Two-loop QCD corrections to vector boson pair production at the LHC

Lorenzo Tancredi

TTP, KIT - Karlsruhe

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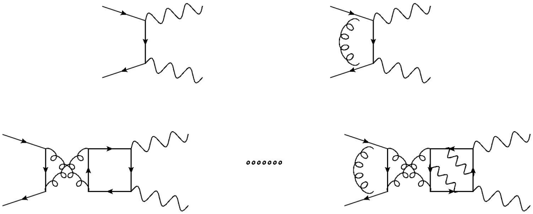
Based on collaboration with *Thomas Gehrmann* and *Andreas von Manteuffel*
See: [arXiv:1503.04812], [arXiv:1503.08835]

Introduction

What do we mean with vector boson pair production?

We mean the production of **pairs** of **electroweak vector bosons** V_1, V_2:

$$V_1\,V_2 = \{\,\gamma\,\gamma\,, \qquad Z\,\gamma\,,\;\; W^{\pm}\,\gamma\,, \qquad Z\,Z\,,\;\; W^{+}W^{-}\,, \qquad W^{\pm}Z\,\}$$



$V_1\, V_2$ production is extremely important **phenomenologically** since it allows to test directly and indirectly the electroweak sector of the Standard Model

a) Background for **Higgs production**:
$H \to \gamma\gamma\,, \quad H \to ZZ\,, \quad H \to W^+W^-$

b) **Anomalous** triple gauge boson couplings ($WW\gamma$, WWZ, ...) as indirect probe for *new physics*

c) Precise prediction for $gg \to ZZ$ can be used to constrain the **total Higgs decay width** at LHC [Caola, Melnikov '13]

Moreover

c) At LHC very good **experimental** control of vector boson pair final states

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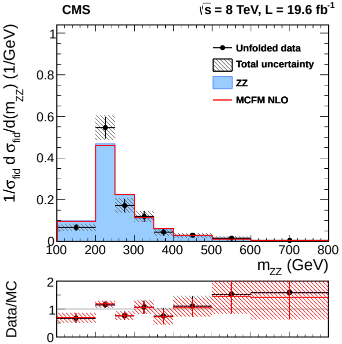
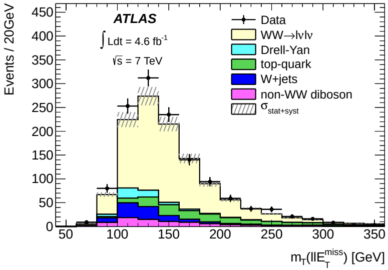
Experimental status

First LHC run 2010-2013. **Run 2** is starting right now!
Already many events for $\{\gamma\gamma, Z\gamma, W^{\pm}\gamma, ZZ, W^{+}W^{-}, \dots\}$ production:



ZZ production at *CMS*



WW production at ATLAS

Experimental results demand high theoretical precision $\approx$ 5%

At the **LHC** $V_1 V_2$ pairs are produced through **two main partonic subchannels**. Do we really need two-loop amplitudes for both?

$q\,\bar{q} \to V_1 V_2$ **NNLO**

$$\sigma^{qqVV}_{NNLO} = \int d\phi_{VV}\,|\mathcal{M}^{(0)*}_{qqVV}\mathcal{M}^{(2)}_{qqVV}|$$

$$+ \int d\phi_{VVj}\,|\mathcal{M}^{(0)*}_{qqVVj}\mathcal{M}^{(1)}_{qqVVj}|$$

$$+ \int d\phi_{VVjj}\,|\mathcal{M}^{(0)*}_{qqVVjj}\mathcal{M}^{(0)}_{qqVVjj}|$$

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N^3LO in $pp \to VV$

a) High gluon luminosity at LHC

b) Needed to have **theoretical uncertainty** under control!

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   1a) NLO gg channel included! ✔ Z.Bern et al. [hep-ph/0206194]

2) $pp \to Z\gamma$ / $W\gamma$ NNLO + LO gg ✔
   M.Grazzini et al. [arXiv:1309.7000, arXiv:1504.01330]

3) $pp \to ZZ$ (*on shell, fully inclusive only*) NNLO + LO gg ✔
   F.Cascioli et al. [arXiv:1405.2219]

3) $pp \to WW$ (*on shell, fully inclusive only*) NNLO + LO gg ✔
   T.Gehrmann et al. [arXiv:1408.5243]

4) $pp \to ZZ^*$ / WW^* / ZW *...on going...* (see Dirk Rathlev's talk)

⇓

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Last **missing piece** were the two-loop amplitudes

Two-loop amplitudes for a $2 \to 2$ process with **4 different scales**.
Probably *the most complicated amplitudes calculated so far...*

qqVV) F.Caola, J.Henn, K.Melnikov, A.V.Smirnov, V.A.Smirnov [arXiv:1408.6409]
T.Gehrmann, A.von Manteuffel, **L.T.** [arXiv:1503.04812]

ggVV) F.Caola, J.Henn, K.Melnikov, A.V.Smirnov, V.A.Smirnov [arXiv:1503.08759]
A.von Manteuffel, **L.T.** [arXiv:1503.08835]

1) **How** did we make it?

2) Are we reaching **limits** of present technology?

Integration by Parts (IBPs) is the only **fully developed** tool we have to deal with **multi-loop** amplitudes.

1) Generate the two-loop **Feynman diagrams**

2) Perform "Dirac algebra" and reduce each of them to **scalar integrals**

$$I = \int \Pi_i d^d k_i \, \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \,, \qquad S_n = \{k_i \cdot p_j, etc...\}$$

3) Once we have scalar integrals in this form, use IBPs to reduce them to a subset of **Master Integrals**

$$\int \Pi_i d^d k_i \left(\frac{\partial}{\partial k_i^\mu} v_\mu \frac{S_1^{j_1} \cdots S_m^{j_m}}{D_1^{r_1} \cdots D_n^{r_n}} \right) = 0, \qquad v^\mu = (k_i^\mu, p_i^\mu)$$

4) Try to compute the masters...

First problem: how to write all **Feynman diagrams** in scalar integrals?

a) In the case of $q\bar{q} \to V_1 V_2$ we could contract with **tree-level** and sum over external polarizations...

But

b) We want to do phenomenology! We lose control on **decay products**:

$$q\bar{q} \to V_1 V_2 \to l_1 \bar{l}_2 \; l_3 \bar{l}_4$$

c) What about $gg \to V_1 V_2$?
It has **no tree-level**! We should contract with **one-loop diagrams**!

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Attempt to study the most general decomposition into invariant **Form Factors** and use it to compute **Helicity Amplitudes**!

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Attempt to study the most general decomposition into invariant **Form Factors** and use it to compute **Helicity Amplitudes**!

Based solely on **Lorentz invariance** and **gauge invariance** one can determine the most general tensor structure for $q\bar{q} \to V_1 V_2$ and $gg \to V_1 V_2$

$q\,\bar{q} \to V_1 V_2$

$$S^{\mu\nu}_{q\bar{q}} = \sum_{j=1}^{10} A^{(j)}_{q\bar{q}}\, t^{\mu\nu}_j$$

$$t^{\mu\nu}_1 = \bar{v}(p_2)\, \not{p}_3\, u(p_1)\, p_1^\mu\, p_1^\nu$$

$$\ldots$$

$$t^{\mu\nu}_{10} = \bar{v}(p_2)\, \gamma^\nu \not{p}_3 \gamma^\mu\, u(p_1)$$

$g\,g \to V_1 V_2$

$$S^{\mu\nu}_{gg} = \sum_{j=1}^{20} A^{(j)}_{gg}\, \tau^{\mu\nu}_j$$

$$\tau^{\mu\nu}_1 = \epsilon_1 \cdot \epsilon_2\, g^{\mu\nu}$$

$$\ldots$$

$$\tau^{\mu\nu}_{20} = \epsilon_1 \cdot p_3\, \epsilon_2 \cdot p_3\, p_2^\mu\, p_2^\nu$$

This decomposition is **non-perturbative**, valid at any number of loops! No assumption on the number of dimensions d $\to$ valid for **d-continuous dimensions**!

All **perturbative dependence** in the **scalar** coefficients $A^{(j)}_X$, for initial state X.

Build up **d-dimensional** projectors to extract the coefficients, expanded in the same **basis of tensors**

$q\,\bar{q} \to V_1\,V_2$

$$P^{\mu\nu}_{q\bar{q},k} = \sum_{j=1}^{10} a^{(j)}_{q\bar{q},k}\, t^{\mu\nu}_j$$

10 Projectors

$g\,g \to V_1\,V_2$

$$P^{\mu\nu}_{gg,k} = \sum_{j=1}^{20} a^{(j)}_{gg,k}\, \tau^{\mu\nu}_j$$

20 Projectors

Such that for *initial state* X we have

$$\sum_{pol} P^{\mu\nu}_{X,k} \left(g_{\mu\rho} - \frac{p_{3\mu}\, p_{3\rho}}{p_3^2} \right) \left(g_{\nu\sigma} - \frac{p_{4\nu}\, p_{4\sigma}}{p_4^2} \right) S^{\rho\sigma}_X = A^{(k)}_X$$

Where we included sum over polarizations of external vector bosons using *transversality of decay currents*.

This **fixes completely** the coefficients $a^{(k)}_{X,j}$!

Once we have the projectors we proceed as follows[1]:

1) Generate all Feynman Diagrams with **QGRAF** [P.Nogueira].

2) Apply the 10 or 20 projectors on **each diagram**.

3) Use **FORM** [J.Vermaseren] to perform Dirac algebra, traces, sums over polarizations

   - Each diagram is written in terms of **scalar Feynman integrals**

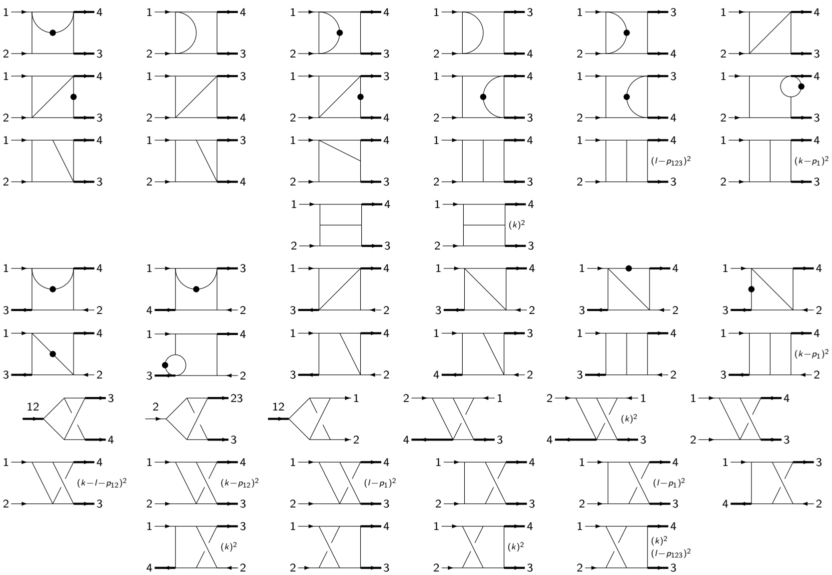
4) Collect Feynman integrals into **Topologies**, perform reduction to MIs with **Reduze 2** [A.von Manteuffel, C. Studerus].

$$\Downarrow$$

Two-loop four-point functions with two legs off-shel $\rightarrow$ **4 independent scales!**

[1] This would work, *in principle*, for any number of loops...

(Some of) The **Master Integrals** for $q\bar{q} \to V_1 V_2$ and $gg \to V_1 V_2$



Including (*almost*) all *crossings* and *trivial topologies* = **111 master integrals**

1) Computed for **equal external masses**
[Gehrmann, LT, Weihs '13; Gehrmann, von Manteuffel, LT, Weihs '14]
2) For **different external masses**
[Henn, Melnikov, Smirnov '14; Caola, Henn, Melnikov, Smirnov '14]
   - Recomputed and **optimized** in
   [Gehrmann, von Manteuffel, LT '15]

$\Downarrow$

- Method of **differential equations** for MIs
[Kotikov '91; Remiddi '97; Gehrmann, Remiddi '99]
- Augmented by choice of a **Canonical Basis**
[Kotikov '10; Henn '13]

All 111 masters $m^{(k)}$ fulfil canonical diff. equations in the external invariants s_j

$$\frac{\partial}{\partial s_j} m^{(k)}(\epsilon;\, s_j) = \epsilon\, A^{(j)}_{k\,l}(s_j)\, m^{(l)}(\epsilon;\, s_j)$$

a) Dependence from ϵ **factored out.**

b) Matrix $A(s_j)$ is in d-*log* form.

$\Downarrow$

Can be expressed in terms of **Multiple polylogarithms** (MPLs)
[Remiddi,Vermaseren; Gehrmann,Remiddi; Goncharov]

$$G(a_1, a_2, \ldots, a_n; x) = \int_0^x \frac{dt}{t - a_1} G(a_2, \ldots, a_n; x)\,, \qquad G(\underbrace{0, \ldots, 0}_{n}; x) = \frac{1}{n!} \ln^n x\,.$$

From the matrices A^j_{kl} we can read off **the alphabet**
(i.e. ≈ the "**arguments**" of the **MPLs**!)

$$\{\bar{l}_1, \ldots, \bar{l}_{20}\} = \{2, \bar{x}, 1+\bar{x}, 1-\bar{y}, \bar{y}, 1+\bar{y}, 1-\bar{x}\bar{y}, 1+\bar{x}\bar{y}, 1-\bar{z}, \bar{z},$$
$$1+\bar{y}-2\bar{y}\bar{z}, 1-\bar{y}+2\bar{y}\bar{z}, 1+\bar{x}\bar{y}-2\bar{x}\bar{y}\bar{z}, 1-\bar{x}\bar{y}+2\bar{x}\bar{y}\bar{z},$$
$$1+\bar{y}+\bar{x}\bar{y}+\bar{x}\bar{y}^2-2\bar{y}\bar{z}-2\bar{x}\bar{y}\bar{z}, 1+\bar{y}-\bar{x}\bar{y}-\bar{x}\bar{y}^2-2\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z},$$
$$1-\bar{y}-\bar{x}\bar{y}+\bar{x}\bar{y}^2+2\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z}, 1-\bar{y}+\bar{x}\bar{y}-\bar{x}\bar{y}^2+2\bar{y}\bar{z}-2\bar{x}\bar{y}\bar{z},$$
$$1-2\bar{y}-\bar{x}\bar{y}+\bar{y}^2+2\bar{x}\bar{y}^2-\bar{x}\bar{y}^3+4\bar{y}\bar{z}+2\bar{x}\bar{y}\bar{z}+2\bar{x}\bar{y}^3\bar{z},$$
$$1-\bar{y}-2\bar{x}\bar{y}+2\bar{x}\bar{y}^2+\bar{x}^2\bar{y}^2-\bar{x}^2\bar{y}^3+2\bar{y}\bar{z}+4\bar{x}\bar{y}\bar{z}+2\bar{x}^2\bar{y}^3\bar{z}\},$$

where, in order to **rationalise** the Källen function

$$\kappa\left(s, p_3^2, p_4^2\right) \equiv \sqrt{s^2+p_3^4+p_4^4-2(s\,p_3^2+p_3^2\,p_4^2+p_4^2\,s)}$$

we employ [Caola, Henn, Melnikov, Smirnov '14]:

$$s = \bar{m}^2(1+\bar{x})^2, \qquad p_3^2 = \bar{m}^2\bar{x}^2(1-\bar{y}^2),$$
$$t = -\bar{m}^2\bar{x}((1+\bar{y})(1+\bar{x}\bar{y})-2\bar{z}\bar{y}(1+\bar{x})), \qquad p_4^2 = \bar{m}^2(1-\bar{x}^2\bar{y}^2),$$

In this way we get all **scalar coefficients** for gg and $q\bar{q}$ initial states, in terms of the **111 masters** (+ crossings), which are in turn integrated in terms of **MPLs**

For example the two-loop amplitude of $q\bar{q} \to V_1 V_2$ reads

$$
\begin{aligned}
S_{q\bar{q}}^{\mu\nu} = {} & A_{q\bar{q}}^{(1)}\, \bar{u}(p_2)\, \not{p}_3 u(p_1)\, p_1^\mu p_1^\nu + A_{q\bar{q}}^{(2)}\, \bar{u}(p_2)\, \not{p}_3 u(p_1)\, p_1^\mu p_2^\nu \\
& + A_{q\bar{q}}^{(3)}\, \bar{u}(p_2)\, \not{p}_3 u(p_1)\, p_2^\mu p_1^\nu + A_{q\bar{q}}^{(4)}\, \bar{u}(p_2)\, \not{p}_3 u(p_1)\, p_2^\mu p_2^\nu \\
& + A_{q\bar{q}}^{(5)}\, \bar{u}(p_2)\, \gamma^\mu u(p_1)\, p_1^\nu \qquad + A_{q\bar{q}}^{(6)}\, \bar{u}(p_2)\, \gamma^\mu u(p_1)\, p_2^\nu \\
& + A_{q\bar{q}}^{(7)}\, \bar{u}(p_2)\, \gamma^\nu u(p_1)\, p_1^\mu \qquad + A_{q\bar{q}}^{(8)}\, \bar{u}(p_2)\, \gamma^\nu u(p_1)\, p_2^\mu \\
& + A_{q\bar{q}}^{(9)}\, \bar{u}(p_2)\, \gamma^\mu \not{p}_3 \gamma^\nu u(p_1) + A_{q\bar{q}}^{(10)}\, \bar{u}(p_2)\, \gamma^\nu \not{p}_3 \gamma^\mu u(p_1)\,.
\end{aligned}
$$

$$\Downarrow$$

Each of the coefficients $A_{q\bar{q}}^{(j)}$ is a **huge** linear combination of **rational functions** and **MPLs** of the external invariants s_j

$$1\,\mathrm{MB} \lesssim A_{q\bar{q}}^{(j)} \lesssim 7\,\mathrm{MB} \quad \to \quad S_{q\bar{q}}^{\mu\nu} \approx 60\,\mathrm{MB}\,.$$

Further simplifications can be achieved by assuming **4-dimensional** external states, contracting by left- or right-leptonic currents and **fixing the helicities**.

$$\begin{aligned}\mathrm{M}_{LLL}(p_1,p_2;p_5,p_6,p_7,p_8) = [1\,\not{p}_3\,2\rangle \Big\{ & E_1 \langle 15\rangle\langle 17\rangle[16][18] \\ & + E_2 \langle 15\rangle\langle 27\rangle[16][28] + E_3 \langle 25\rangle\langle 17\rangle[26][18] \\ & + E_4 \langle 25\rangle\langle 27\rangle[26][28] + E_5\langle 57\rangle[68]\Big\} \\ & + E_6 \langle 15\rangle\langle 27\rangle[16][18] + E_7 \langle 25\rangle\langle 27\rangle[26][18] \\ & + E_8 \langle 25\rangle\langle 17\rangle[16][18] + E_9 \langle 25\rangle\langle 27\rangle[16][28]\,,\end{aligned}$$

$$\begin{aligned}\mathrm{M}_{RLL}(p_1,p_2;p_5,p_6,p_7,p_8) = [2\,\not{p}_3\,1\rangle \Big\{ & E_1 \langle 15\rangle\langle 17\rangle[16][18] \\ & + E_2 \langle 15\rangle\langle 27\rangle[16][28] + E_3 \langle 25\rangle\langle 17\rangle[26][18] \\ & + E_4 \langle 25\rangle\langle 27\rangle[26][28] + E_5\langle 57\rangle[68]\Big\} \\ & + E_6 \langle 15\rangle\langle 17\rangle[16][28] + E_7 \langle 25\rangle\langle 17\rangle[26][28] \\ & + E_8 \langle 15\rangle\langle 17\rangle[26][18] + E_9 \langle 15\rangle\langle 27\rangle[26][28]\,,\end{aligned}$$

where the 9 E_j are **simple 4-dim linear combinations** of the 10 $A^{(j)}_{q\bar{q}}$

Very similar (*formally identical*) expressions can be obtained for $gg \to V_1 V_2$.
Again, **two independent helicity configurations**

$$
\begin{aligned}
\mathrm{M}_{\lambda_1\lambda_2 LL}(p_1, p_2; p_5, p_6, p_7, p_8) = C_{\lambda_1\lambda_2} \Bigg[& [2\, \not{p}_3\, 1\rangle \Big\{ E_1^{\lambda_1\lambda_2} \langle 57\rangle[68] \\
& + E_2^{\lambda_1\lambda_2} \langle 15\rangle\langle 17\rangle[16][18] + E_3^{\lambda_1\lambda_2} \langle 15\rangle\langle 27\rangle[16][28] \\
& + E_4^{\lambda_1\lambda_2} \langle 25\rangle\langle 17\rangle[26][18] + E_5^{\lambda_1\lambda_2} \langle 25\rangle\langle 27\rangle[26][28] \Big\} \\
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\end{aligned}
$$

$$
C_{LL} = [1\, \not{p}_3\, 2\rangle \frac{\langle 12\rangle}{[12]}, \qquad C_{LR} = [2\, \not{p}_3\, 1\rangle,
$$

In this case we go from 20 d-dimensional coefficients $A_{gg}^{(j)}$ to the 18 helicity coefficients $E_j^{(\lambda_1,\lambda_2)}$. All in all $\approx$ 80 MB.

Issues

Expressions for the E_j and the $E_j^{\lambda_1 \lambda_2}$ are **huge**:

1. Very large **rational prefactors**
2. Complicated combinations of **MPLs**, involved **analytic structure**

$$\Downarrow$$

a) **GiNaC** can evaluate **MPLs** on the whole complex plane [Vollinga, Weinzierl '04]

b) If arguments don't satisfy special **constraints**, GiNaC must perform rather complicated re-mappings in order to evaluate the **MPLs**.

c) **Rational prefactors** contain denominators raised to very high powers.

Instabilities and long evaluation time, especially close to (pseudo-)thresholds.

Improvements

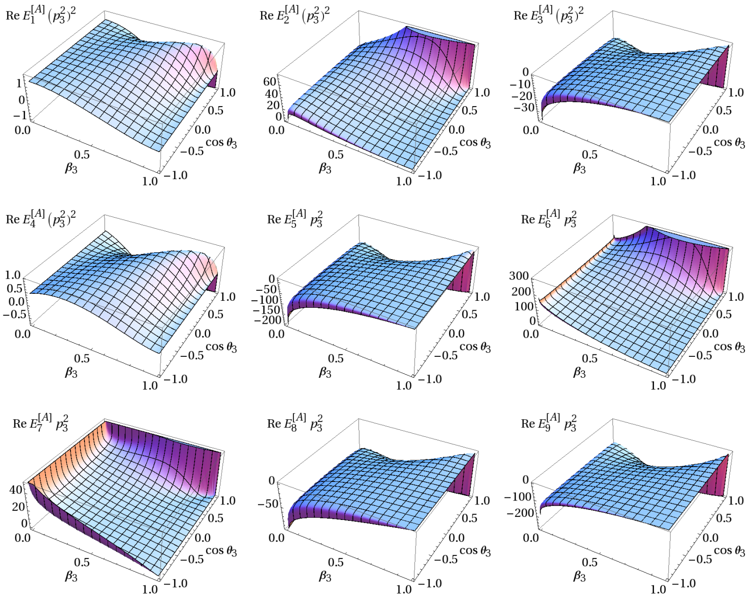
1. Simplifications of MPLs:
   Choose **real-valued** $\ln(l_j)$, $\mathrm{Li}_n(R_i)$, $\mathrm{Li}_{2,2}(R_i, S_j)$ such that
   a) R_i and S_j are power products of the letters above.
   b) $l_j \geq 0$, $|R_i| \leq 1$, $|R_i\, S_j| < 1 \to$ **convergent series expansions**

$$\mathrm{Li}_n(R_i) = -\sum_{k=1}^{\infty} \frac{R_i^k}{k^n}\,, \qquad \mathrm{Li}_{2,2}(R_i, S_j) = \sum_{k,l=1}^{\infty} \frac{R_i^k}{(k+l)^2} \frac{(R_i S_j)^l}{l^2}\,.$$

   Based on [Duhr, Gangl, Rodes '11]

2. **Stable and fast** numerical evaluation ,
   2.1 $\mathcal{O}(150ms)$ for full $q\bar{q}$ amplitude!!
   2.2 $\mathcal{O}(600ms)$ for full gg amplitude!!
3. Stability over the **whole phase space** $\to$ precision control system!
4. **Public code** at: http://vvamp.hepforge.org/

Helicity coefficients for $q\,\bar{q} \to V_1 V_2$



Summary and Conclusions

1. We computed the **two-loop** helicity amplitudes for $pp \to V_1 V_2$.

2. Put together **cutting edge technology** for multiloop calculations

   a) differential equations and canonical basis
   b) co-product augmented symbol calculus

3. We are able to handle such a huge complexity and generate code that evaluates the amplitudes in $\mathcal{O}(150ms)$.

$$\Downarrow$$

$2 \to 2$ processes with 4 independent scales seem to be within reach.

4. Extension to $2 \to 3$ processes looks unfortunately **out of reach**.
   $gg \to V_1 V_2 \, g$ has 7 scales and $\mathcal{O}(200)$ tensor structures!!!

Unitarity at two loops??? [see Kaspar Larsen's talk]

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Thanks !

Back-Up Slides

Simplification of the MPLs

1) Use a different **parametrization**

$$s = m^2(1+x)(1+xy), \quad t = -m^2 xz, \quad p_3^2 = m^2, \quad p_4^2 = m^2 x^2 y$$

which generates the **alphabet**

$$\begin{aligned}\{l_1, \dots, l_{17}\} = \{&x, 1+x, y, 1-y, z, 1-z, -y+z, 1+y-z, 1+xy, 1+xz, xy+z,\\ &1+y+xy-z, 1+x+xy-xz, 1+y+2xy-z+x^2yz,\\ &2xy+x^2y+x^2y^2+z-x^2yz, 1+x+y+xy+xy^2-z-xz-xyz,\\ &1+y+xy+y^2+xy^2-z-yz-xyz\}\end{aligned}$$

(which cannot be used to integrate MPLs in the common way)

2) Argument construction based on [Duhr, Gangle, Rodes '11]

$$\Downarrow$$

Generate arguments which are rational functions of x, y, z and don't generate spurious letters $\rightarrow$ **factorize** into **letters** and their **inverse**.

Helicity coefficients for $g\,g \to V_1 V_2$