An EFT Description of Forward Scattering and Factorization Violation

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work with Ira Rothstein (to appear soon)

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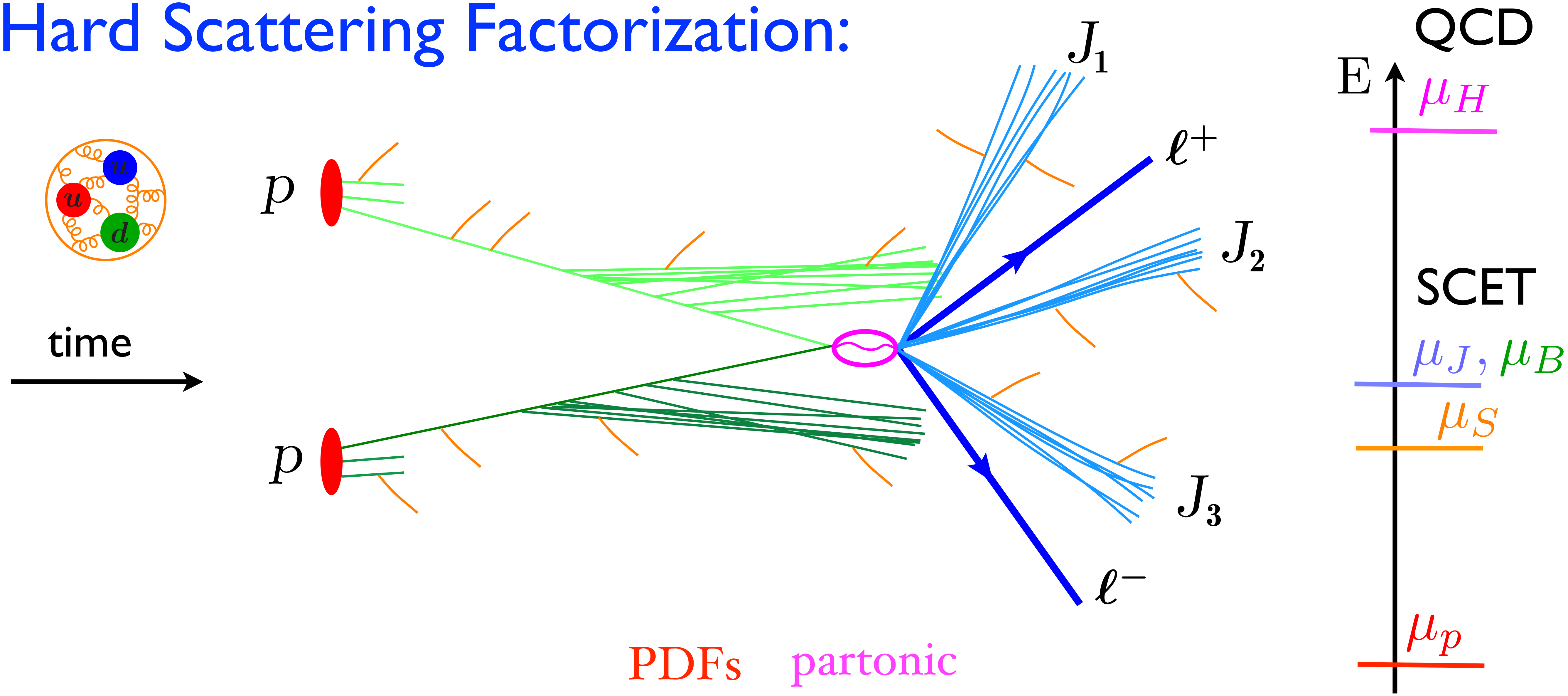
Outline

- Introduction: Forward Scattering and Factorization (Violation), why EFT?
- Glauber Interaction Lagrangian in Soft-Collinear Effective Theory
  - Operators, interactions between 2 or 3 rapidity sectors
  - Rapidity Regulator
  - Subtractions
- One-Loop Graphs, Eikonal Scattering, Reggeization for Octet Ops.
- Forward Scattering and BFKL
  - Rapidity RG equations for Collinear and Soft functions: BFKL
- Glaubers in Hard Scattering, one and two loop examples
- Summary

Hard Scattering Factorization:



PDFs partonic

Nonperturbative: $d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$ ← hadronization

$\mu_p \simeq \Lambda_{\mathrm{QCD}}$

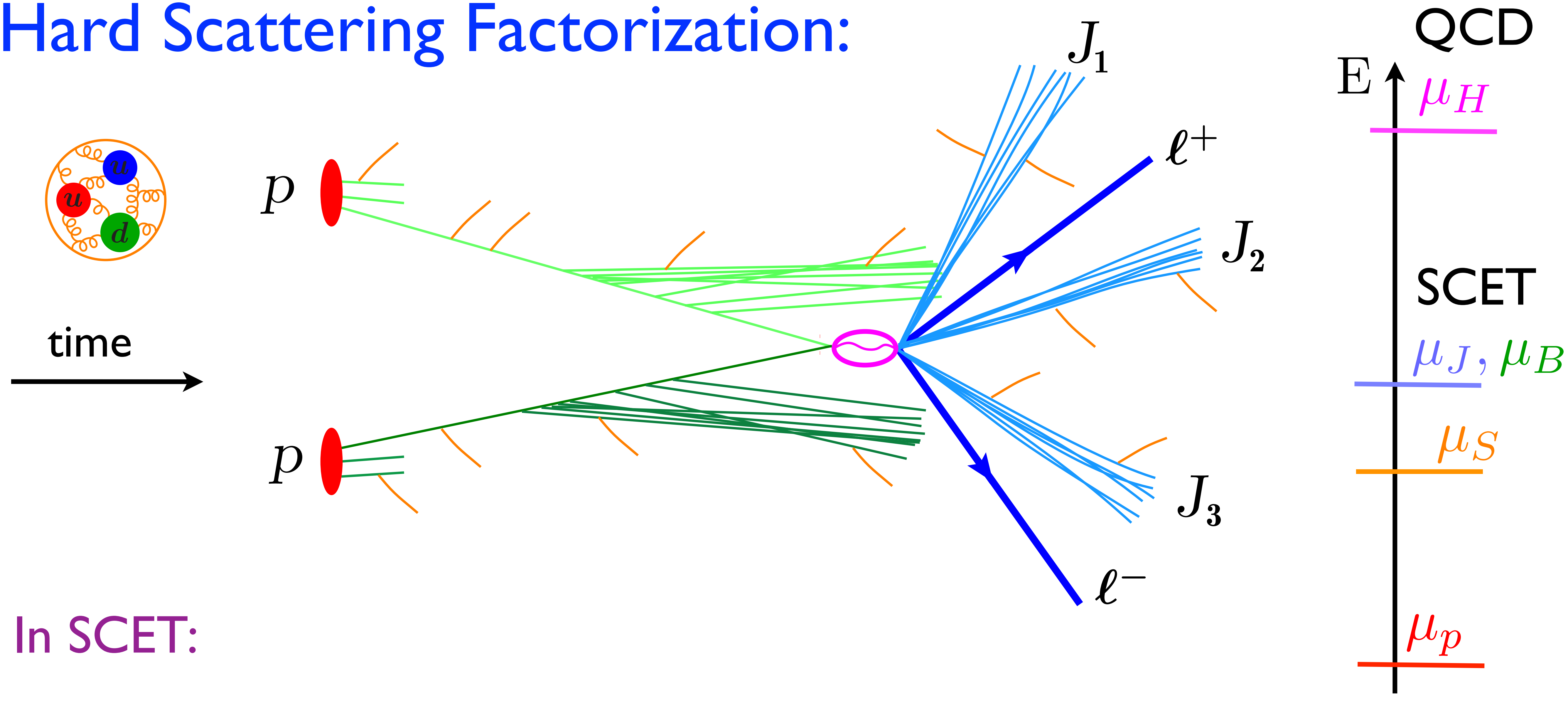
(In some cases by Operators,
or is power suppressed)

eg. Perturbative: $\hat{\sigma}_{\mathrm{fact}} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$

Used to Sum Logs

μ_B	μ_H	μ_J	μ_S
beam	hard	jet	pert. soft

Hard Scattering Factorization:



In SCET:

factorized Lagrangian: $\mathcal{L}^{(0)}_{\mathrm{SCET_{II},S,\{n_i\}}} = \mathcal{L}^{(0)}_S(\psi_S, A_S) + \sum_{n_i} \mathcal{L}^{(0)}_{n_i}(\xi_{n_i}, A_{n_i})$

factorized Hard Ops: $C \otimes (\mathcal{B}_{n_a\perp})(\mathcal{B}_{n_b\perp})(\mathcal{B}_{n_1\perp})(\bar{\chi}_{n_2})(\chi_{n_3})(\mathcal{S}_{n_a}\mathcal{S}_{n_b}\mathcal{S}_{n_1}\mathcal{S}_{n_2}\mathcal{S}_{n_3})$

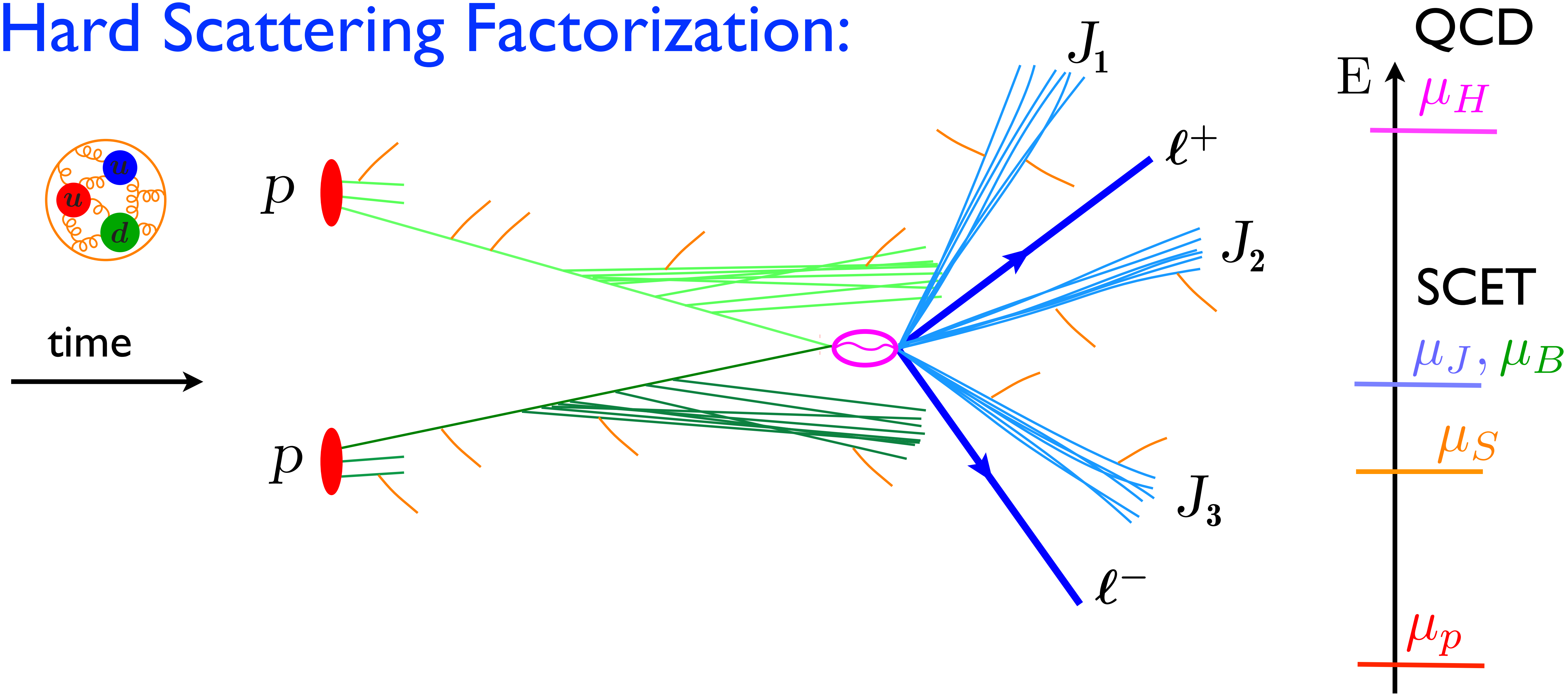
Soft Wilson Lines

Collinear Wilson Lines in quark & gluon operator building blocks:

$$\chi_n = W_n^\dagger \xi_n \qquad g\mathcal{B}^\mu_{n\perp} = [W_n^\dagger i D^\mu_{n\perp} W_n]$$

→ factorized matrix elements defining jet, soft, … functions

Hard Scattering Factorization:



Factorization will be broken if:

- it is not possible to identify a finite basis of leading power operators

  (examples studied by Collins & Qiu, Aybat & Rogers, …)
- there are non-factorizable leading power Lagrangian interactions between soft & collinear sectors

Compatible with violation of Collinear factorization (Catani,de Florian,Rodrigo), Regge factorization (Del Duca,Glover,Falcioni,Magnea,Vernazza,Duhr,White), Cross-section factorization (Collins,Soper,Sterman=CSS,…), …

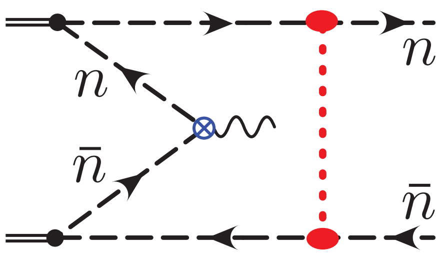
Relevant Modes

$\lambda \ll 1$ large Q

Infrared Structure of Amplitudes (CSS, ...)
Method of Regions (Beneke & Smirnov)

mode	fields	p^μ momentum scaling	physical objects	type
n_a-collinear	ξ_{n_a}, $A^\mu_{n_a}$	$(n_a\cdot p, \bar n_a \cdot p, p_{\perp a}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet a	onshell
n_b-collinear	ξ_{n_b}, $A^\mu_{n_b}$	$(n_b\cdot p, \bar n_b \cdot p, p_{\perp b}) \sim Q(\lambda^2, 1, \lambda)$	collinear initial state jet b	onshell
n_j-collinear	ξ_{n_j}, $A^\mu_{n_j}$	$(n_j\cdot p, \bar n_j \cdot p, p_{\perp j}) \sim Q(\lambda^2, 1, \lambda)$	collinear final state jet in $\hat n_j$	onshell
soft	ψ_S, A^μ_S	$p^\mu \sim Q(\lambda, \lambda, \lambda)$	soft virtual/real radiation	onshell
ultrasoft	ψ_{us}, A^μ_{us}	$p^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda)$, $a+b>2$	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Glauber's (Coulomb gluons) may break factorization:



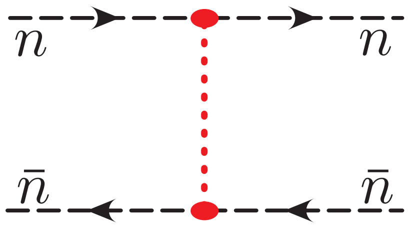
spectator-spectator

(CSS proof for Drell-Yan)

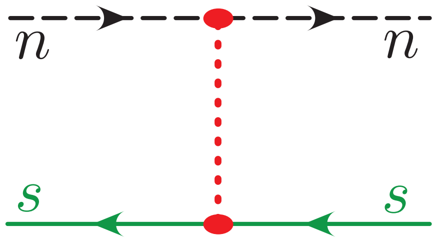
- $\dfrac{1}{k_\perp^2}$ potential
- instantaneous in x^+, x^- (t and z)

Glauber's dominate Forward Scattering:

n-$\bar n$
fwd. scattering



n-s
fwd. scattering



(small-x logs, reggeization, BFKL, BK/BJMWLK, ...)

Plan: Add Glaubers to SCET

$$\mathcal{L}^{(0)}_{\rm SCET_{II}} = \mathcal{L}^{(0)}_{\rm SCET_{II},S,\{n_i\}} + \mathcal{L}^{(0)}_{G}(\psi_S, A_S, \xi_{n_i}, A_{n_i})$$

Focus in this talk on $\rm SCET_{II}$

Goals & Possible Advantages for EFT approach

- Hard Scattering and Forward Scattering in single framework
- Operator based: Can exploit symmetries, Gauge invariant
- $\overline{\rm MS}$ style renormalization for rapidity divergences (counterterms, renormalization group equations, …)
- Distinct Infrared Modes in Feyn. Graphs + Power Counting → derive when eikonal approximation is relevant
- Factorization violating interactions also obey factorization theorems
- Valid to all orders in α_s & clear path using this formalism to study subleading power amplitudes (subleading ops & Lagrangians)
- Potential method to derive factorization results for less inclusive collider processes, predict things about UE, etc.

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
n-collinear	$\xi_n,\ A_n^\mu$	$(n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2,1,\lambda)$	n-collinear "jet"	onshell
$\bar n$-collinear	$\xi_{\bar n},\ A_{\bar n}^\mu$	$(\bar n\cdot p, \bar n\cdot p, p_\perp) \sim Q(\lambda^2,1,\lambda)$	$\bar n$-collinear "jet"	onshell
soft	$\psi_{\rm S},\ A_{\rm S}^\mu$	$p^\mu \sim Q(\lambda,\lambda,\lambda)$	soft virtual/real radiation	onshell
ultrasoft	$\psi_{\rm us},\ A_{\rm us}^\mu$	$p^\mu \sim Q(\lambda^2,\lambda^2,\lambda^2)$	ultrasoft virtual/real radiation	onshell
Glauber	–	$p^\mu \sim Q(\lambda^a,\lambda^b,\lambda),\ a+b>2$ (here $\{a,b\}=\{2,2\},\{2,1\},\{1,2\}$)	forward scattering potential	offshell
hard	–	$p^2 \gtrsim Q^2$	hard scattering	offshell

Power Counting formula for graph (any loop order, any power):

$$\sim \lambda^\delta$$

$$\delta = 6 - N^n - N^{\bar n} - N^{nS} - N^{\bar n S} + 2u\,,$$

$$+\sum_k (k-8)\underbrace{V_k^{us} + (k-4)}_{\text{standard SCET}}\big(V_k^n + V_k^{\bar n} + V_k^S\big) + (k-3)\big(V_k^{nS} + V_k^{\bar n S}\big) + (k-2)V_k^{n\bar n}$$

need $\sim \lambda^3$ $\sim \lambda^2$

Glauber

operators at leading power

(gauge invariant)

Construction:

$\lambda \ll 1$ large Q

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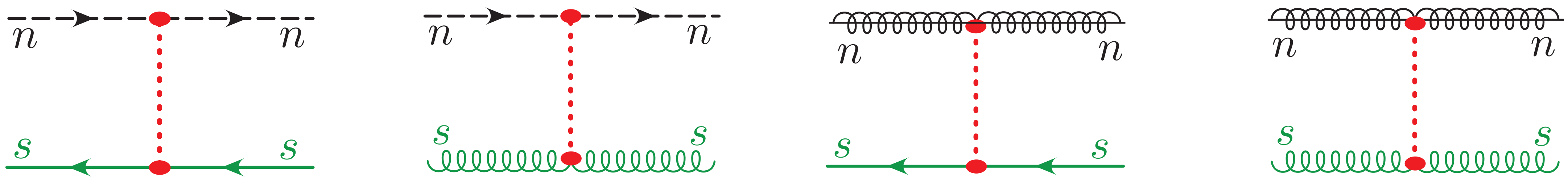
integrated out

n-s fwd. scattering (2 rapidity sectors) $s \gg t$



$$\mathcal{O}(\lambda^3): \quad \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

$$\lambda^2 = \frac{t}{s} \ll 1$$

with bilinear octet operators

$$\psi_s^n = S_n^\dagger \psi_s$$

$$\mathcal{O}(\lambda^2): \quad \mathcal{O}_n^{qB} = \bar{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n\,,$$

$$\mathcal{O}_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}_{n\perp}^{D\mu}$$

$$\mathcal{O}(\lambda^3): \quad \mathcal{O}_s^{q_n B} = 8\pi\alpha_s \left(\bar{\psi}_S^n T^B \frac{n\!\!\!/}{2} \psi_S^n\right),$$

$$\mathcal{O}_s^{g_n B} = 8\pi\alpha_s \left(\frac{i}{2} f^{BCD} \mathcal{B}_{S\perp\mu}^{nC} \frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}_{S\perp}^{nD\mu}\right)$$

Construction:

$\lambda \ll 1$ large Q

mode	fields	p^μ momentum scaling	physical objects	type
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		(here $\{a,b\} = \{2,2\}, \{2,1\}, \{1,2\}$)		
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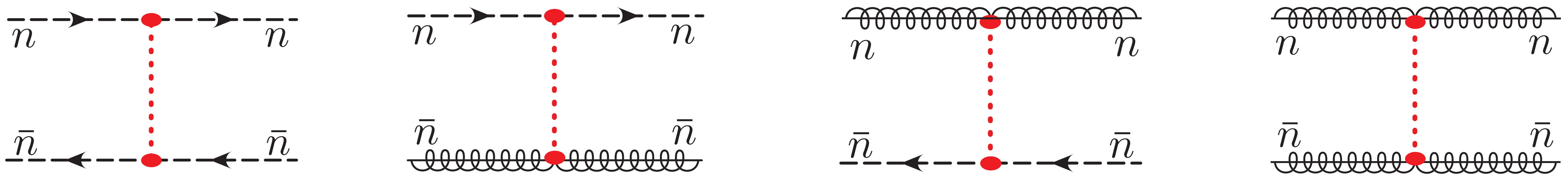
n-$\bar n$ fwd. scattering $s \gg t$



$\mathcal{O}(\lambda^2)$:

(3 rapidity sectors)

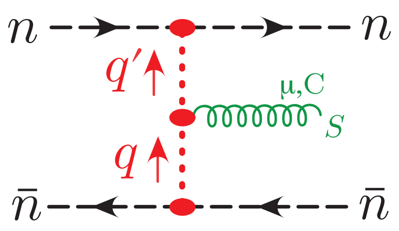
$$\sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar n}^{jC}$$

same $\mathcal{O}_n^{iB}$

analogous $\mathcal{O}_{\bar n}^{jC}$

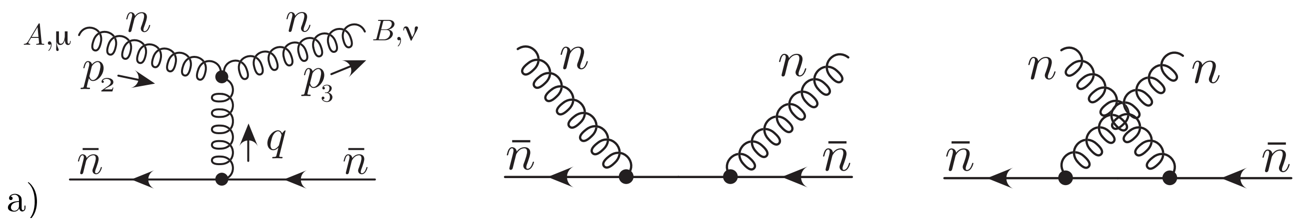
must allow for soft emission from **between** the rapidity sectors:

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \mathcal{P}_\perp^2 \delta^{BC} \boxed{+\ ....}$$

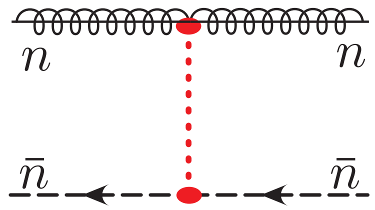


Gluon Operators include Compton graphs in fwd.limit:

QCD



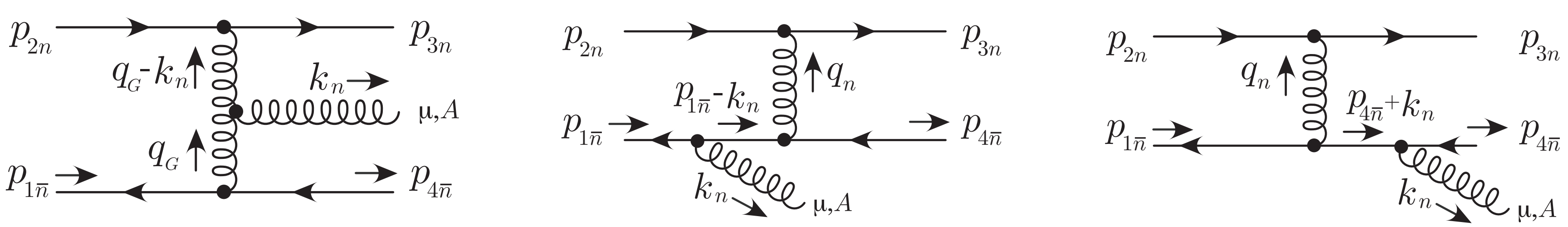
EFT



Wilson Lines in the operators are obtained from Matching:

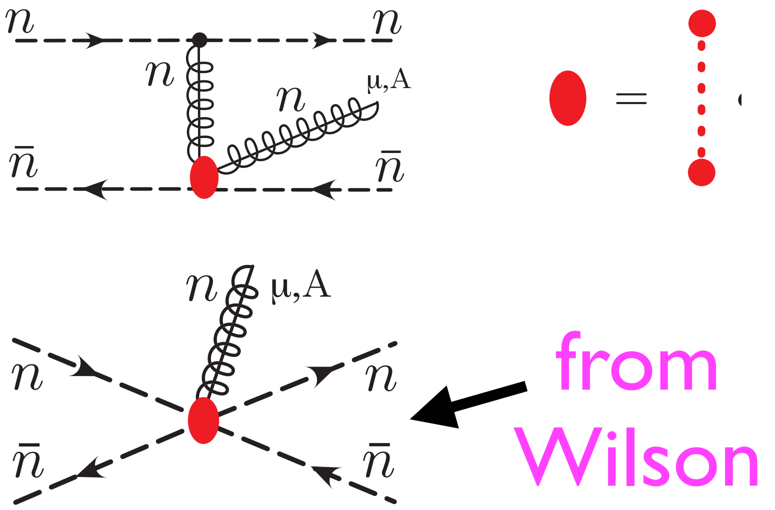
eg. W_n in χ_n

QCD



similar for other operators

EFT



from Wilson line

Full Leading Power Glauber Lagrangian:

$$\mathcal{L}_G^{\mathrm{II}(0)} = \sum_{n,\bar{n}} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{BC} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_{\bar{n}}^{jC} + \sum_{n} \sum_{i,j=q,g} \mathcal{O}_n^{iB} \frac{1}{\mathcal{P}_\perp^2} \mathcal{O}_s^{j_n B}$$

Soft $\mathcal{O}_s^{BC}$ Operator

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \sum_i C_i O_i^{BC}$$

basis of $\mathcal{O}(\lambda^2)$ operators allowed by symmetries:

$$O_1 = \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu}, \qquad O_2 = \mathcal{P}_\perp^\mu \mathcal{S}_{\bar{n}}^T \mathcal{S}_n \mathcal{P}_{\perp\mu},$$

$$O_3 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{n})(\mathcal{S}_n^T \mathcal{S}_{\bar{n}}) + (\mathcal{S}_n^T \mathcal{S}_{\bar{n}})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}}) \cdot \mathcal{P}_\perp, \qquad O_4 = \mathcal{P}_\perp \cdot (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}})(\mathcal{S}_{\bar{n}}^T \mathcal{S}_n) + (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n}) \cdot \mathcal{P}_\perp,$$

$$O_5 = \mathcal{P}_\mu^\perp (\mathcal{S}_n^T \mathcal{S}_{\bar{n}})(g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})(\mathcal{S}_n^T \mathcal{S}_{\bar{n}})\mathcal{P}_\mu^\perp, \qquad O_6 = \mathcal{P}_\mu^\perp (\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)(g\widetilde{\mathcal{B}}_{S\perp}^{n\mu}) + (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu})(\mathcal{S}_{\bar{n}}^T \mathcal{S}_n)\mathcal{P}_\mu^\perp,$$

$$O_7 = (g\widetilde{\mathcal{B}}_{S\perp}^{n\mu})\mathcal{S}_n^T \mathcal{S}_{\bar{n}}(g\widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}}), \qquad O_8 = (g\widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu})\mathcal{S}_{\bar{n}}^T \mathcal{S}_n(g\widetilde{\mathcal{B}}_{S\perp\mu}^{n}),$$

$$O_9 = \mathcal{S}_n^T n_\mu \bar{n}_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_{\bar{n}}, \qquad O_{10} = \mathcal{S}_{\bar{n}}^T n_\mu \bar{n}_\nu (ig\widetilde{G}_s^{\mu\nu})\mathcal{S}_n,$$

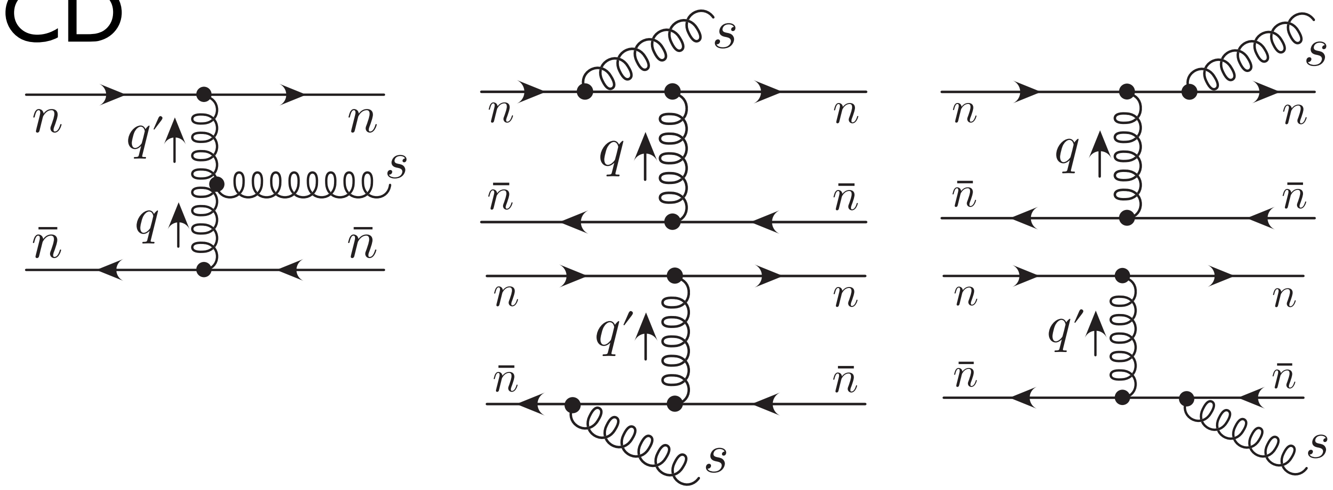
octet Wilson line

octet reps

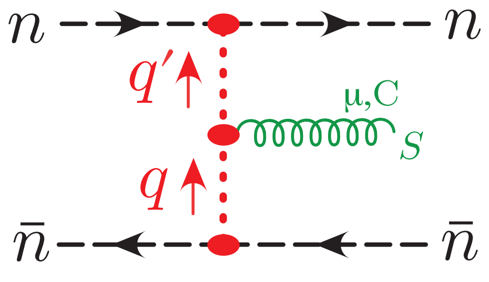
Matching with up to 2 soft gluons fixes all coefficients

One Soft Gluon:

QCD



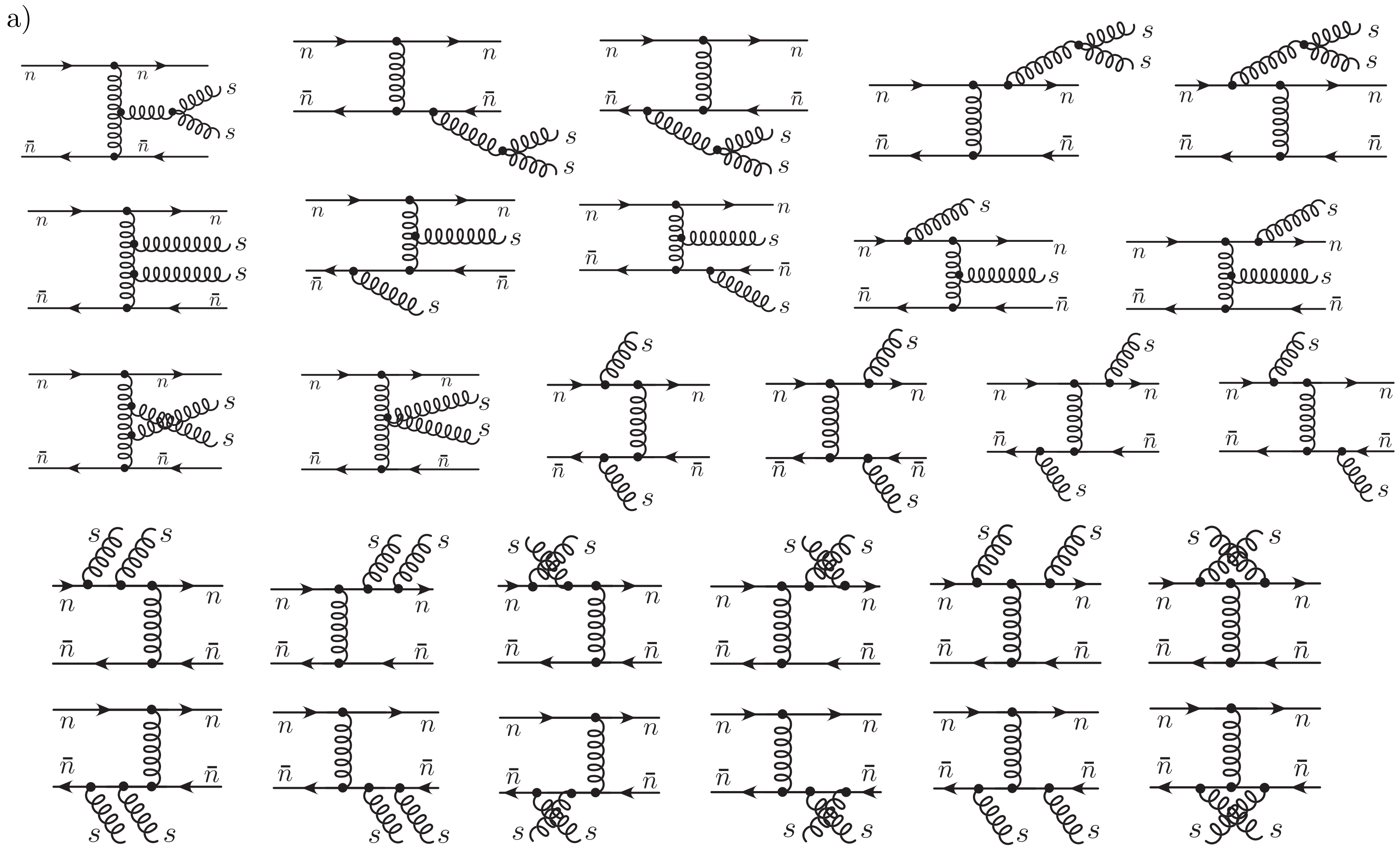
EFT



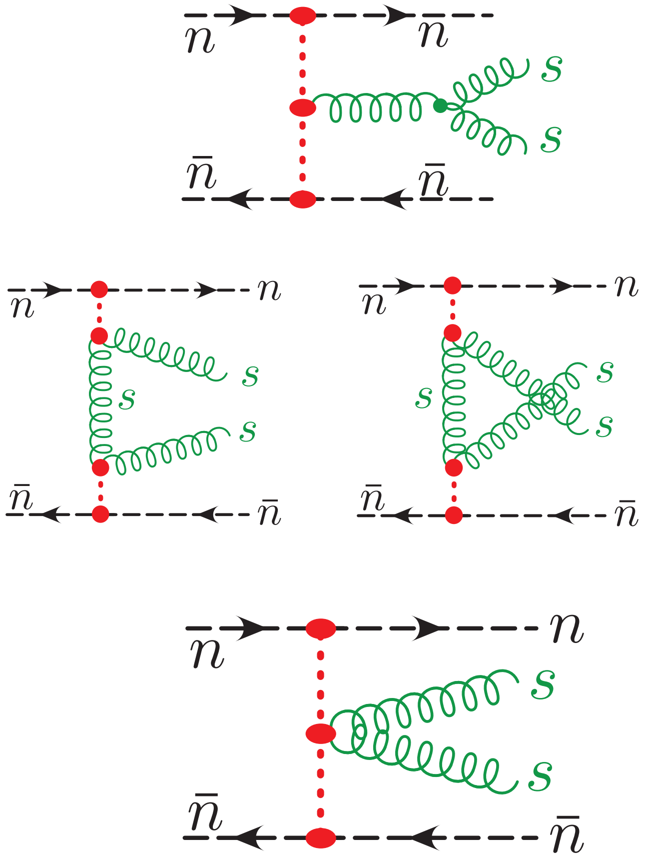
Lipatov vertex

Two Soft Gluons:

QCD



EFT



new vertex

Find:

$$\boxed{C_2 = C_4 = C_5 = C_6 = C_8 = C_{10} = 0}\,,$$

$$\boxed{C_1 = -C_3 = -C_7 = +1\,, \qquad C_9 = -\frac{1}{2}}$$

$$\mathcal{O}_s^{BC} = 8\pi\alpha_s \Big\{ \mathcal{P}_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_\mu^\perp g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp}^{\bar{n}\mu} \mathcal{P}_\mu^\perp - g \widetilde{\mathcal{B}}_{S\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{S\perp\mu}^{\bar{n}}$$
$$- \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T ig \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC} \,.$$

Form is unique to all loops since there are no hard α_s corrections to this matching (more later)

One Loop EFT graphs

- QCD topologies appear more than once (soft, collinear, …)
- Each dominated by one invariant mass scale & one rapidity
- Require invariant mass regulator (dim.reg.)

Requires rapidity regulator for Glauber potential $|2k^z|^{-\eta}\nu^{\eta}$
and for Wilson lines

$$S_n = \sum_{\rm perms} \exp\left\{ \frac{-g}{n\cdot\mathcal{P}} \left[\frac{w|2\mathcal{P}^z|^{-\eta/2}}{\nu^{-\eta/2}} n\cdot A_s \right] \right\}$$

$$W_n = \sum_{\rm perms} \exp\left\{ \frac{-g}{\bar{n}\cdot\mathcal{P}} \right] \frac{w^2|\bar{n}\cdot\mathcal{P}|^{-\eta}}{\nu^{-\eta}} \bar{n}\cdot A_n \left] \right\}$$

(use Chieu et.al., works like $\overline{\rm MS}$)

$$\frac{1}{\eta} \qquad \ln(\nu)$$

- Zero-bin subtractions, avoid double counting IR regions

1-loop graphs:

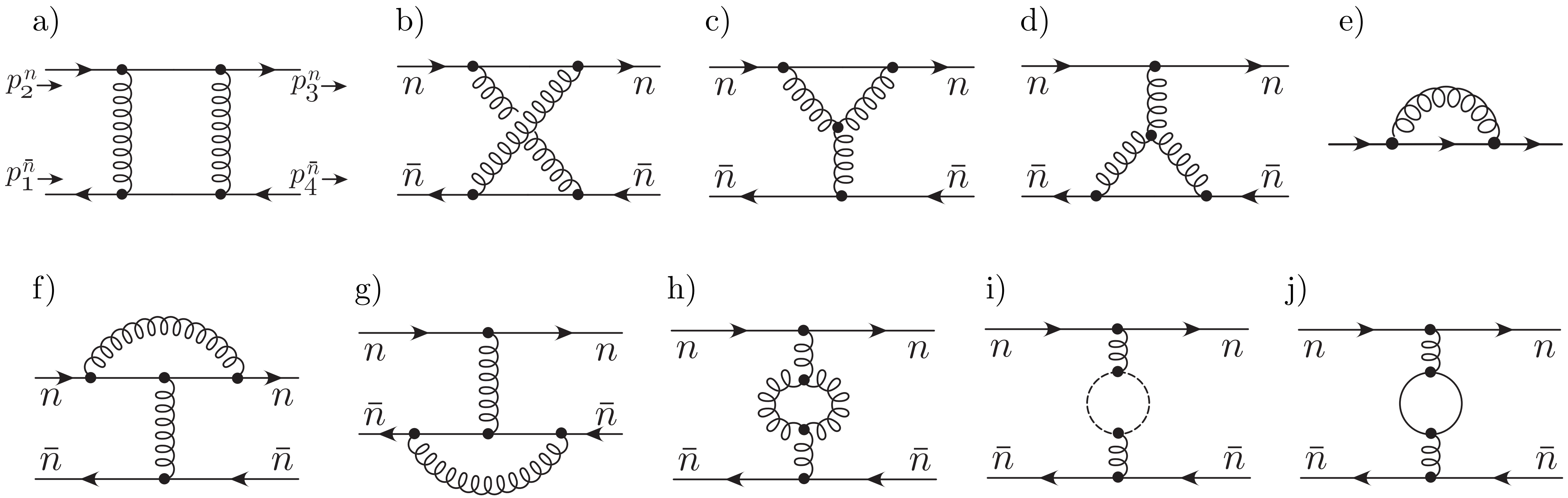
$$S = \tilde{S} - S^{(G)}$$

$$C_n = \tilde{C}_n - C_n^{(S)} - C_n^{(G)} + C_n^{(GS)}$$

(construction ala
Manohar & IS)

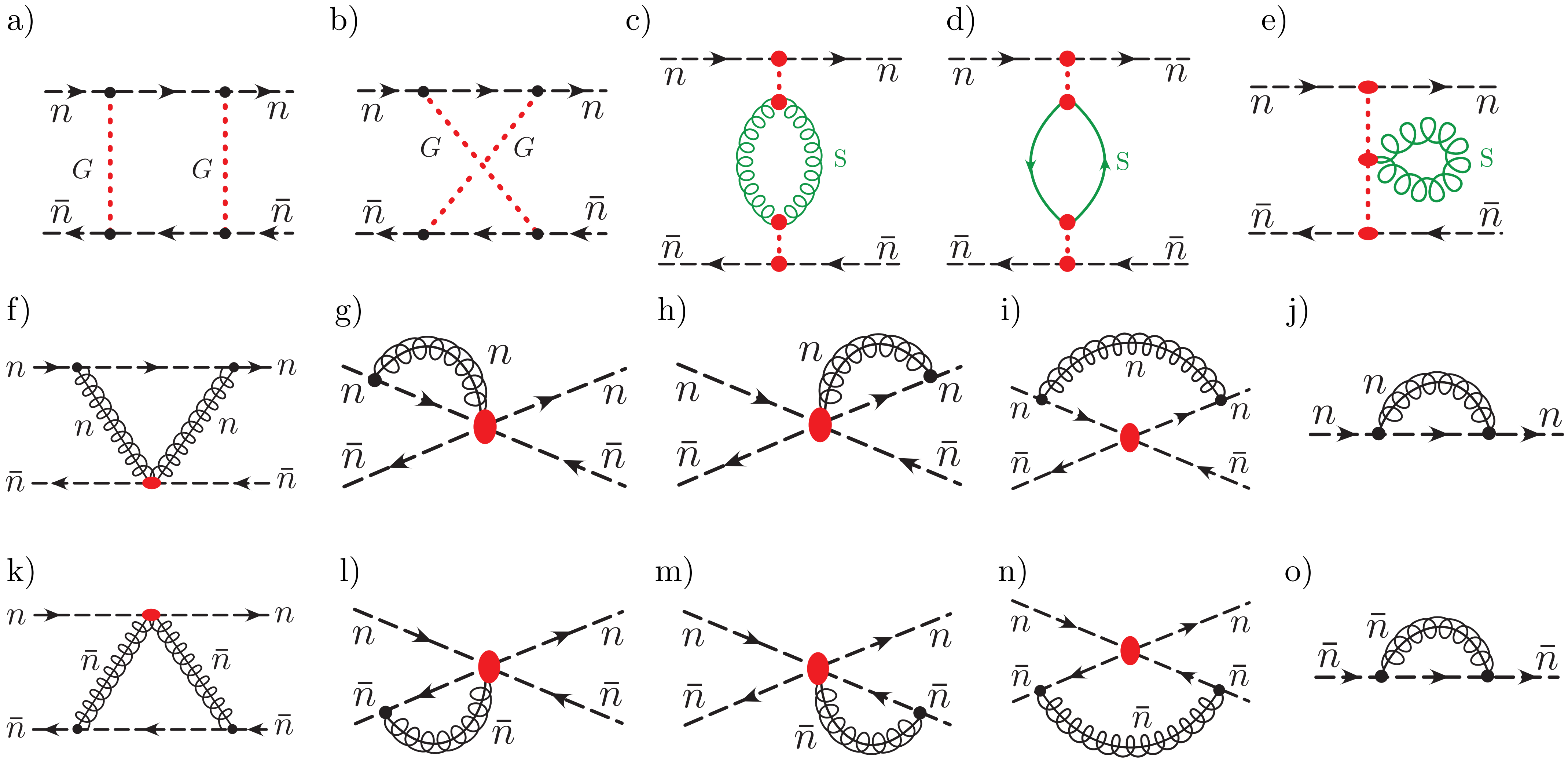
eq. One Loop $q\bar{q}$ scattering

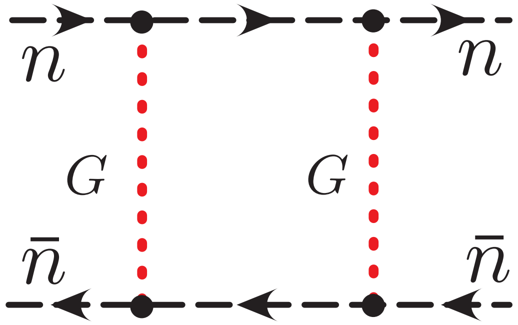
QCD graphs with leading power contributions, $s \gg t$



eq. One Loop $q\bar{q}$ scattering

Leading Power EFT graphs (Glauber, Soft, & Collinear Loops)

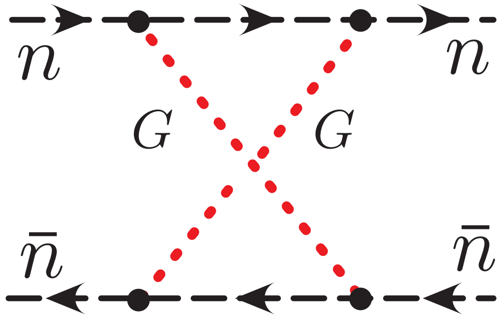




$$I_{\rm Gbox} = \int \frac{\text{đ}^{d-2}k_\perp \;\, \text{đ}k^0 \;\, \text{đ}k^z \;\, |k^z|^{-2\eta}\,(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2\Big(k^+ - \Delta_1(k_\perp)+i0\Big)\Big(-k^- - \Delta_2(k_\perp)+i0\Big)}$$

$$= \left(\frac{-i}{4\pi}\right)\int \frac{\text{đ}^{d-2}k_\perp}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2}\Big[-i\pi+\mathcal{O}(\eta)\Big]$$

building up
Glauber phase

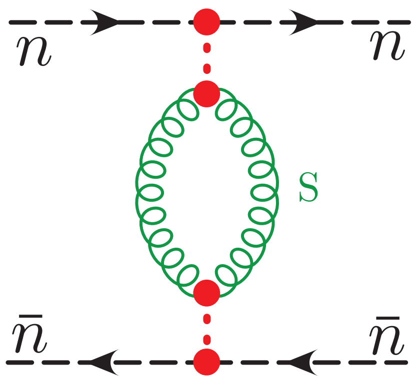


$$I_{\rm Gcbox} = \int \frac{\text{đ}^{d-2}k_\perp \;\, \text{đ}k^0 \;\, \text{đ}k^z \;\, |k^z|^{-2\eta}\,(\nu/2)^{2\eta}}{(\vec{k}_\perp^{\,2})(\vec{k}_\perp+\vec{q}_\perp)^2\Big(k^+ - \Delta_1(k_\perp)+i0\Big)\Big(k^- - \Delta_2(k_\perp)+i0\Big)}$$

$$= 0$$

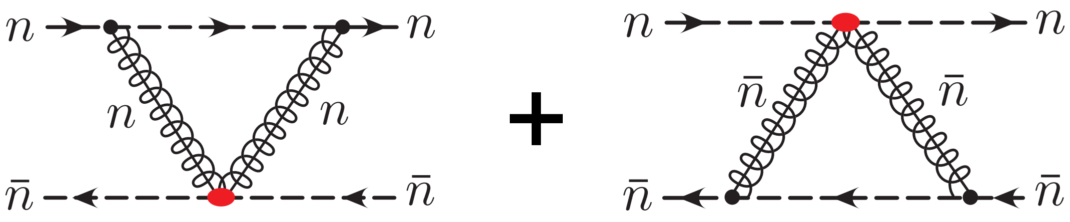
$\mu^2 \sim \nu^2 \sim -t$

rapidity divergent



$$= -\frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\Big\{\frac{8}{\eta}\,g(\epsilon,\mu^2/t)+\frac{4}{\epsilon^2}+\frac{4}{\epsilon}\ln\left(\frac{\mu^2}{\nu^2}\right)+\boxed{4\ln\left(\frac{\mu^2}{\nu^2}\right)\ln\left(\frac{\mu^2}{-t}\right)}-2\ln^2\left(\frac{\mu^2}{-t}\right)+\frac{\pi^2}{3}$$
$$+\,2\Big(-\frac{11}{3\epsilon}-\frac{11}{3}\ln\frac{\mu^2}{-t}-\frac{67}{9}\Big)\Big\}$$

β -function without ghosts



$$= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\Big[\Big\{\frac{4}{\eta}\,g(\epsilon,\mu^2/t)-\frac{4}{\epsilon}\ln\left(\frac{\nu}{\bar n\cdot p_3}\right)-4\ln\left(\frac{\nu}{\bar n\cdot p_3}\right)\ln\left(\frac{\mu^2}{-t}\right)-\frac{3}{\epsilon}-3\ln\left(\frac{\mu^2}{-t}\right)-6+\frac{4\pi^2}{3}\Big\}$$
$$+\Big\{\frac{4}{\eta}\,g(\epsilon,\mu^2/t)-\frac{4}{\epsilon}\ln\left(\frac{\nu}{n\cdot p_4}\right)-4\ln\left(\frac{\nu}{n\cdot p_4}\right)\ln\left(\frac{\mu^2}{-t}\right)-\frac{3}{\epsilon}-3\ln\left(\frac{\mu^2}{-t}\right)-6+\frac{4\pi^2}{3}\Big\}\Big]$$
$$= \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\Big\{\frac{8}{\eta}\,g(\epsilon,\mu^2/t)-\frac{4}{\epsilon}\ln\left(\frac{\nu^2}{s}\right)-\boxed{4\ln\left(\frac{\nu^2}{s}\right)\ln\left(\frac{\mu^2}{-t}\right)}-\frac{6}{\epsilon}-6\ln\left(\frac{\mu^2}{-t}\right)-12+\frac{8\pi^2}{3}\Big\}.$$

opposite sign for
rapidity divergence

$\mu^2 \sim -t$

$\nu^2 \sim s$

One Loop Results & Matching

$$\mathcal{S}_1^{n\bar n} = \Big[\bar u_n T^A T^B \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} T^B T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,, \qquad \mathcal{S}_2^{n\bar n} = C_F\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,,$$

$$\mathcal{S}_3^{n\bar n} = C_A\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,, \qquad \mathcal{S}_4^{n\bar n} = T_F n_f\Big[\bar u_n T^A \frac{\bar{n}\!\!\!/}{2} u_n\Big]\Big[\bar v_{\bar n} T^A \frac{n\!\!\!/}{2} v_{\bar n}\Big]\,.$$

m = gluon mass IR regulator

$$\text{Glauber Loops } = \frac{i\alpha_s^2}{t}\,\mathcal{S}_1^{n\bar n}\Big[8i\pi\ln\Big(\frac{-t}{m^2}\Big)\Big]$$

$$\begin{aligned}\text{Soft Loops} = \ & \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{-\frac{8}{\eta}h(\epsilon,\mu^2/m^2)-\frac{8}{\eta}\,g(\epsilon,\mu^2/t)-4\ln\Big(\frac{\mu^2}{\nu^2}\Big)\ln\Big(\frac{m^2}{-t}\Big)\\ &-2\ln^2\Big(\frac{\mu^2}{m^2}\Big)+2\ln^2\Big(\frac{\mu^2}{-t}\Big)-\frac{2\pi^2}{3}+\frac{22}{3}\ln\frac{\mu^2}{-t}+\frac{134}{9}\bigg\}\\ &+\frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar n}\Big[-\frac{8}{3}\ln\Big(\frac{\mu^2}{-t}\Big)-\frac{40}{9}\Big].\end{aligned}$$

← no $1/\epsilon$ poles

(after coupling renormalization)

$$\begin{aligned}\text{Collinear Loops} = \ & \frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar n}\bigg\{\frac{8}{\eta}h\Big(\epsilon,\frac{\mu^2}{m^2}\Big)+\frac{8}{\eta}\,g\Big(\epsilon,\frac{\mu^2}{-t}\Big)+4\ln\Big(\frac{\nu^2}{s}\Big)\ln\Big(\frac{-t}{m^2}\Big)+2\ln^2\Big(\frac{m^2}{-t}\Big)+4+\frac{4\pi^2}{3}\bigg\}\\ &+\frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar n}\Big[-4\ln^2\Big(\frac{m^2}{-t}\Big)-12\ln\Big(\frac{m^2}{-t}\Big)-14\Big]\end{aligned}$$

$$\text{Total SCET} \;= \frac{i\alpha_s^2}{t}\,\mathcal{S}_1^{n\bar{n}}\left[8i\pi\ln\left(\frac{-t}{m^2}\right)\right] + \frac{i\alpha_s^2}{t}\,\mathcal{S}_2^{n\bar{n}}\left[-4\ln^2\left(\frac{m^2}{-t}\right) - 12\ln\left(\frac{m^2}{-t}\right) - 14\right]$$
$$+\frac{i\alpha_s^2}{t}\,\mathcal{S}_3^{n\bar{n}}\left\{-4\ln\left(\frac{s}{-t}\right)\ln\left(\frac{-t}{m^2}\right) + \frac{22}{3}\ln\frac{\mu^2}{-t} + \frac{170}{9} + \frac{2\pi^2}{3}\right\}$$
$$+\frac{i\alpha_s^2}{t}\,\mathcal{S}_4^{n\bar{n}}\left[-\frac{8}{3}\ln\left(\frac{\mu^2}{-t}\right) - \frac{40}{9}\right]$$

rapidity divergences cancel

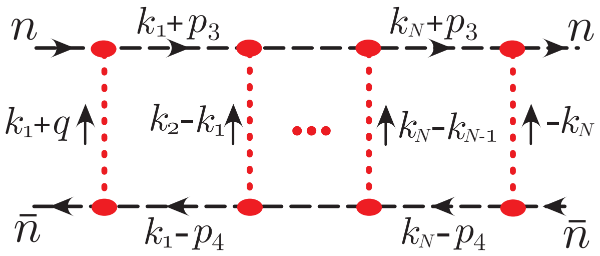
$$\text{Total SCET} \;= \text{Total QCD}\;(s \gg t)$$

IR divergences are all reproduced

no hard matching here (no loops with momenta $\sim s$)

Eikonal Scattering

Rapidity regulator consistent with eikonal phase



$$= i(-2g^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}\, I^{(N)}(q_\perp)\int\frac{{\mathchar'26\mkern-12mu d}k_1^z\cdots{\mathchar'26\mkern-12mu d}k_N^z\,\left|2k_1^z(2k_1^z-2k_2^z)\cdots(2k_{N-1}^z-2k_N^z)2k_N^z\right|^{-\eta}\nu^{N\eta}}{2^N(-k_1^z+\Delta_1+i0)\cdots(-k_N^z+\Delta_N+i0)}$$

$$= -2(ig^2)^{N+1}\mathcal{S}^{n\bar{n}}_{(N+1)}I_\perp^{(N)}(q_\perp)\;\frac{1}{(N+1)!}\Big[1+\mathcal{O}(\eta)\Big]$$

Sum of Glauber Boxes

$$G(q_\perp) \;= (2\pi)^2\delta^2(q_\perp) + \frac{i4\pi C_F\alpha_s(\mu)}{(-t)}\,\frac{\Gamma\big(1-iC_F\alpha_s(\mu)\big)}{\Gamma\big(1+iC_F\alpha_s(\mu)\big)}\Big(\frac{-t}{m^2e^{2\gamma_E}}\Big)^{iC_F\alpha_s(\mu)}$$

classic eikonal scattering result

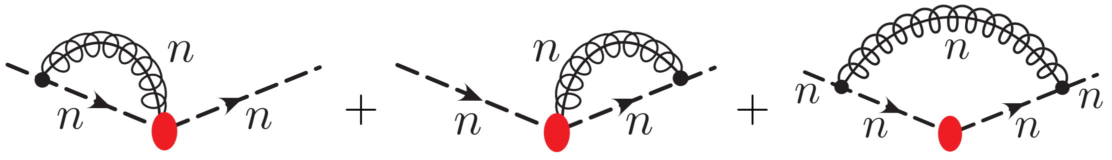
$$\tilde{G}(b_\perp) = e^{i\phi(b_\perp)}$$

Gluon Reggeization

Consider separate rapidity renormalization of soft & collinear component operators

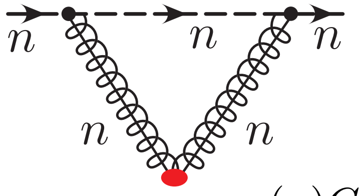
eg. $\mathcal{O}_n^{iB}$ $\quad \vec{\mathcal{O}}_n^{A\text{bare}} = \hat{V}_{\mathcal{O}_n} \cdot \vec{\mathcal{O}}_n^{A}(\nu,\mu)$

anom.dim. $\hat{\gamma}_{n\nu} = -\hat{V}_n^{-1} \cdot \nu \dfrac{\partial}{\partial\nu} \hat{V}_n$ $\quad$ 1-loop: $\gamma_{n\nu}^{ij} = -(\nu d/d\nu)\delta V_n^{ij}$

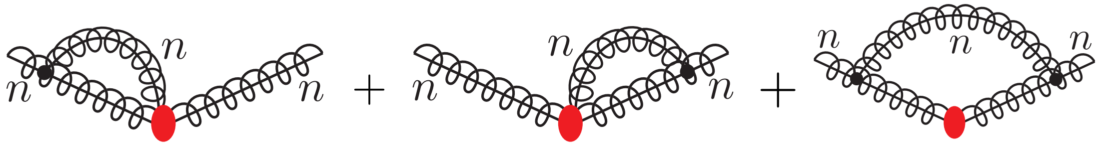


$+$ w.fn. renorm

$$\gamma_{n\nu}^{qq} = \frac{\alpha_s(\mu)C_A}{2\pi} \ln\left(\frac{\mu^2}{m^2}\right)$$

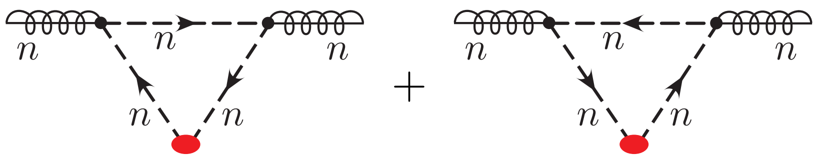


$$\gamma_{n\nu}^{gq} = \frac{\alpha_s(\mu)C_A}{2\pi} \ln\left(\frac{-t}{\mu^2}\right)$$



$+$ gluon w.fn.

$$\gamma_{n\nu}^{gg} = \frac{\alpha_s(\mu)C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$



$$\gamma_{n\nu}^{gq} = 0$$

$$\nu \frac{d}{d\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA}) = \gamma_{n\nu}(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})$$

$$\gamma_{n\nu} \equiv \gamma_{n\nu}^{qq} + \gamma_{n\nu}^{gq} = \gamma_{n\nu}^{gg} + \gamma_{n\nu}^{qg} = \frac{\alpha_s(\mu)C_A}{2\pi} \ln\left(\frac{-t}{m^2}\right)$$

(IR divergent since no real
emission graphs in Reggeization)

Standard RGE form: $\mathcal{O}(\nu_1) = U_{n\nu}(\nu_1,\nu_2)\mathcal{O}(\nu_2)$

$$(\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{-t}) = \left(\frac{s}{-t}\right)^{-\gamma_{n\nu}/2} (\mathcal{O}_n^{qA} + \mathcal{O}_n^{gA})(\nu = \sqrt{s})$$

same factor from $\bar{n}$

soft: no large logs for $\nu = \sqrt{-t}$

Forward Scattering & BFKL

Expand time evolution, do soft-collinear factorization term by term:

$$T \exp\, i\int d^4x\, \mathcal{L}_G^{\mathrm{II}(0)}(x) = \left[1 + i\int d^4y_1\, \mathcal{L}_G^{\mathrm{II}(0)}(y_1) + \frac{i^2}{2!}\int d^4y_1\, d^4y_2\, \mathcal{L}_G^{\mathrm{II}(0)}(y_1)\mathcal{L}_G^{\mathrm{II}(0)}(y_2) + \ldots\right]$$

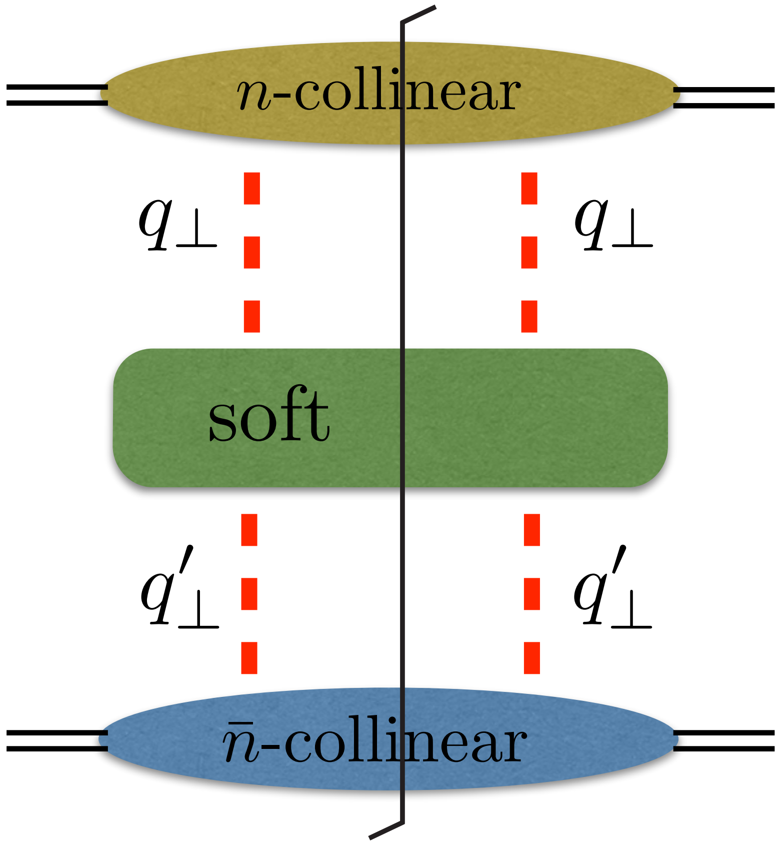
$$\sim 1 + T\sum_{k=1}^{\infty}\sum_{k'=1}^{\infty}\Big[\mathcal{O}_n^{jA_i}(q_{i\perp})\Big]^k \Big[\mathcal{O}_{\bar n}^{j'B_{i'}}(q_{i'\perp})\Big]^{k'} \otimes O_{s(k,k')}^{A_1\cdot A_k,B_1\cdots B_{k'}}(q_{\perp 1},\ldots,q_{\perp k'})$$

$$\equiv 1 + \sum_{k=1}^{\infty}\sum_{k'=1}^{\infty} U_{(k,k')}$$

As a traditional approximation, consider forward scattering with just the first term (linearization which leads to BFKL equation):

$$T_{(1,1)} = \frac{1}{V_4}\sum_X \langle pp'|U_{(1,1)}^\dagger|X\rangle\langle X|U_{(1,1)}|pp'\rangle \quad = \ldots$$

$$= \int d^2q_\perp d^2q'_\perp\, C_n(q_\perp, p^-) S_G(q_\perp, q'_\perp) C_{\bar n}(q'_\perp, p'^+)$$

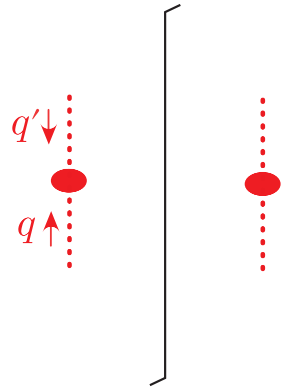


after rapidity renormalization:

$$T_{(1,1)} = \int d^2q_\perp d^2q'_\perp\, C_n(q_\perp, p^-, \nu) S_G(q_\perp, q'_\perp, \nu) C_{\bar n}(q'_\perp, p'^+, \nu)$$

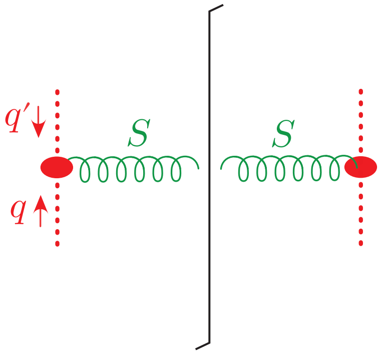
Consider rapidity renormalization for soft function that appears here:

$$S_G(q_\perp, q'_\perp) = \frac{1}{V_2}\frac{\delta^{AA'}\delta^{BB'}}{(\vec{q}_\perp^{\,2}\,\vec{q}_\perp^{\,\prime 2})^2}\sum_X \langle 0|O^{AB}_{s(1,1)}(q_\perp, q'_\perp)|X\rangle\langle X|O^{\dagger A'B'}_{s(1,1)}(q_\perp, q'_\perp)|0\rangle$$

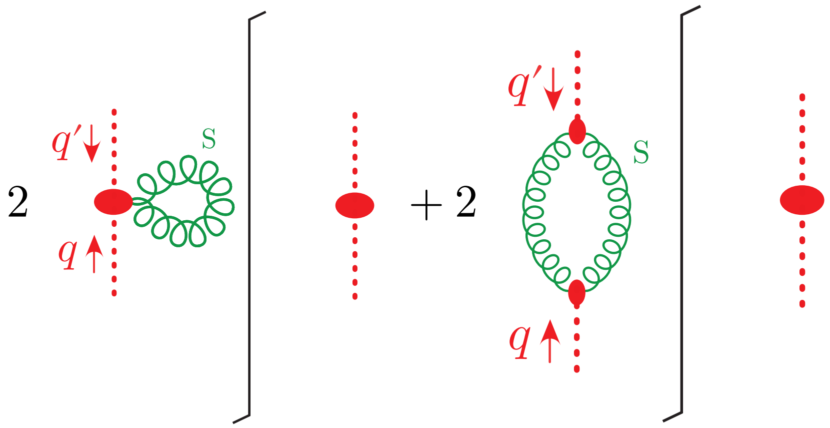


$$= S_G^{(0)}(q_\perp, q'_\perp)$$

$$= \left(\frac{8\pi\alpha_s}{\vec{q}_\perp^{\,2}}\right)^2 \delta^{AA}\,(2\pi)^2\delta^2(\vec{q}_\perp + \vec{q}\,'_\perp)$$



$$= \frac{C_A\alpha_s}{\pi^2}\,w^2\Gamma\left(\frac{\eta}{2}\right)\int \frac{d^2k_\perp\,\vec{k}_\perp^{\,2}}{(\vec{k}_\perp - \vec{q}_\perp)^2\,\vec{q}_\perp^{\,2}}\,S_G^{(0)}(k_\perp, q'_\perp)$$



$$= -\frac{C_A\alpha_s}{2\pi^2}\,w^2\Gamma\left(\frac{\eta}{2}\right)\int d^2k_\perp \frac{\vec{q}_\perp^{\,2}}{\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2}\,S_G^{(0)}(q_\perp, q'_\perp)$$

$$\widetilde{S}_G(q_\perp, q'_\perp) = \vec{q}_\perp^{\,2}\,S_G(q_\perp, q'_\perp)\,\vec{q}_\perp^{\,\prime 2}$$

$$\widetilde{S}_G(\vec{q}_\perp, \vec{q}\,'_\perp, \nu) = \int d^2k_\perp\,Z(q_\perp, k_\perp)\,\tilde{S}_G^{\rm bare}(k_\perp, q'_\perp)$$

To cancel the $1/\eta$ divergence we require

$$Z(q_\perp, k_\perp) = \delta^2(\vec{q}_\perp - \vec{k}_\perp) - \frac{2C_A\alpha_s(\mu)w^2(\nu)}{\pi^2\,\eta}\left[\frac{1}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \delta^2(\vec{q}_\perp - \vec{k}_\perp)\int\frac{d^2k_\perp\vec{q}_\perp^{\,2}}{2\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2}\right]$$

$$0 = \nu\frac{d}{d\nu}\widetilde{S}_G^{\rm bare}(q_\perp, q'_\perp)$$

➡

$$\nu\frac{d}{d\nu}\widetilde{S}_G(q_\perp, q'_\perp, \nu) = \int d^2k_\perp\,\gamma_{S_G}(q_\perp, k_\perp)\,\widetilde{S}_G(k_\perp, q'_\perp, \nu)$$

$$= \frac{2C_A\alpha_s(\mu)}{\pi^2}\int d^2k_\perp\left[\frac{\widetilde{S}_G(k_\perp, q'_\perp, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2}\,\widetilde{S}_G(q_\perp, q'_\perp, \nu)}{2\vec{k}_\perp^{\,2}(\vec{k}_\perp - \vec{q}_\perp)^2}\right]$$

evolution is
BFKL equation

(see also work by S. Fleming)

RGE consistency of linearized amplitude at LL order implies

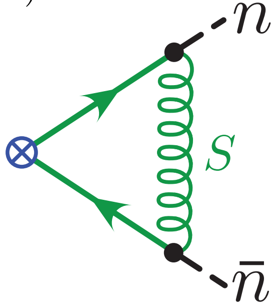
$$\nu \frac{d}{d\nu} C_n(q_\perp, p^-, \nu) = -\frac{C_A \alpha_s}{\pi^2} \int d^2 k_\perp \left[\frac{C_n(k_\perp, p^-, \nu)}{(\vec{k}_\perp - \vec{q}_\perp)^2} - \frac{\vec{q}_\perp^{\,2}\, C_n(q_\perp, p^-, \nu)}{2\vec{k}_\perp^2 (\vec{k}_\perp - \vec{q}_\perp)^2} \right] \qquad -\frac{1}{2}(\mathrm{BFKL})$$

same for $C_{\bar{n}}$

Hard Scattering $\otimes$

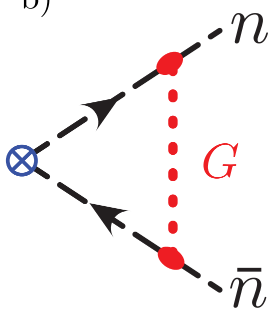
Active-Active and Soft Overlap

a)



$$S = \tilde S - S^{(G)}$$

b)



$$G = S^{(G)} \quad = \bar u_n \Gamma v_{\bar n} \frac{C_F \alpha_s}{2\pi}\Big[(i\pi)\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\Big)\Big]$$

Glaubers give $(i\pi)$ terms

with physical directions for soft Wilson lines in hard scattering

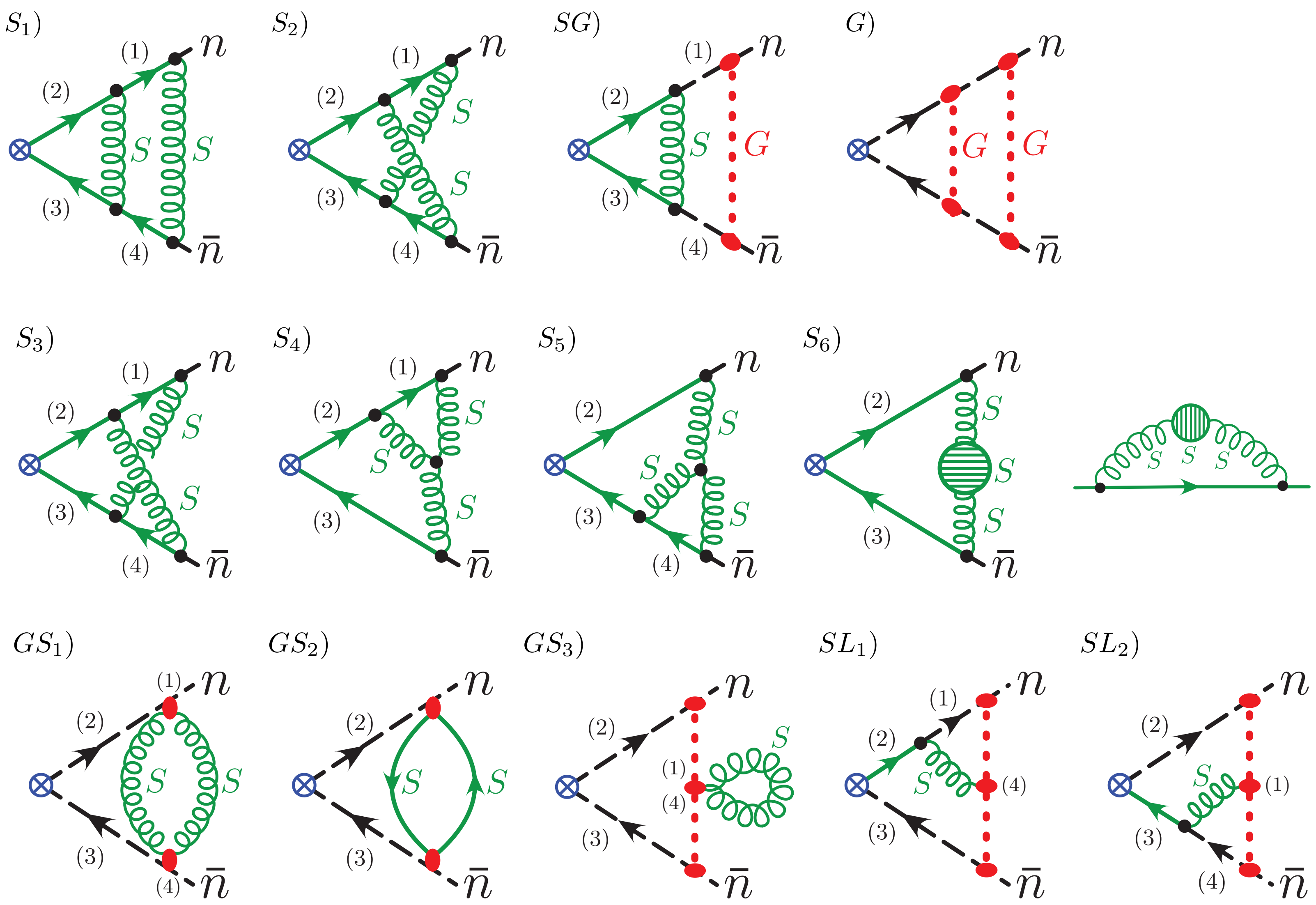
$$(\tilde S - S^{(G)}) + G = \tilde S$$

so we don't see Glauber in Hard Matching

can absorb this Glauber into Soft Wilson lines if they have proper directions

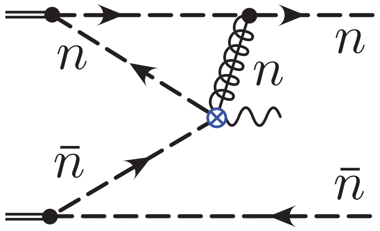
(see Bauer, Lange, Ovanesyan for analog in $\mathrm{SCET_I}$)

This continues at higher orders:

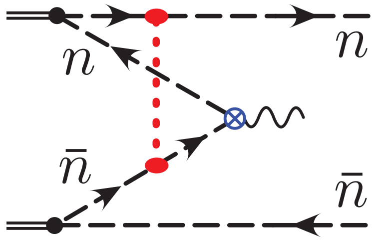


This overlap with the subtractions is the analog in the EFT of the CSS statement that one can deform the contour from the Glauber into Soft region for active-active graphs.

Active-Spectator and the Collinear Overlap



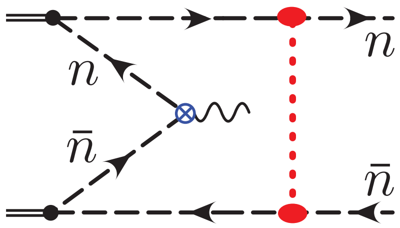
$$C_n = \tilde{C}_n - C_n^{(G)}$$



$$G = C_n^{(G)}$$

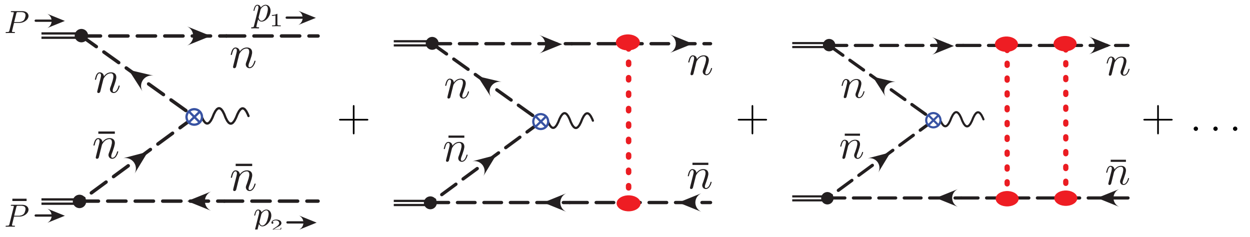
the correspondence here requires an incoming collinear Wilson line in this hard scattering operator

Spectator-Spectator



no soft or collinear analogs ($\mathrm{SCET_{II}}$)

Recall:



$$= \mathcal{S}^{\gamma} \int d^{d-2}b_\perp \; e^{-i\Delta\vec{p}_\perp\cdot\vec{b}_\perp} \; \tilde{E}'(b_\perp, q_\perp) \; e^{i\phi(b_\perp)}$$

rescattering phases can cancel in cross section

Conclusion

- Constructed an EFT for $s \gg t$, Fwd. Scattering & Fact. Violation
- Universal Operators that can be used for many processes & problems
- Reggeization, BFKL, Soft-Glauber & Collinear-Glauber overlaps, …

Near Future

- Amplitude Collinear and Regge Fact. Violation
- Joint DGLAP(μ) and BFKL(ν) resummation for small-x DIS
- Reproduce classic CSS proof of factorization in Drell-Yan

Future Directions

- Study and prove or disprove factorization for less inclusive processes
- Improve theoretical description of Underlying Event
- ….