The NNLO counterterm contributions to $\mathcal{B}(\bar{B} \to X_s \gamma)$ for an arbitrary charm quark mass

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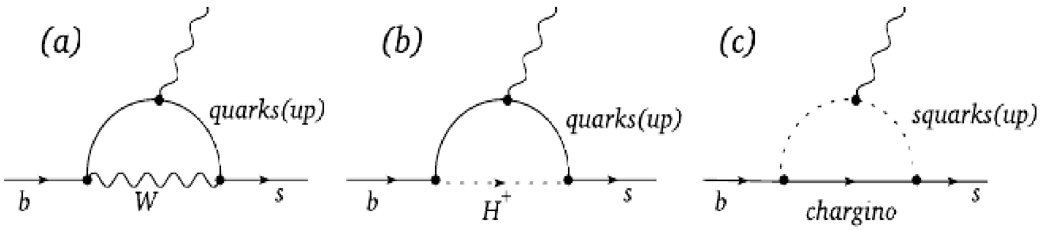
- **Introduction: radiative B-decays**
- **Interpolation in the charm quark mass**
- **NNLO counterterms: no interpolation**
- **Outlook of bare NNLO calculations**
- **Summary**

Motivation

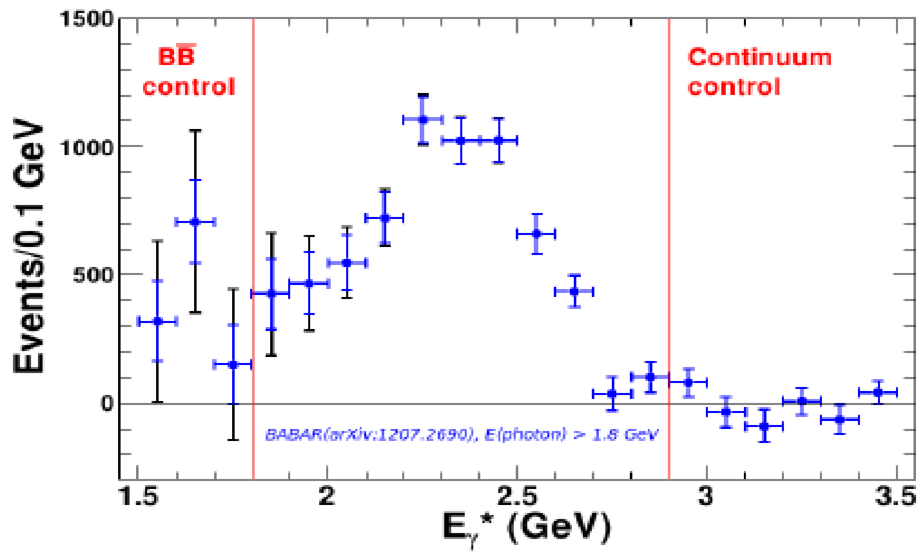
FCNC process $\Rightarrow$ bounds on beyond-SM physics



Photon energy spectrum in inclusive measurements

First observation of $b \to s\gamma$ penguin mediated by CLEO 1994.

BABAR [arXiv:1207.2690] ; $E_\gamma > 1.8$ GeV



Peak is around $m_b/2 \sim 2.35$ GeV.

Background grows for smaller E_0.

Similar data from Belle with $E_\gamma > 1.7$ GeV [arXiv:0907.1384].

Belle-II

Better accuracy is expected using hadronic tagging.

Recent bounds on $M_{H^\pm}$ [arXiv:1503.01789] in 2HDM II

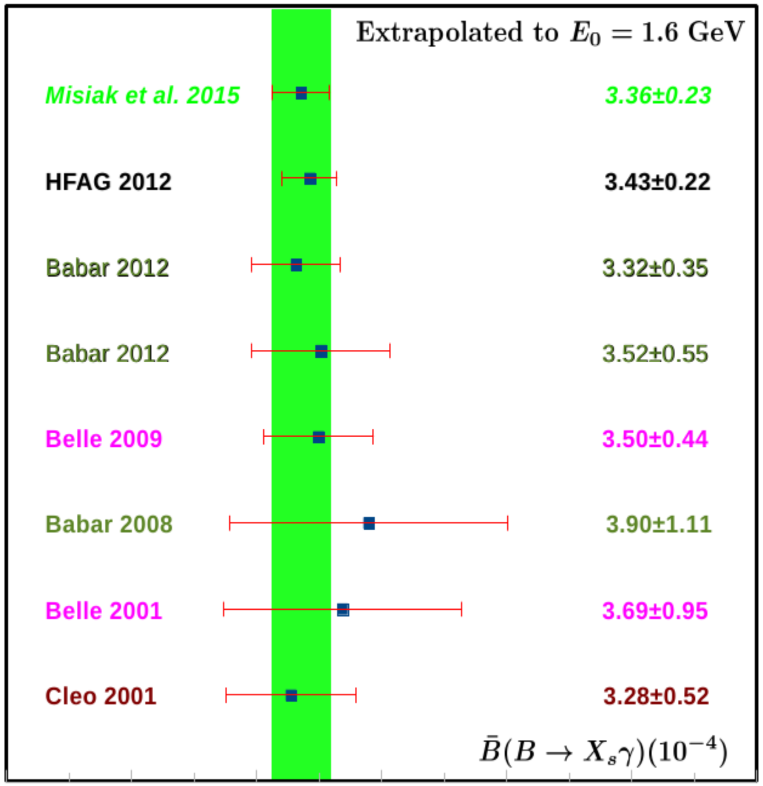
$$M_{H^\pm} > 480 \text{ GeV at } 95\%\text{C.L.}$$

$$M_{H^\pm} > 358 \text{ GeV at } 99\%\text{C.L.}$$

Branching fraction; HFAG [arXiv:1412.7515]

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{Exp}_{E_\gamma > 1.6\text{GeV}} = (3.43 \pm 0.21 \pm 0.07) \cdot 10^{-4}$$

uncertainity 6.5%



Theoretical framework

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{8} C_i(\mu_b)\mathcal{Q}_i$$

Higher-order EW and/or CKM-suppressed effects ($|V_{ub}V_{us}^*/V_{tb}V_{ts}^*| < 0.02$) bring other operators.

Chetyrkin, Misiak, Münz, 1996

$$\mathcal{Q}_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L)$$
$$\mathcal{Q}_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$
$$\mathcal{Q}_7 = \frac{e}{16\pi^2} m_b\,(\bar{s}_L\sigma_{\mu\nu}b_R)\,F^{\mu\nu}$$
$$\mathcal{Q}_8 = \frac{g}{16\pi^2} m_b\,(\bar{s}_L\sigma_{\mu\nu}T^a b_R)\,G^{a\mu\nu}$$

$$\mathcal{Q}_3 = (\bar{s}_L\gamma_\mu b_L)\sum\nolimits_q(\bar{q}\gamma^\mu q)$$
$$\mathcal{Q}_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum\nolimits_q(\bar{q}\gamma^\mu T^a q)$$
$$\mathcal{Q}_5 = (\bar{s}_L\gamma_{\mu 1}\gamma_{\mu 2}\gamma_{\mu 3} b_L)\sum\nolimits_q(\bar{q}\gamma^{\mu 1}\gamma^{\mu 2}\gamma^{\mu 3} q)$$
$$\mathcal{Q}_6 = (\bar{s}_L\gamma_{\mu 1}\gamma_{\mu 2}\gamma_{\mu 3} T^a b_L)\sum\nolimits_q(\bar{q}\gamma^{\mu 1}\gamma^{\mu 2}\gamma^{\mu 3} T^a q)$$

current-current	photonic dipole	gluonic dipole	penguin
$\mathcal{Q}_{1,2}$	$\mathcal{Q}_7$	$\mathcal{Q}_8$	$\mathcal{Q}_{3,4,5,6}$
$C_{1,2}(m_b) \sim 1$	$C_7(m_b) \sim -0.3$	$C_8(m_b) \sim -0.15$	$C_{3,4,5,6}(m_b) \sim 0.07$

$$|C_7| : |C_{1,2}| : |C_8| \simeq 1 : 3 : 1/2$$

Branching ratio at NNLO

The matrix elements can be effectively evaluated in perturbation theory

$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} + \begin{pmatrix} \text{Non-perturbative} \\ \sim (\pm 5)\% \\ \text{arXiv} : 1003.5012 \end{pmatrix}$$

$b \in \bar{B} = \bar{B}^0 \text{or} B^-$

Provided that E_0 is large ($\sim m_b/2$) but not close to endpoint ($m_b - 2E_0 \gg \Lambda_{QCD}$).
$E_0 \sim m_b/3 \simeq 1.6$ GeV is now conventional.

$$\Gamma(b \to X_s^p\gamma)_{E_\gamma > E_0} = N \sum_{i,j=1}^{8} C_i^{\text{eff}}(\mu_b) C_j^{\text{eff}}(\mu_b) \widetilde{G}_{ij}(E_0, \mu_b)$$

At NNLO $K_{77}^{(2)}$, $K_{17}^{(2)}$, $K_{27}^{(2)}$ depend on z. The central value of z is 0.056724126.

$$N = \frac{G_F^2 m_b^5}{32\pi^3}\left(\frac{\alpha_{em}}{\pi}\right)|V_{tb}V_{ts}^*|^2$$

$$K_{ij} \equiv \widetilde{G}_{ij}/G_u^{\text{semi}}$$

$$\mu_b \sim m_b/2$$

$$z = m_c^2/m_b^2$$

$$\delta = 1 - 2E_0/m_b$$

uncertainity 7.0%

The SM prediction

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{SM}_{E_\gamma > 1.6\text{GeV}} = (3.36 \pm 0.23) \cdot 10^{-4}$$ [arXiv:1503.01789]

M. Misiak, H. M. Asatrian, R. Boughezal, M. Czakon, T. Ewerth, A. Ferroglia, P. Fiedler, P. Gambino, C. Greub, U. Haisch, T. Huber, M. Kaminski, G. Ossola, M. Poradzinski, A. Rehman, T. Schutzmeier, M. Steinhauser, J. Virto

Non-perturbative	5%	mostly $\mathcal{O}(\alpha_s\Lambda/m_b)$
Parametric	2%	$\alpha_s(M_Z)(0.75\%)$, $\mathcal{B}_{SL}^{Exp}(1.49\%)$, CKM(0.12%), ...
Charm mass dependence	3%	Q_1, Q_2 matrix elements
Higher order NNNLO	3%	$\mu_b(2.0\text{GeV})$, $\mu_c(2.0\text{GeV})$, $\mu_0(160\text{GeV})$

Only parametric uncertainties go down from 3% (2006) to 2% (2015).

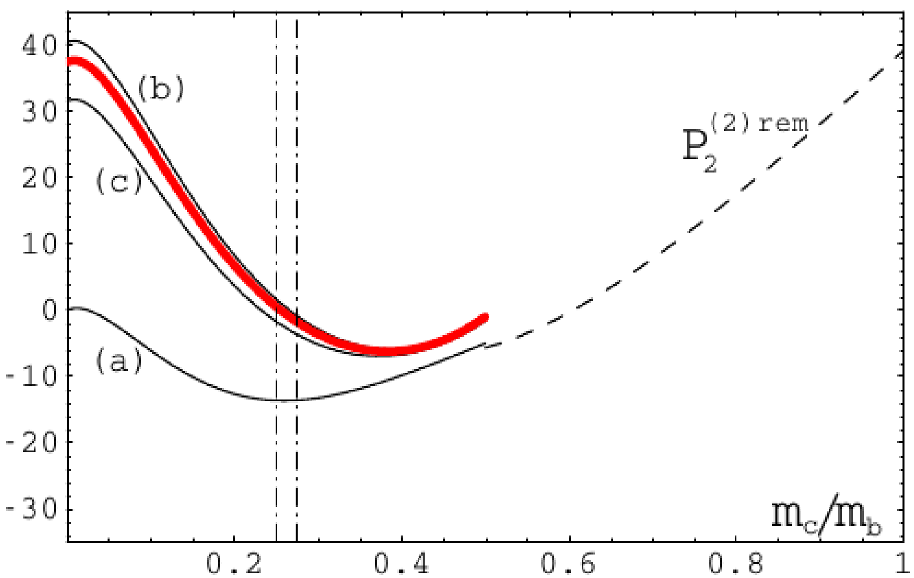
Interpolation in charm quark mass

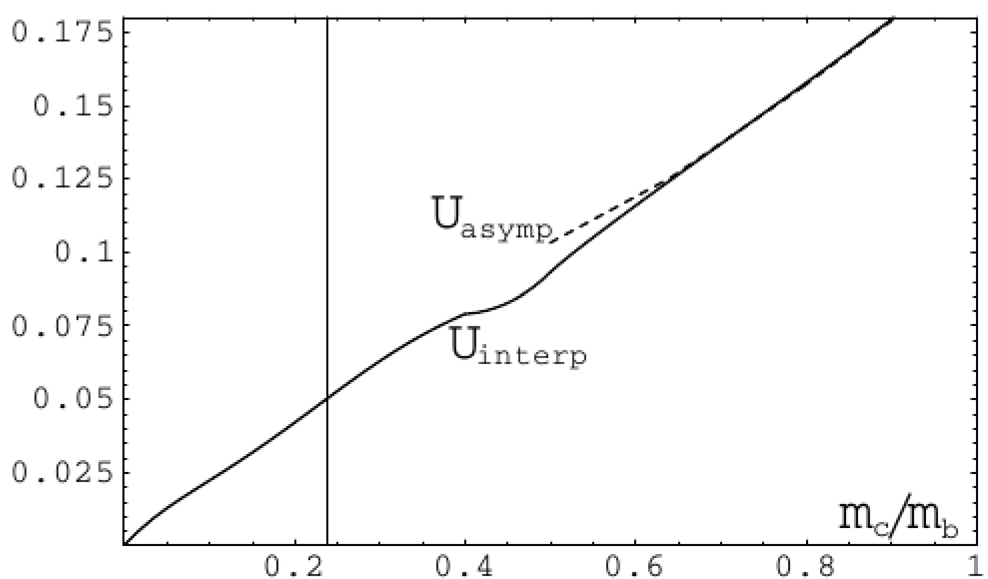
$$\frac{G_{27,17}}{m_c = 0}$$

$$C_i(\mu_b) = \sum_{n=0} \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^{(n)} C_i^{(n)}(\mu_b) \quad \textbf{and} \quad K_{ij} = \sum_{n=0} \left(\frac{\alpha_s(\mu_b)}{4\pi}\right)^{(n)} K_{ij}^{(n)}$$

$$\sum_{i,j=1}^{8} C_i^{(0)}(\mu_b) C_j^{(0)}(\mu_b) K_{ij}^{(2)}(E_0, \mu_b) \equiv P_2^{(2)} = \underbrace{P_2^{(2)\beta^{(0)}}}_{\text{BLM}} + \underbrace{P_2^{(2)\text{rem}}}_{\text{Non-BLM}}$$

- **BLM with arbitrary charm quark mass** — Bieri, Greub, Steinhauser, 2003; Ligeti, Luke, Manohar, Wise, 1999
- **Non-BLM by interpolation in m_c** assuming BLM at $m_c = 0$ — M. Steinhauser, M. Misiak, 2006
- **Non-BLM by interpolation in m_c** with explicit calculation at $m_c = 0$ — M. Czakon, P. Fiedler, T. Huber, M. Misiak, T. Schutzmeier, M. Steinhauser; 2015





Still interpolation uncertainity is 3%

NNLO Counterterms: no interpolation

$\frac{G_{27,17}}{m_c \neq 0}$

Renormalization [arXiv:1503.01791]

$$\widetilde{\alpha}_s \widetilde{G}^{(1)}_{27} + \widetilde{\alpha}_s^2 \widetilde{G}^{(2)}_{27} = Z_b^{OS} Z_m^{OS} \bar{Z}_{77} \Big\{ \widetilde{\alpha}_s^2 s^{3\varepsilon} \widetilde{G}^{(2)\text{bare}}_{27} + (Z_m^{OS} - 1) s^{\varepsilon} \big[\bar{Z}_{24} \hat{G}^{(0)m}_{47} + \widetilde{\alpha}_s s^{\varepsilon} \hat{G}^{(1)m}_{27} \big]$$
$$+ \widetilde{\alpha}_s (Z_G^{OS} - 1) s^{2\varepsilon} \hat{G}^{(1)3P}_{27} + \bar{Z}_{27} Z_m^{OS} \big[\hat{G}^{(0)}_{77} + \widetilde{\alpha}_s s^{\varepsilon} \hat{G}^{(1)\text{bare}}_{77} \big]$$
$$+ \widetilde{\alpha}_s \bar{Z}_{28} s^{\varepsilon} \hat{G}^{(1)\text{bare}}_{78} + \sum_{j=1,\ldots,6,11,12} \bar{Z}_{2j} s^{\varepsilon} \big[\hat{G}^{(0)}_{j7} + \widetilde{\alpha}_s s^{\varepsilon} Z_g^2 \hat{G}^{(1)\text{bare}}_{j7} \big] \Big\}$$
$$+ 2 \widetilde{\alpha}_s \widetilde{s}^{2\varepsilon} (\bar{Z}_m - 1) z \frac{d}{dz} \hat{G}^{(1)}_{27} + \mathcal{O}(\widetilde{\alpha}_s^3)$$

$$\hat{G}^{(1)}_{27} = \hat{G}^{(1)2P}_{27} + \hat{G}^{(1)3P}_{27}$$

$$\widetilde{\alpha}_s = \alpha_s / 4\pi = g_s^2 / 16\pi^2$$

$$s = \frac{4\pi \mu_b^2}{m_b^2} e^{\gamma}, \quad \widetilde{s} = \frac{4\pi \mu_c^2}{m_c^2} e^{\gamma}$$

$\hat{G}^{(0)}_{j7}$ vanish for $j = 1, 2, 11, 12$

$$Q_{11} = (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} T^a c_L)(\bar{c}_L \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} T^a b_L) - 16 Q_1$$
$$Q_{12} = (\bar{s}_L \gamma_{\mu 1} \gamma_{\mu 2} \gamma_{\mu 3} c_L)(\bar{c}_L \gamma^{\mu 1} \gamma^{\mu 2} \gamma^{\mu 3} b_L) - 16 Q_2$$

The $\hat{G}_{ij}$'s correspond to $\widetilde{G}_{ij}$'s once we replace C^{eff}'s with C's.

Now, $m_c = 0$ counterterms are known to all order in ϵ except $\hat{G}^{(1)}_{47}$, [AR, PhD thesis].

$$\hat{G}^{(1)3P}_{27}(z) = g_0(z) + \epsilon\, g_1(z) + \mathcal{O}(\epsilon^2) \qquad \hat{G}^{(1)2P}_{27}(z) = -\frac{92}{81\epsilon} + f_0(z) + \epsilon\, f_1(z) + \mathcal{O}(\epsilon^2)$$

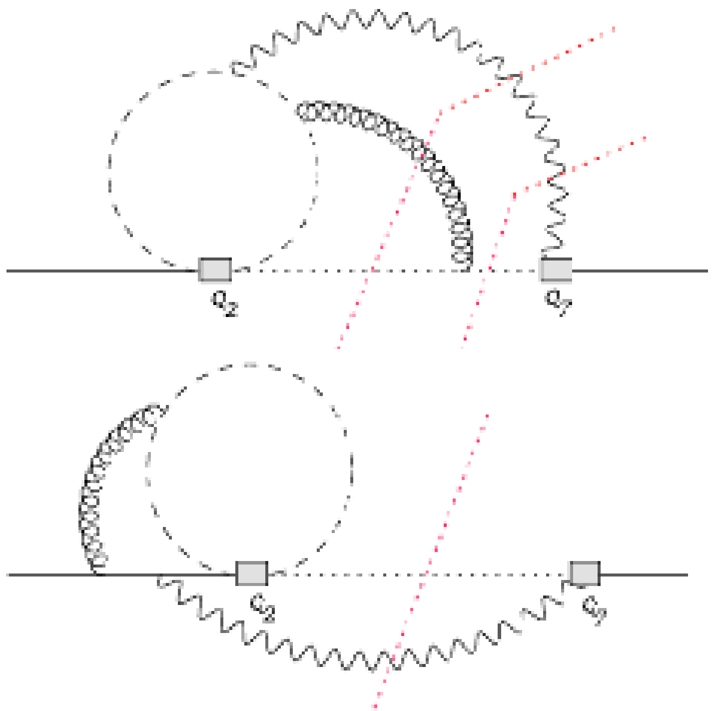
$$\hat{G}^{(1)3P}_{7(12)}(z) = 0 - \epsilon\,(20\, g_0(z)) + \mathcal{O}(\epsilon^2) \qquad \hat{G}^{(1)2P}_{7(12)}(z) = \frac{2096}{81} + \epsilon\, e_1(z) + \mathcal{O}(\epsilon^2)$$

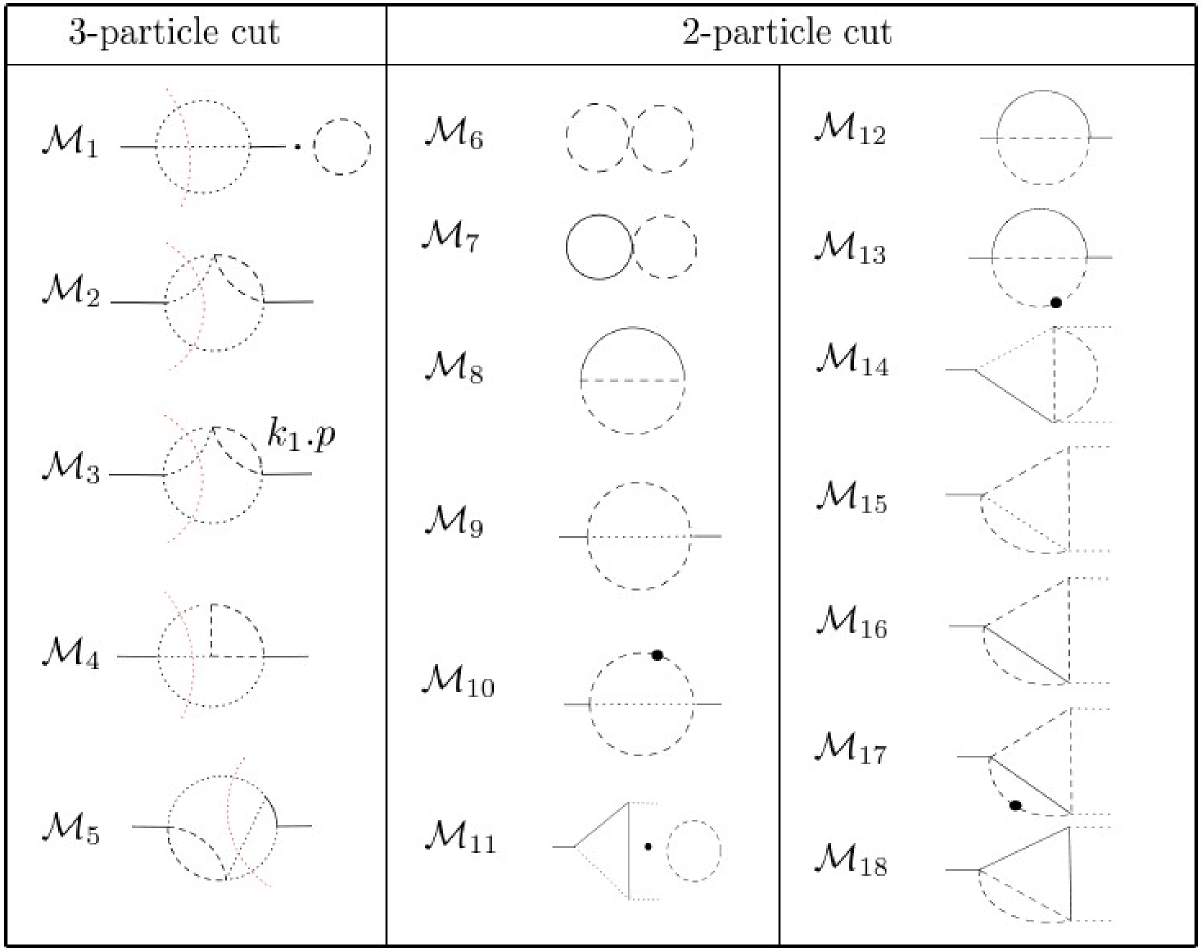
$$\hat{G}^{(1)m,3P}_{27}(z) = j_0(z) + \epsilon\, j_1(z) + \mathcal{O}(\epsilon^2) \qquad \hat{G}^{(1)m,2P}_{27}(z) = -\frac{1}{3\epsilon^2} + \frac{1}{\epsilon} r_{-1}(z) + r_0(z) + \epsilon\, r_1(z) + \mathcal{O}(\epsilon^2)$$

$$\hat{G}^{(1)}_{47}(z) = \hat{G}^{(1)}_{47}(z=0) + 2\, \mathcal{R}e \left(b(z) + \epsilon\, \widetilde{b}(z) \right) + \mathcal{O}(\epsilon^2)$$

Calculational method

Such contributions are obtained from three-loop propagators with unitarity cuts.





Reduction to master integrals

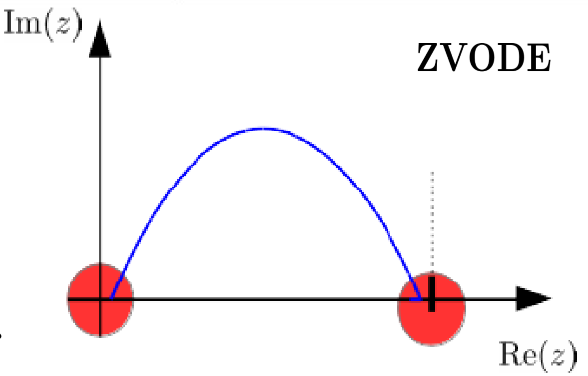
FIRE	[arXiv:1408.2372v2]
Reduze	[arXiv:1201.4330v1]

Reverse unitarity

$$\int \frac{d^D k}{(2\pi)^{D-1}} \theta(k^0)\delta(k^2 - m^2)$$

$$\frac{i}{k^2 - m^2 - i0} - \frac{i}{k^2 - m^2 + i0}$$

$$\frac{d}{dz}\mathcal{M}_n(z,\epsilon) = \sum_m R_{nm}(z,\epsilon)\mathcal{M}_m(z,\epsilon)$$



Boundary conditions at large z

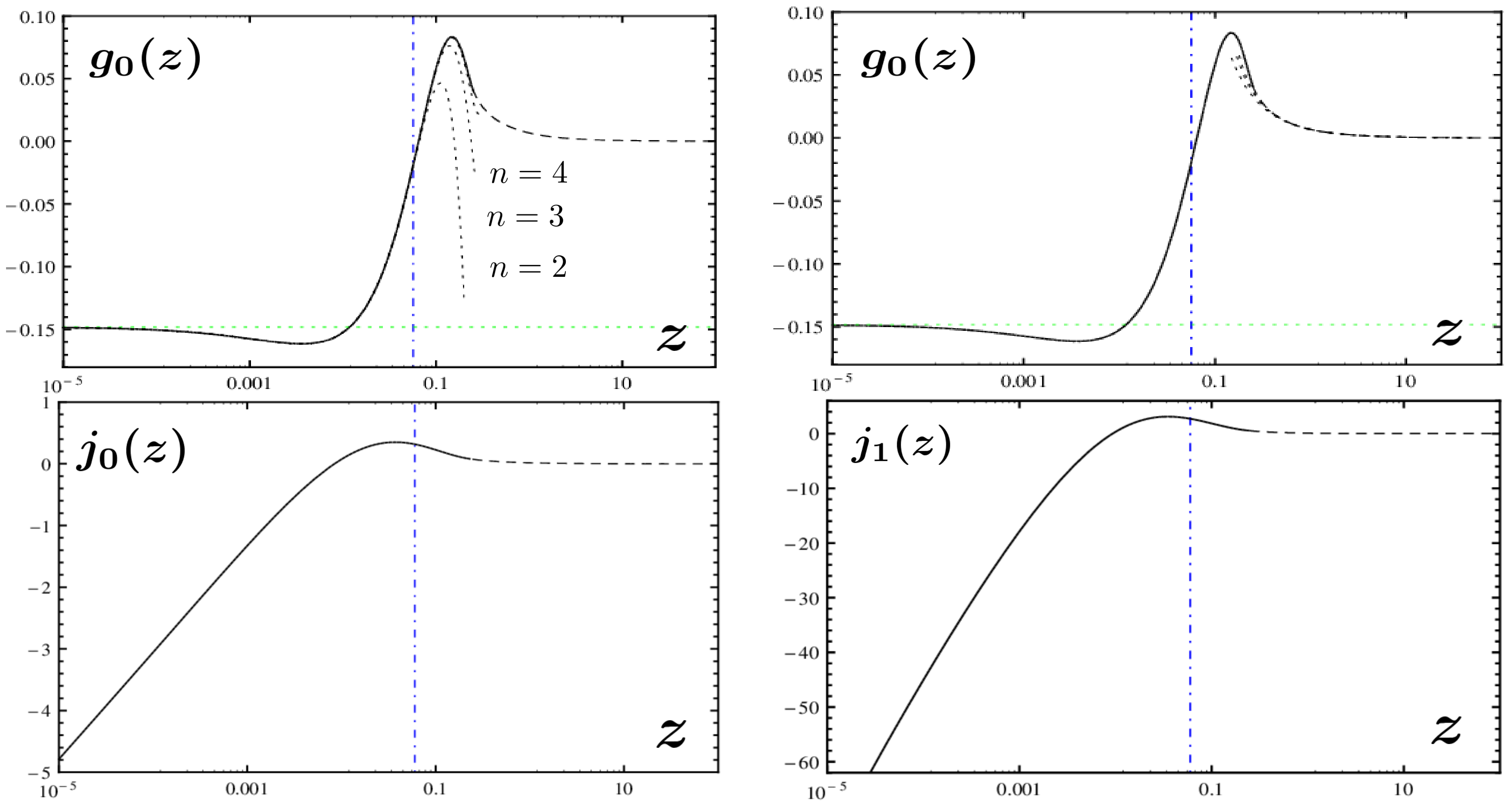
MB method, MB.m, M. Czakon
Hard mass expansion, exp, M. Steinhauser

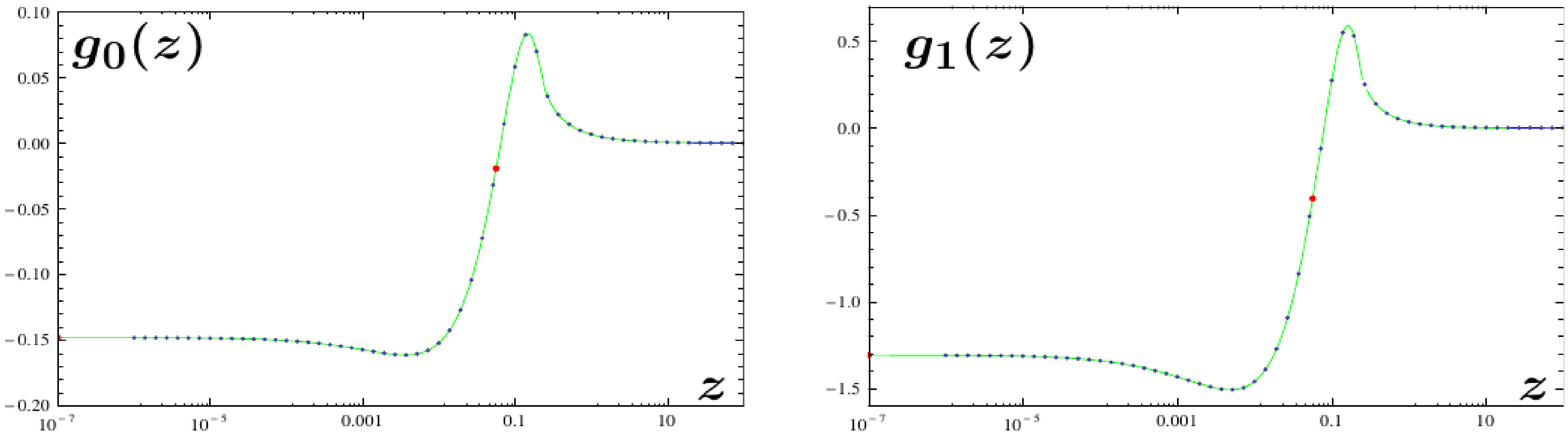
Results: 3-body

for $\delta = 1$

$$
g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\,s\,L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \le \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\,t\,A + \frac{16}{9}z(6z^2-4z+1)\,A^2, & \text{for } z > \frac{1}{4}, \end{cases}
$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, and $A = \arctan(1/t)$.





Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.

Boundary condition for the numerical DE's is at $z = 20$.

At $z = 1/4$ we have the charm production threshold, and the DE's have a singular point there.

Agreement with numerical solution of differential equations over wide range of z.

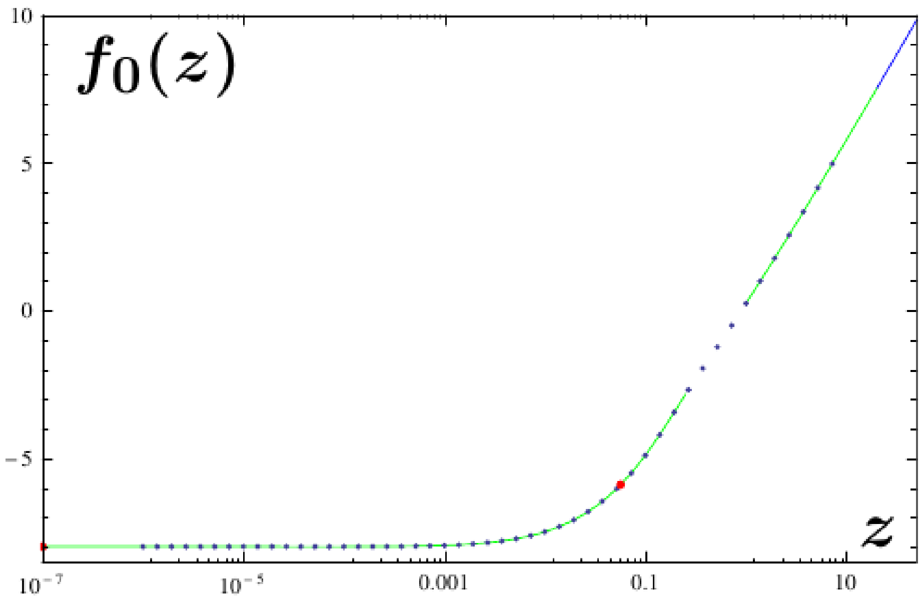
Blue curves: large-z asymptotic expansions above $z = 20$.

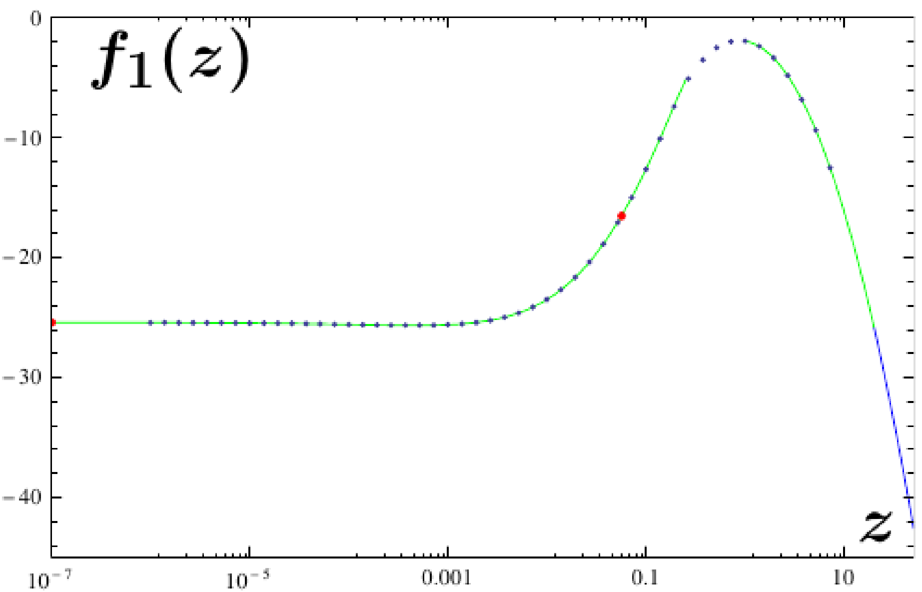
Red dots: Exact $z = 0$ results and numerical results from the DE's at the physical z.

Green curves: known asymptotic expansions either at large z or at small z.

This provides a test of our DE algorithm that is aimed at to be used in bare NNLO calculation where no analytic expansion at small z is going to be available.

Results: 2-body





$$\hat{G}_{27}^{(1)2P} = -\frac{92}{81\epsilon} + f_0(z) + \epsilon\, f_1(z) + \mathcal{O}(\epsilon^2) \qquad f_0(z) = -\frac{1942}{243} + 2\,\mathcal{R}e\,[a(z) + b(z)]$$

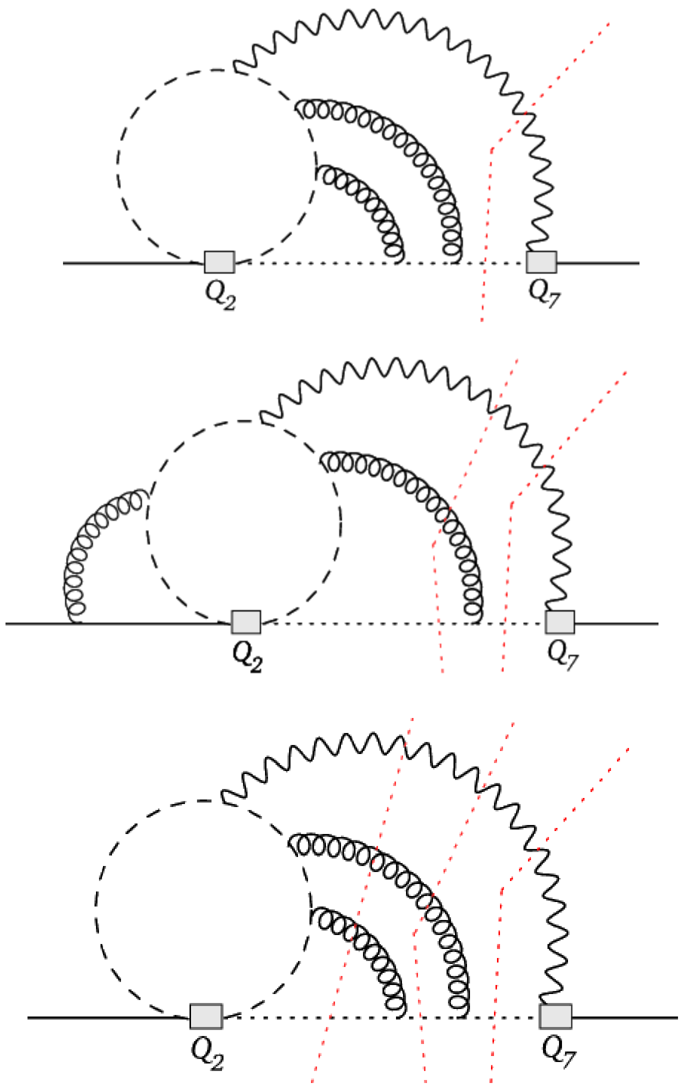
$$\hat{G}_{7(12)}^{(1)2P} = \frac{2096}{81} + \epsilon\, e_1(z) + \mathcal{O}(\epsilon^2) \qquad e_1(z) = \frac{39112}{243} - 8\,\mathcal{R}e\,[5\,a(z) + b(z)]$$

$$\hat{G}_{27}^{(1)m,2P} = -\frac{1}{3\epsilon^2} + \frac{1}{\epsilon} r_{-1}(z) + r_0(z) + \epsilon\, r_1(z) + \mathcal{O}(\epsilon^2)$$

This provides a test of our DE algorithm that is aimed at to be used in bare NNLO calculation where no analytic expansion at small z is going to be available.

Outlook of bare NNLO Contribution

in progress



$+ \cdots\cdots$

$s\gamma, s\gamma g, s\gamma gg, s\gamma q\bar{q}$

Required same techniques as used/mentioned above.

but much more complex.

BCs using automatized asymptotic expansions at $m_c \gg m_b$.

Higher order terms using power-log ansatz

$$I_i(w,\epsilon) = \sum_{n,m,k} c_{inmk}\epsilon^n w^m \mathrm{Log}^k(w) \qquad w = 1/z$$

2-body

number of scalar integrals $\sim 20,000$

number of masters ~ 500

Summary

- **The $B \to X_s\gamma$ process constrains extensions of the the SM in a strong manner. At present, its observed branching ratio agrees with the SM at better than 1σ.**

- **However, $K_{27}^{(2)}$ are currently included with the help of interpolation in the charm quark mass. This causes about $\pm\,3\%$ uncertainty.**

- **Completing the calculation of $K_{27}^{(2)}$ for arbitrary $z = m_c^2/m_b^2$ to remove $\pm 3\%$ uncertainty is necessary.**

- **A calculation of the counterterm contribution to $K_{27}^{(2)}(z)$ has been presented. Such contributions are obtained by evaluating three-loop propagator diagrams with unitarity cuts.**