Calculation of multi-loop integrals with SecDec-3.0

Johannes Schlenk

Max Planck Institute for Physics

jschlenk@mpp.mpg.de

SecDec collaboration: S. Borowka, G. Heinrich, S. Jones, M. Kerner, JS, T. Zirke

RADCOR-Loopfest 2015
Los Angeles, USA

June 19, 2015

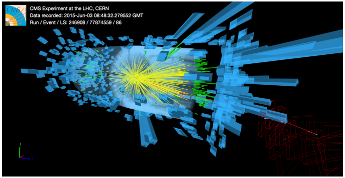
Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Overview

1. The sector decomposition method
2. New features of SecDec-3.0
3. Application to the process $gg \to HH$
4. Conclusions

Motivation

- Run II of the LHC at 13 TeV started recently
- In the absence of new physics signals, precision measurements become more important
- Need for precise predictions at higher loop orders involving multiple scales
- Master integrals are an important building block for these calculations



OPEN-PHO-ACCEL-2015-007-11, CERN

Sector Decomposition

$$I \sim \int_0^\infty \mathrm{d}^N x \, \delta(1 - x_N) \frac{\mathcal{U}^{N-(L+1)D/2}}{\mathcal{F}^{N-LD/2}} = \frac{c_{-2L}}{\epsilon^{2L}} + \frac{c_{-2L+1}}{\epsilon^{2L-1}} + \dots$$

- Decompose integration region into sectors with simple singularity structure
- Subtract divergences and expand in the regularization parameter ϵ
- Integrate the finite coefficients (numerically)
- Applicable to loop integrals and more general parametric integrals

Hepp (1966); Denner, Roth (1996); Binoth, Heinrich (2000)

Decomposition algorithms

Authors	# of sectors	infinite recursion	Description
Binoth, Heinrich	small	possible	heuristic algorithm
Bogner, Weinzierl	large	no	based on Hironaka's polyhedra game
Smirnov, Smirnov, Tentyukov	smallish	no	hybrid of the first two
Kaneko, Ueda	small	no	Geometric strategy

SecDec

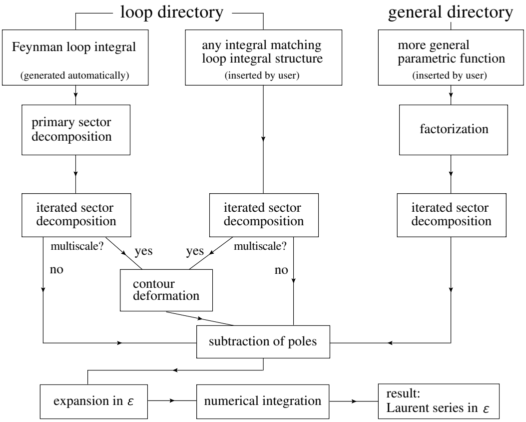
- Public implementation of the sector decomposition method Carter, Heinrich (2010); Borowka, Heinrich (2011)

http://secdec.hepforge.org/

SecDec-3.0

Borowka, Heinrich, Jones, Kerner, JS, Zirke (2015)

- Improved user interface
- New decomposition strategies



Other codes

- sector_decomposition Bogner, Weinzierl (2007)
- Fiesta Smirnov, Smirnov, Tentyukov ('08,'09,'13)

SecDec - Installation

Installation:

```
tar -xvf SecDec-3.0.7.tar.gz
cd SecDec-3.0.7
make
(make check)
```

Included programs:

Cuba Hahn, Bases Kawabata, CQuad Gonnet (2010)

Dependencies:

Mathematica version 7 or higher, Perl, C/Fortran compiler, (Normaliz Bruns, Ichim, Roemer, Soeger for geometric decomposition)

Improved user interface

```
secdec -p param.input -m math.m -k kinem.input
```

param.input

```
graph=2Lbox
epsord=0
contourdef=True
integrator=3
epsrel=1.e-3,1.e-2,1.e-2,2.e-2
epsabs=1.e-5,1.e-4,1.e-4,1.e-4
```

General options and parameters for numerical integration

kinem.input

```
p1 3 -2
p2 2 -1
```

List of kinematic points

math.m

```
momlist={k1,k2};
proplist={k1^2,(k1+p2)^2,(k1-p1)^2,(k1-k2)^2,
         (k2+p2)^2,(k2-p1)^2,(k2+p2+p3)^2,(k1+p3)^2};
numerator={1};
powerlist={0,1,1,1,1,1,1,-1};
ExternalMomenta={p1,p2,p3,p4};
externallegs=4;
prefactor=Gamma[1+eps]^2;
KinematicInvariants = {s,t};
Masses={};
ScalarProductRules = {
  SP[p1,p1]->0,
  SP[p2,p2]->0,
  SP[p3,p3]->0,
  SP[p4,p4]->0,
  SP[p1,p2]->s/2,
  SP[p2,p3]->t/2,
  SP[p1,p3]->-s/2-t/2};
Dim=4-2*eps;
```

Graph data and kinematics

New features

- Support for linear propagators including contour deformation (assumes $+i\delta$ prescription)

$$\frac{1}{q\cdot n+i\,\delta},\ \cancel{\frac{1}{q\cdot n+i\,\delta\,\mathrm{sign}(q\cdot n^*)}}$$

- Propagators with nonpositive exponents allowed
- Simplified scans over parameter ranges
- Option to run compilation and numerical integration on a cluster (Condor, PBS, LSF)
- Numerical integration with CQuad (1D) and Mathematica possible
- General parametric integrals: input can contain ϵ-dependent symbolic functions (masked during the decomposition stage)

Decomposition algorithms in SecDec-3.0

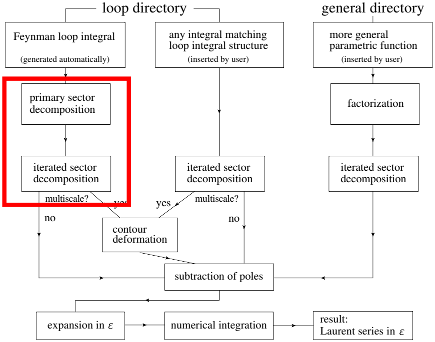
Implemented strategies:

- X: Heuristic strategy Binoth, Heinrich
- G1: Original geometric strategy Kaneko, Ueda
- G2: Improved geometric decomposition

Flag `strategy` in `param.input` to switch between strategies:

```
strategy=G2
```

Normaliz Bruns, Ichim, Roemer, Soeger is used for the calculation of convex polytopes



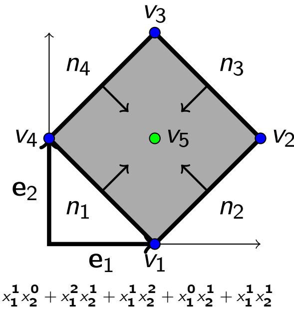
Geometric Method - General concept

- Eliminate x_N using the Cheng-Wu theorem:

$$I \sim \int_0^\infty \mathrm{d}^N x\, \delta(1 - x_N) \mathcal{N} \frac{\mathcal{U}^{N-(L+1)D/2}}{\mathcal{F}^{N-LD/2}}$$

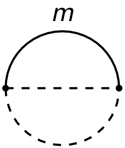
- Calculate Newton polytope of $\mathcal{U} \cdot \mathcal{F} \cdot \mathcal{N}$

$$\Delta = \begin{cases} \mathrm{ConvHull}(\{\mathbf{v}_i\}) \\ \bigcap_F \{\mathbf{m} \in \mathbb{R}^{N-1} \mid \langle \mathbf{m}, \mathbf{n}_F \rangle + a_F \geq 0\} \end{cases}$$



- Change of variables in sector j: $x_i = \prod_{F \in S_j} y_F^{\langle \mathbf{e}_i, \mathbf{n}_F \rangle}$
- For each vertex S_j contains facets incident to it (triangulation)

Geometric Method - Vacuum integral



$$\sim \int_0^\infty \mathrm{d}x_1 \mathrm{d}x_2 \frac{1}{P(x_1, x_2)^{2-\epsilon}}$$

$$P(x_1, x_2) = x_1 + x_1 x_2 + x_2$$

Geometric Method - Vacuum integral

Vertices:

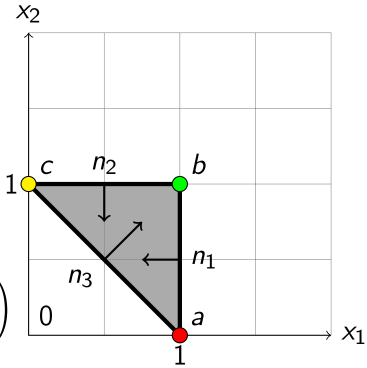
$$P(x_1, x_2) = \underline{x_1^1 x_2^0} + \underline{x_1^1 x_2^1} + \underline{x_1^0 x_2^1}$$

$$\underline{\mathbf{v}_a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \underline{\mathbf{v}_b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \underline{\mathbf{v}_c} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Facet normals:

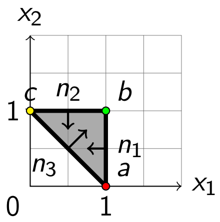
$$\mathbf{n}_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad \mathbf{n}_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad \mathbf{n}_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$a_1 = 1 \quad a_2 = 1 \quad a_3 = -1$$



Change of variables

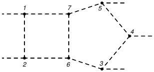
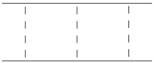
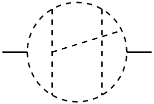
$$x_i = \prod_{F=1}^{3} y_F^{\langle \mathbf{e}_i, \mathbf{n}_F \rangle} \rightarrow \begin{array}{rcl} x_1 & = & y_1^{-1} y_3 \\ x_2 & = & y_2^{-1} y_3 \end{array}$$



$$\begin{array}{rcl} P(x_1, x_2) & = & x_1 + x_1 x_2 + x_2 \\ & = & y_1^{-1} y_2^{-1} y_3^1 \left(y_1 + y_2 + y_3 \right) \end{array}$$

- Include Jacobi determinant
- Result for all three sectors ($S_a = \{3, 1\}$, $S_b = \{1, 2\}$, $S_c = \{2, 3\}$):

$$\int_0^1 \mathrm{d}y_1 \mathrm{d}y_2 \mathrm{d}y_3 \frac{y_1^{\epsilon} y_2^{\epsilon} y_3^{-1-\epsilon}}{(y_1 + y_2 + y_3)^{2-\epsilon}} \left[\delta(1 - y_1) + \delta(1 - y_2) + \delta(1 - y_3) \right]$$

Diagram	X	G1	G2
	282 1 s	266 8 s	166 4 s
	368 1 s	360 9 s	235 5 s
	548 3 s	506 15 s	304 4 s
	infinite recursion	72 5 s	76 1 s
	27336 5510 s	32063 11856 s	27137 443 s

Number of produced sectors and timings for the decomposition

Application: $gg \to HH$

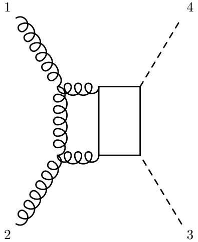
- Process $gg \to HH$ with full top mass dependence known to LO (loop-induced) Glover, van der Bij (1988)
- NLO in $m_t \to \infty$ limit Plehn, Spira, Zerwas (1996); Dawson, Dittmaier, Spira (HPAIR 1998)
  - $+ \frac{1}{m_t}$ expansion Grigo, Hoff, Melnikov, Steinhauser (2013)
  - $+$ full m_t dependence in real radiation and parton shower Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro (2014); Maltoni, Vryonidou, Zaro (2014)
- NNLO in $m_t \to \infty$ limit De Florian, Mazzitelli (2013); Grigo, Melnikov, Steinhauser (2014).
- Approximation is poor above production threshold
- Need for NLO calculation with full top mass dependence
- Requires computation of unknown two-loop integrals

Application: $gg \to HH$

Borowka, Heinrich, Greiner, Jones,

Kerner, Luisoni, Mastrolia, JS,

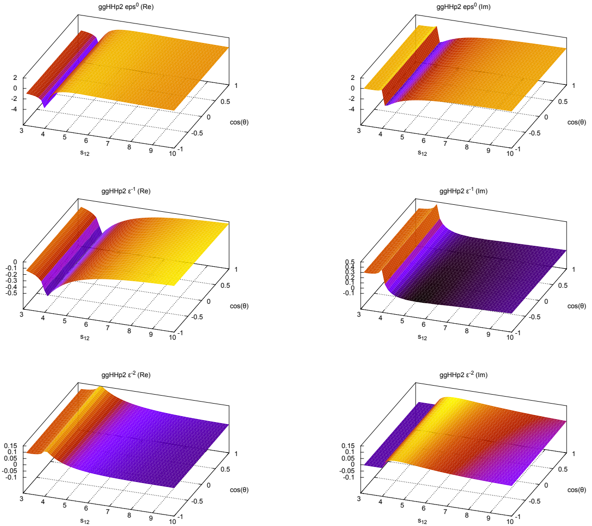
Schubert, Stoyanov, Di Vita, Zirke



$$C_0 + \frac{C_{-1}}{\epsilon} + \frac{C_{-2}}{\epsilon^2}$$

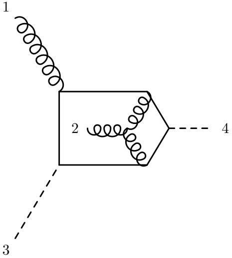
$$m_t^2 = 1$$

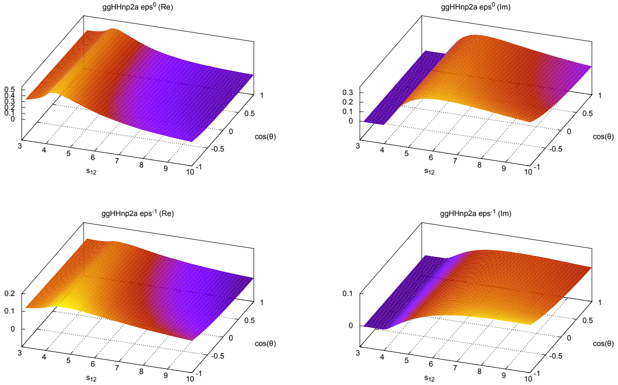
$$m_H^2 = 0.75$$



Plots by Anton Stoyanov

Application: $gg \to HH$





$$C_0 + \frac{C_{-1}}{\epsilon}$$

$$m_t^2 = 1$$

$$m_H^2 = 0.75$$

Plots by Anton Stoyanov

Conclusions and Outlook

- SecDec: Public implementation of the sector decomposition algorithm

  http://secdec.hepforge.org/

- New version SecDec-3.0:
  - Improved user interface
  - New geometric decomposition strategies
  - Linear propagators
  - Cluster mode

Outlook

- Improvement of geometric decomposition algorithm
- Interface to Gosam-2Loop
- Application to phenomenologically relevant processes

Thank you for your attention

Conclusions and Outlook

- SecDec: Public implementation of the sector decomposition algorithm

  http://secdec.hepforge.org/

- New version SecDec-3.0:
  - Improved user interface
  - New geometric decomposition strategies
  - Linear propagators
  - Cluster mode

Outlook

- Improvement of geometric decomposition algorithm
- Interface to Gosam-2Loop
- Application to phenomenologically relevant processes

Thank you for your attention