LOOP-TREE DUALITY AND QUANTUM FIELD THEORY IN 4D

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RadCor/LoopFest (Los Ángeles-USA) – 06/18/2015

Content

- Introduction (motivation and notation)
- Loop-tree duality (LTD) (quick review)
- IR regularization
  - Threshold and IR singularities
  - Finite real+virtual integration
- UV renormalization
- Conclusions

Based in:

Catani *et al.*, *JHEP 09 (2008) 065*
Bierenbaum et al., JHEP 1010 (2010) 073; JHEP 03 (2013) 025
Buchta *et al.*, *JHEP 11 (2014) 014*
Hernandez-Pinto, Rodrigo and GS, *arXiv:2015.04617 [hep-ph]*
Rodrigo *et al., in preparation* (to be published soon…*)*

Introduction

Theoretical motivation

- Collider physics are dominated by QCD effects
- Exact solutions for scattering amplitudes in gauge theories are unknown (i.e. **no exact solutions for QCD…**) → Use perturbation theory!
- We need to reach higher orders in pQCD (increase **theoretical accuracy**)
- Deal with **ill-defined expressions** in intermediate steps → **DREG!!!**
    - Proposed by {Giambiagi&Bollini, t'Hooft&Veltman, Cicuta&Montaldi, Ashmore,…}, it becomes a standard in HEP since it preserves gauge invariance
    - Change the number of space-time dimensions from 4 to $d = 4 - 2\varepsilon$
    - Singularities regularized as poles in ε (meromorphic functions)
- Final **physical predictions** must be **FINITE** → **Singularities MUST cancel!**

Introduction

Theoretical motivation

- Two kind of physical singularities:
  - **Ultraviolet singularities** coming from the high-energy region of the loop momentum, due to the pointlike structure of the theory

    Add proper counterterms obtained from RENORMALIZATION procedure. These counterterms have EXPLICIT ε–poles and are proportional to lower-order amplitudes.

  - **Infrared singularities** associated with degenerate configurations: extra-particle radiation in soft (i.e. low energy) or collinear (i.e. parallel) configurations

    Kinoshita-Lee-Nauenberg theorem states that adding real-emission processes and computing IR-safe observables guarantees the cancellation of all the IR poles present in renormalized virtual amplitudes and INTEGRATED real-radiation contributions. ADD suitable SUBTRACTION TERMS to virtual and real corrections independently

- **Threshold singularities** are **integrable;** countour deformation techniques can be applied to obtain numerically stable integrals.

Loop-tree duality (LTD)

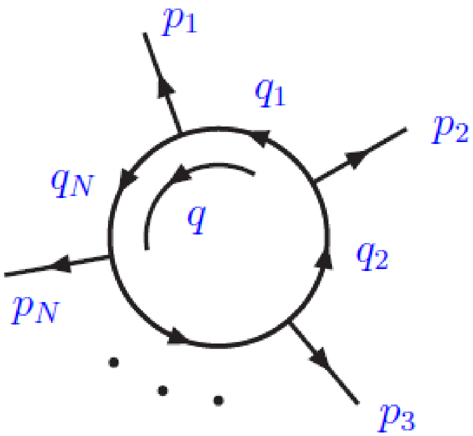
Introduction and motivation

- KLN theorem suggests that **virtual and real** contributions have the **same IR divergent structure** (because they cancel in IR-safe observables)
- **Cut contributions** are similar to **tree-level** scattering amplitudes, if all the loops are cut. At one-loop, **1-cuts are tree-level objects** (higher-cuts are products of unconnected graphs)
- **Objetive:** Combine real and virtual contributions at **integrand level** and perform the **computation in four-dimensions** (take ε to 0 with DREG)
  - Write virtual contributions as real radiation phase-space integrals of *«tree-level» objects* → *1-cut = sum over «tree level» contributions*
  - *Loop measure is related with extra-radiation phase-space*

→ ***Loop-Tree Duality*** Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Feynman integrals and propagators



$$L_R^{(N)}(p_1, p_2, \cdots, p_N) = -\imath \int \frac{d^d\ell}{(2\pi)^d} \prod_{i=1}^{N} G_R(q_i)$$

Generic one-loop
Feynman integral

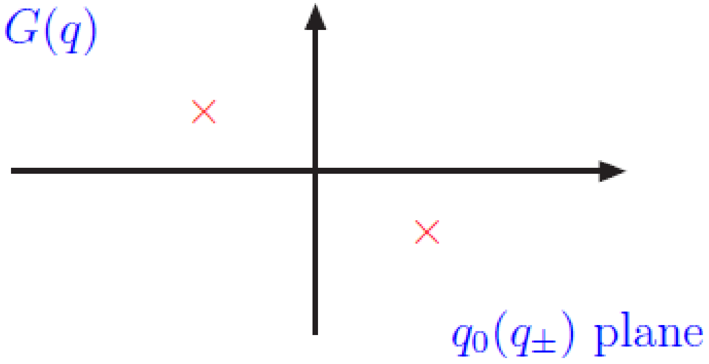
$$q_i = \ell + \sum_{k=1}^{i} p_k$$

Momenta definition

+i0 prescription guarantees that positive (negative) frecuencies are propagated forward (backward) in time

Feynman propagator

$$G(q) \equiv \frac{1}{q^2 + \imath 0}$$

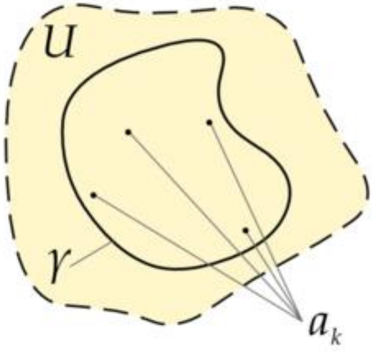


Loop-tree duality (LTD)

Feynman integrals and propagators

Cauchy Residue theorem

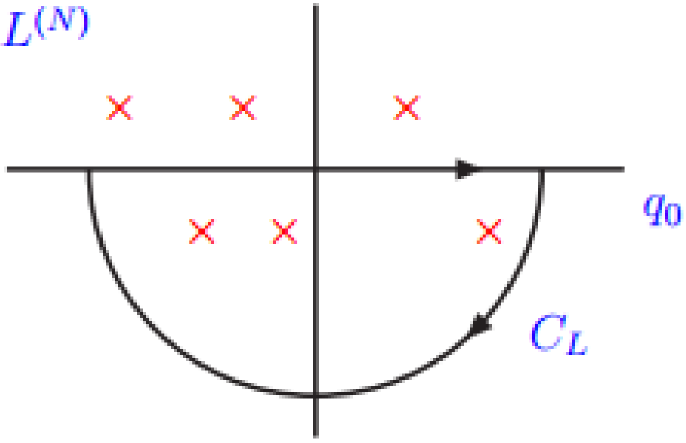
$$\oint_\gamma f(z)\,dz = 2\pi i \sum \mathrm{Res}(f, a_k)$$



«If f is a holomorphic function in U/{a_i}, and γ a simple positively oriented curve, then the integral is given by the sum of the residues at each singular point a_i»

Feynman propagator's poles

$$[G(q)]^{-1} = 0 \quad \Longrightarrow \quad q_0 = \pm\sqrt{\mathbf{q}^2 - i0}$$



Location of Feynman's integrand poles

Loop-tree duality (LTD)

Derivation

- **Idea:** *«Sum over all possible 1-cuts»* (but with a **modified i0 prescription**…)
  - Apply Cauchy residue theorem to the Feynman integral:

$$L^{(N)}(p_1, p_2, \ldots, p_N) = \int_{\mathbf{q}} \int dq_0 \prod_{i=1}^{N} G(q_i) = \int_{\mathbf{q}} \int_{C_L} dq_0 \prod_{i=1}^{N} G(q_i) = -2\pi i \int_{\mathbf{q}} \sum \mathrm{Res}_{\{\mathrm{Im}\, q_0 < 0\}} \left[\prod_{i=1}^{N} G(q_i) \right]$$

  - Select the residue of the poles with negative imaginary part:

$$\mathrm{Res}_{\{i\mathrm{-th\ pole}\}} \left[\prod_{j=1}^{N} G(q_j) \right] = \left[\mathrm{Res}_{\{i\mathrm{-th\ pole}\}}\, G(q_i) \right] \left[\prod_{\substack{j=1 \\ j \neq i}}^{N} G(q_j) \right]_{\{i\mathrm{-th\ pole}\}}$$

$$\left[\mathrm{Res}_{\{i\mathrm{-th\ pole}\}} \frac{1}{q_i^2 + i0} \right] = \int dq_0\, \delta_+(q_i^2)$$

Set internal propagators on-shell

$$\left[\prod_{j \neq i} G(q_j) \right]_{\{i\mathrm{-th\ pole}\}} = \prod_{j \neq i} \frac{1}{q_j^2 - i0\, \eta(q_j - q_i)}$$

Introduction of «dual propagators» (η prescription, a future- or light-like vector)

Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Derivation

- *It is crucial to keep track of the i0 prescription!* Duality relation involves the presence of dual propagators:

$$L^{(N)}(p_1, p_2, \dots, p_N) = - \int_q \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} \frac{1}{q_j^2 - i0\, \eta(q_j - q_i)} \qquad \tilde{\delta}\,(q_i) \equiv 2\pi\, i\, \theta(q_{i,0})\, \delta(q_i^2 - m_i^2)$$

  - The prescription involves a future- or light-like vector (arbitrary) and could depend on the loop momenta (at 1-loop is always independent of *q*). It is related with the finite value of **I0** in intermediate steps…
  - Implement a shift in each term of the sum to have the same measure: the loop integral becomes a phase-space integral!

$$\begin{aligned} \widetilde{L}^{(N)}(p_1, p_2, \dots, p_N) &= I^{(N-1)}(p_1, p_1 + p_2, \dots, p_1 + p_2 + \dots + p_{N-1}) + \text{cyclic perms.} \\ &= \sum_{i=1}^{N} I^{(N-1)}(p_i, p_i + p_{i+1}, \dots, p_i + p_{i+1} + \dots + p_{i+N-2}) \end{aligned}$$

$$I^{(n)}(k_1, k_2, \dots, k_n) = \int_q \tilde{\delta}(q)\ \mathcal{I}^{(n)}(q; k_1, k_2, \dots, k_n) = \int_q \tilde{\delta}(q) \prod_{j=1}^{n} \frac{1}{2qk_j + k_j^2 - i0\, \eta k_j}$$

Basic dual phase-space integral

Catani et al, JHEP 09 (2008) 065

Loop-tree duality (LTD)

Summary for one-loop processes

Loop (Feynman integral)

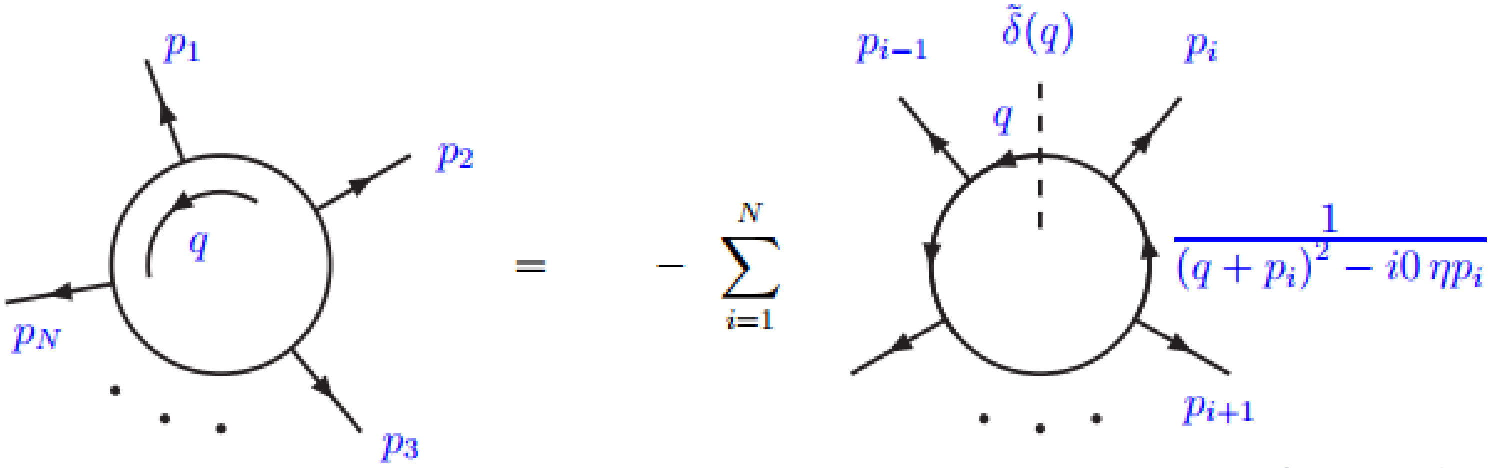
$$L^{(N)}(p_1, p_2, \dots, p_N) = - \widetilde{L}^{(N)}(p_1, p_2, \dots, p_N)$$

Dual integral

Dual integral

$$\widetilde{L}^{(N)}(p_1, p_2, \dots, p_N) = \int_q \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j \neq i}}^{N} \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}$$

Sum of phase-space integrals!

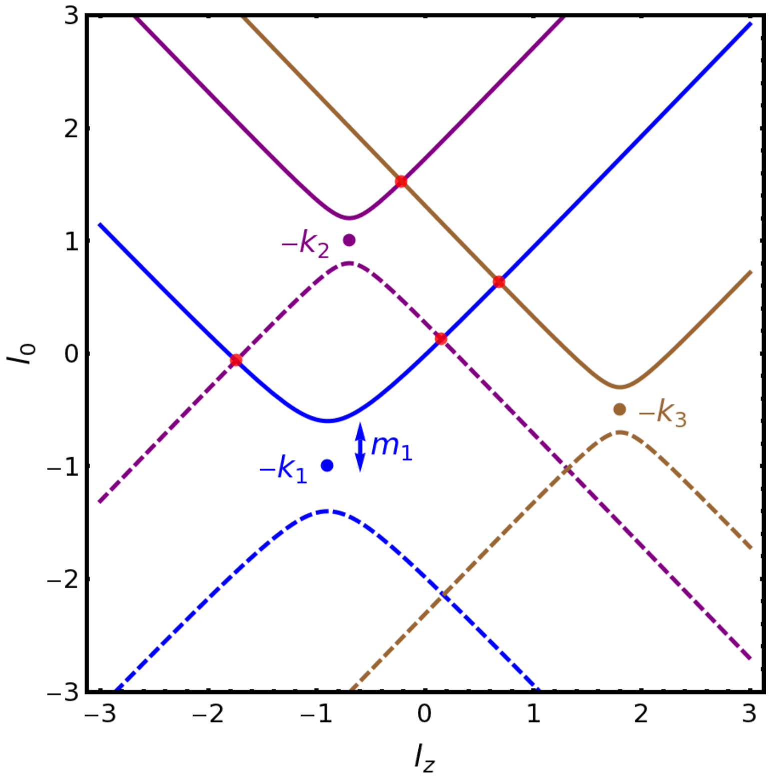


Catani et al, JHEP 09 (2008) 065

LTD at one-loop

Threshold and IR singularities

- Feynman integrands develop singularities when propagators go on-shell. LTD allows to understand it as soft/collinear divergences of real radiation.



- Forward (backward) on-shell hyperboloids associated with possitve (negative) energy solutions.

$$\begin{aligned} G_F(q_i)^{-1} &= q_i^2 - m_i^2 + \imath 0 = 0 \\ q_{i,0}^{(\pm)} &= \pm\sqrt{\vec{q}_i^{\,2} + m_i^2 \mp \imath 0} \end{aligned}$$

- LTD equivalent to integrate along the forward on-shell hyperboloids.
- Degenerate to light-cones for massless propagators.
- Dual integrands become singular at intersections (two or more on-shell propagators)

Buchta et al, JHEP 11 (2014) 014

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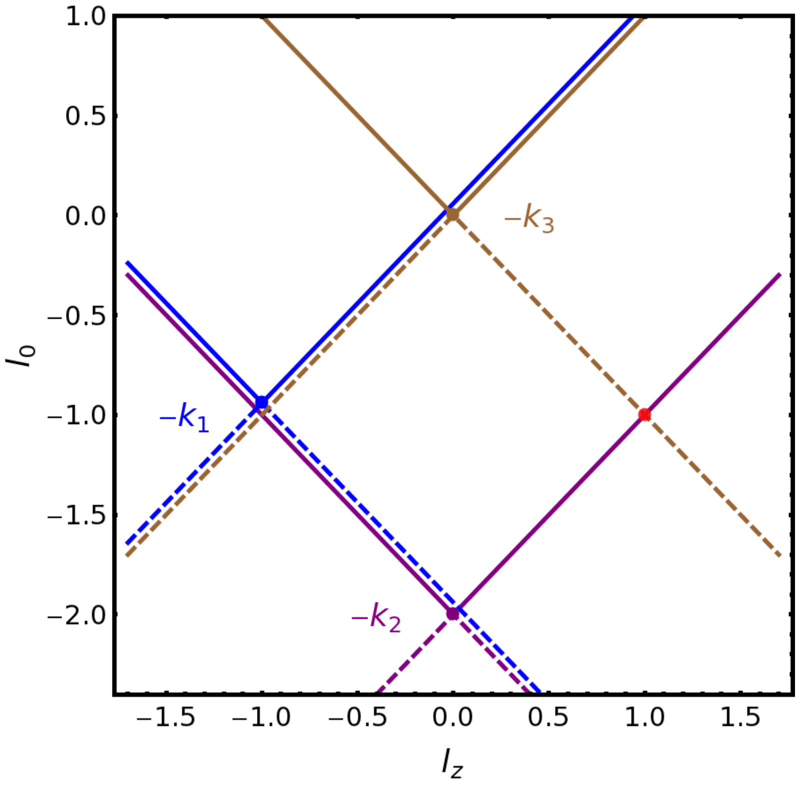

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Buchta et al, JHEP 11 (2014) 014

IR regularization in LTD

IR singularities

- *Reference example: Massless triangle in the time-like region*

$$L^{(1)}(p_1, p_2, -p_3) \; = \; \int_l \prod_{i=1}^{3} \frac{1}{q_i} = c_\Gamma \frac{\mu^{2\epsilon}}{\epsilon^2} (-s_{12} - \imath 0)^{-1-\epsilon} = \sum_{1}^{3} I_i$$

LTD

$$I_1 \; = \; - \int_\ell \frac{\tilde{\delta}(q_1)}{(2q_1 \cdot p_2 - \imath 0)(-2q_1 \cdot p_1 + \imath 0)}$$

$$I_2 \; = \; - \int_\ell \frac{\tilde{\delta}(q_2)}{(-2q_2 \cdot p_2 + \imath 0)(-2q_2 \cdot p_{12} + s_{12} + \imath 0)}$$

$$I_3 \; = \; - \int_\ell \frac{\tilde{\delta}(q_3)}{(2q_3 \cdot p_1 - \imath 0)(2q_3 \cdot p_{12} + s_{12} - \imath 0)}$$

To regularize threshold singularity

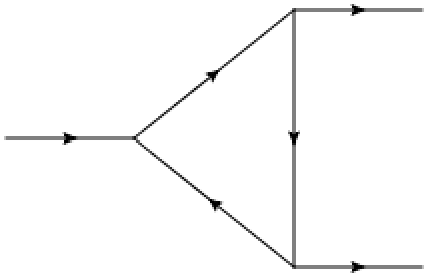
- This integral is UV-finite
- But, there are IR-singularities, associated to soft and collinear regions
- **OBJECTIVE:** Define a *IR-regularized* triangle integral by adding real corrections in 4D (i.e. no epsilon should appear at integrand level…)

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

IR singularities

- In order to analyze IR divergences and their origins, we parametrize the momenta involved in the dual representation.



$$\left\{ \begin{aligned} p_1^\mu &= \frac{\sqrt{s_{12}}}{2}(1,0,0,1) \\ p_2^\mu &= \frac{\sqrt{s_{12}}}{2}(1,0,0,-1) \\ q_i^\mu &= \xi_{i,0}\frac{\sqrt{s_{12}}}{2}\left(1, \sqrt{1-y^2}\hat{e}_T^i, y\right) \end{aligned} \right.$$

$$y \in [-1,1]$$
$$\xi_{i,0} \in [0,\infty)$$

$$\Downarrow$$

$$y = 1-2v.$$

- Then, we rewrite dual integrals by using the new variables:

$$\begin{aligned} I_1 &= \frac{1}{4s_{12}} \int d[\xi_{1,0}]\, d[v_1]\, \xi_{1,0}^{-1}\, (v_1(1-v_1))^{-1} \\ I_2 &= \frac{1}{4s_{12}} \int d[\xi_{2,0}]\, d[v_2]\, \frac{(1-v_2)^{-1}}{1-\xi_{2,0}+\imath 0} \\ I_3 &= -\frac{1}{4s_{12}} \int d[\xi_{3,0}]\, d[v_3]\, \frac{v_3^{-1}}{1+\xi_{3,0}-\imath 0} \end{aligned}$$

$$\boxed{\begin{aligned} d[\xi_{i,0}] &= 4\tilde{c}_\Gamma \left(\frac{s_{12}}{\mu^2}\right)^{-\epsilon} \xi_{i,0}^{-2\epsilon}\, d\xi_{i,0} \\ d[v_i] &= (v_i(1-v_i))^{-\epsilon}\, dv_i \end{aligned}}$$

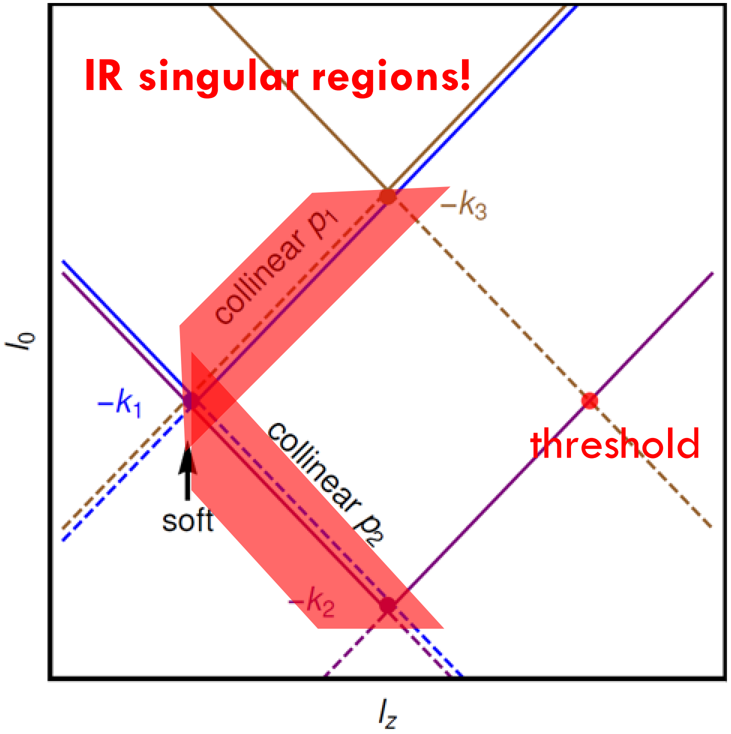
Measure in DREG (epsilon dependence comes from this…)

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

IR singularities

- ☐ Analize the integration region. Application of LTD converts loop-integrals into PS: integrate in forward light-cones.



- Only **forward-backward** interference originate **threshold or IR poles.**
- **Forward-forward** cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a compact region)
- No threshold or IR singularity at large loop momentum

- ☐ This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable, but numerically inestable…)

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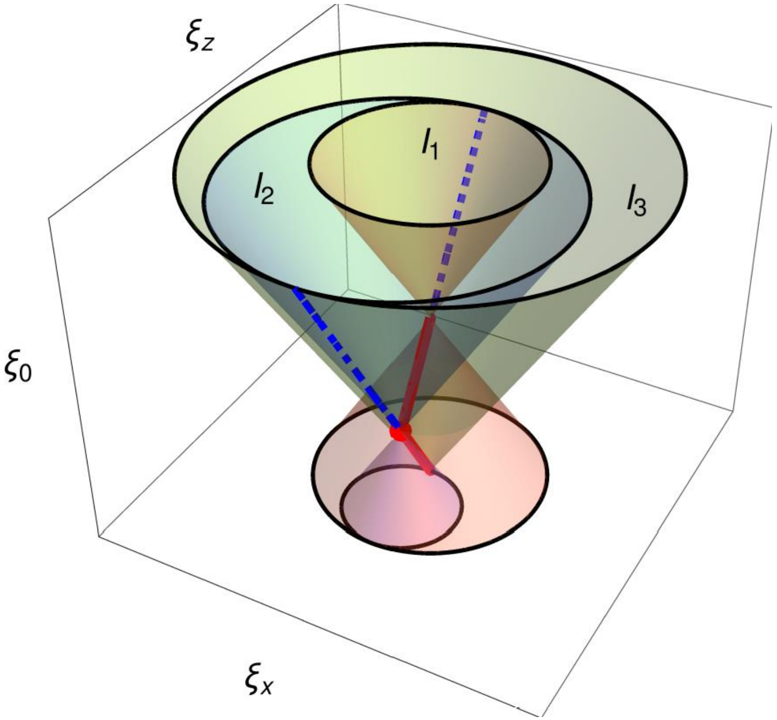

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IR regularization in LTD

IR singularities

☐ From the previous plots, we define three contributions:

IR-divergent contributions (ξ_0<1+w)

- *Originated in a **finite region** of the loop three-momentum*
- *All the IR singularities of the original loop integral*

$$I^{\mathrm{IR}} = I_1^{(\mathrm{s})} + I_1^{(\mathrm{c})} + I_2^{(\mathrm{c})} = \frac{c_\Gamma}{s_{12}} \left(\frac{-s_{12} - \imath 0}{\mu^2} \right)^{-\epsilon} \times \left[\frac{1}{\epsilon^2} + \left(\ln(2) \ln(w) - \frac{\pi^2}{3} - 2\mathrm{Li}_2\left(-\frac{1}{w}\right) + \imath\pi \ln(2) \right) \right] + \mathcal{O}(\epsilon)$$

Forward integrals (v<1/2,ξ_0>1)

- *Free of IR/UV poles*
- *Integrable in 4-dimensions!*

$$I^{(\mathrm{f})} = \sum_{i=1}^{3} I_i^{(\mathrm{f})} = c_\Gamma \frac{1}{s_{12}} \left[\frac{\pi^2}{3} - \imath\pi \log(2) \right] + \mathcal{O}(\epsilon)$$

Backward integrals (v>1/2,ξ_0>1+w)

- *Free of IR/UV poles*
- *Integrable in 4-dimensions!*

$$I^{(\mathrm{b})} = c_\Gamma \frac{1}{s_{12}} \left[2\mathrm{Li}_2\left(-\frac{1}{w}\right) - \ln(2)\ln(w) \right] + \mathcal{O}(\epsilon)$$

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

IR singularities

- Let's stop and make some remarks about the structure of these expressions:
  - Introduction of an **arbitrary cut** w to **include threshold regions.**
  - Forward and backward integrals can be performed in 4D because the sum does not contain poles.
  - Presence of extra Log's in (F) and (B) integrals. They are originated from the expansion of the measure in DREG, i.e.

$$\xi_r^{-1-2\epsilon} = -\frac{Q_S^{-2\epsilon}}{2\epsilon}\delta(\xi_r) + \left(\frac{1}{\xi_r}\right)_C - 2\epsilon\left(\frac{\ln(\xi_r)}{\xi_r}\right)_C + \mathcal{O}(\epsilon^2)$$

for both v and ξ (keep finite terms only). **It is possible to avoid them!**

  - IR-poles isolated in I^{IR}! → **IR divergences originated in compact region of the three-loop momentum!!!**

$$\underbrace{L^{(1)}(p_1,p_2,-p_3) \;=\; I^{\mathrm{IR}}}_{\textit{Explicit poles still present…}} + \underbrace{I^{(\mathrm{b})} + I^{(\mathrm{f})}}_{\textit{Can be done in 4D!}}$$

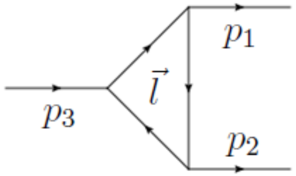
Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

Finite real+virtual integration

- Now, we must add **real** contributions. Suppose **one-loop** scalar scattering amplitude given by the triangle

Virtual

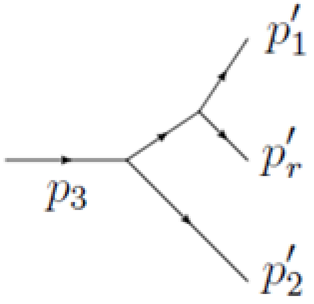


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3) \qquad \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** → **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p'_1,p'_2,p'_r;p_3)\rangle = -ig^2/s'_{ir} \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s'_{ir}\,s'_{jr}}$$

- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

IR regularization in LTD

Finite real+virtual integration

- **Mapping of momenta:** generate **1->3 real** emission kinematics (**3 external on-shell momenta**) starting from the variables available in the dual description of **1->2 virtual** contributions (**2 external on-shell momenta and 1 free three-momentum**)

$$p'^{\mu}_r = q^{\mu}_1 \qquad p'^{\mu}_1 = -q^{\mu}_3 + \alpha_1\, p^{\mu}_2 = p^{\mu}_1 - q^{\mu}_1 + \alpha_1\, p^{\mu}_2$$

$$p'^{\mu}_2 = (1-\alpha_1)\, p^{\mu}_2 \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2} \qquad q_1 = \ell + p_1$$

$$p_3 \to p_1 + p_2 \;\Rightarrow\; p_3 \to p'_1 + p'_2 + p'_r$$
$$+\ \vec{l}$$

  - Mapping optimized for $y'_{1r} < y'_{2r}$; analogous expression in the complement
  - Express interference terms using this map → **Real and virtual contributions are described using the same integration variables!**

Only required for I_1 and I_2 (I_3 singularities cancel among dual terms)

$$\tilde{\sigma}_{i,R} = \sigma_0^{-1}\, 2\mathrm{Re} \int d\Phi_{1\to 3}\, \langle \mathcal{M}^{(0)}_{2r} | \mathcal{M}^{(0)}_{1r} \rangle\, \theta(y'_{jr} - y'_{ir})$$

$$\tilde{\sigma}_{i,V} = \sigma_0^{-1}\, 2\mathrm{Re} \int d\Phi_{1\to 2}\, \langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)}_{i} \rangle\, \theta(y'_{jr} - y'_{ir})$$

$$\tilde{\sigma}_1 = \tilde{\sigma}_{1,V} + \tilde{\sigma}_{1,R} = \mathcal{O}(\epsilon)$$

$$\tilde{\sigma}_2 = \tilde{\sigma}_{2,V} + \tilde{\sigma}_{2,R} = -c_\Gamma \frac{g^2}{s_{12}} \frac{\pi^2}{6} + \mathcal{O}(\epsilon)$$

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

UV renormalization in LTD

UV singularities

- *Reference example: bubble with massless propagators*

$$L^{(1)}(p,-p) = \int_\ell \prod_{i=1}^{2} G_F(q_i) = \frac{c_\Gamma}{\epsilon\,(1-2\epsilon)} \left(\frac{-p^2 - \imath 0}{\mu^2} \right)^{-\epsilon} = \sum_{i=1}^{2} I_i$$

LTD

$$I_1 = -\int_\ell \tilde{\delta}\left(q_1\right) G_D(q_2; q_1) = -\int_\ell \frac{\tilde{\delta}\left(q_1\right)}{p^2 - 2\, q_1 \cdot p + \imath 0}$$

$$I_2 = -\int_\ell \tilde{\delta}\left(q_2\right) G_D(q_1; q_2) = -\int_\ell \frac{\tilde{\delta}\left(q_2\right)}{p^2 + 2\, q_2 \cdot p - \imath 0}$$

To regularize threshold singularity

- In this case, the integration regions of dual integrals are two energy-displaced forward light-cones. This integral contains UV poles only
- **OBJETIVE:** Define a *UV-regularized* triangle integral by adding unintegrated UV counter-terms, and find a purely 4-dimensional integral that represents this regularized bubble!

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

UV renormalization in LTD

UV counter-term

- Divergences arise from the high-energy region (UV poles) and must be cancelled with a suitable renormalization counter-term. For the scalar case, we use

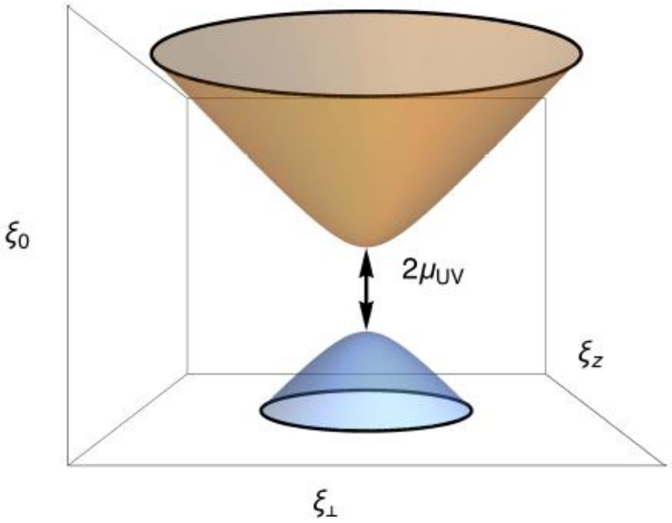
$$I_{\mathrm{UV}}^{\mathrm{cnt}} = \int_\ell \frac{1}{\left(q_{\mathrm{UV}}^2 - \mu_{\mathrm{UV}}^2 + \imath 0\right)^2}$$

Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

- Dual representation (**new: double poles in the loop energy**)

$$I_{\mathrm{UV}}^{\mathrm{cnt}} = \int_\ell \frac{\tilde{\delta}\left(q_{\mathrm{UV}}\right)}{2\left(q_{\mathrm{UV},0}^{(+)}\right)^2} \qquad q_{\mathrm{UV},0}^{(+)} = \sqrt{\mathbf{q}_{\mathrm{UV}}^2 + \mu_{\mathrm{UV}}^2 - \imath 0}$$

Bierenbaum et al., JHEP 03 (2013) 025



- Loop integration for loop energies larger than μ_{UV}

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

UV renormalization in LTD

Cancellation of UV singularities

- Using the standard parametrization we define

Regularized bubble

$$L^{(1)}(p,-p) - I_{\rm UV}^{\rm cnt} = c_\Gamma \left[-\log\left(-\frac{p^2}{\mu_{\rm UV}^2} - i0\right) + 2\right] + \mathcal{O}(\epsilon)$$

- Since it is finite, we can express the regularized bubble in terms of 4-dimensional quantities (i.e. no epsilon required!!)

- ***Physical interpretation of renormalization scales***: Separation between on-shell hyperboloids in UV-counterterm is $2\mu_{\rm UV}$. To avoid intersections with forward light-cones associated with l_1 and l_2, the renormalization scale has to be larger or of the order of the hard scale. So, the minimal choice that fulfills this agrees with the standard choice (i.e. ½ of *the hard scale*).

Hernández-Pinto, Rodrigo and GS, arXiv:2015.04617 [hep-ph], Rodrigo et al, in preparation

Conclusions

- Introduced new method based on the Loop-Tree Duality (LTD) that allows to treat **virtual** and **real** contributions in the **same way:** simultaneous implementation and no need of IR subtraction
- Physical interpretation of **IR/UV singularities** in loop integrals
- **Presented proof of concept of LTD with reference examples**

- **Perspectives:**
  - Apply the technique to compute full NLO physical observables
  - Extend the procedure to higher orders: NNLO and beyond

Thanks!!!