Non-renormalization Theorems without Supersymmetry

Chia-Hsien Shen

Walter Burke Institute for Theoretical Physics, Caltech

based on 1505.01844 with Clifford Cheung

Miraculous Cancellation in SMEFT

- Full 59×59 anomalous dimension matrix of dimension-6 operators

□ =non-zero

×=no diagram

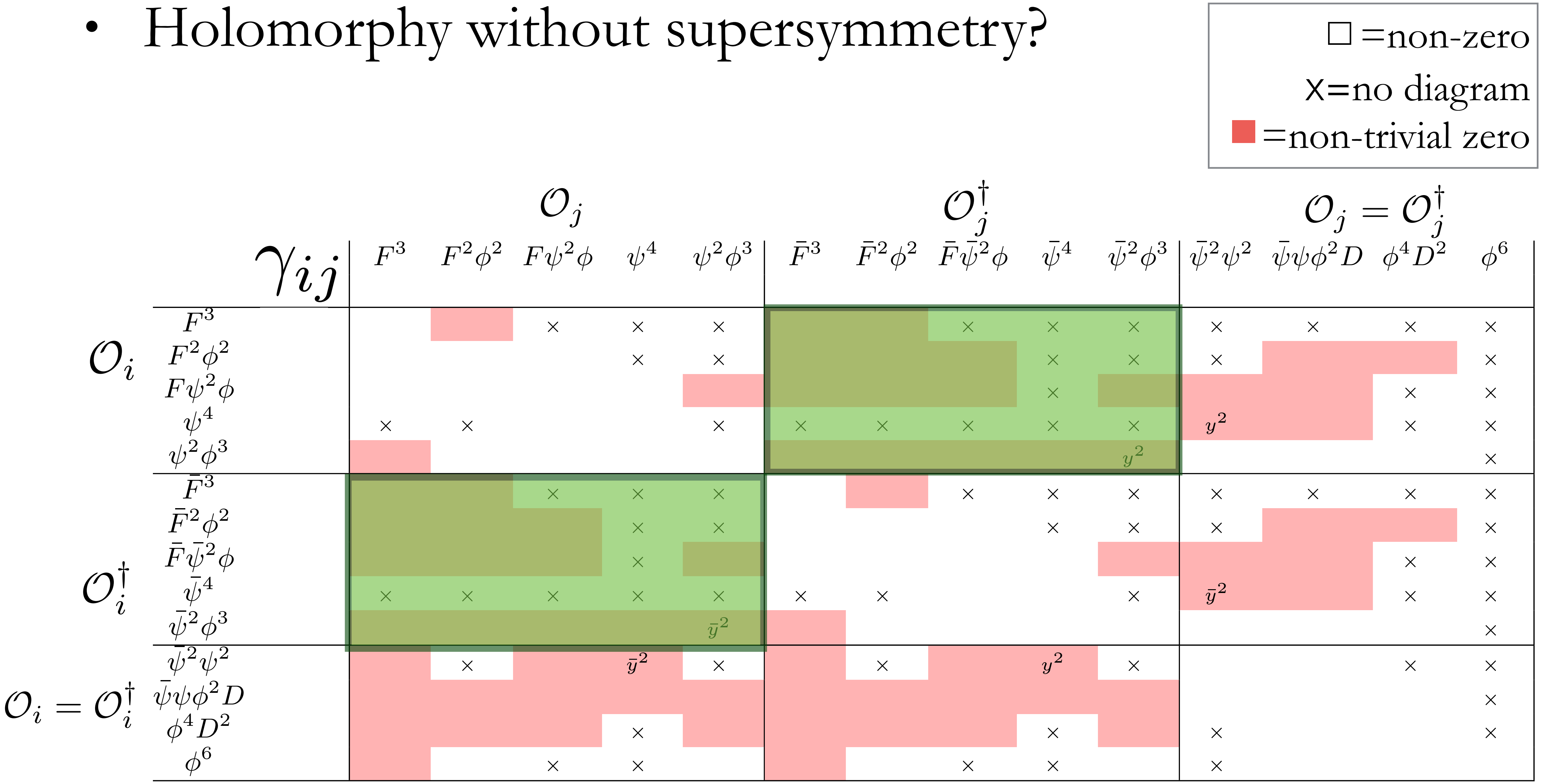
■ =non-trivial zero

γ_{ij}	F^3	$F^2\phi^2$	$F\psi^2\phi$	ψ^4	$\psi^2\phi^3$	$\bar{F}^3$	$\bar{F}^2\phi^2$	$\bar{F}\bar{\psi}^2\phi$	$\bar{\psi}^4$	$\bar{\psi}^2\phi^3$	$\bar{\psi}^2\psi^2$	$\bar{\psi}\psi\phi^2 D$	$\phi^4 D^2$	ϕ^6
F^3			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$F^2\phi^2$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$F\psi^2\phi$									$\times$				$\times$	$\times$
ψ^4	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	y^2		$\times$	$\times$
$\psi^2\phi^3$										y^2				$\times$
$\bar{F}^3$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\bar{F}^2\phi^2$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$\bar{F}\bar{\psi}^2\phi$				$\times$									$\times$	$\times$
$\bar{\psi}^4$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$	$\bar{y}^2$		$\times$	$\times$
$\bar{\psi}^2\phi^3$					$\bar{y}^2$									$\times$
$\bar{\psi}^2\psi^2$		$\times$		$\bar{y}^2$	$\times$		$\times$		y^2	$\times$			$\times$	$\times$
$\bar{\psi}\psi\phi^2 D$														$\times$
$\phi^4 D^2$				$\times$					$\times$		$\times$			$\times$
ϕ^6			$\times$	$\times$				$\times$	$\times$		$\times$			

Alonso, Jenkins, Manohar, Trott: 1409.0868

Miraculous Cancellation in SMEFT

- Holomorphy without supersymmetry?



Alonso, Jenkins, Manohar, Trott: 1409.0868

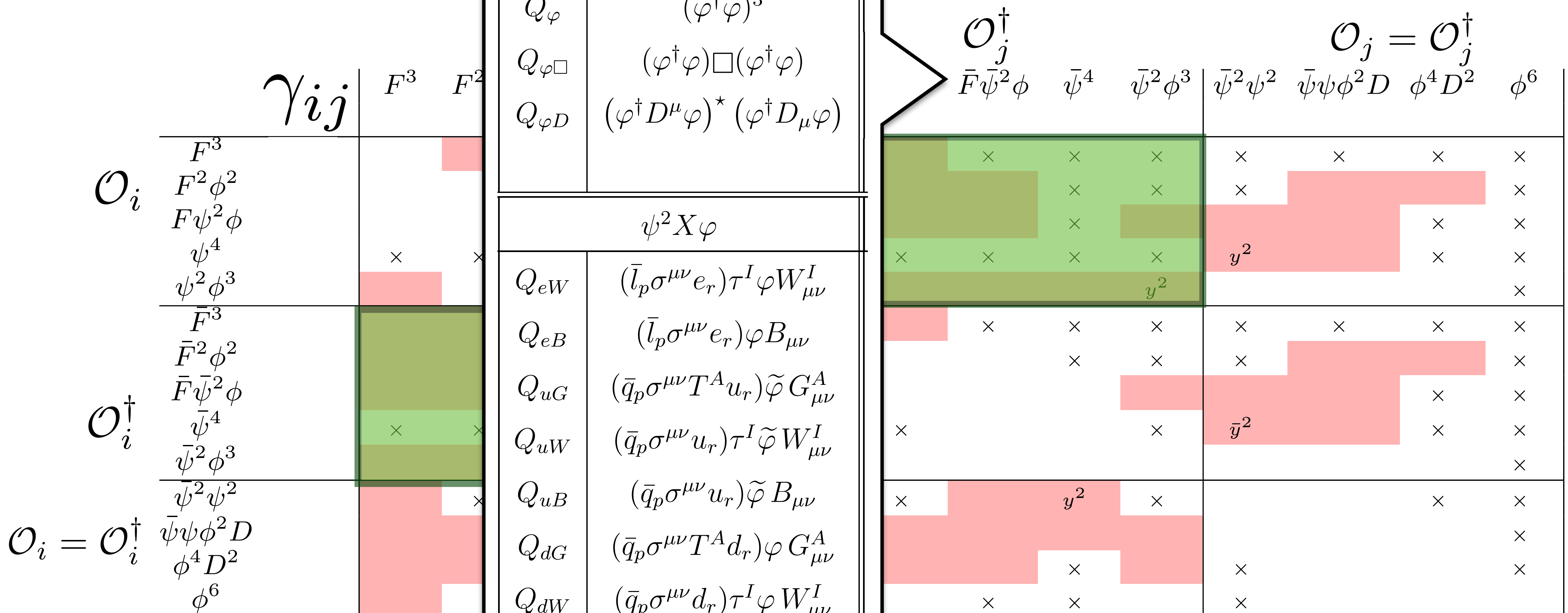
Miraculous Cancellation in SMEFT

- Holomorphy without supersymmetry?

□ =non-zero

X=no diagram

■ =non-trivial zero



φ^6 and $\varphi^4 D^2$	
Q_φ	$(\varphi^\dagger\varphi)^3$
$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$
$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$
$\psi^2 X\varphi$	
Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$
Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$
Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$
Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde\varphi\, W^I_{\mu\nu}$
Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde\varphi\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)\varphi\, G^A_{\mu\nu}$
Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W^I_{\mu\nu}$
Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$

Alonso, Jenkins, Manohar, Trott: 1409.0868

Outline

- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Outline

- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Theory space

- EFTs: **massless** theories with **marginal** interactions, deformed by **irrelevant operators** of the same dimension.

$$\mathcal{L} = \mathcal{L}_{d=4} + \sum_i c_i \mathcal{O}_i$$

massless particles with marginal interactions

higher dimensional operators of the same mass dimension

- Operator mixing at 1-loop: $\dfrac{1}{(4\pi)^2}\dfrac{dc_i}{d\log\mu} = \sum_j \gamma_{ij} c_j$
- Suitable for the anomalous dimension of SMEFT.
- No assumption on matter content or gauge symmetries!

Independent Basis of Operators

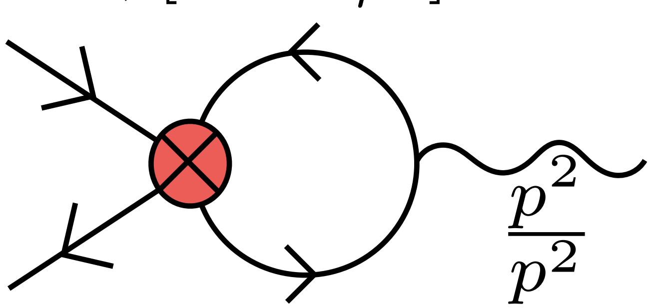
- Field Redefinitions ⇔ Equations of Motion:

$$\phi \to \phi + \frac{1}{\Lambda}\delta\phi$$

$$\mathcal{L} \to \mathcal{L} + \frac{1}{\Lambda}\delta\phi \times \mathrm{EoM} + \mathcal{O}(\frac{1}{\Lambda^2})$$

- Independent basis: no operators related by EoM
- Applying equations of motion at loop level
  - local operator from non-local diagram!

$$\bar\psi^2\psi^2 \xrightarrow{\text{loop}} \bar\psi\sigma^\mu\psi[D_\nu, F_{\mu\nu}] \xrightarrow{\text{EoM}} \bar\psi\sigma^\mu\psi\bar\psi\sigma_\mu\psi$$



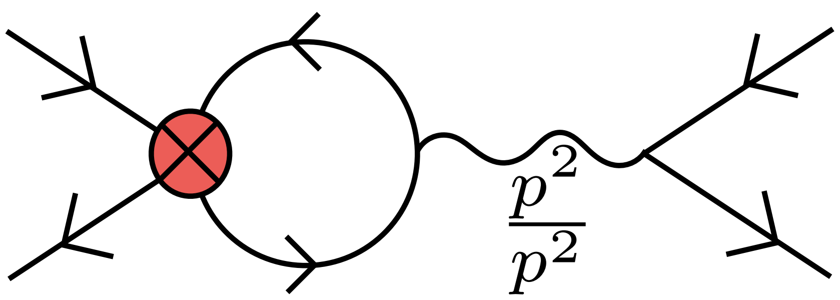
Independent Basis of Operators

- Field Redefinitions ⇔ Equations of Motion:

$$\phi \to \phi + \frac{1}{\Lambda}\delta\phi$$
$$\mathcal{L} \to \mathcal{L} + \frac{1}{\Lambda}\delta\phi \times \mathrm{EoM} + \mathcal{O}(\frac{1}{\Lambda^2})$$

- Independent basis: no operators related by EoM
- Applying equations of motion at loop level
  - local operator from non-local diagram!

$$\bar{\psi}^2\psi^2 \xrightarrow{\text{loop}} \bar{\psi}\sigma^\mu\psi[D_\nu, F_{\mu\nu}] \xrightarrow{\text{EoM}} \bar{\psi}\sigma^\mu\psi\bar{\psi}\sigma_\mu\psi$$



Off-shell Lagrangian v.s. On-shell Amplitudes

- **Off-shell** Lagrangian: **unphysical**, *e.g.*, gauge choice and field redefinition
- **On-shell** amplitudes: **all physical** information!
  - Use multiplicity & helicity; factorization & unitarity

Off-shell Lagrangian v.s. On-shell Amplitudes

- **Off-shell** Lagrangian: **unphysical**, *e.g.*, gauge choice and field redefinition
- **On-shell** amplitudes: **all physical** information!
  - Use multiplicity & helicity; factorization & unitarity

Zvi Bern:

TASI 2014

Off-shell Lagrangian v.s. On-shell Amplitudes

- **Off-shell** Lagrangian: **unphysical**, *e.g.*, gauge choice and field redefinition
- **On-shell** amplitudes: **all physical** information!
  - Use multiplicity & helicity; factorization & unitarity

Zvi Bern:

On-shell: Good

Off-shell: Bad

TASI 2014

Off-shell Lagrangian v.s. On-shell Amplitudes

Off-shell	**On-shell**
Higher dimensional operator $\mathcal{O}_i$	Contact amplitude A_i with $\mathcal{O}_i$ insertion
Anomalous dimension γ_{ij}	UV divergence of the loop amplitude A_i^{loop} from $\mathcal{O}_j$.
Field redefinitions/Equations of motion	Full amplitudes including non-1PI diagrams
Non-renormalization	UV finiteness of loop amplitudes

Outline

- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Definition of Weights

- Weight/Anti-weight for an **amplitude**

$$\boxed{w(A) = n(A) - h(A), \qquad \overline{w}(A) = n(A) + h(A)}$$

  - $n(A)$: total number of particles; $h(A)$: sum over helicities

- Weight/Anti-weight for an **operator**

$$\boxed{w(\mathcal{O}) = \min\{w(A)\}, \qquad \overline{w}(\mathcal{O}) = \min\{\overline{w}(A)\}}$$

  - Minimum weights for all contact amplitudes with this operator $\mathcal{O}$
  - Weights of amplitudes are greater or equal to $\mathcal{O}$ (to be proven…)

Weights 101

- Physical: free from field redefinition/gauge choice
- Non-negative: $(w, \overline{w}) \geq 0$ (n ≥ h for spin ≤ 1 theories)
- In practice, sum over each field component in an operator

$\mathcal{O}$	$F_{\alpha\beta}$	ψ_α	ϕ	$\bar{\psi}_{\dot{\alpha}}$	$\bar{F}_{\dot{\alpha}\dot{\beta}}$
h	$+1$	$+1/2$	0	$-1/2$	-1
$(w, \overline{w})$	$(0,2)$	$(1/2, 3/2)$	$(1,1)$	$(3/2, 1/2)$	$(2,0)$

  - Using spinor indices: *e.g.* $F_{\alpha\dot{\alpha}\beta\dot{\beta}} = F_{\alpha\beta}\bar{\epsilon}_{\dot{\alpha}\dot{\beta}} + \bar{F}_{\dot{\alpha}\dot{\beta}}\epsilon_{\alpha\beta}$
  - Covariant derivative D does not contribute (*i.e.* taking D=∂)
  - Example: $F^3 \to (0,6)$, $\bar{\psi}\psi\phi^2 D \to (4,4)$
  - same as k-charge in *N=4* sYM.

New Non-renormalization Theorems

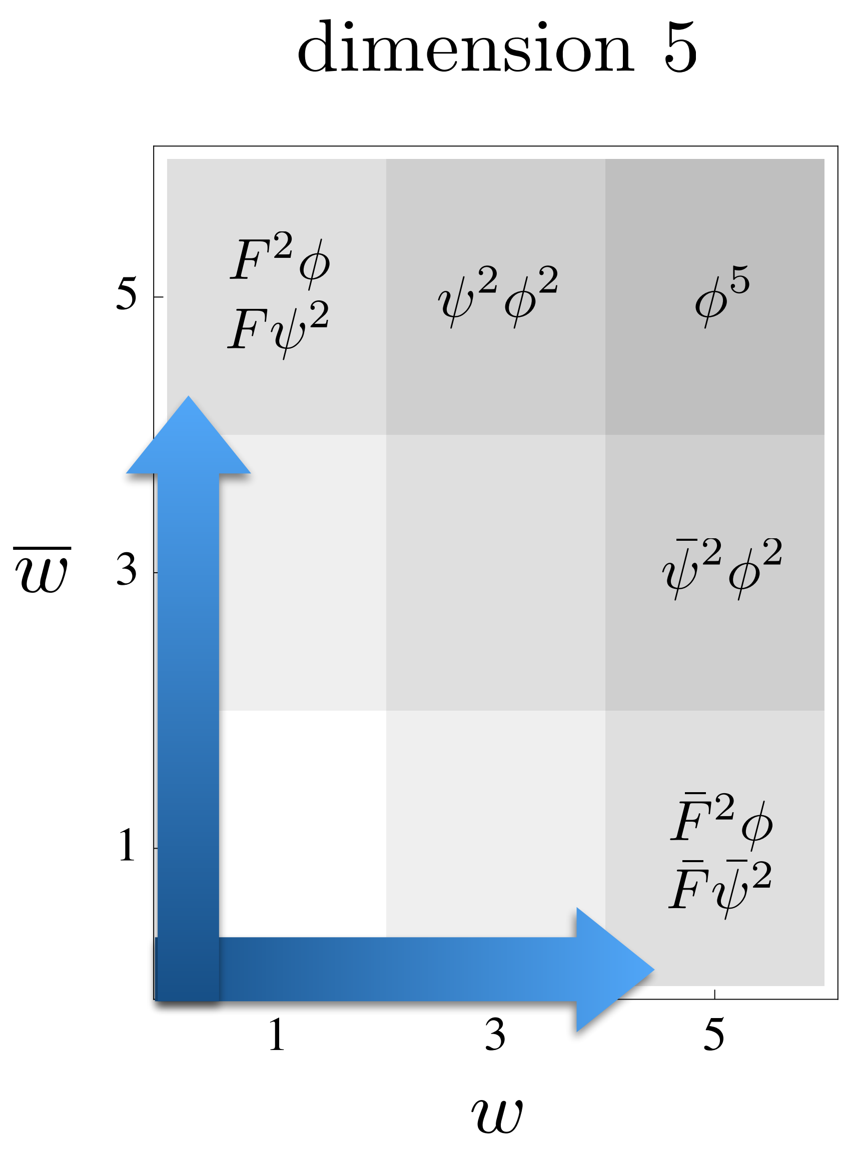
- An operator $\mathcal{O}_j$ **CANNOT renormalize** operator $\mathcal{O}_i$ at 1-loop if $\mathcal{O}_j$ has **higher weight** or anti-weight than $\mathcal{O}_i$.

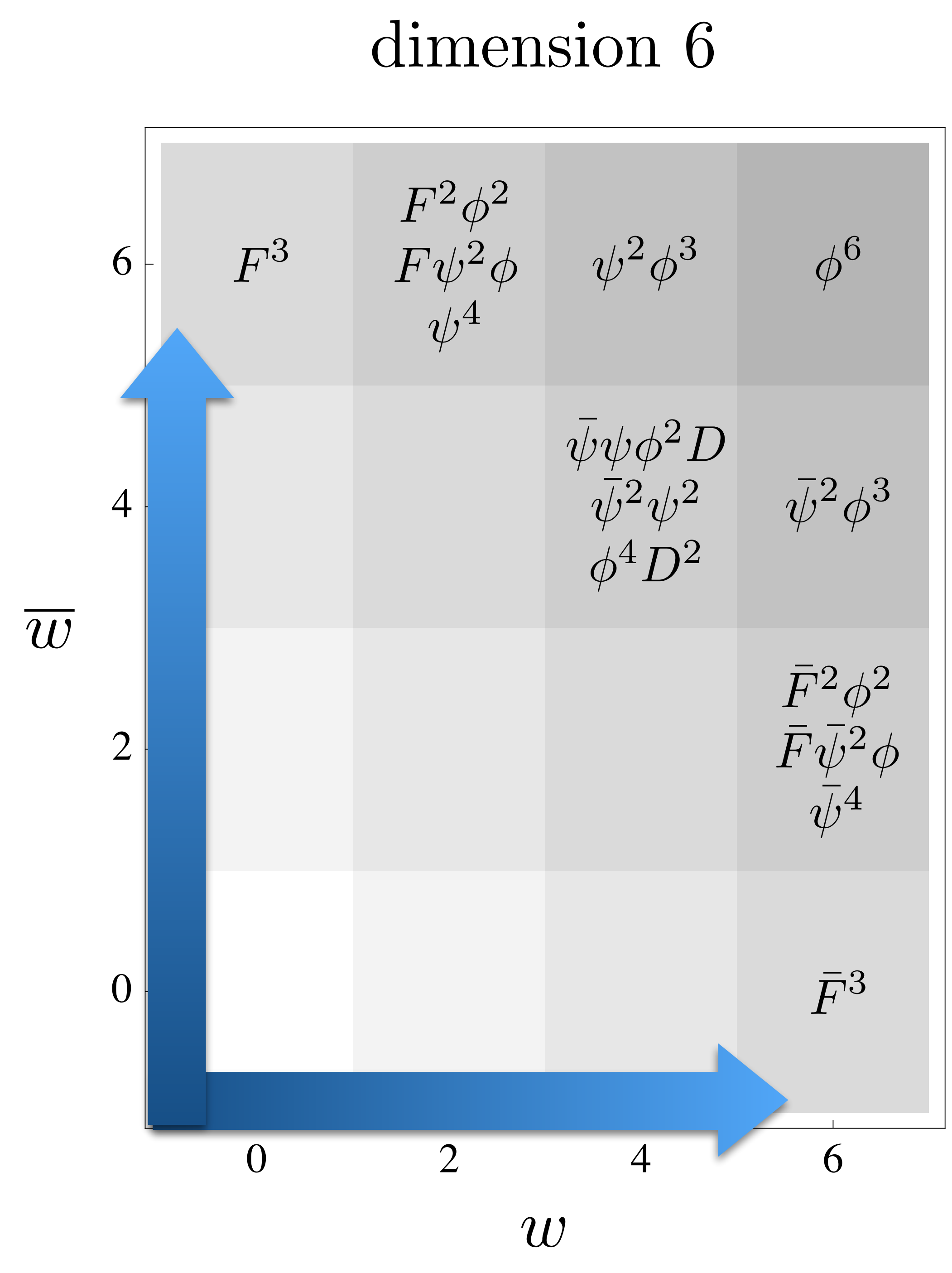
$$\boxed{\gamma_{ij} = 0 \quad \text{if} \quad w_i < w_j \quad \text{or} \quad \overline{w}_i < \overline{w}_j\,,}$$

  only violated by non-holomorphic Yukawa interactions.

- The direction of RG has to go **positive** (non-negative) in weights and anti-weights.

Positivity in RG Running





Outline

- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Three-point Amplitudes

- Universal formula for massless particles
  [Benincasa & Cachazo,0705.4305]

$$A(1^{h_1}2^{h_2}3^{h_3}) = g \begin{cases} \langle 12\rangle^{r_3}\langle 23\rangle^{r_1}\langle 31\rangle^{r_2}, & \sum_i h_i \leq 0 \quad (\mathrm{MHV}) \\ [12]^{\overline{r}_3}[23]^{\overline{r}_1}[31]^{\overline{r}_2}, & \sum_i h_i \geq 0 \quad (\overline{\mathrm{MHV}}) \end{cases}$$

- Little group+dimensional analysis

$$(w_3, \overline{w}_3) = (4-[g], 2+[g]),\ (2+[g], 4-[g])$$

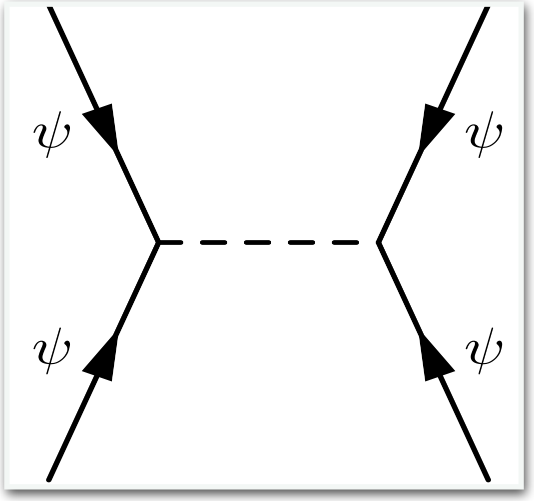
- Lower bound for marginal interactions:

$$\boxed{w_3, \overline{w}_3 \geq 2}$$

Four-point Amplitudes

- Most of $w < 4$ amplitudes vanish
  - $w = 1, 3$: no diagram
  - $w = 0, 2$: have diagrams, but vanish on-shell, e.g., all-plus/all-but-one plus gluon amplitudes
- Lower bound for four-point amplitudes:

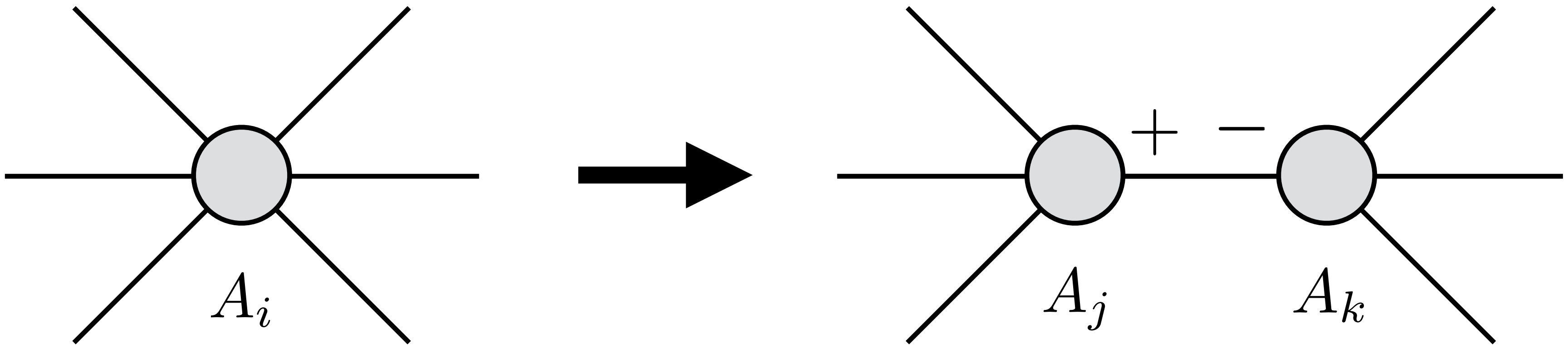
$$\boxed{w_4, \overline{w}_4 \geq 4}$$ modulo



- Exceptional amplitude $A(\psi^+\psi^+\psi^+\psi^+)$: $w = 2$
  - Non-holomorphic Yukawa: $\psi^2\phi + \psi^2\phi^\dagger$

Tree Rule

- Approach **factorization limit** of an amplitude



$$A_i \to A_j \cdot A_k \frac{1}{p^2} \qquad \begin{cases} n_i = n_j + n_k - 2 \\ h_i = h_j + h_k \end{cases}$$

$$\boxed{\text{Tree rule:} \begin{cases} w_i = w_j + w_k - 2 \\ \overline{w}_i = \overline{w}_j + \overline{w}_k - 2 \end{cases}}$$

General Tree Amplitudes

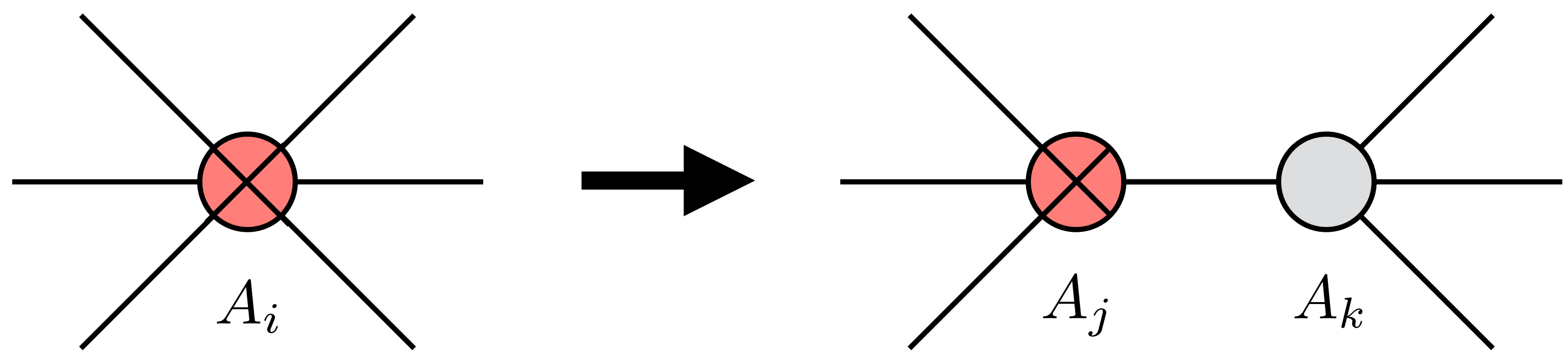
- Factorize higher point amplitudes with renormalizable interactions: $w_3 \geq 2,\ w_4 \geq 4$ & $w_i = w_j + w_k - 2$
  5pt → 3pt × 4pt: $(w_5, \overline{w}_5) \geq (4, 4)$…

$$\boxed{w_n, \overline{w}_n \quad \geq \quad \begin{cases} 2, & n = 3 \\ 4, & n > 3 \end{cases}}$$

- modulo exceptional amplitudes with w_n or $\overline{w}_n = 2$ from non-holomorphic Yukawa interactions.

General Tree Amplitudes

- Factorize an amplitude built from a single insertion higher dimensional operator: higher dim'l operator × renormalizable



$$\begin{aligned} w_i &= w_j + w_k - 2 \\ \overline{w}_i &= \overline{w}_j + \overline{w}_k - 2 \end{aligned} \longrightarrow \quad w_i \geq w_j\,, \ \overline{w}_i \geq \overline{w}_j$$

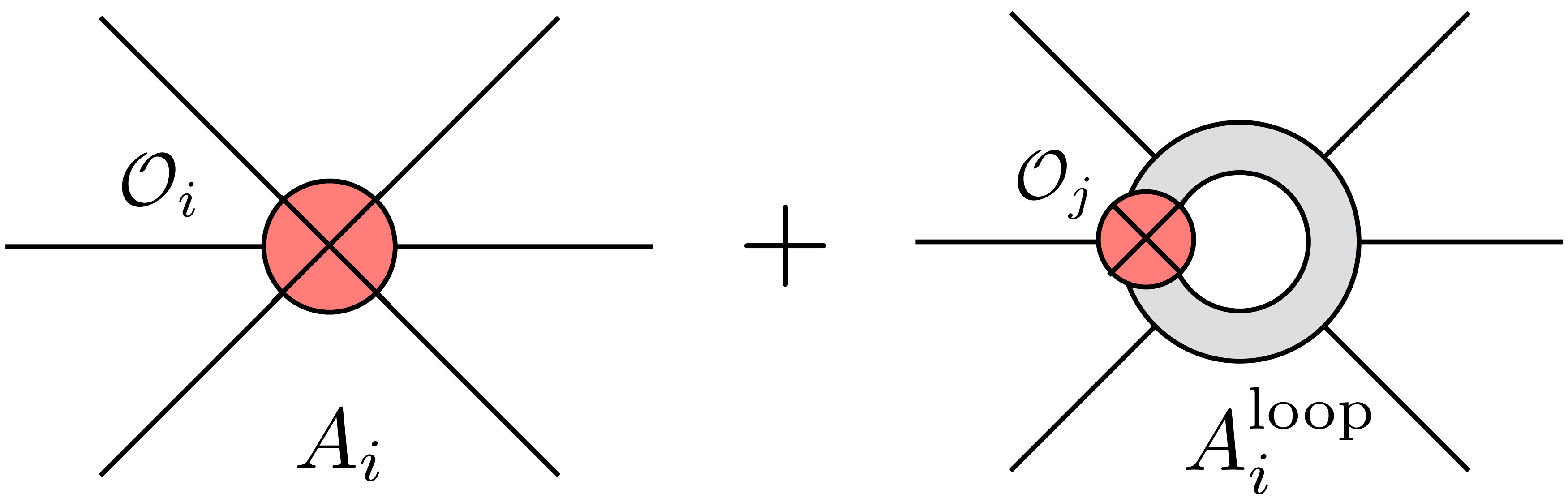
- Contact amplitudes have minimal weights among all amplitudes with the same operator insertion.

Outline

- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Anomalous Dimensions and Loop Amplitudes

- Anomalous dimension v.s. loop correction to A_i from $\mathcal{O}_j$

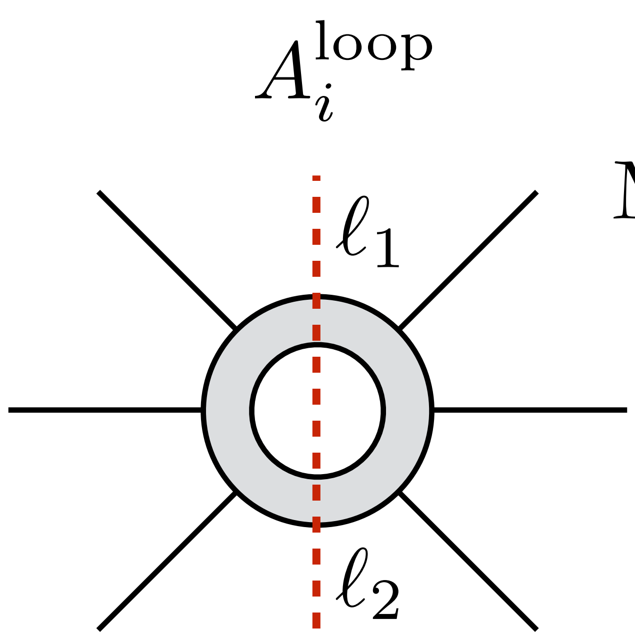


Generalized Unitarity

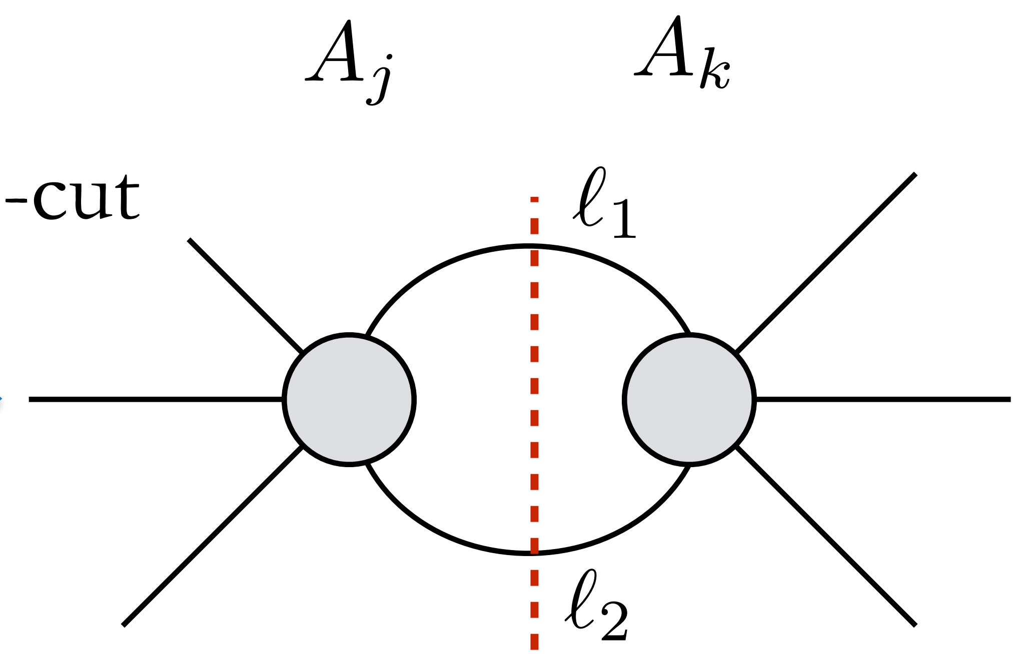
- "Cut" = locate internal propagators on-shell

$$\frac{1}{\ell^2+i\epsilon} \xrightarrow{\text{cut}} \delta^+(l^2)$$

- This breaks 1-loop amplitudes into products of tree amplitudes

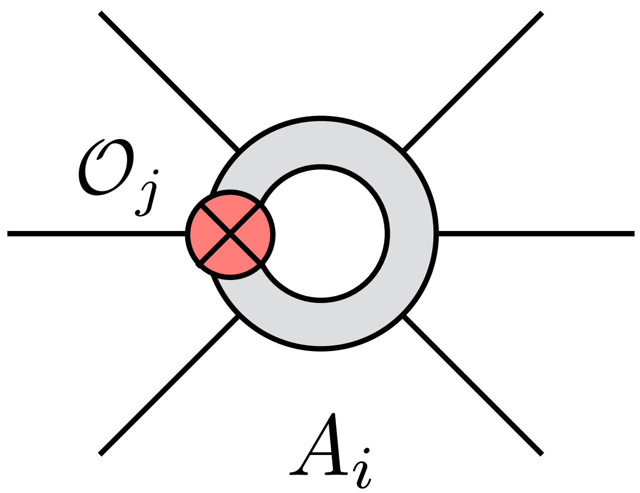


Massive two-cut

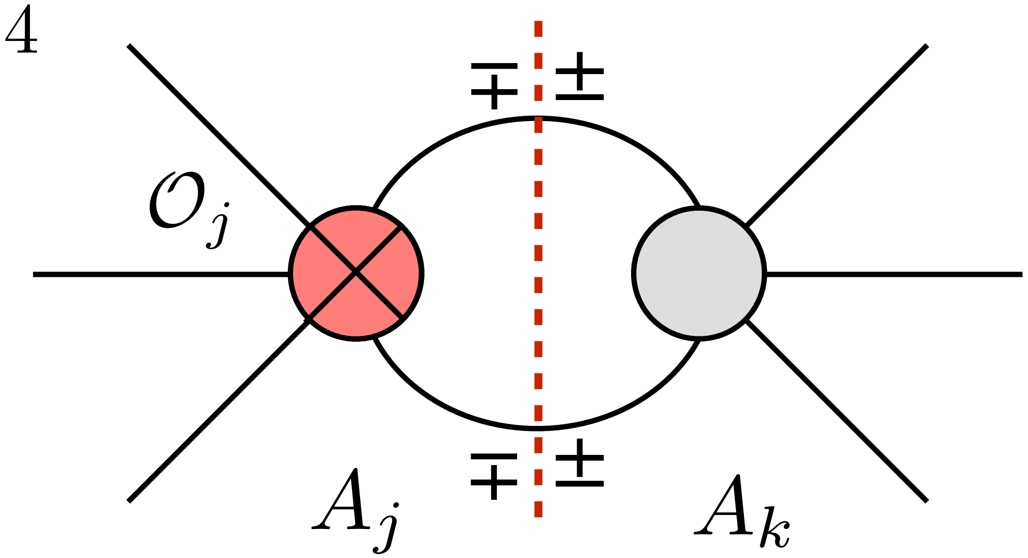


One-loop Rule

- Selection rule via massive two-cut on a 1-loop amplitude



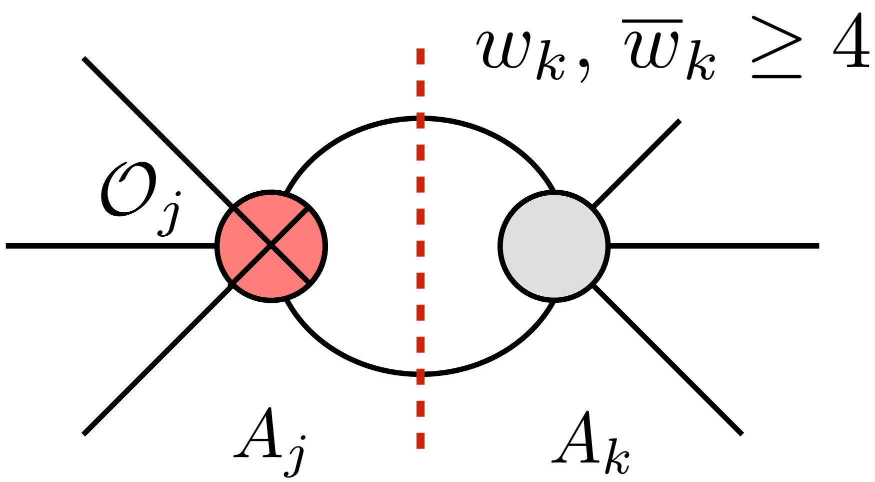
$$\begin{cases} n_i = n_j + n_k - 4 \\ h_i = h_j + h_k \end{cases}$$



One-Loop rule: $\begin{cases} w_i = w_j + w_k - 4 \\ \overline{w}_i = \overline{w}_j + \overline{w}_k - 4 \end{cases}$

Non-renormalization Theorems (?)

- 1-loop rule+
  lower bound on tree:
  $$\begin{cases} w_i = w_j + w_k - 4 \\ \overline{w}_i = \overline{w}_j + \overline{w}_k - 4 \end{cases}$$



- Massive two-cut is non-trivial only if the operators satisfy

$$\boxed{w_i \geq w_j \quad \text{and} \quad \overline{w}_i \geq \overline{w}_j}$$

  - Necessary for massive bubble/triangle/box integrals.

- Finite 1-loop amplitude in dimensional regularization.

$$w_i < w_j \qquad \text{or} \qquad \overline{w}_i < \overline{w}_j$$

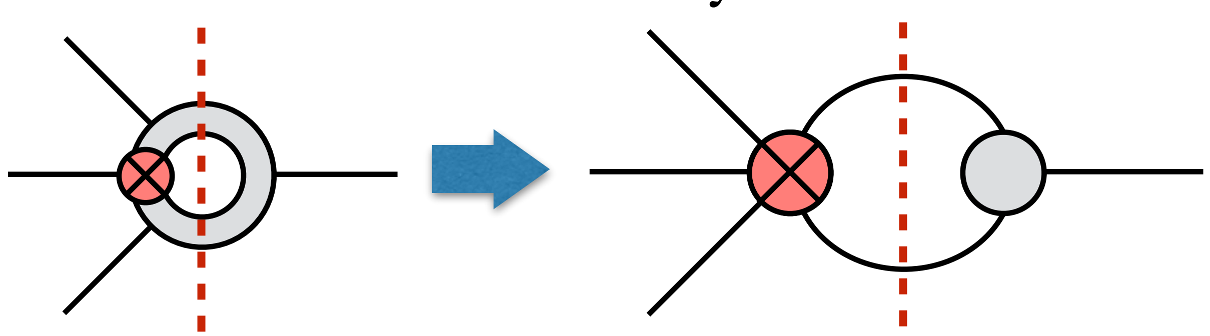
"Non-renormalization?"

Massless Bubbles

- Known result: find QCD beta function if stop here.
  [Dixon; Arkani-Hamed, Cachazo, and Kaplan 0808.1446; Huang, McGady, Peng 1205.5606]
- Scaleless massless bubble integrals vanish in dimensional regularization as UV-IR cancellation.

$$I_2(p^2=0) = \frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right) \xrightarrow{\epsilon_{\rm UV}=\epsilon_{\rm IR}} 0$$

- **Massless bubbles DO contribute to UV** divergence despite being invisible under unitarity cut!



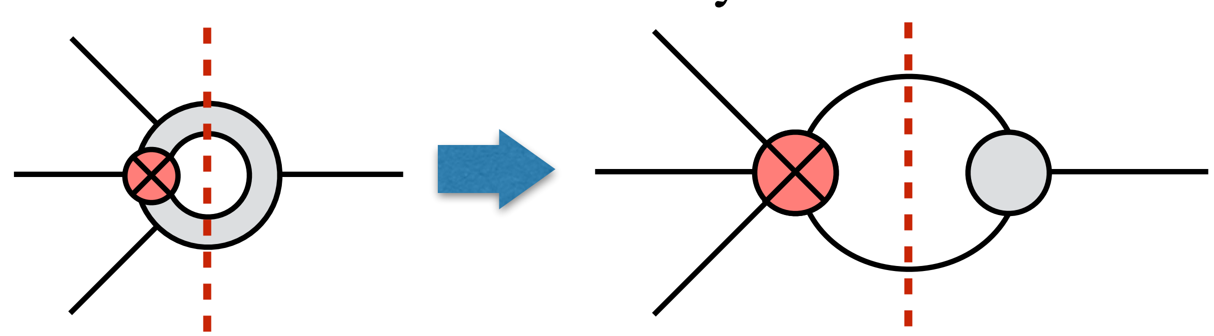
Massless Bubbles

- Known result: find (-1)×QCD beta function if stop here.
  [Dixon; Arkani-Hamed, Cachazo, and Kaplan 0808.1446; Huang, McGady, Peng 1205.5606]

- Scaleless massless bubble integrals vanish in dimensional regularization as UV-IR cancellation.

$$I_2(p^2=0) = \frac{1}{(4\pi)^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right) \xrightarrow{\epsilon_{\rm UV}=\epsilon_{\rm IR}} 0$$

- **Massless bubbles DO contribute to UV** divergence despite being invisible under unitarity cut!



Outline

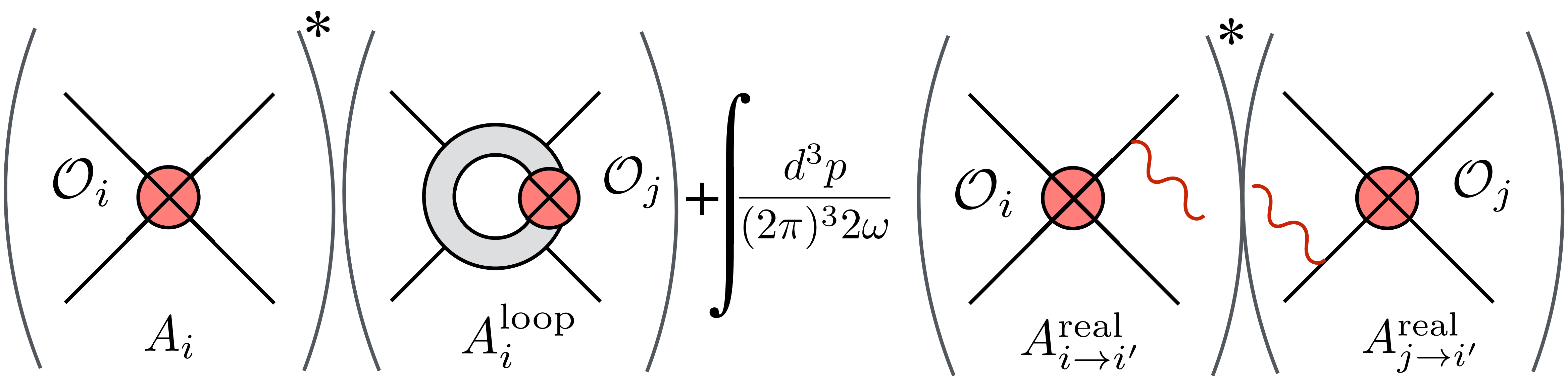
- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Weightlifting from IR to UV

- Kinoshita-Lee-Nauenberg (KLN) Theorem:
  - IR divergence in (1-loop) virtual correction is cancelled by inclusive sum over (a single) unresolved final state(s)!—cross section level statement.

=−IR virtual div. =+IR real div.

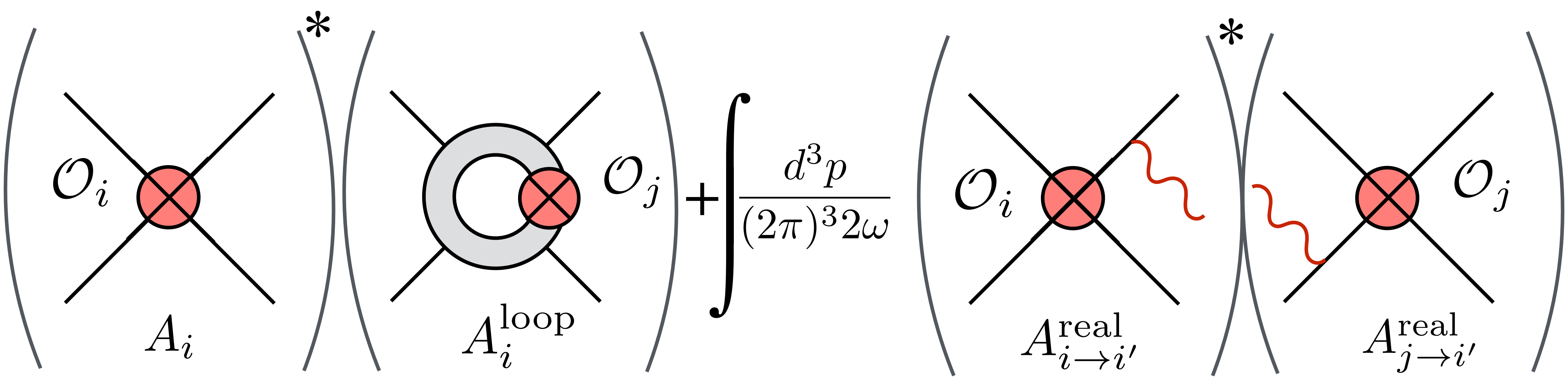
(KLN)



Weightlifting from IR to UV

- Kinoshita-Lee-Nauenberg (KLN) Theorem:
  - IR divergence in (1-loop) virtual correction is cancelled by inclusive sum over (a single) unresolved final state(s)!—cross section level statement.
- Focus on $w_i < w_j$, divergence only from massless bubbles

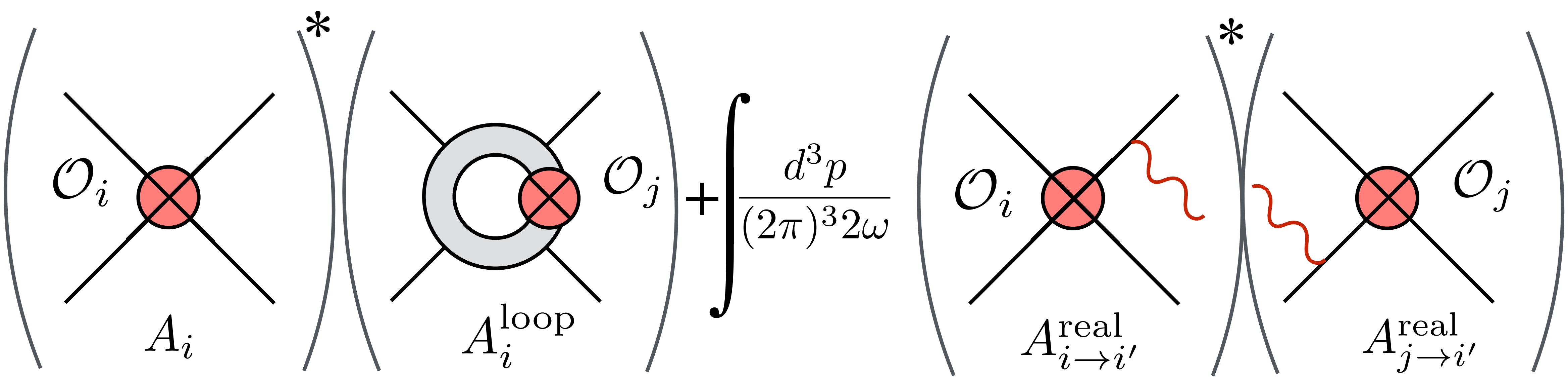
UV div.	=−IR virtual div.	=+IR real div.
	(massless bubble)	(KLN)



Weightlifting from IR to UV

- Kinoshita-Lee-Nauenberg (KLN) Theorem:
  - IR divergence in (1-loop) virtual correction is cancelled by inclusive sum over (a single) unresolved final state(s)!—cross section level statement.
- Focus on $w_i < w_j$, divergence only from massless bubbles

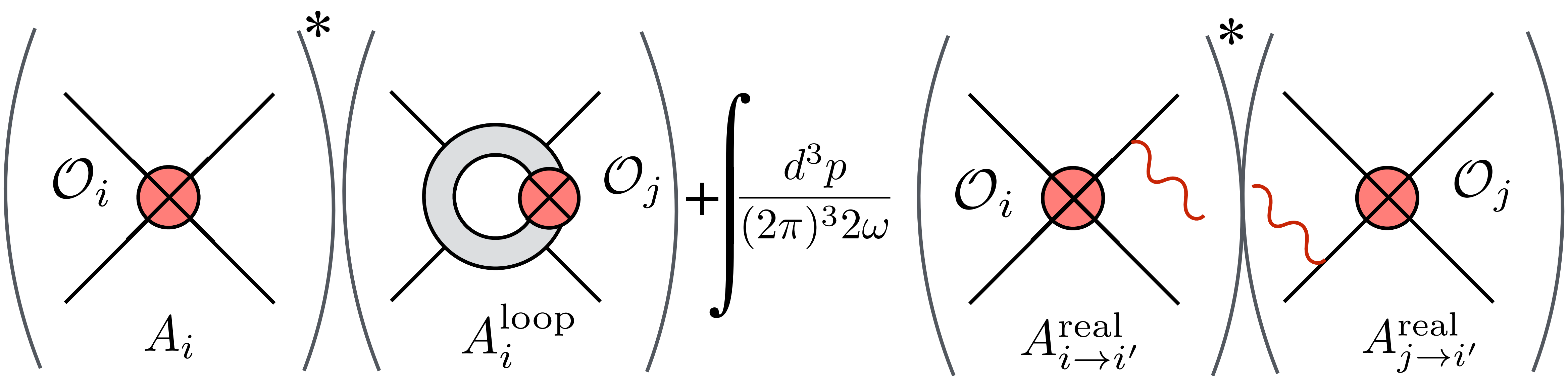
UV div.	=−IR virtual div.	=+IR ~~real~~ div.
	(massless bubble)	(KLN)



Weightlifting from IR to UV

- Kinoshita-Lee-Nauenberg (KLN) Theorem:
  - IR divergence in (1-loop) virtual correction is cancelled by inclusive sum over (a single) unresolved final state(s)!—cross section level statement.
- Focus on $w_i < w_j$, divergence only from massless bubbles

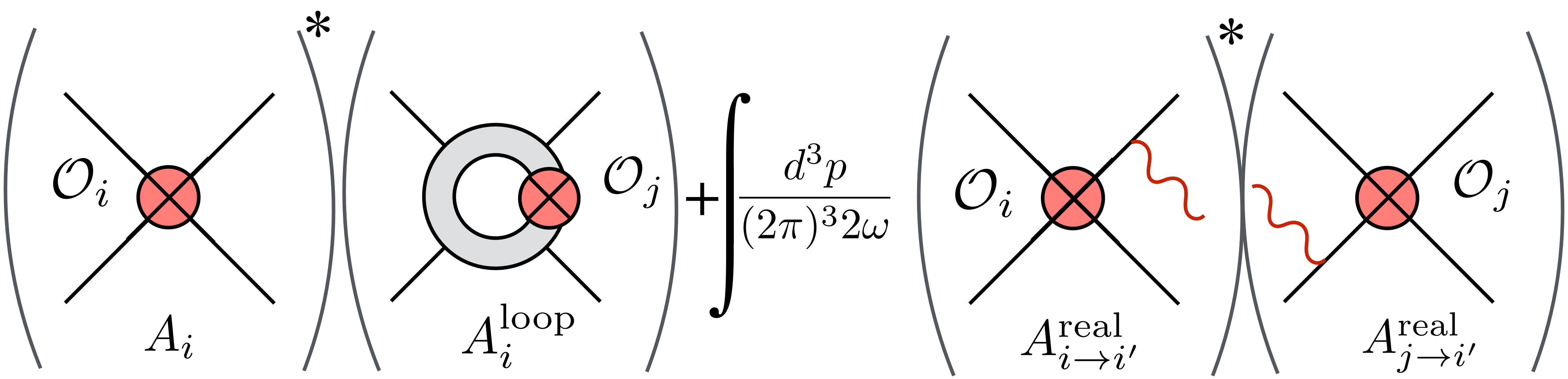
UV div.	=−~~IR virtual div.~~ (massless bubble)	=+~~IR real div.~~ (KLN)



Weightlifting from IR to UV

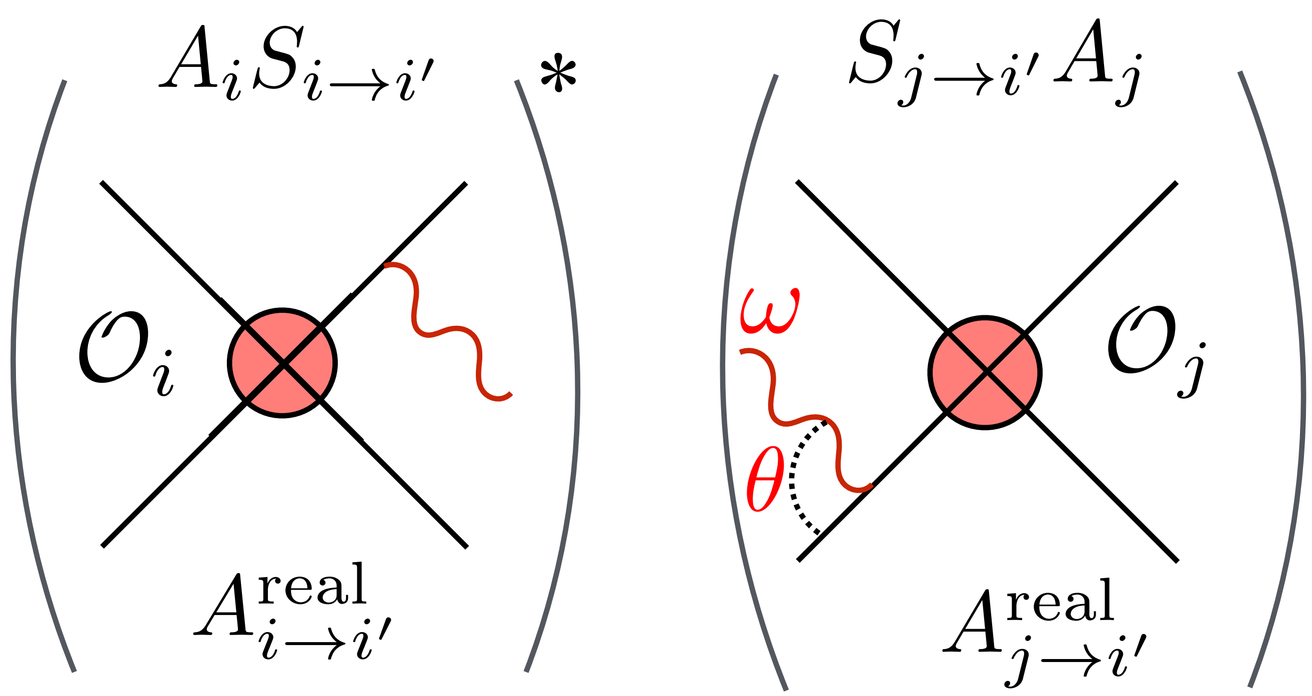
- Kinoshita-Lee-Nauenberg (KLN) Theorem:
  - IR divergence in (1-loop) virtual correction is cancelled by inclusive sum over (a single) unresolved final state(s)!—cross section level statement.
- Focus on $w_i < w_j$, divergence only from massless bubbles

~~UV div.~~	=−~~IR virtual div.~~ (massless bubble)	=+~~IR real div.~~ (KLN)



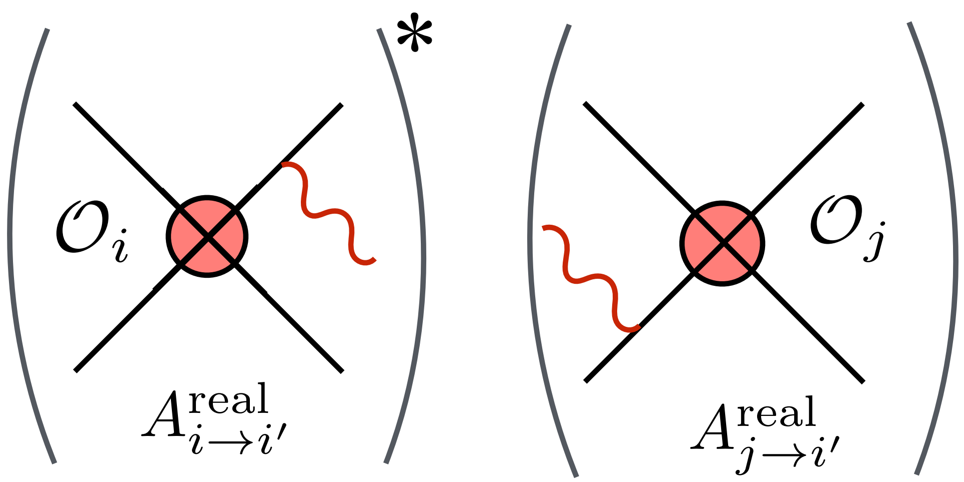
Soft-Collinear Singularity

- Singular terms also factorized universally
  - soft/collinear behavior $S \propto \frac{1}{\omega} \propto \frac{1}{\sqrt{1-\cos\theta}}$
- Upon integration $\int d\omega\omega \, d\cos\theta \, |S|^2 \propto \ln\omega \, \ln\cos\theta$
- Focus **leading soft and collinear singularity**!



Soft Divergence?

- Soft particle does not change helicities of hard process to leading order [Weinberg 1965]
- The two amplitudes start with the same weight to interfere. Stripping the soft particle unchanged the weights.
- $w_i = w_j$ for hard processes; impossible for operators with $w_i < w_j$

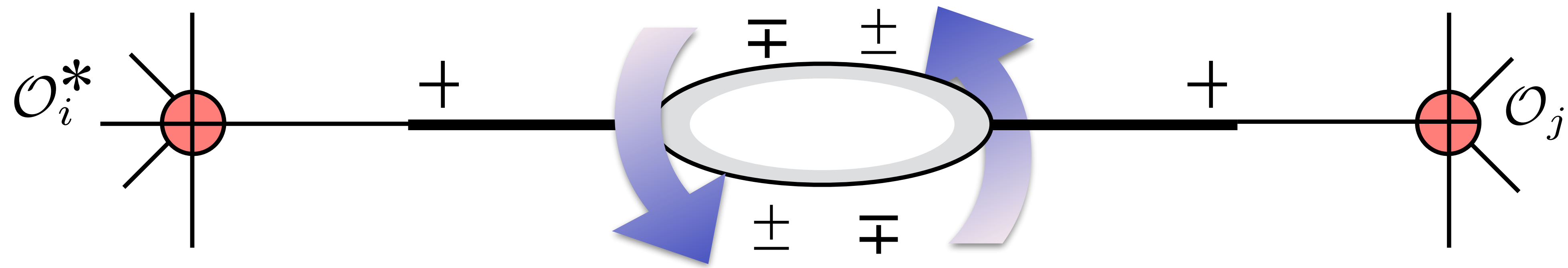


No soft divergence when

$w_i < w_j$ or $\overline{w}_i < \overline{w}_j$

Collinear Divergence?

- Collinear on both sides: TWO BACK-TO-BACK "JETS"



- Rotation around jet direction= little group!
- Different weights of hard processes ($w_i < w_j$) are balanced by the two collinear functions—produce net helicity/phase after conjugation, $\int_0^{2\pi} d\phi \, S^*_{i\to i'} S_{j\to i'} \propto \int_0^{2\pi} d\phi \, e^{2i\phi} = 0$.

No collinear divergence:

$w_i < w_j$ or $\overline{w}_i < \overline{w}_j$

Outline

- Introduction
- New Non-renormalization Theorems
- Weighting Tree Amplitudes
- Weighting 1-loop Amplitudes
- Weightlifting from IR to UV
- Application to the Standard Model
- Conclusion

Anomalous Dimension of Dim-6 SMEFT

- How many can be explained?

Alonso, Jenkins, Manohar, Trott: 1409.0868

□ =non-zero
×=no diagram
=non-trivial zero
=zero by theorems

	F^3	$F^2\phi^2$	$F\psi^2\phi$	ψ^4	$\psi^2\phi^3$	$\bar{F}^3$	$\bar{F}^2\phi^2$	$\bar{F}\bar{\psi}^2\phi$	$\bar{\psi}^4$	$\bar{\psi}^2\phi^3$	$\bar{\psi}^2\psi^2$	$\bar{\psi}\psi\phi^2 D$	$\phi^4 D^2$	ϕ^6
F^3			×	×	×			×	×	×	×	×	×	×
$F^2\phi^2$				×	×				×	×	×			×
$F\psi^2\phi$									×				×	×
ψ^4	×	×			×	×	×	×	×	×	y^2		×	×
$\psi^2\phi^3$										y^2				×
$\bar{F}^3$			×	×	×			×	×	×	×	×	×	×
$\bar{F}^2\phi^2$				×	×				×	×	×			×
$\bar{F}\bar{\psi}^2\phi$				×									×	×
$\bar{\psi}^4$	×	×	×	×	×	×	×			×	$\bar{y}^2$		×	×
$\bar{\psi}^2\phi^3$					$\bar{y}^2$									×
$\bar{\psi}^2\psi^2$		×		$\bar{y}^2$	×		×		y^2	×			×	×
$\bar{\psi}\psi\phi^2 D$														×
$\phi^4 D^2$				×					×		×			×
ϕ^6			×	×				×	×		×			

Anomalous Dimension of Dim-6 SMEFT

- Got almost all **zeros for free!**
- Explain the breaking via Yukawa

Alonso, Jenkins, Manohar, Trott: 1409.0868

□ =non-zero
×=no diagram
=non-trivial zero
=zero by theorems

	$(w,\bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$F^2\phi^2$	$(2,6)$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$F\psi^2\phi$	$(2,6)$									$\times$				$\times$	$\times$
ψ^4	$(2,6)$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	y^2		$\times$	$\times$
$\psi^2\phi^3$	$(4,6)$										y^2				$\times$
$\bar{F}^3$	$(6,0)$			$\times$	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
$\bar{F}^2\phi^2$	$(6,2)$				$\times$	$\times$				$\times$	$\times$	$\times$			$\times$
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				$\times$									$\times$	$\times$
$\bar{\psi}^4$	$(6,2)$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$	$\bar{y}^2$		$\times$	$\times$
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$									$\times$
$\bar{\psi}^2\psi^2$	$(4,4)$		$\times$		$\bar{y}^2$	$\times$		$\times$		y^2	$\times$			$\times$	$\times$
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														$\times$
$\phi^4 D^2$	$(4,4)$				$\times$					$\times$		$\times$			$\times$
ϕ^6	$(6,6)$			$\times$	$\times$				$\times$	$\times$		$\times$			

Anomalous Dimension of Dim-6 SMEFT

- Got almost all **zeros for free!**
- Explain the breaking via Yukawa

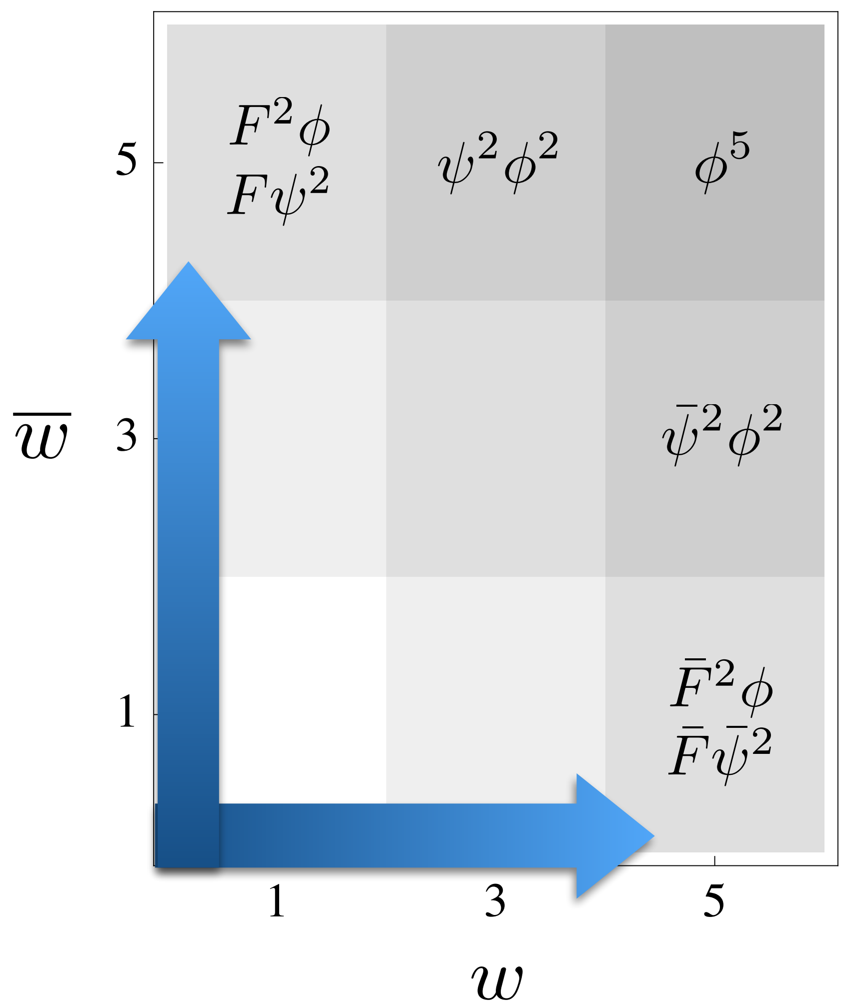
Alonso, Jenkins, Manohar, Trott: 1409.0868

□ =non-zero
×=no diagram
=non-trivial zero
=zero by theorems

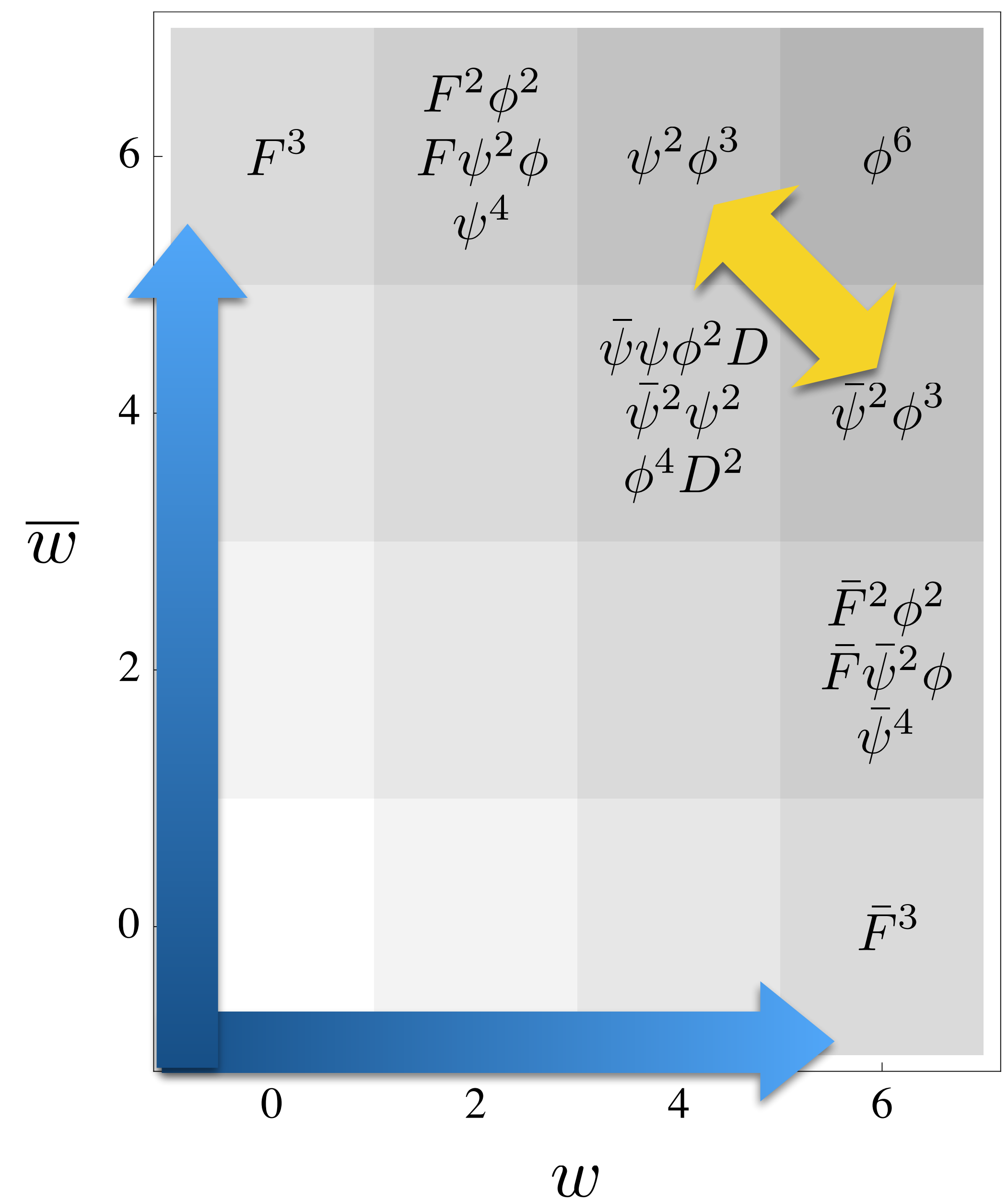
	$(w,\bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			×	×	×			×	×	×	×	×	×	×
$F^2\phi^2$	$(2,6)$				×	×				×	×	×			×
$F\psi^2\phi$	$(2,6)$									×				×	×
ψ^4	$(2,6)$	×	×			×	×	×	×	×		y^2		×	×
$\psi^2\phi^3$	$(4,6)$										y^2				×
$\bar{F}^3$	$(6,0)$			×	×	×			×	×		×	×	×	×
$\bar{F}^2\phi^2$	$(6,2)$				×	×				×	×	×			×
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				×									×	×
$\bar{\psi}^4$	$(6,2)$	×	×	×	×	×	×	×			×	$\bar{y}^2$		×	×
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$									×
$\bar{\psi}^2\psi^2$	$(4,4)$		×		$\bar{y}^2$			×		y^2	×			×	×
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														×
$\phi^4 D^2$	$(4,4)$				×					×		×			×
ϕ^6	$(6,6)$			×	×				×	×		×			

Holomorphy from Positivity

dimension 5

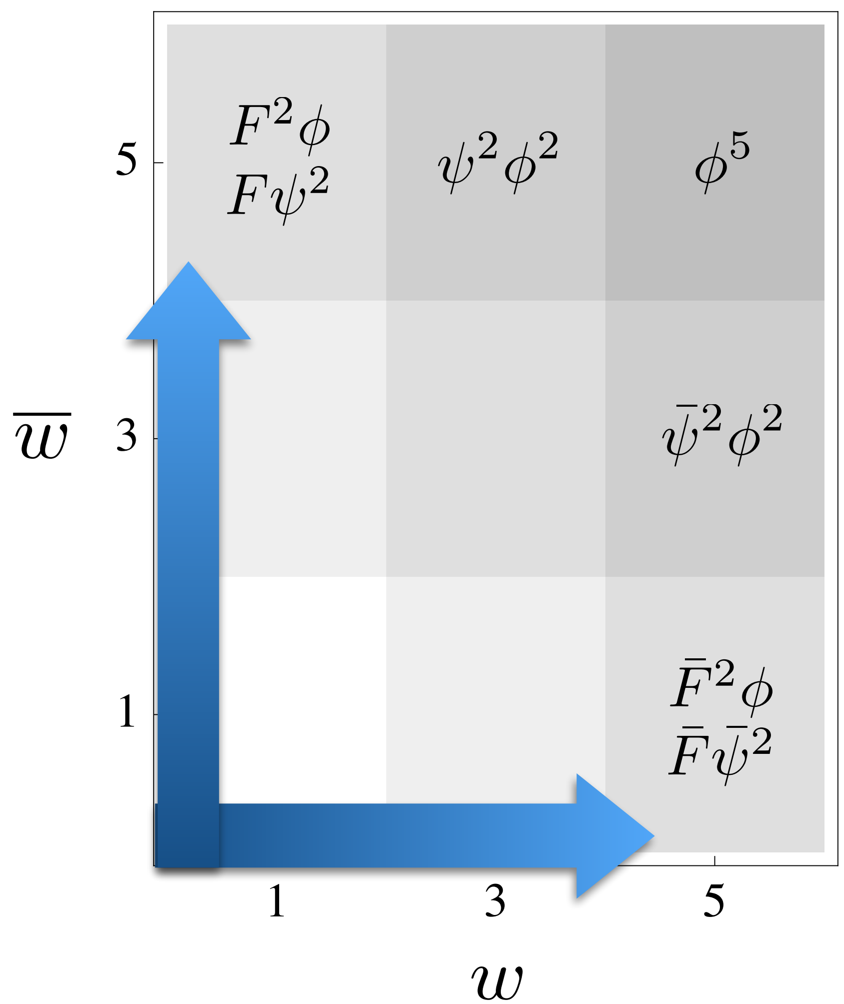


dimension 6

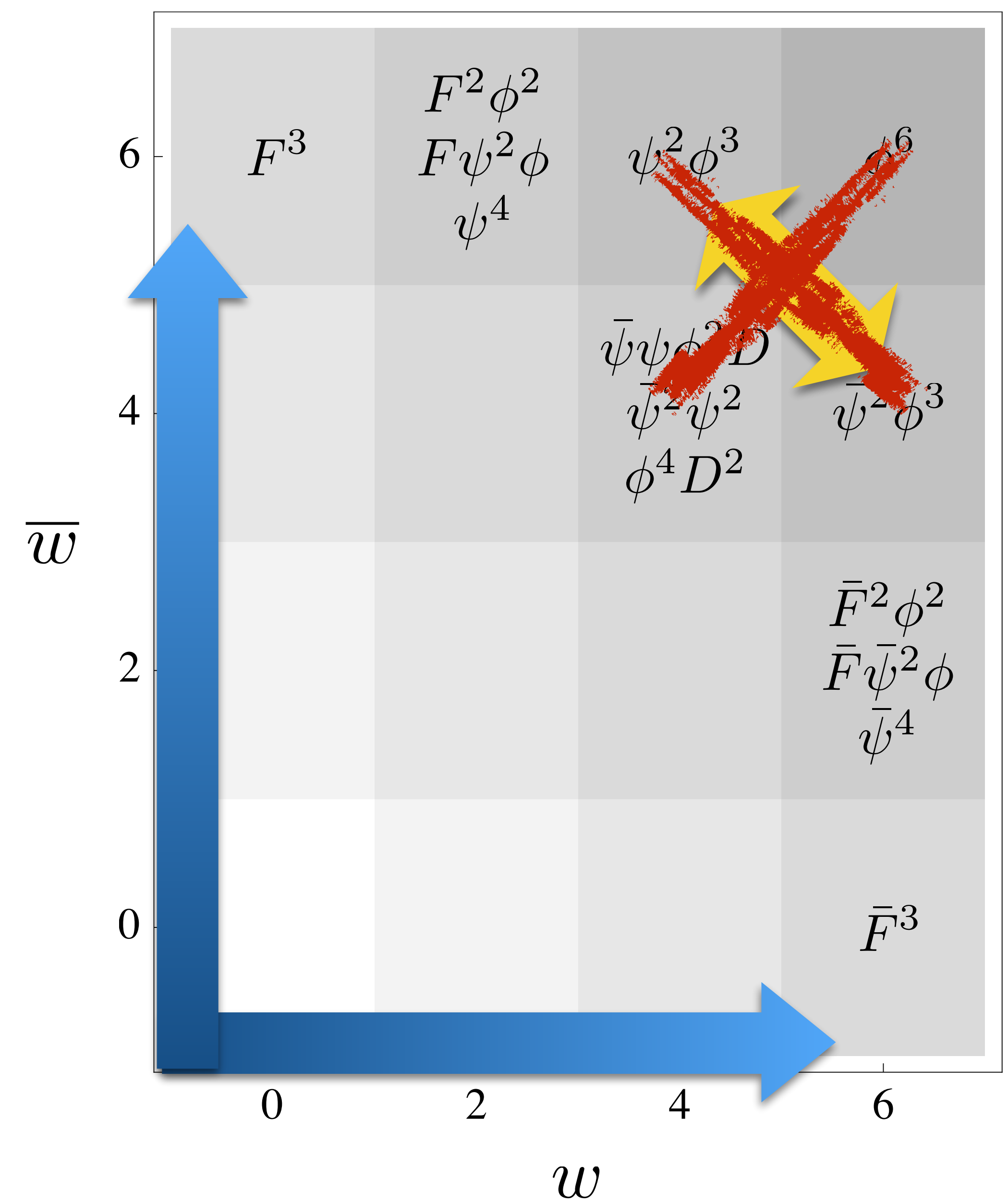


Holomorphy from Positivity

dimension 5



dimension 6



Conclusion

- We prove a new class of non-renormalization theorems at 1-loop for any 4d massless theory with marginal interactions deformed by leading irrelevant operators.
- Explained the non-trivial cancellations in the anomalous dimensions between different class of operators in dimension six SMEFT.
- No symmetry is assumed. Only use unitarity and helicity selection rule in derivation.

Outlook

- Generalization to higher loop?
  - Seems to fail by finite part of 1-loop amplitudes
  - Lower bound for marginal interactions is broken as well (e.g. all plus amplitudes non-zero at 1-loop)
- Generalization to other space-time dimension?
- Underlying symmetry explanation?
- Special case in N=4 sYM? [Chen, Huang, Wen; 1505.07093]
- Other phenomenological implication? [SM+DM EFT? Crivellin, Francesco, Massimiliano 1402.1173]

Backup Slides

Standard Model Effective Field Theories (SMEFT)

- 59 dim-6 operators in SMEFT:
  [Grzadkowski, Iskrzynski, Misiak, and Rosiek, 2010]

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde W B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Standard Model Effective Field Theories (SMEFT)

- 59 dim-6 operators in SMEFT:

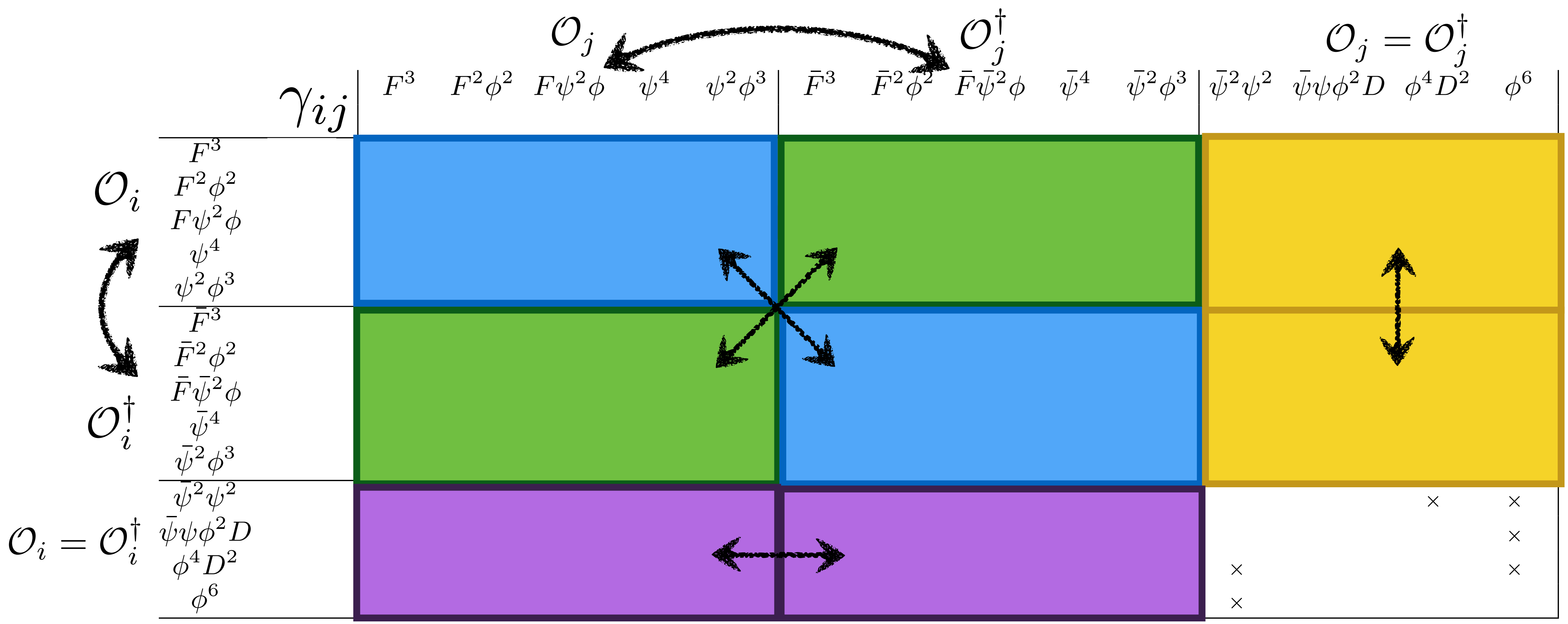
[Grzadkowski, Iskrzynski, Misiak, and Rosiek, 2010]

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Miraculous Cancellation in SMEFT

- Full 59×59 anomalous dimension matrix—holomorphy without supersymmetry?



Alonso, Jenkins, Manohar, Trott: 1409.0868

Operator Mixing

- Operators in general mix under loop correction
  - Crucial for constraints in SMEFT

    [Grojean, Alonso, Jenkins, Manohar, Trott: 1301.25881308.2627, 1310.4838, 1312.2014; Elias-Miro, Espinosa, Masso, Pomarol, 1302.5661, 1308.1879; Elias-Miro, Grojean, Gupta, Marzocca 1312.2928]
- Anomalous dimension:

$$\frac{1}{(4\pi)^2}\frac{dc_i}{d\log\mu} = \sum_j \gamma_{ij} c_j$$

γ_{ij} is dimensionless

$\Rightarrow$ depends only on marginal couplings in massless theories

new physics $\Lambda\sim$ TeV

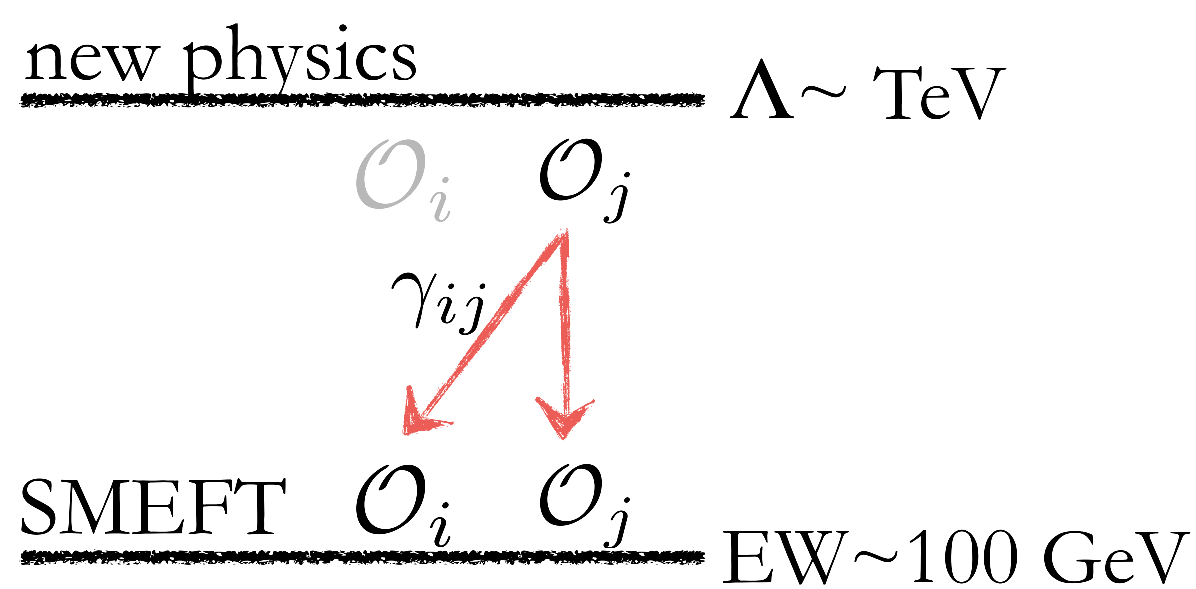
$\mathcal{O}_i$ $\mathcal{O}_j$

SMEFT EW$\sim$100 GeV

Operator Mixing

- Operators in general mix under loop correction
  - Crucial for constraints in SMEFT
    [Grojean, Alonso, Jenkins, Manohar, Trott: 1301.25881308.2627, 1310.4838, 1312.2014; Elias-Miro, Espinosa, Masso, Pomarol, 1302.5661, 1308.1879; Elias-Miro, Grojean, Gupta, Marzocca 1312.2928]
- Anomalous dimension:

$$\frac{1}{(4\pi)^2}\frac{dc_i}{d\log\mu} = \sum_j \gamma_{ij} c_j$$



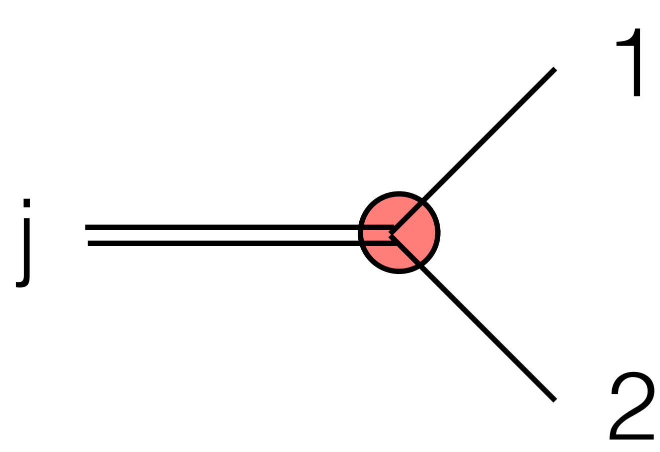
γ_{ij} is dimensionless

$\Rightarrow$ depends only on marginal couplings in massless theories

General Soft-Collinear Singularity

- General soft-collinear function S:

$$S(1^{h_1}, 2^{h_2})_{\mathrm{MHV}} = \frac{1}{[12]} \left(\frac{[\eta j]}{[\eta 1]}\right)^{-2h_1-1} \left(\frac{[\eta j]}{[\eta 2]}\right)^{-2h_2-1} \times \left(\frac{\langle 12\rangle [\eta 1][\eta 2]}{[\eta j]^2}\right)^{-[g]}$$



IR singularity is suppressed in higher dimensional operator [g]<0.

Passarino-Veltman Decomposition

- Recycling known scalar integrals:

$$A_i^{\text{loop}} = \sum_{\text{box}} d_4 I_4 + \sum_{\text{triangle}} d_3 I_3 + \sum_{\text{bubble}} d_2 I_2 + \text{rational},$$

- UV (log) divergence only from bubble integrals:

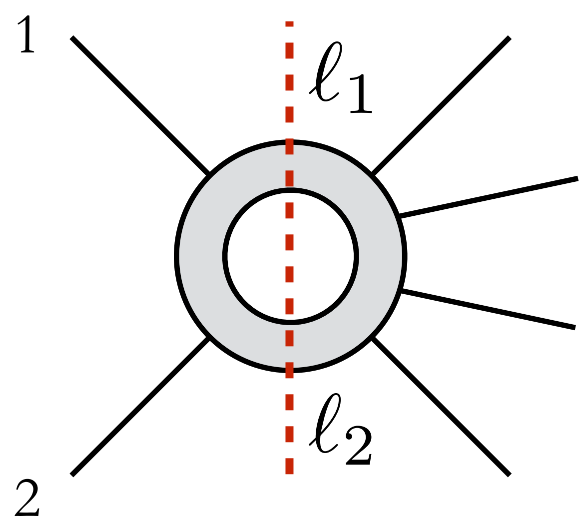
$$I_2(p^2) = \int d^{4-\epsilon}\ell \,/\ell^2(\ell+p)^2 = 1/(4\pi)^2\epsilon + \ldots$$

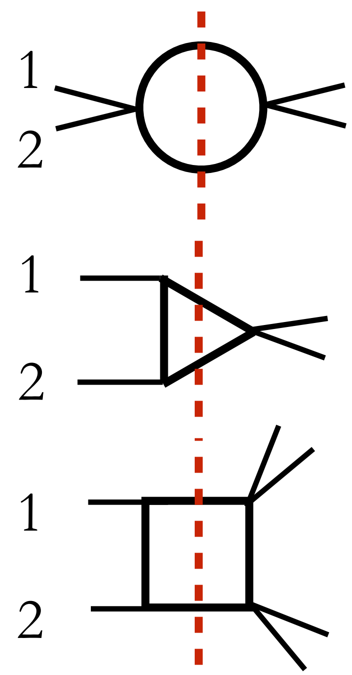
- Tadpole and "massless" bubble, $I_2(p^2=0)$, are scaleless.
  - vanish in dimensional regularization
  - Important caveat in massless bubble

Generalized Unitarity

- Cutting on the integral basis side

$$A_i^{\text{loop}} \qquad \text{cut}[A_i^{\text{loop}}] = d_2 + f(\ell_1, \ell_2)$$





$$\frac{1}{\ell^2(\ell+p)^2} \xrightarrow{\text{cut}} 1$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2} \xrightarrow{\text{cut}} \frac{1}{(\ell+p')^2}$$

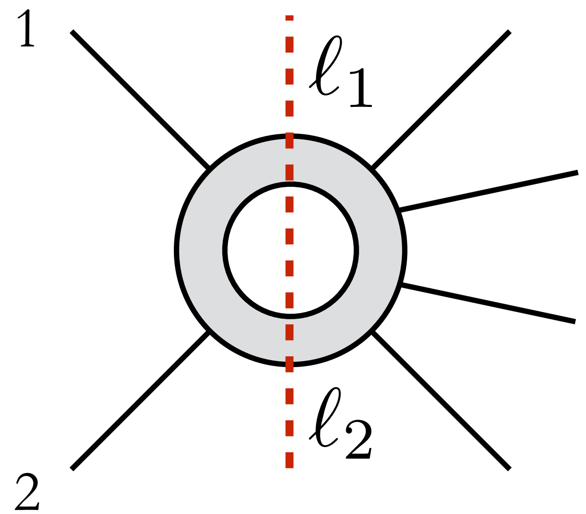
$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2(\ell+p'')^2} \xrightarrow{\text{cut}} \frac{1}{(\ell+p')^2(\ell+p'')^2}$$

- If the product of tree amplitude vanishes identically in massive 2-cut, no contribution from the associated bubble, triangle and box.

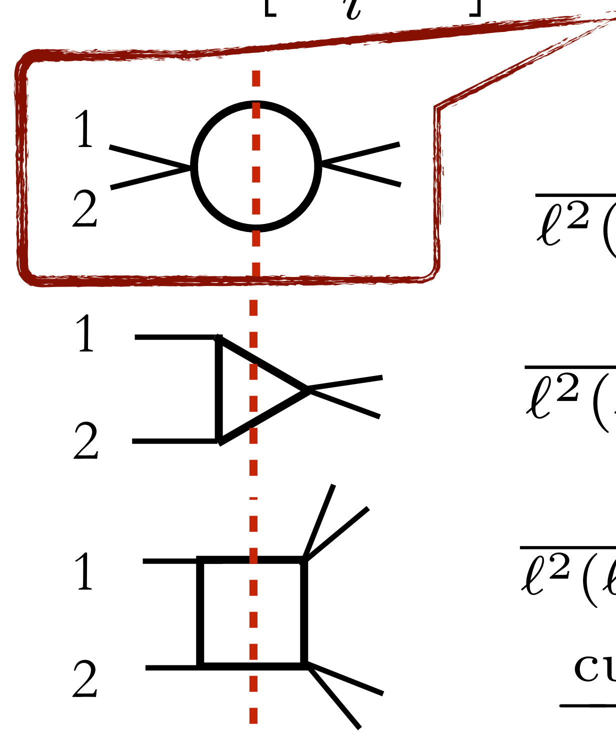
Generalized Unitarity

- Cutting on the integral basis side

A_i^{loop}



$$\text{cut}[A_i^{\text{loop}}] = d_2 + f(\ell_1, \ell_2)$$



$$\frac{1}{\ell^2(\ell+p)^2} \xrightarrow{\text{cut}} 1$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2} \xrightarrow{\text{cut}} \frac{1}{(\ell+p')^2}$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2(\ell+p'')^2}$$

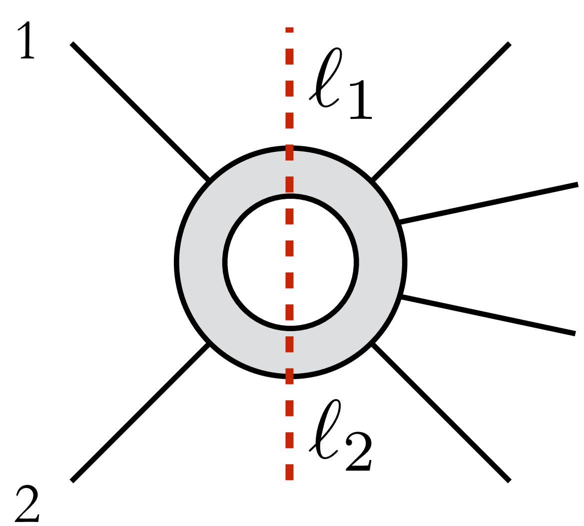
$$\xrightarrow{\text{cut}} \frac{1}{(\ell+p')^2(\ell+p'')^2}$$

- If the product of tree amplitude vanishes identically in massive 2-cut, no contribution from the associated bubble, triangle and box.

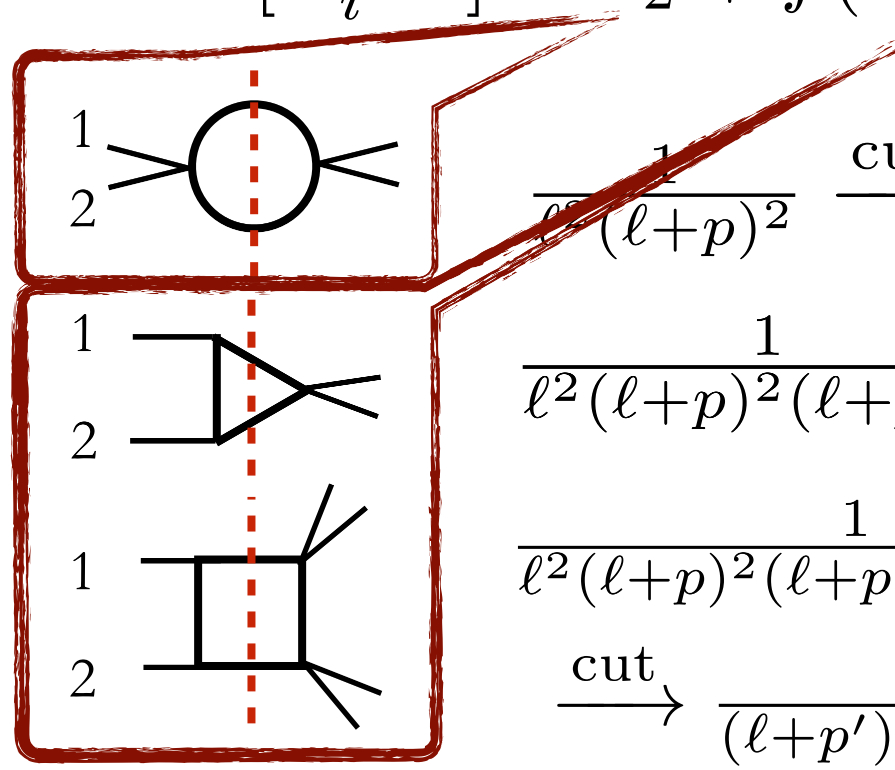
Generalized Unitarity

- Cutting on the integral basis side

A_i^{loop}



$$\mathrm{cut}[A_i^{\text{loop}}] = d_2 + f(\ell_1, \ell_2)$$



$$\frac{1}{\ell^2(\ell+p)^2} \xrightarrow{\text{cut}} 1$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2} \xrightarrow{\text{cut}} \frac{1}{(\ell+p')^2}$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2(\ell+p'')^2}$$

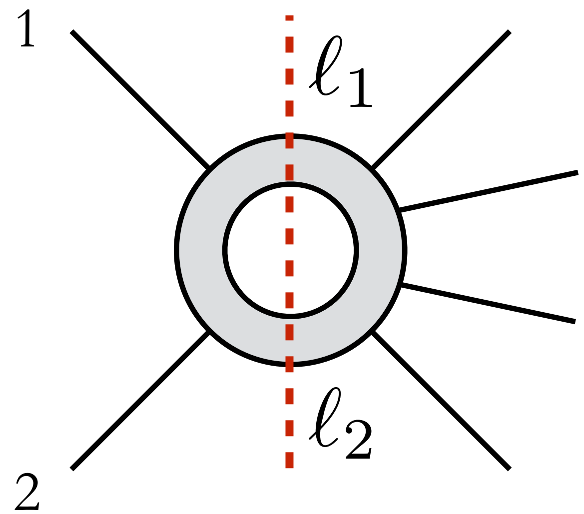
$$\xrightarrow{\text{cut}} \frac{1}{(\ell+p')^2(\ell+p'')^2}$$

- If the product of tree amplitude vanishes identically in massive 2-cut, no contribution from the associated bubble, triangle and box.

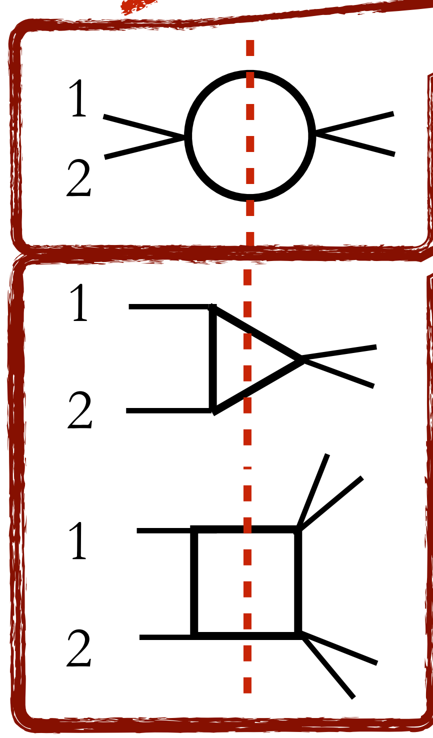
Generalized Unitarity

- Cutting on the integral basis side

A_i^{loop}



$$\cancel{\mathrm{cut}[A_i^{\mathrm{loop}}]} = d_2 + f(\ell_1, \ell_2)$$



$$\frac{1}{\ell^2(\ell+p)^2} \xrightarrow{\mathrm{cut}} 1$$

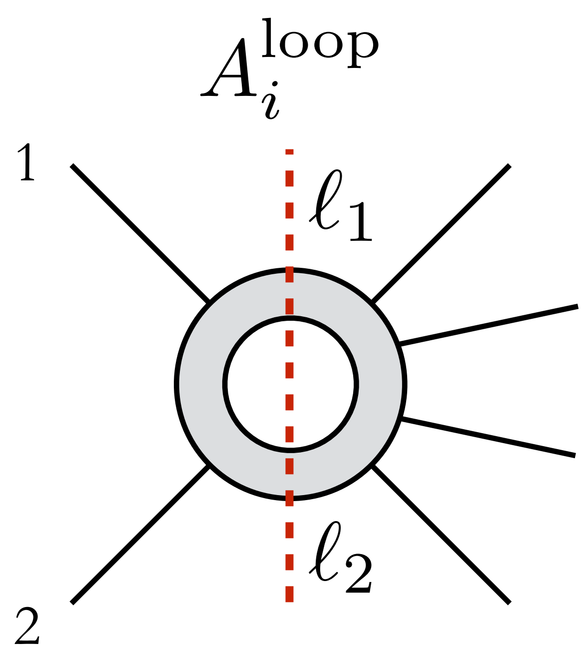
$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2} \xrightarrow{\mathrm{cut}} \frac{1}{(\ell+p')^2}$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2(\ell+p'')^2} \xrightarrow{\mathrm{cut}} \frac{1}{(\ell+p')^2(\ell+p'')^2}$$

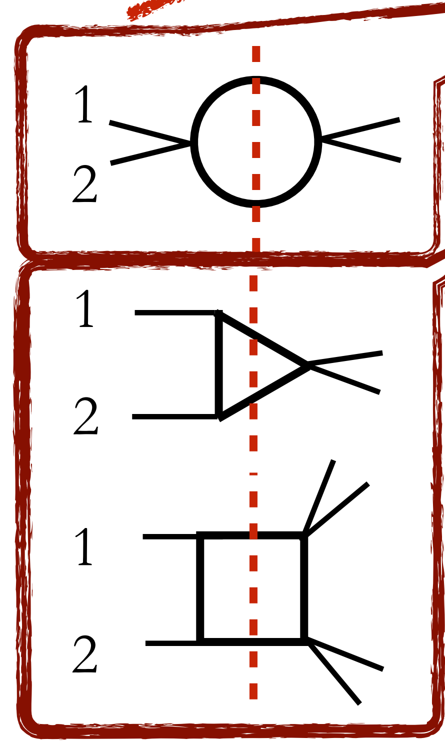
- If the product of tree amplitude vanishes identically in massive 2-cut, no contribution from the associated bubble, triangle and box.

Generalized Unitarity

- Cutting on the integral basis side



$$\cancel{\mathrm{cut}[A_i^{\mathrm{loop}}] = d_2 + f(\ell_1, \ell_2)}$$



$$\frac{1}{\ell^2(\ell+p)^2} \xrightarrow{\mathrm{cut}} 1$$

$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2} \xrightarrow{\mathrm{cut}} \frac{1}{(\ell+p')^2}$$

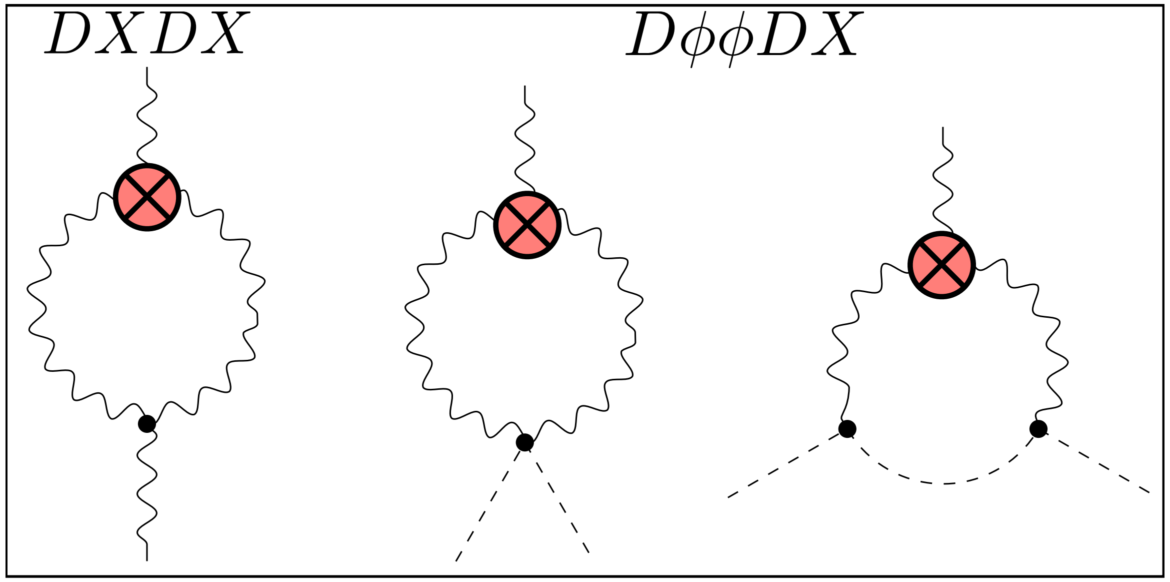
$$\frac{1}{\ell^2(\ell+p)^2(\ell+p')^2(\ell+p'')^2} \xrightarrow{\mathrm{cut}} \frac{1}{(\ell+p')^2(\ell+p'')^2}$$

- If the product of tree amplitude vanishes identically in massive 2-cut, no contribution from the associated bubble, triangle and box.

Accidental Cancellation in $\psi^2\phi^3/\phi^6 - F^3$

- Counterterm analysis for $\phi^4 D^2, \psi^2\phi^3, \phi^6$ from $F^3/\bar{F}^3$:
  - Recasting counterterms from 1PI diagrams only generates
    $j_\phi^{\mu a}\,[D_\nu, X^{\nu\mu}]^a \xrightarrow{\text{EoM}} j_\phi^{\mu a} j_{\phi\mu}^a$ where scalar current $j_\phi^\mu = \phi^\dagger \overleftrightarrow{D}_\mu^a \phi$
  - In standard basis, $j_\phi^{\mu a} j_{\phi\mu}^a = \phi^4 D^2 \oplus \psi^2\phi^3 \oplus \phi^6$
- $j_\phi^{\mu a} j_{\phi\mu}^a$ has a non-zero 4-scalar amplitude—forbidden to be generated by $F^3/\bar{F}^3$ according to our theorem!

$$\boxed{\begin{aligned} 0 &= \gamma_{\phi^4 D^2 - F^3} \\ &\propto \gamma_{\psi^2\phi^3 - F^3} \propto \gamma_{\phi^6 - F^3} \end{aligned}}$$



[fig. from Alonso, Jenkins, Manohar, Trott: 1312.2014]