- Mass and dimensional regularization supported for IR-singularities
- Complex masses supported (unstable particles)
- Cache-system to avoid recalculation of identical integrals
- Output: coefficients $T^N_{0\cdots 0 i_1\cdots i_k}$ or tensors $(T^N)^{\mu_1\cdots\mu_P}$
- Two independent implementations: COLI and DD

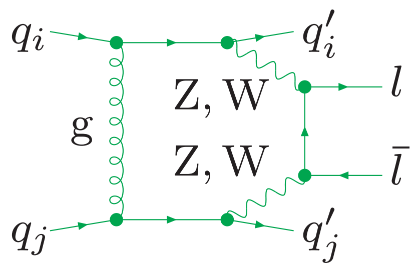
Virtual corrections:

of loop diagrams contributing at $\mathcal{O}(\alpha_s^2\alpha^3)$

	$\mathrm{q\,g} \to \mathrm{q\,g}\,\bar{l}\,l$	$\mathrm{q}_i\,\bar{\mathrm{q}}_i \to \mathrm{q}_j\,\bar{\mathrm{q}}_j\,\bar{l}\,l$	$q_i\,q_j \to q'_i\,q'_j\,\bar{l}\,l$
order	$\mathcal{O}(g_s^2 e^4)$	$\mathcal{O}(g_s^4 e^2) + \mathcal{O}(g_s^2 e^4)$	$\mathcal{O}(g_s^2 e^4)$
loop diagrams	~ 1200	$\sim 150 + 800$	~ 120



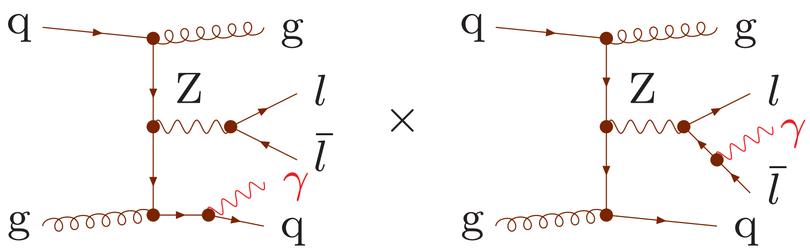




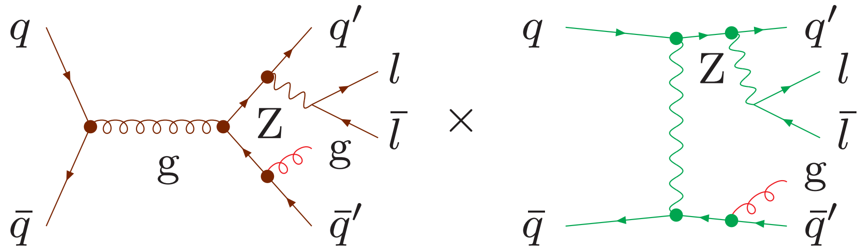
- Most complicated topology: hexagon of rank 4
- Finite top-quark-mass effects:
  - Fully included in closed fermion loops
  - Contributions with external bottom quarks neglected at NLO

Real corrections:

Photon emission from QCD×QCD



Gluon emission from QCD×EW



Crossing of gluon: new partonic channels $\mathrm{g\,q} \to \mathrm{q\,q'\,\bar{q}'}\,\bar{l}\,l$ (IR-finite)

Soft and collinear singularities:

- Catani-Seymour dipole subtraction [Catani, Seymour ’96; Nagy, Trocsanyi ’98; Nagy ’03; Campbell, Ellis, Tramontano ’04]
- Recombination of collinear parton-photon pairs:
  - Cut recombined parton-photon jets with hard photons
    ⇒ cancel singularity from (soft gluon)-(hard photon) jets
  - Quark-photon fragmentation function
    ⇒ cancel singularity from collinear quark-photon