Colorful NNLO – Completely local subtractions for fully differential predictions at NNLO

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Radcor-Loopfest 2015, UCLA
June 18, 2015

Message

Message: can compute NNLO cross sections like you always thought you would

1. Compute relevant IR factorization formulae
2. Use them to construct general, explicit, local subtractions
3. Integrate subtractions once and for all, verify pole cancellation
4. Apply the generic scheme to specific process

Outline

1. The problem
2. The recipe
3. Integrating the subtractions
4. Cancellation of poles
5. Application: $H \to b\bar{b}$
6. Conclusions and outlook

The problem

The problem

Consider the NNLO correction to a generic m-jet observable

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \int_{m+2} \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} + \int_{m+1} \mathrm{d}\sigma^{\mathrm{RV}}_{m+1} J_{m+1} + \int_{m} \mathrm{d}\sigma^{\mathrm{VV}}_{m} J_{m} \,.$$

- matrix elements for $\sigma^{\mathrm{RR}}_{m+2}$ (tree) and $\sigma^{\mathrm{RV}}_{m+1}$ (1-loop) known for many processes
- σ^{VV}_{m} (2-loop) know for 4 parton, V+3 parton processes, higher multiplicities are on the horizon
- the three contributions are separately infrared divergent in $d = 4$ dimensions

Double real

- kin. singularities as one or two partons unresolved: up to $\mathrm{O}(\epsilon^{-4})$ poles from PS integration
- no explicit ϵ poles

Real-virtual

- kin. singularities as one parton unresolved: up to $\mathrm{O}(\epsilon^{-2})$ poles from PS integration
- explicit ϵ poles up to $\mathrm{O}(\epsilon^{-2})$

Double virtual

- kin. singularities screened by jet function: PS integration finite
- explicit ϵ poles up to $\mathrm{O}(\epsilon^{-4})$

The problem

Consider the NNLO correction to a generic m-jet observable

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \int_{m+2} {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} + \int_{m+1} {\rm d}\sigma^{\rm RV}_{m+1} J_{m+1} + \int_{m} {\rm d}\sigma^{\rm VV}_{m} J_{m} \,.$$

KLN theorem

Infrared singularities cancel between real and virtual quantum corrections at the same order in perturbation theory, for sufficiently inclusive (i.e. IR safe) observables.

However

How to make this cancellation explicit, so that the various contributions can be computed numerically? Need a method to deal with implicit poles.

Several approaches – why this one?

Colorful NNLO: Completely Local subtRactions for Fully differential predictions at NNLO

- general and explicit expressions, including color and flavor (automation, color space notation is used)
- fully local counterterms, taking account of all color and spin correlations (mathematical rigor, efficiency)
- analytic cancellation of explicit ϵ poles in loop amplitudes (mathematical rigor)
- option to constrain subtractions to near singular regions (efficiency, important check)
- very algorithmic construction (valid at any order in perturbation theory)

The recipe

Structure of the NNLO correction

Rewrite the NNLO correction as a sum of three terms

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

each integrable in four dimensions

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right] \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

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$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left[\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right] J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left[\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

1. $\mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2}$ regularizes the doubly-unresolved limits of $\mathrm{d}\sigma^{\mathrm{RR}}_{m+2}$

Structure of the NNLO correction

Rewrite the NNLO correction as a sum of three terms

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$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left[{\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right] \right\}$$

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4. $\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1}$ regularizes the singly-unresolved limits of $\mathrm{d}\sigma^{\mathrm{RV}}_{m+1}$

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4. ${\rm d}\sigma^{\rm RV,A_1}_{m+1}$ regularizes the singly-unresolved limits of ${\rm d}\sigma^{\rm RV}_{m+1}$
5. $(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2})^{\rm A_1}$ regularizes the singly-unresolved limit of $\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2}$

Collinear and soft factorization of QCD matrix elements at NNLO known

- Tree level 3-parton splitting functions and double soft gg and $q\bar{q}$ currents

(Campbell, Glover 1997; Catani, Grazzini 1998; Del Duca, Frizzo, Maltoni 1999; Kosower 2002)

- One-loop 2-parton splitting functions and soft gluon current

(Bern, Dixon, Dunbar, Kosower 1994; Bern, Del Duca, Kilgore, Schmidt 1998-9; Kosower, Uwer 1999; Catani, Grazzini 2000; Kosower 2003)

Use known ingredients

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But note

- unresolved regions in phase space overlap
- quantities appearing in factorization formulae are only well-defined in the strict limit

Colorful NNLO - general features

Construction based on universal IR limit formulae

- Altarelli-Parisi splitting functions, soft currents (tree and one-loop, triple AP functions)
- simple and general procedure for matching of limits using physical gauge
- extension based on momentum mappings that can be generalized to any number of unresolved partons

Fully local in color $\otimes$ spin space

- no need to consider the color decomposition of real emission ME's
- azimuthal correlations correctly taken into account in gluon splitting
- can check explicitly that the ratio of the sum of counterterms to the real emission cross section tends to unity in any IR limit

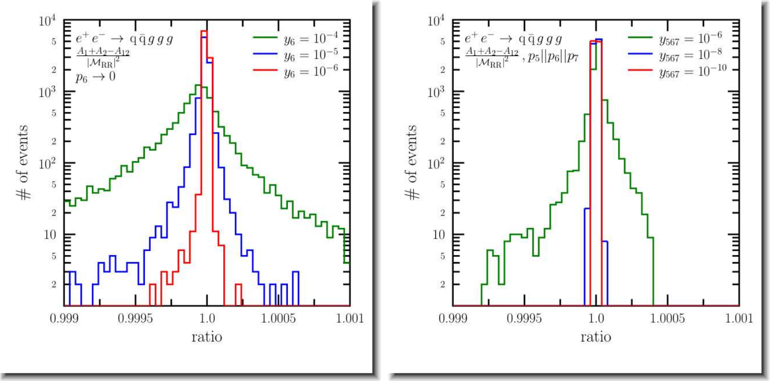
Straightforward to constrain subtractions to near singular regions

- gain in efficiency
- independence of physical results on phase space cut is strong check

Given completely explicitly for any process with non colored initial state

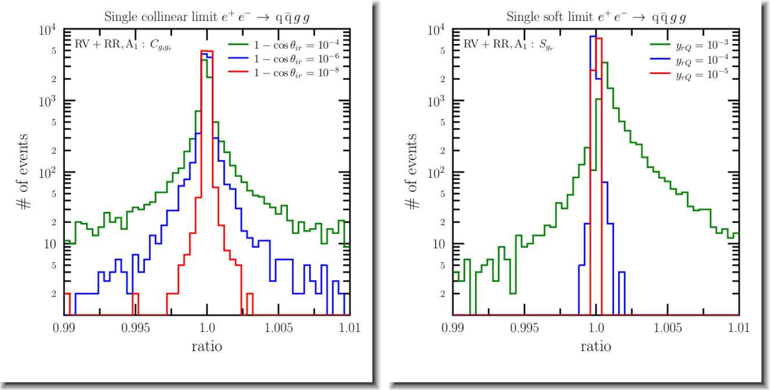
Kinematic singularities cancel in RR

Can check the ratio of the double real emission matrix element and the sum of all subtractions for all IR limits tends to one.



ratio = subtractions/RR

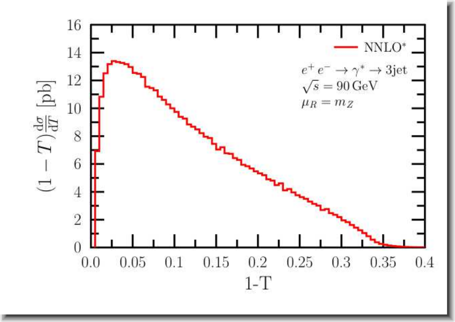
Can check the ratio of the real-virtual matrix element and the sum of all subtractions for all IR limits tends to one.

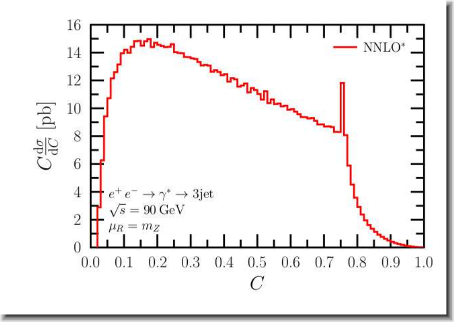


ratio = subtractions/(RV + RR,A_1)

RR and RV contributions finite

Regularized RR and RV contributions finite, can be computed by standard MC techniques. Implementation for general m in progress ⇒ see Ádám Kardos' talk





$$\text{NNLO}^* = \text{B} + \text{R} + \text{V} + \text{RR} + \text{RV}$$

Integrating the subtractions

Integrating the subtractions

Momentum mappings used to define the counterterms

$$\{p\}_{n+p} \xrightarrow{R} \{\tilde{p}\}_n \quad \Rightarrow \quad \mathrm{d}\phi_{n+p}(\{p\};Q) = \mathrm{d}\phi_n(\{\tilde{p}\}_n^{(R)};Q)[\mathrm{d}p_{p,n}^{(R)}]$$

- implement exact momentum conservation, recoil distributed democratically (can be generalized to any p)
- different collinear and soft mappings (R labels precise limit)
- exact factorization of phase space

Counterterms are products (in color and spin space) of

- factorized ME's independent of variables in $[\mathrm{d}p_{p,n}^{(R)}]$
- singular factors (AP functions, soft currents), to be integrated over $[\mathrm{d}p_{p,n}^{(R)}]$

$$\mathcal{X}_R(\{p\}_{n+p}) = (8\pi\alpha_s\mu^{2\epsilon})^p \, Sing_R(p_p^{(R)}) \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

Can compute once and for all the integral over unresolved partons

$$\int_p \mathcal{X}_R(\{p\}_{n+p}) = (8\pi\alpha_s\mu^{2\epsilon})^p \left[\int_p Sing_R(p_p^{(R)})\right] \otimes |\mathcal{M}_n^{(0)}(\{\tilde{p}\}_n^{(R)})|^2$$

List of basic integrals

Int	status
$\mathcal{I}^{(k)}_{1\mathcal{C},0}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},3}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},4}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{1\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{C},7}$	✔
$\mathcal{I}_{1\mathcal{C},8}$	✔

Int	status
$\mathcal{I}_{1\mathcal{S},0}$	✔
$\mathcal{I}_{1\mathcal{S},1}$	✔
$\mathcal{I}_{1\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{S},3}$	✔
$\mathcal{I}_{1\mathcal{S},4}$	✔
$\mathcal{I}_{1\mathcal{S},5}$	✔
$\mathcal{I}_{1\mathcal{S},6}$	✔
$\mathcal{I}_{1\mathcal{S},7}$	✔

Int	status
$\mathcal{I}_{1\mathcal{CS},0}$	✔
$\mathcal{I}_{1\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{1\mathcal{CS},2}$	✔
$\mathcal{I}_{1\mathcal{CS},3}$	✔
$\mathcal{I}_{1\mathcal{CS},4}$	✔

Int	status
$\mathcal{I}^{(k,l)}_{12\mathcal{C},1}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},3}$	✔
$\mathcal{I}^{(k,l)}_{12\mathcal{C},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},5}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},6}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},7}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},8}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},9}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{C},10}$	✔

Int	status
$\mathcal{I}_{2\mathcal{S},1}$	✔
$\mathcal{I}_{2\mathcal{S},2}$	✔
$\mathcal{I}_{2\mathcal{S},3}$	✔
$\mathcal{I}_{2\mathcal{S},4}$	✔
$\mathcal{I}_{2\mathcal{S},5}$	✔
$\mathcal{I}_{2\mathcal{S},6}$	✔
$\mathcal{I}_{2\mathcal{S},7}$	✔
$\mathcal{I}_{2\mathcal{S},8}$	✔
$\mathcal{I}_{2\mathcal{S},9}$	✔
$\mathcal{I}_{2\mathcal{S},10}$	✔
$\mathcal{I}_{2\mathcal{S},11}$	✔
$\mathcal{I}_{2\mathcal{S},12}$	✔
$\mathcal{I}_{2\mathcal{S},13}$	✔
$\mathcal{I}_{2\mathcal{S},14}$	✔
$\mathcal{I}_{2\mathcal{S},15}$	✔
$\mathcal{I}_{2\mathcal{S},16}$	✔
$\mathcal{I}_{2\mathcal{S},17}$	✔
$\mathcal{I}_{2\mathcal{S},18}$	✔
$\mathcal{I}_{2\mathcal{S},19}$	✔
$\mathcal{I}_{2\mathcal{S},20}$	✔
$\mathcal{I}_{2\mathcal{S},21}$	✔
$\mathcal{I}_{2\mathcal{S},22}$	✔
$\mathcal{I}_{2\mathcal{S},23}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{S},1}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},2}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},3}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},4}$	✔
$\mathcal{I}^{(k)}_{12\mathcal{S},5}$	✔
$\mathcal{I}_{12\mathcal{S},6}$	✔
$\mathcal{I}_{12\mathcal{S},7}$	✔
$\mathcal{I}_{12\mathcal{S},8}$	✔
$\mathcal{I}_{12\mathcal{S},9}$	✔
$\mathcal{I}_{12\mathcal{S},10}$	✔
$\mathcal{I}_{12\mathcal{S},11}$	✔
$\mathcal{I}_{12\mathcal{S},12}$	✔
$\mathcal{I}_{12\mathcal{S},13}$	✔

Int	status
$\mathcal{I}^{(k)}_{12\mathcal{CS},1}$	✔
$\mathcal{I}_{12\mathcal{CS},2}$	✔
$\mathcal{I}_{12\mathcal{CS},3}$	✔

Int	status
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},1}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},2}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},3}$	✔
$\mathcal{I}^{(j,k,l,m)}_{2\mathcal{C},4}$	✔
$\mathcal{I}^{(-1,-1,-1,-1)}_{2\mathcal{C},5}$	✔
$\mathcal{I}^{(k,l)}_{2\mathcal{C},6}$	✔

Int	status
$\mathcal{I}^{(k)}_{2\mathcal{CS},1}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(2)}_{2\mathcal{CS},2}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},3}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},4}$	✔
$\mathcal{I}^{(k)}_{2\mathcal{CS},5}$	✔

✔: pole coefficients known analytically, finite numerically

An example

The double soft subtraction term leads to the following integral, among others:

$$\mathcal{I}_{2\mathcal{S},2}(Y_{ik,Q};\epsilon,y_0,d_0') = -\frac{4\Gamma^4(1-\epsilon)}{\pi\Gamma^2(1-\epsilon)}\,\frac{B_{y_0}(-2\epsilon,d_0')}{\epsilon}\,Y_{ik,Q}\int_0^{y_0}\mathrm{d}y\,y^{-1-2\epsilon}(1-y)^{d_0'-1+\epsilon}$$
$$\times\int_{-1}^{1}\mathrm{d}(\cos\vartheta)\,(\sin\vartheta)^{-2\epsilon}\int_{-1}^{1}\mathrm{d}(\cos\varphi)\,(\sin\varphi)^{-1-2\epsilon}\big[f(\vartheta,\varphi;0)\big]^{-1}\big[f(\vartheta,\varphi;Y_{ik,Q})\big]^{-1}$$
$$\times\big[Y(y,\vartheta,\varphi;Y_{ik,Q})\big]^{-\epsilon}{}_2F_1\big(-\epsilon,-\epsilon,1-\epsilon,1-Y(y,\vartheta,\varphi;Y_{ik,Q})\big)$$

where

$$f(\vartheta,\varphi;Y_{ik,Q}) = 1-2\sqrt{Y_{ik,Q}(1-Y_{ik,Q})}\sin\vartheta\cos\varphi-(1-2Y_{ik,Q})\chi\cos\vartheta$$

$$Y(y,\vartheta,\varphi;\chi) = \frac{4(1-y)Y_{ik,Q}}{[2(1-y)+y\,f(\vartheta,\varphi;0)][2(1-y)+y\,f(\vartheta,\varphi;Y_{ik,Q})]}$$

An example

This integral is equal to $\left(y_0 = 1,\ d_0' = 3 - 3\epsilon\right)$

$$
\begin{aligned}
&\mathcal{I}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon) = \\
&= \frac{1}{2\epsilon^4} - \frac{1}{\epsilon^3}\Big[\ln(Y) - 3\Big] + \frac{1}{\epsilon^2}\Big[2\,\mathrm{Li}_2(1-Y) + \ln^2(Y) - \pi^2 - \Big(\frac{2}{1-Y} \\
&- \frac{1}{2(1-Y)^2} + \frac{9}{2}\Big)\ln(Y) + \frac{1}{2(1-Y)} + 16\Big] + \frac{1}{\epsilon}\Big[\frac{5}{3}\Big(\frac{18\,\mathrm{Li}_3(1-Y)}{5} + \frac{6\,\mathrm{Li}_3(Y)}{5} \\
&- \frac{6\,\mathrm{Li}_2(1-Y)\ln(Y)}{5} - \frac{2}{5}\ln^3(Y) + \frac{3}{5}\ln(1-Y)\ln^2(Y) + \pi^2\ln(Y) - \frac{78\zeta_3}{5}\Big) \\
&+ \Big(\frac{3}{1-Y} - \frac{3}{4(1-Y)^2} + \frac{15}{4}\Big)\Big(2\,\mathrm{Li}_2(1-Y) + \ln^2(Y)\Big) - 6\pi^2 - \Big(\frac{27}{2(1-Y)} \\
&- \frac{13}{4(1-Y)^2} + \frac{91}{4}\Big)\ln(Y) + \frac{19}{4(1-Y)} + \frac{163}{2}\Big] + \mathrm{O}(\epsilon^0)
\end{aligned}
$$

- Note the $Y \to 1$ limit is finite

$$
\lim_{Y\to 1}\mathcal{I}_{2\mathcal{S},2}(Y;\epsilon,1,3-3\epsilon) = \frac{1}{2\epsilon^4} + \frac{3}{\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{71}{4} - \pi^2\right) + \frac{1}{\epsilon}\left(\frac{393}{4} - 6\pi^2 - 24\zeta_3\right) + \mathrm{O}(\epsilon^0)
$$

- Finite term is computed numerically

Solving the integrals

Strategy for computing the master integrals

1. write phase space in terms of angles and energies
2. angular integrals in terms of Mellin-Barnes representations
3. resolve the ϵ poles by analytic continuation
4. MB integrals to Euler-type integrals, pole coefficients are finite parametric integrals

1. choose explicit parametrization of phase space
2. write the parametric integral representation in chosen variables
3. resolve the ϵ poles by sector decomposition
4. pole coefficients are finite parametric integrals

5. evaluate the parametric integrals in terms of multiple polylogs
6. simplify result (optional)

Cancellation of poles

Recall the NNLO correction is a sum of three terms

$$\sigma^{\rm NNLO} = \sigma^{\rm RR}_{m+2} + \sigma^{\rm RV}_{m+1} + \sigma^{\rm VV}_{m} = \sigma^{\rm NNLO}_{m+2} + \sigma^{\rm NNLO}_{m+1} + \sigma^{\rm NNLO}_{m}$$

each integrable in four dimensions

$$\sigma^{\rm NNLO}_{m+2} = \int_{m+2} \left\{ {\rm d}\sigma^{\rm RR}_{m+2} J_{m+2} - {\rm d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left[{\rm d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right] \right\}$$

$$\sigma^{\rm NNLO}_{m+1} = \int_{m+1} \left\{ \left[{\rm d}\sigma^{\rm RV}_{m+1} + \int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right] J_{m+1} - \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}$$

$$\sigma^{\rm NNLO}_{m} = \int_{m} \left\{ {\rm d}\sigma^{\rm VV}_{m} + \int_2 \left[{\rm d}\sigma^{\rm RR,A_2}_{m+2} - {\rm d}\sigma^{\rm RR,A_{12}}_{m+2} \right] + \int_1 \left[{\rm d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 {\rm d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\} J_m$$

Integrated approximate cross sections

- After summing over unobserved flavors, all integrated approximate cross sections can be written as products (in color space) of various insertion operators with lower point cross sections.
- Can be computed once and for all (though admittedly lots of tedious work).
- Poles are computed analytically, finite part numerically.

Poles cancel: VV contribution finite

After adding all integrated approximate cross sections the double virtual contribution must be finite in ϵ.

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

- Have checked the cancellation of the $\frac{1}{\epsilon^4}$ and $\frac{1}{\epsilon^3}$ poles analytically for any number of jets (i.e., with m symbolic).
- Have checked $m = 2$ ($e^+e^- \to q\bar{q}$, $H \to b\bar{b}$) explicitly and we find that all poles cancel.
- Have checked $m = 3$ ($e^+e^- \to q\bar{q}g$) explicitly and we find that all poles cancel.

Example: $H \to b\bar{b}$

The double virtual contribution has the following pole structure ($\mu^2 = m_H^2$)

$$
\begin{aligned}
\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma^{\mathrm{B}}_{H\to b\bar{b}} \Bigg\{ & \frac{2C_{\mathrm{F}}^2}{\epsilon^4} + \left(\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} + 6C_{\mathrm{F}}^2 - \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2}\right)\frac{1}{\epsilon^3} \\
& + \left[\left(\frac{8}{9} + \frac{\pi^2}{12}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2\right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2} \\
& + \left[\left(-\frac{961}{216} + \frac{13\zeta_3}{2}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{109}{8} - 2\pi^2 - 14\zeta_3\right) C_{\mathrm{F}}^2 + \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108}\right]\frac{1}{\epsilon} \Bigg\}
\end{aligned}
$$

(Anastasiou, Herzog, Lazopoulos, `arXiv:0111.2368`)

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$$+ \left[\left(\frac{8}{9} + \frac{\pi^2}{12}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(\frac{17}{2} - 2\pi^2\right) C_{\mathrm{F}}^2 - \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2}$$
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(Anastasiou, Herzog, Lazopoulos, `arXiv:0111.2368`)

The sum of the integrated approximate cross sections gives ($\mu^2 = m_H^2$)

$$\sum \int \mathrm{d}\sigma^{\mathrm{A}} = \left(\frac{\alpha_{\mathrm{s}}(\mu^2)}{2\pi}\right)^2 \mathrm{d}\sigma^{\mathrm{B}}_{H\to b\bar{b}} \Bigg\{ \frac{-2C_{\mathrm{F}}^2}{\epsilon^4} + \left(-\frac{11 C_{\mathrm{A}} C_{\mathrm{F}}}{4} - 6C_{\mathrm{F}}^2 + \frac{C_{\mathrm{F}} n_{\mathrm{f}}}{2}\right)\frac{1}{\epsilon^3}$$
$$+ \left[\left(-\frac{8}{9} - \frac{\pi^2}{12}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(-\frac{17}{2} + 2\pi^2\right) C_{\mathrm{F}}^2 + \frac{2 C_{\mathrm{F}} n_{\mathrm{f}}}{9}\right]\frac{1}{\epsilon^2}$$
$$+ \left[\left(\frac{961}{216} - \frac{13\zeta_3}{2}\right) C_{\mathrm{A}} C_{\mathrm{F}} + \left(-\frac{109}{8} + 2\pi^2 + 14\zeta_3\right) C_{\mathrm{F}}^2 - \frac{65 C_{\mathrm{F}} n_{\mathrm{f}}}{108}\right]\frac{1}{\epsilon} \Bigg\}$$

(Del Duca, Duhr, GS, Tramontano, Trócsányi,
`arXiv:1501.07226`)

Example: $e^+e^- \to 3$ jets

The double virtual contribution has the following pole structure ($\mu^2 = s$)

$$\mathrm{d}\sigma_3^{\mathrm{VV}} = \mathcal{P}oles\Big(A_3^{(2\times 0)} + A_3^{(1\times 1)}\Big) + \mathcal{F}inite\Big(A_3^{(2\times 0)} + A_3^{(1\times 1)}\Big)$$

where

$$\begin{aligned}\mathcal{P}oles\Big(A_3^{(2\times 0)} + A_3^{(1\times 1)}\Big) = 2\Bigg[&-\Big(I_{q\bar{q}g}^{(1)}(\epsilon)\Big)^2 - \frac{\beta_0}{\epsilon} I_{q\bar{q}g}^{(1)}(\epsilon) \\ &+ e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right) I_{q\bar{q}g}^{(1)}(2\epsilon) + H_{q\bar{q}g}^{(2)}\Bigg] A_3^0(1_q, 3_g, 2_{\bar{q}}) \\ &+ 2 I_{q\bar{q}g}^{(1)}(\epsilon) A_3^{1\times 0}(1_q, 3_g, 2_{\bar{q}})\end{aligned}$$

with

$$\begin{aligned}H_{q\bar{q}g}^{(2)} = \frac{e^{\epsilon\gamma}}{4\epsilon\Gamma(1-\epsilon)}\Bigg[&\left(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72}\right)N_c + \left(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48}\right) \\ &+ \left(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4}\right)\frac{1}{N_c} + \left(-\frac{19}{18} + \frac{\pi^2}{36}\right)N_c n_f + \left(-\frac{1}{54} - \frac{\pi^2}{24}\right)\frac{n_f}{N_c} + \frac{5}{27}n_f^2\Bigg]\end{aligned}$$

(Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, `arXiv:0710.0346`)

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Adding the sum of the integrated approximate cross sections gives

$$\mathcal{P}oles\Big(A_3^{(2\times0)} + A_3^{(1\times1)}\Big) + \mathcal{P}oles \sum \int \mathrm{d}\sigma^{\mathrm{A}} = 117\text{k terms}$$

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- zero numerically in any phase space point

```
In[35]:= N[PolesVV3 /. {y13 → 2 / 10, y23 → 3 / 10}, 40]
```

Out[35]=

$$\frac{0.\times 10^{-387} + \frac{0.\times 10^{-388}}{\mathrm{Nc}^2} + 0.\times 10^{-388}\,\mathrm{Nc}^2 + \frac{0.\times 10^{-438}\,\mathrm{nf}}{\mathrm{Nc}} + 0.\times 10^{-439}\,\mathrm{Nc}\,\mathrm{nf}}{\mathrm{e}^2} + \frac{1}{\mathrm{e}}\left(\left(0.\times 10^{-384} + 0.\times 10^{-438}\,\mathrm{i}\right) + \frac{0.\times 10^{-385}}{\mathrm{Nc}^2} + \left(0.\times 10^{-384} + 0.\times 10^{-438}\,\mathrm{i}\right)\mathrm{Nc}^2 + \frac{0.\times 10^{-437}\,\mathrm{nf}}{\mathrm{Nc}} + 0.\times 10^{-437}\,\mathrm{Nc}\,\mathrm{nf}\right) + \mathrm{O}[\mathrm{e}]^0$$

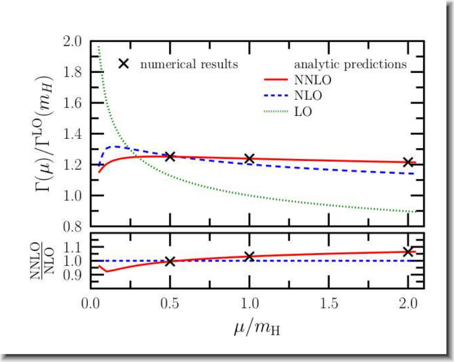
- zero analytically after simplification using symbol technology (C. Duhr)

Application: $H \to b\bar{b}$

Consider $H \to b\bar{b}$ decay at NNLO:

- admittedly the simplest case
- but this just amounts to having to sum less terms in general formulae

Inclusive decay rate



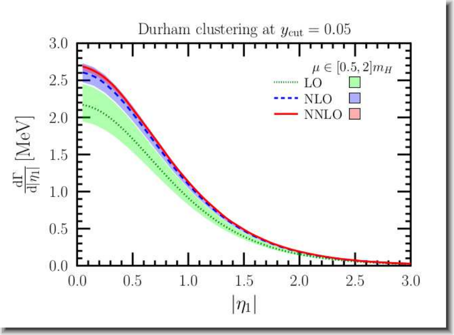
Higgs decay to b-quarks

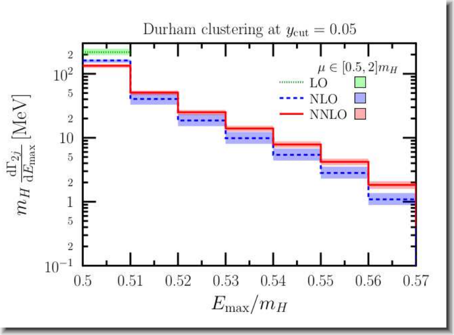
Consider $H \to b\bar{b}$ decay at NNLO:

- admittedly the simplest case
- but this just amounts to having to sum less terms in general formulae

Differential distributions

- pseudorapidity of highest energy jet (right) and leading jet energy (left)





Constrained subtractions

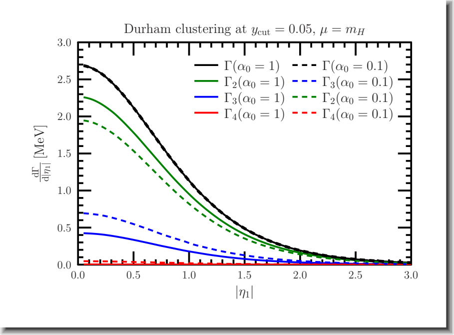
We can constrain subtractions to near singular regions: $\alpha_0 \in (0, 1]$

- poles cancel numerically ($\alpha_0 = 0.1$)

$$\mathrm{d}\sigma^{\mathrm{VV}}_{H\to b\bar{b}} + \sum \int \mathrm{d}\sigma^{\mathrm{A}} = \frac{5.4 \times 10^{-8}}{\epsilon^4} + \frac{3.9 \times 10^{-5}}{\epsilon^3} + \frac{3.3 \times 10^{-3}}{\epsilon^2} + \frac{6.7 \times 10^{-3}}{\epsilon} + \mathcal{O}(1)$$

$$Err\left(\sum \int \mathrm{d}\sigma^{\mathrm{A}}\right) = \frac{3.1 \times 10^{-5}}{\epsilon^4} + \frac{5.0 \times 10^{-4}}{\epsilon^3} + \frac{8.1 \times 10^{-3}}{\epsilon^2} + \frac{7.7 \times 10^{-2}}{\epsilon} + \mathcal{O}(1)$$

- results unchanged



Constrained subtractions

We can constrain subtractions to near singular regions: $\alpha_0 \in (0, 1]$

- improved efficiency

α_0	1	0.1
timing (rel.)	1	0.40
$\langle N_{\text{sub}} \rangle$	52	14.5

$\langle N_{\text{sub}} \rangle$ is the average number of subtraction terms computed

Conclusions and outlook

Conclusions and outlook

Colorful NNLO framework

- Completely Local subtRactions for Fully differential predictions at NNLO
- construction of subtraction terms based on IR limit formulae
- analytic integration of subtraction terms is feasible with modern integration techniques
- demonstrated cancellation of ϵ poles for $m = 2$ and $m = 3$
- worked out in full detail for processes with no colored particles in the initial state

First application: Higgs boson decay into a b and anti-b quark

Next steps

- $e^+e^- \to 3$ jets is almost finished
- extension to hadronic initial states conceptually understood and on the way