$W^+W^- +$ jet – compact analytic results

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based on

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J. Campbell, D. Miller, TR
(arXiv:1506.xxxxx)

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WW + jet: Motivation from experiment

WW [+jet(s)] at the LHC

- Measurement of **WW production cross section** [e.g. ATLAS, JHEP01(2015)049; CMS, Phys. Lett. B 721 (2013)]
- $h \to W\,W$ measurement [e.g. ATLAS, arXiv:1503.01060; CMS, JHEP01 (2014) 096]
- **spin-/ parity determination** of Higgs [e.g. ATLAS, EPJC75 (2015) 231; CMS, arXiv:1411.3441]
- limits on **anomalous couplings** [e.g. ATLAS, Phys. Rev. D 87, 112001 (2013); CMS, arXiv: 1411.3441]
- background for **BSM searches** (e.g. heavy scalars) [e.g. ATLAS, ATLAS-CONF-2013-067; CMS, arXiv:1504.00936]
- ...
- **K-factors** $\sim 1.2 - 1.8$ [depending on analysis details, cuts, etc...]

[listed are most recent publications]

in more detail...

Process we are interested in

$$p\,p \to W^+ W^- \textbf{jet} \to (\ell \bar{\nu}_\ell)\ (\bar{\ell}'\,\nu_{\ell'})\ \textbf{jet}$$

at NLO, offshell W's, spin correlations

omitting g g induced contributions

obviously, not the first calculation...

- **previous results:** Campbell, Ellis, Zanderighi [CEZ] (2007); Dittmaier, Kallweit, Uwer [DKU] (2008/ 2010), Sanguinetti/ Karg [BGKKS] (2008)
- **together with shower merging/ matching:** Cascioli ea (2014) in Sherpa/ OpenLoops framework
- also **"ad hoc" available from automatized tools** (personally tested: MG5/aMC@NLO, others probably similar...)

Why (yet) another calculation ??

- Main motivation: want to have a **fast and stable code**

$\Rightarrow$ important as **ingredient for NNLO calculations**

$\Rightarrow$ a lot of (recent) progress here, Chachamis ea (2008), Gehrmann ea (2014/ 2015), von Manteuffel, Tancredi (2015), Caola ea (2015),...]

$\Rightarrow$ our approach: use **unitarity-based techniques**, derive **completely analytic expressions**

- tool/ user-interface: $\Rightarrow$ **implementation in MCFM**

Unitarity methods: a brief recap

Unitary methods: basic idea

$$\mathcal{A}(\{p_i\}) = \sum_j d_j I_4^j + \sum_j c_j I_3^j + \sum_j b_j I_2^j + R \,.$$

$\Rightarrow$ know that all one-loop calculations can be reduced to **integral basis, + rational terms** [Passarino, Veltman, '78]

$\Rightarrow$ idea: **project out coefficients in front of basis integrals by putting momenta in the loop on mass shell**

(Bern, Dixon, Dunbar, Kosower ('94); Britto, Cachazo, Feng ('04))

- putting $2/3/4$ particles on their mass shell projects out coefficients of a **bubble/ triangle/ box** contribution

Other (related) (reduction) methods and implementations

- purely analytic: **generalized unitarity** [Britto, (Buchbinder), Cachazo, Feng (2005, 2006); Britto, Feng, Mastrolia (2006), Forde (2007), Badger (2009), Mastrolia (2009), ...]
- other widely used approach: **reduction on the amplitude level** [del Aguila, Pitta (2004); Ossola, Papadopoulos, Pittau (2006)]
- **implemented** in many (publicly available) codes: CutTools (OPP, 2007), Samurai (Mastrolia ea, 2010), Gosam (Cullen ea, 2011), MadLoop/aMC@NLO (Hirschi ea, 2011, Frederix ea, 2011), Helac-NLO (van Hameren ea, 2010; Bevilacqua ea, 2013)
- **many other important generic NLO codes and tools,** build around reduction/ recursion principles: Blackhat (Berger ea, 2008), Rocket (Giele, Zanderighi, 2008), Golem (Binoth ea, 2008) NGluon (Badger ea, 2011), OpenLoops (Cascioli ea, 2011), PJFry (Fleischer ea, 2011), Collier (Denner ea, 2014), Ninja (Peraro, 2014),...

Unitarity methods: purely analytic approaches

- as you start with a $d(4)$-dimensional loop integral, **cutting 4 legs is easier than cutting 2**
- **boxes** $\Rightarrow$ straightforward, using **quadruple cuts with complex momenta (BCF)**
- **triangles** $\Rightarrow$ relatively straightforward, using **Fordes method**
- **bubbles** $\Rightarrow$ can get quite complicated, use **spinor integration (BBCF)**
- **rational parts** $\Rightarrow$ long but OK, use **effective mass term (Badger)**

Previous NLO calculations in the SM using analytic expressions from unitarity methods in MCFM

... on the amplitude level ...

- $e\,e \rightarrow 4$ **quarks**: Bern, Dixon, Kosower, Weinzierl (1996); Bern, Dixon, Kosower (1997)
- **Higgs and four partons** (in various configurations): Dixon, Sofianatos (2009); Badger, Glover, Mastrolia, Williams (2009); Badger, Campbell, Ellis, Williams (2009)
- $t\,\bar{t}$ **production**: Badger, Sattler, Yundin (2011)

... generalized unitarity implemented ...

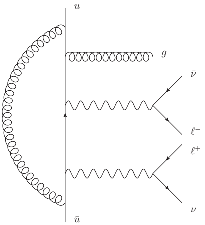
- **Higgs + 2 jets** Campbell, Ellis, Williams (2010)
- **W + 2 b-jets** Badger, Campbell, Ellis (2011)
- $g\,g \rightarrow W\,W$ Campbell, Ellis, Williams (2011, 2014)
- $\gamma\gamma\gamma$ Campbell, Williams (2014)
- $\gamma\gamma\gamma\gamma$ Dennen, Williams (2014)

WWj @ NLO in more detail

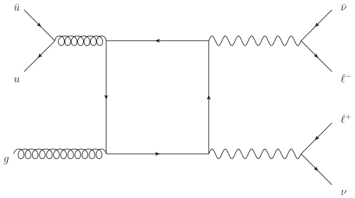
We consider

$$q\,\bar{q} \;\rightarrow\; W^+\,W^-\,g \quad [+\text{ permutations}]$$

diagram classes



(a)



(b)

[+ diagrams for $q\,\bar{q} \;\rightarrow\; (Z/\gamma)\,g \;\rightarrow\; ...$]

WWj @ NLO w/ unitarity: complexity

	(a)	(b)
boxes	13	1
triangles	8	4
bubbles	18	2
rational	13	5

Table : **Number of independent** (via singularity structure and/ or symmetries) **coefficients** [neglecting contributions from Z/γ current]

- involving **1,2,3-mass boxes and triangles**,
- **bubbles: 16 different underlying structures**, involving (0/1/2) quadratic poles, e.g. [terms before spinor integration]

$$\frac{[\ell a]^2\,[\ell b]\,[\ell c]}{[\ell d][\ell e]\,\langle\ell|P|\ell]^4},\quad \frac{[\ell a]\,[\ell b]\,[\ell c]}{\langle\ell|P|\ell]^4\,\langle\ell|Q|\ell]},\quad \frac{[\ell a]\,[\ell b]\,[\ell c]\,[\ell d]}{\langle\ell|P|\ell]^4\,\langle\ell|Q|\ell]\,\langle\ell|Q_2|\ell]},\quad \ldots$$

Coefficients: three-massive box

$$d_4\,(s_{56}, s_{34}, 0, s_{17}; s_{127}, s_{234}) = \frac{1}{s_{34}-m_W^2}\,\frac{1}{s_{56}-m_W^2}\,\frac{\langle 12\rangle^2\,[2|P|2\rangle}{\langle 27\rangle\,\langle 17\rangle}\times$$
$$\left([42]-\frac{\langle 2|P|4]}{D_1}\right)\left(\langle 3|2+4|6]-\frac{\langle 23\rangle\langle 2|P|6]}{D_1}\right)\left(\frac{[71]\langle 15\rangle}{\langle 2|P|7]}+\frac{\langle 25\rangle}{D_1}\right)$$

$$P \;=\; s_{17}\,p_{34}+s_{234}\,p_{17},\; D_2 \;=\; [2|(3+4)\,(1+7)|2],\; D_1 \;=\; \langle 2|(3+4)\,(1+7)|2\rangle.$$

- in principle: also contributions with second denimonator $D_2 \;=\; [2|(3+4)\,(1+7)|2]$ (here: $=0$)
- $D_1\,D_2 \;\sim\;$ Gram determinant

Coefficients: easiest bubble and triangle

$$\mathbf{I}_2^{\mathbf{LC}}(\mathbf{s_{156}}) \sim$$

$$\frac{\langle 65\rangle\,[43]}{\langle 27\rangle} \times \left\{ \frac{\langle 73\rangle^2\,\langle 7|P|6]\,[76]}{\langle 7|P|1]\,\langle 7|P|7]} \left[\frac{1}{\langle 7|P|7]} \left(\frac{[7|P|3\rangle}{\langle 37\rangle} + \frac{[76]\,s_{156}}{2\,\langle 7|P|6]} \right) + \frac{[1|P|3\rangle}{\langle 37\rangle\,[1|P|7\rangle} \right. \right.$$

$$\left. \left. - \frac{\langle 15\rangle[56]\,[1|P|3\rangle^2}{s_{156}\,\langle 1|P|1]\,[1|P|7\rangle} \left[\frac{\langle 15\rangle[56]}{2\,\langle 1|P|1]} + \frac{\langle 7|P|6]}{[1|P|7\rangle} \right] \right\}, \quad P = p_{156}$$

$$\mathbf{I}_3^{\mathbf{LC}}(\mathbf{s_{34}}, \mathbf{s_{27}}, \mathbf{s_{156}}) \sim \frac{1}{2} \sum_{\gamma=\gamma_{1,2}} \frac{s_{27}[4K_2^\flat][72][65]\langle K_1^\flat 2\rangle\langle K_1^\flat 3\rangle\langle 15\rangle^2}{(\gamma - s_{27})\,[7K_2^\flat]\,\langle K_1^\flat 1\rangle\langle K_1^\flat 7\rangle\,\langle 27\rangle}$$

where

$$K_1^\flat = \frac{\gamma\,[\gamma\,p_{27} + s_{27}\,p_{34}]}{\gamma^2 - s_{27}\,s_{34}}, \quad K_2^\flat = -\frac{\gamma\,[\gamma\,p_{34} + s_{34}\,p_{27}]}{\gamma^2 - s_{27}\,s_{34}},$$

$$\gamma_{1,2} = p_{27}\cdot p_{34} \pm \sqrt{(p_{27}\cdot p_{34})^2 - s_{27}\,s_{34}}$$

Implementation: in practise

⇒ **fully implemented in MCFM framework**, i.e. in combination with Born, real radiation, ...

⇒ **MCFM output** (distributions/ cuts implementation/ interfaces/ etc...)

- in practise: handling of expressions ⇒ S@M [Maitre, Mastrolia, 2007]

  [comment: also implemented in multi-core version [Campbell, Ellis, Giele, 2015]]

Implementation: cross checks

Cross checks

- on the **amplitude as well as coefficient level**, i.e. for several ($\sim 20-30$) single phase space points against code using D-dimensional unitarity (Ellis, Giele, Kunszt, Melnikov, 2009)
- for a **single phase space point as well as total cross section** against comparison in Les Houches proceedings, arXiv:0803.0494 (comparison CEZ, DKU, BGKKS)
- for the latter, also **independent MG5/aMC@NLO run**

overall agreement:
amplitude/ coefficient level: 10^{-6} **or better**
cross section level: always within integration errors
✓

Phenomenology

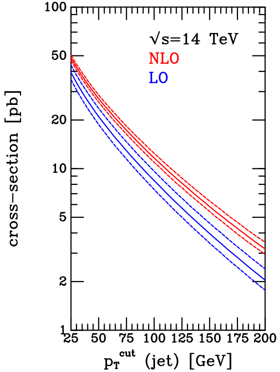
- **total cross section** as a function of $p_{T,\text{jet}}^{\text{cut}}$ for pp collisions **@ 13/ 14/ 100 TeV**
- **differential distributions,** including more specific cuts
- ... for **spin- parity determination of Higgs @ 14 TeV**
- ... for **searches of extra heavy scalars @ 100 TeV**

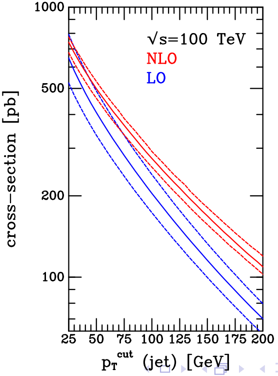
jet definitions: anti-k_T, $p_T^{jet} > 25$ GeV , $|\eta^{jet}| < 4.5$, $R = 0.5$

scales: $\mu_R = \mu_F = \frac{1}{2} \sum_i p_T^i$

Phenomenology: total cross sections, as function of $p_{T,\text{jet}}^{\text{cut}}$

$\sqrt{s}$	σ_{LO} [pb]	σ_{NLO} [pb]
13 TeV	34.9 (-11.0%, +11.4%)	42.9 (-3.7%, +3.7%)
14 TeV	39.5 (-11.0%, +11.7%)	48.6 (-4.0%, +3.8%)
100 TeV	648 (-23.8%, +22.3%)	740 (-9.3%, +4.5%)





More phenomenology: specific studies as background

- e.g. **spin/ parity determination of SM Higgs** (ATLAS, 1503.03643) $\Rightarrow$ @ 14 TeV
- e.g. **searches for additional scalars at high masses** (CMS, 1504.00936) $\Rightarrow$ @ 100 TeV

with cuts roughly following above studies...

Results

order	cm energy	no cuts	K	cuts	K
LO	14 TeV	462.0(2)fb		67.12(4)fb	
NLO	14 TeV	568.4(2)fb	1.23	83.91(5)	1.25
LO	100 TeV	6815(1)fb		1237(1)fb	
NLO	100 TeV	7939(5)	1.16	1471(1)fb	1.19

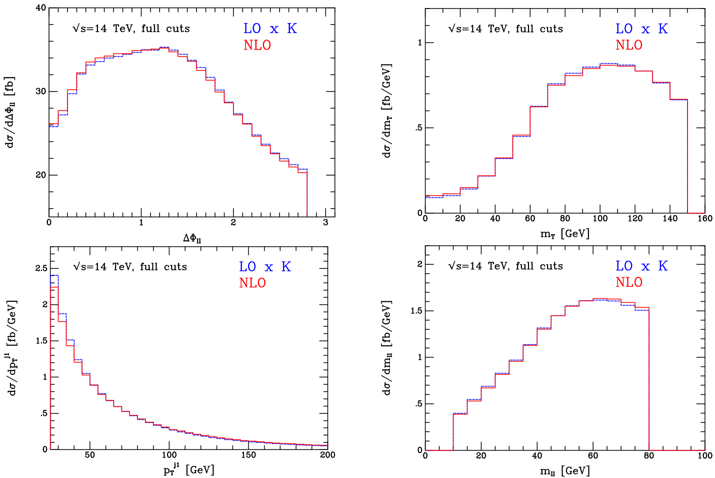
Cuts

variable	14 TeVanalysis	100 TeVanalysis
$p_{\perp,j}$	> 25 GeV	30 GeV
$\lvert\eta_j\rvert$	< 4.5	4.5
η_ℓ	≤ 2.5	2.5
$p_{\perp,\ell_1}$	> 22 GeV	50 GeV
$p_{\perp,\ell_2}$	> 15 GeV	10 GeV
$m_{\ell\ell}$	$\in [10; 80]\,\mathrm{GeV}$	–
$p_\perp^{\text{miss}}$	> 20 GeV	20 GeV
$\Delta\Phi_{\ell\ell}$	< 2.8	
m_T	≤ 150 GeV	$\geq 80\mathrm{GeV}$
$\max[m_T^{\ell_1}, m_T^{\ell_2}]$	> 50 GeV	–

$$\left[m_T^2 \;=\; 2\, p_T^{\ell\ell}\, E_T^{\text{miss}} \left(1 - \cos \Delta\Phi(\vec{p}_T^{\,\ell\ell}, \vec{E}_T^{\,\text{miss}})\right)\right]$$

Results for 14 TeV after all cuts

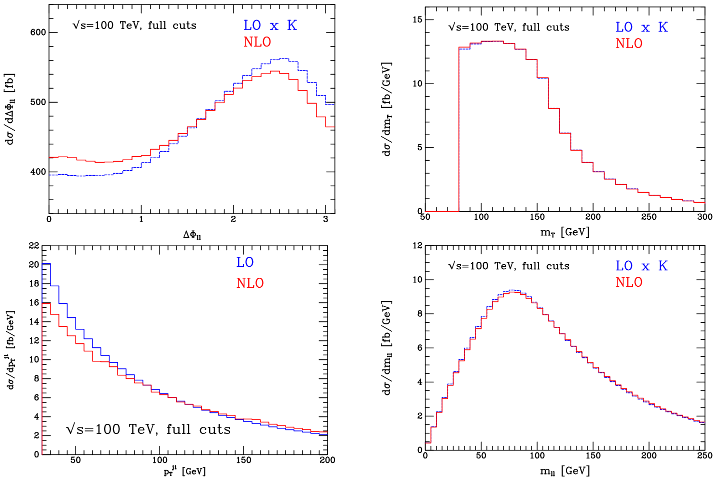
$$\sigma_{\mathrm{LO}} = 67.128(30)\mathrm{fb}; \; \sigma_{\mathrm{NLO}} = 83.923(47)\mathrm{fb}$$



[plot: $\sigma_{\mathrm{LO}} \times K$]

Results for 100 TeV after all cuts

$$\sigma_{\text{LO}} = 1237.2(4)\text{fb}; \; \sigma_{\text{NLO}} = 1472.0(7)\text{fb}$$



[plot: $\sigma_{\text{LO}} \times K$]

One slide of self-commercial

- calculation of **bubble-coefficients: process-independent**

$\Rightarrow$ **mathematica-based library,** with (all) librarised poles for $\sim$ 20 different structures

$\Rightarrow$ can **apply these completely straightforward to any other calculation** where the same structures appear in bubble (and can also obviously extend this)

- current interface: **me**
- future plan: **make public** in librarized format

all tested (✓)

but obviously not every possible structure available at the moment

Summary and outlook

$$q\,\bar{q} \rightarrow W^+ W^- g$$

available and implemented in MCFM, running, rendering stable results

- virtual contributions: calculated using **unitarity methods** $\Rightarrow$ **available in analytic format**

$\Rightarrow$ **extensively tested** on coefficient, amplitude, and cross section level ✓

$\Rightarrow$ **important ingredient for NNLO calculations**, ready to be used

$\Rightarrow$ obviously, **similarly useful for stand alone NLO calculations**

- provided sample applications for typical **Higgs spin/ parity studies @ 14 TeV, heavy scalar searches @ 100 TeVpp colliders**

$\Longrightarrow$ **Thanks for listening** $\Longleftarrow$

Appendix

Basis integrals, type (a)

$D^{(1)}$	$I_4(0,0,s_{56},s_{234};s_{17},s_{156})$
$D^{(2)}$	$I_4(s_{34},s_{56},0,s_{27};s_{127},s_{156})$
$D^{(3)}$	$I_4(0,0,s_{34},s_{156};s_{27},s_{234})$
$D^{(4)}$	$I_4(s_{56},s_{34},0,s_{17};s_{127},s_{234})$
$D^{(5)}$	$I_4(0,0,0,s_{127};s_{17},s_{27})$
$C^{(1)}$	$I_3(0,s_{234},s_{156})$
$C^{(2)}$	$I_3(s_{56},s_{17},s_{234})$
$C^{(3)}$	$I_3(s_{34},s_{27},s_{156})$
$C^{(4)}$	$I_3(0,s_{27},s_{127})$
$C^{(5)}$	$I_3(0,s_{17},s_{127})$
$C^{(6)}$	$I_3(0,0,s_{17})$
$C^{(7)}$	$I_3(0,s_{56},s_{156})$
$C^{(8)}$	$I_3(s_{56},s_{34},s_{127})$
$C^{(9)}$	$I_3(0,0,s_{27})$
$C^{(10)}$	$I_3(0,s_{34},s_{234})$
$B^{(1)}$	$I_2(s_{156})$
$B^{(2)}$	$I_2(s_{234})$
$B^{(3)}$	$I_2(s_{56})$
$B^{(4)}$	$I_2(s_{17})$
$B^{(5)}$	$I_2(s_{34})$
$B^{(6)}$	$I_2(s_{127})$

$D^{(6)}$	$I_4(s_{56},s_{34},0,s_{17};s_{127},s_{234})$
$D^{(7)}$	$I_4(0,0,0,s_{127};s_{27},s_{12})$
$D^{(8)}$	$I_4(0,0,0,s_{127};s_{17},s_{12})$
$D^{(9)}$	$I_4(s_{34},0,0,s_{567};s_{234},s_{12})$
$D^{(10)}$	$I_4(s_{34},s_{12},s_{56},0;s_{567},s_{347})$
$D^{(11)}$	$I_4(0,0,s_{56},s_{347};s_{12},s_{156})$
$D^{(12)}$	$I_4(s_{34},s_{12},0,s_{56};s_{567},s_{127})$
$D^{(13)}$	$I_4(0,s_{12},s_{56},s_{34};s_{127},s_{347})$
$C^{(11)}$	$I_3(0,0,s_{27})$
$C^{(12)}$	$I_3(0,s_{34},s_{234})$
$C^{(13)}$	$I_3(0,s_{34},s_{347})$
$C^{(14)}$	$I_3(0,s_{347},s_{156})$
$C^{(15)}$	$I_3(0,s_{127},s_{12})$
$C^{(16)}$	$I_3(0,0,s_{12})$
$C^{(17)}$	$I_3(s_{34},s_{567},s_{12})$
$C^{(18)}$	$I_3(s_{56},s_{347},s_{12})$
$B^{(7)}$	$I_2(s_{567})$
$B^{(8)}$	$I_2(s_{347})$
$B^{(9)}$	$I_2(s_{12})$

Table : scalar integrals of type (a) *left* leading colour and *right* additional subleading color amplitude.

Basis integrals, type (b)

$D^{(10)}$	$I_4(s_{34}, s_{12}, s_{56}, 0; s_{567}, s_{347})$
$D^{(12)}$	$I_4(s_{34}, s_{12}, 0, s_{56}; s_{567}, s_{127})$
$D^{(13)}$	$I_4(0, s_{12}, s_{56}, s_{34}; s_{127}, s_{347})$
$C^{(8)}$	$I_3(0, s_{56}, s_{567})$
$C^{(10)}$	$I_3(s_{56}, s_{34}, s_{127})$
$C^{(13)}$	$I_3(0, s_{34}, s_{347})$
$C^{(15)}$	$I_3(0, s_{127}, s_{12})$
$C^{(17)}$	$I_3(s_{34}, s_{567}, s_{12})$
$C^{(18)}$	$I_3(s_{56}, s_{347}, s_{12})$
$B^{(3)}$	$I_2(s_{56})$
$B^{(5)}$	$I_2(s_{34})$
$B^{(6)}$	$I_2(s_{127})$
$B^{(7)}$	$I_2(s_{567})$
$B^{(8)}$	$I_2(s_{347})$
$B^{(9)}$	$I_2(s_{12})$

Table : Definitions of the scalar integrals that appear in the calculation of the diagrams of type (b), leading colour only.

Appearance of quadratic poles/ square roots

- have a cut leading to a propagator $\sim \frac{1}{s_{\ell_1\,p}}$, where $p^2 \neq 0$ (e.g. $p = p_1 + p_2$)
- for spinor integration $\ell_1 \rightarrow \frac{P^2}{[\ell|P|\ell\rangle}\,\ell$, where P is momentum over the cut
- i.e., use

$$s_{\ell_1,p} = \langle \ell_1|\not{p}|\ell_1] + p^2 \rightarrow \frac{P^2\,\langle \ell|\not{p}|\ell]}{\langle \ell|\not{P}|\ell]} + p^2 = \frac{\langle \ell|\not{Q}|\ell]}{\langle \ell|\not{P}|\ell]},$$

- contributions often appear together with factors $\sim \frac{1}{[\ell|P|\ell\rangle}$

$\Rightarrow$ contains poles $\sim \frac{1}{\langle \ell|PQ\ell\rangle}$

- leads to two possible solutions for $|\ell\rangle$ where $\langle \ell|PQ|\ell\rangle = 0$ (pole)

Electroweak parameters

m_W	80.385 GeV	Γ_W	2.085 GeV
m_Z	91.1876 GeV	Γ_Z	2.4952 GeV
e^2	0.095032	g_W^2	0.42635
$\sin^2\theta_W$	0.22290	G_F	0.116638×10^{-4}

Effect of neglecting diagrams containing Higgs/ diagrams with top loops

- im MG5/aMC@NLO: top and Higgs included
- check: run with $m_t\, m_H \times 10\,(100)$
- results

calculation	parameters	σ^{NLO} [pb]
MCFM	default	14.571 (18)
MG5	default	14.547 (19)
MG5	$m_h \times 10, m_t \times 10$	14.615 (21)
MG5	$m_h \times 100, m_t \times 100$	14.563 (19)
DKU	default	14.678 (10)

Other (related) (reduction) methods [non-exhaustive listing]

- purely analytic: **generalized unitarity** [Britto, (Buchbinder), Cachazo, Feng (2005, 2006); Britto, Feng, Mastrolia (2006), Forde (2007), Badger (2009), Mastrolia (2009), ...]
- other approaches: **recursion/ reduction methods** [Berends, Giele (1987); del Aguila, Pitta (2004); Ossola, Papadopoulos, Pittau (2006)]
- **numerical implementions** in many (publicly available) codes: (in order of appearance) CutTools (OPP, 2007), Samurai (Mastrolia ea, 2010), Gosam (Cullen ea, 2011), MadLoop/aMC@NLO (Hirschi ea, 2011, Frederix ea, 2011), Helac-NLO (van Hameren ea, 2010; Bevilacqua ea, 2013)
- other numerical implementations (in order of appearance): Blackhat (Berger ea, 2008), Rocket (Giele, Zanderighi, 2008), NGluon (Badger ea, 2011), OpenLoops (Cascioli ea, 2011), Ninja (Peraro, 2014)