Threshold corrections at N^3LO in QCD for Drell-Yan and Higgs productions at the LHC.

V. Ravindran

Institute of Mathematical Sciences, Chennai

- Golden information from N^3LO of Higgs production in gluon fusion
- New results: Threshold corrections at N^3LO for
  - Drell-Yan: $d\sigma/dQ^2$ and $d\sigma/dQ^2dY$
  - $b + \overline{b} \rightarrow Higgs$: σ and $d/\sigma dY$
  - $q + \overline{q} \rightarrow W^{\pm}/Z \rightarrow Higgs$: σ
- Renormalisation group improvement at N^3LO for Higgs production
- Conclusions

Taushif Ahmed, Maguni Mahakhud, Manoj Mandal,M.C. Kumar,Narayan Rana

RADCOR/LOOPFEST at UCLA, LA

Higgs production to N^3LO in QCD at the LHC

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

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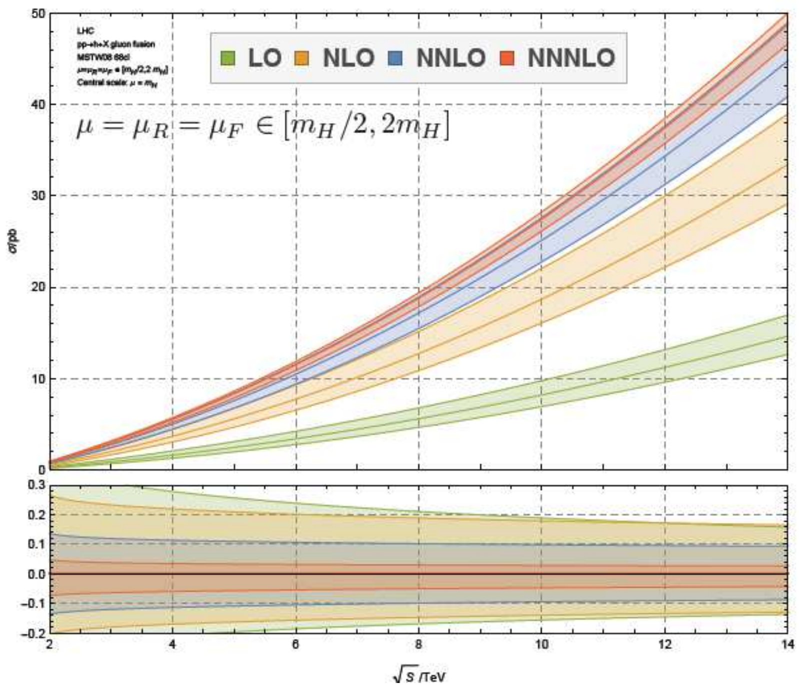
Energy variation



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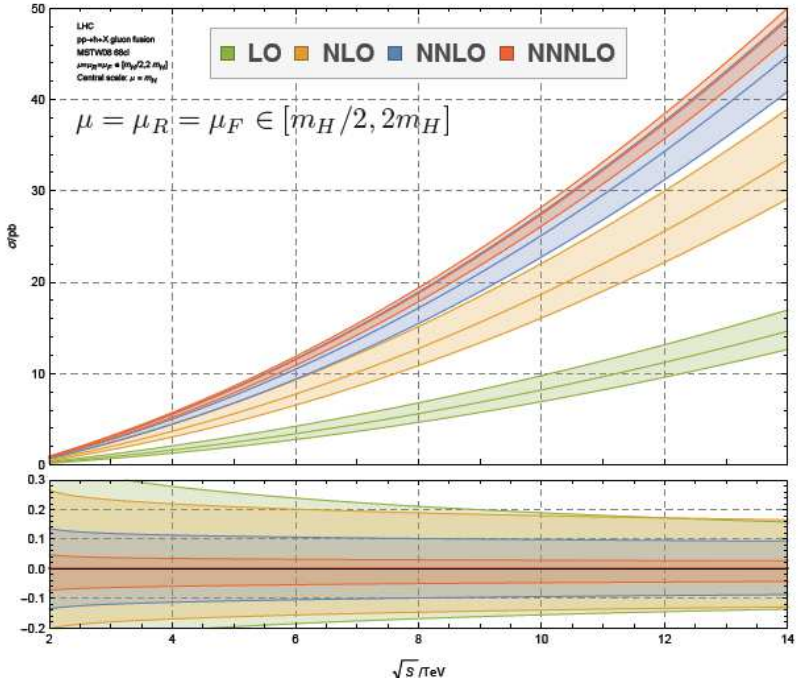
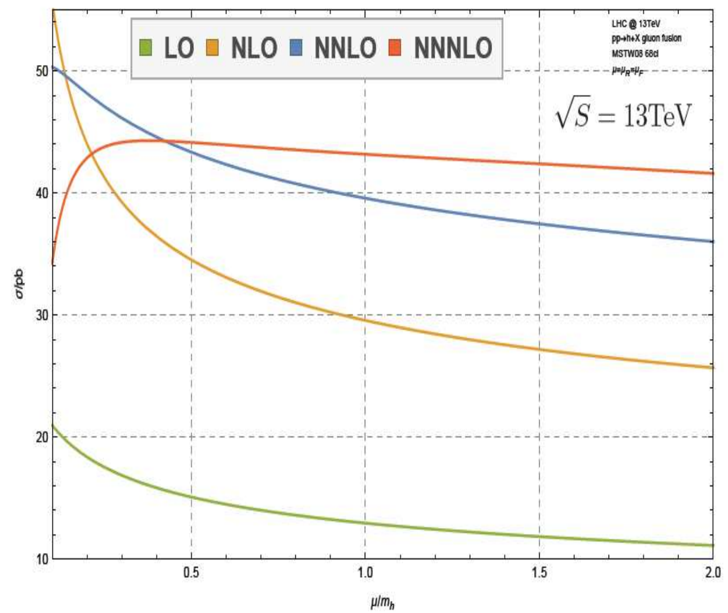


Scale variation



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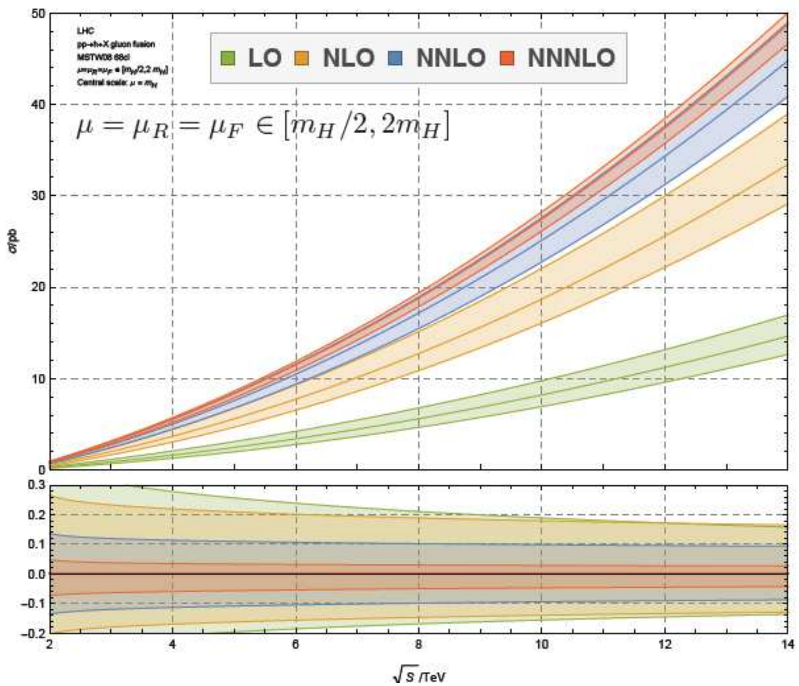

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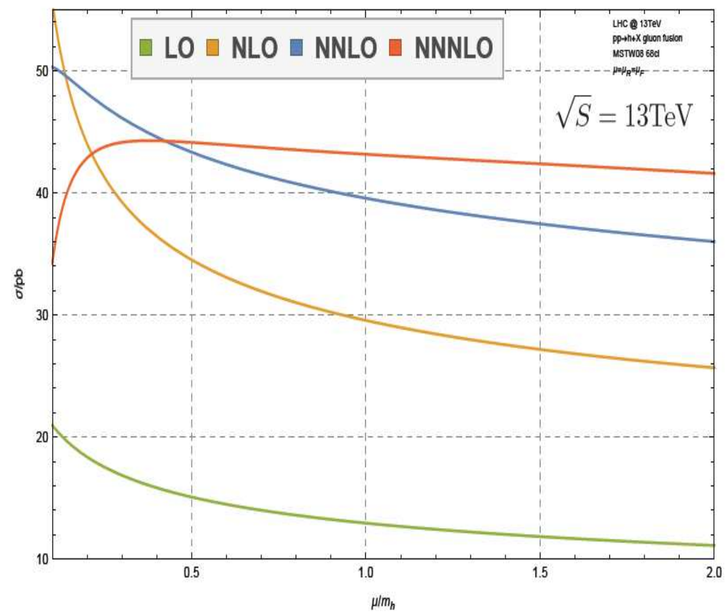

- Computed for the first time in the infinite top mass limit: an effective theory with 5 flavours.
- More than a decade after NNLO.
- Scale uncertainty goes down in the canonical range.

See the talk by C. Duhr

Spin offs from Higgs production at N^3LO for the LHC

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$Ahmed, Mandal, Mahakhud, Rana, Kumar, VR$

- $d\sigma/dQ$ and $d\sigma/dQ/dY$ of Drell-Yan pairs:
  Phys. Rev. Lett. $\mathbf{113}$ (2014) 11, 112002

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- RG improved Higgs boson production in gluon fusion to N^{3}LO in QCD
  arXiv:1505.07422

Why Threshold corrections?

Catani et al, Harlander, Kilgore

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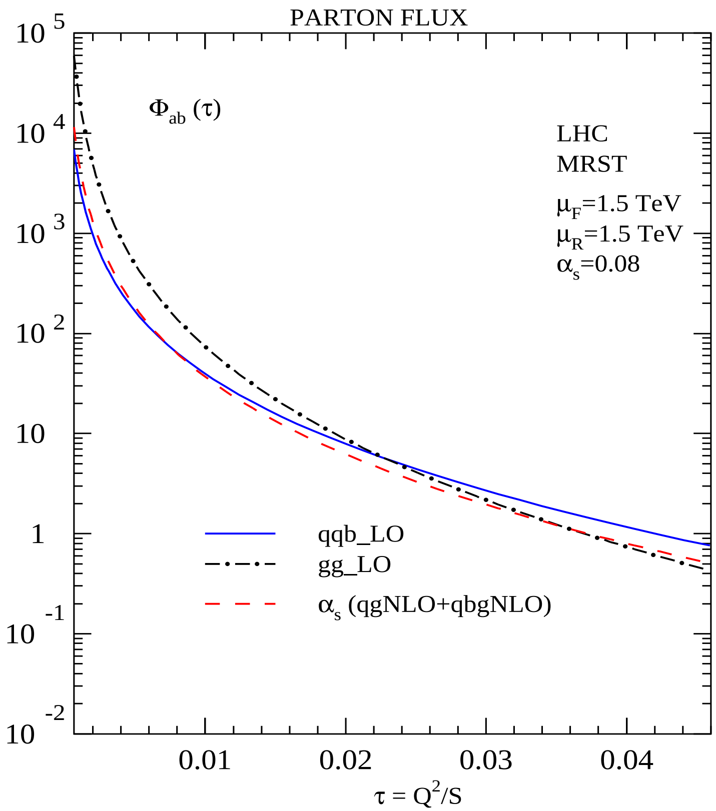
$$2S\, d\sigma^{P_1 P_2}(\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}(x)\;\; 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$



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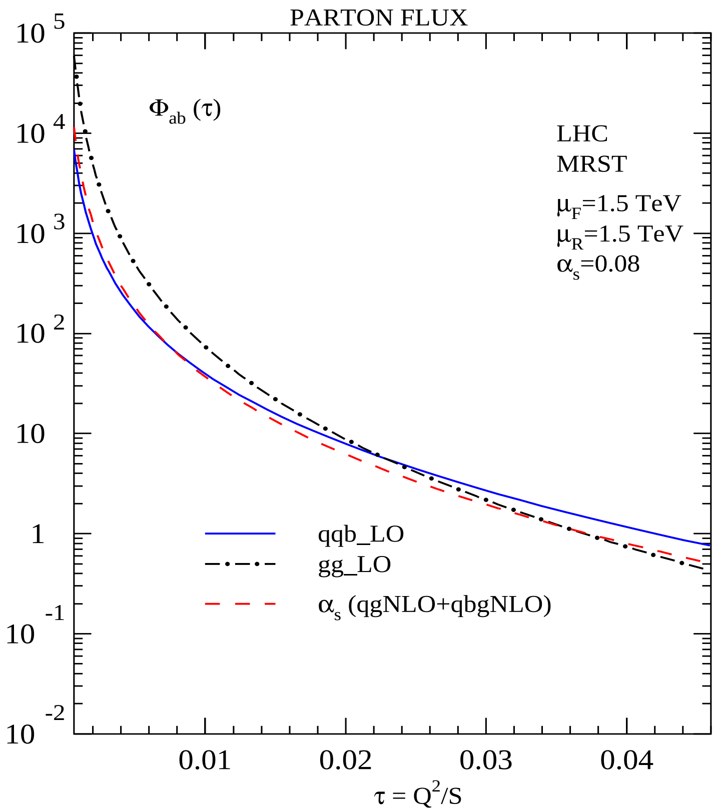

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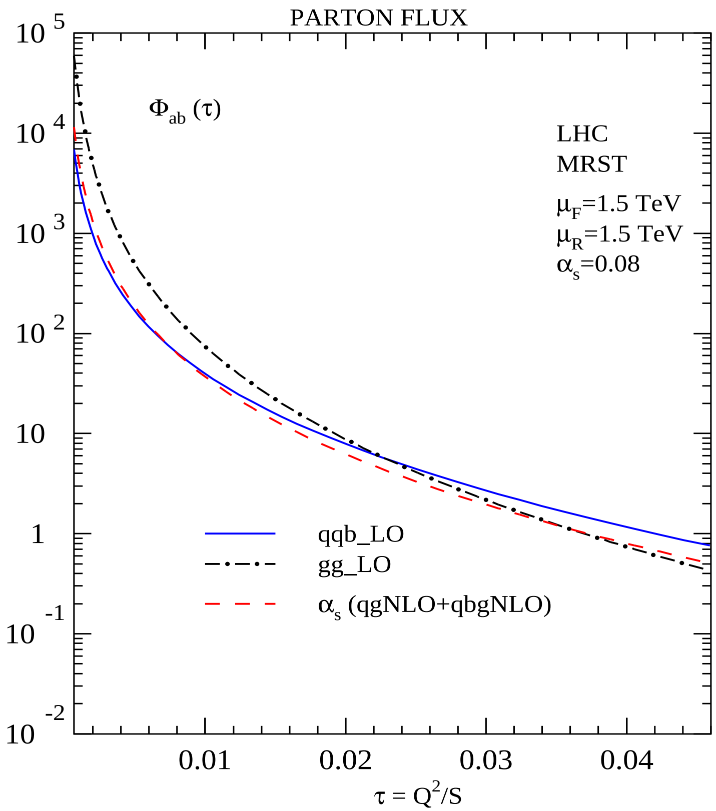

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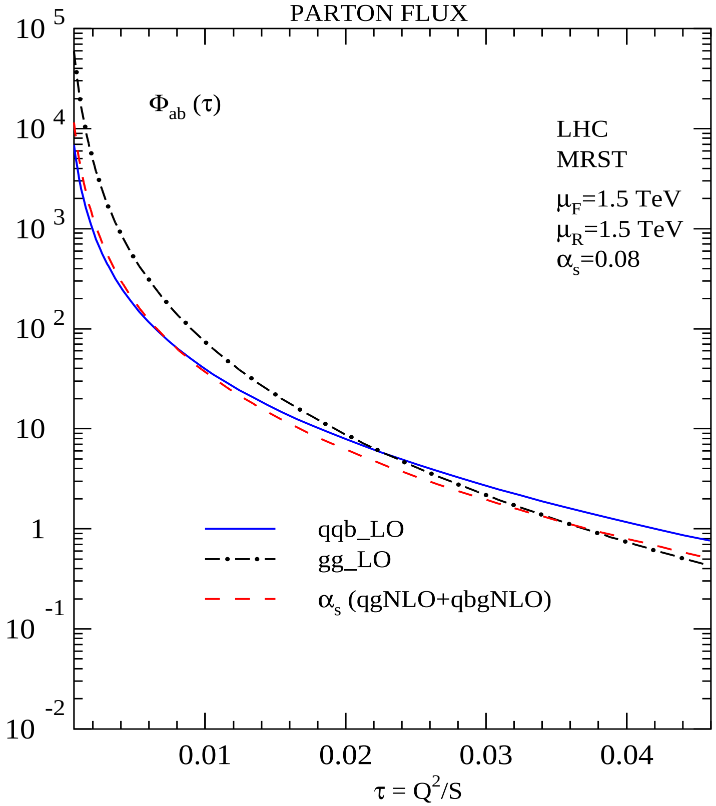

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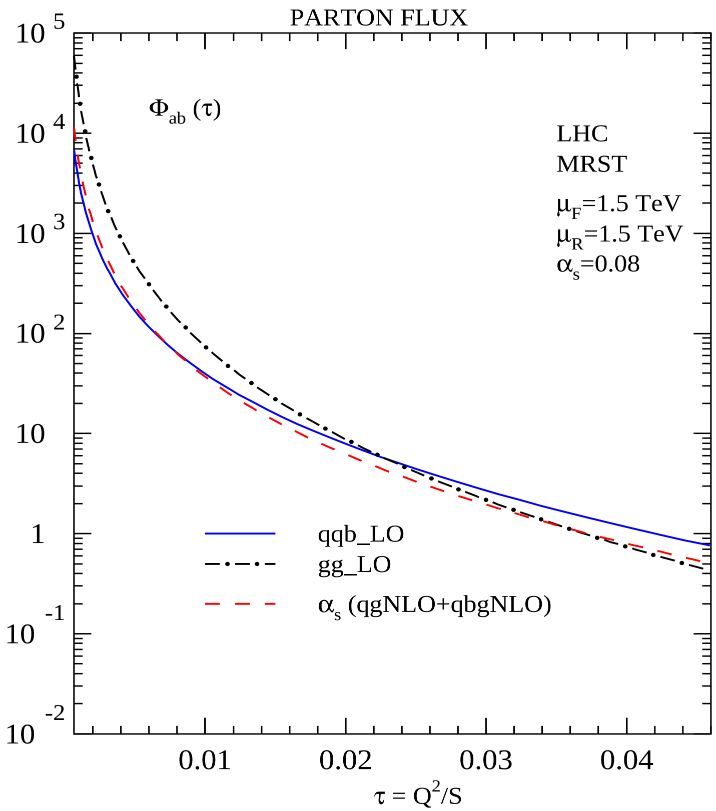

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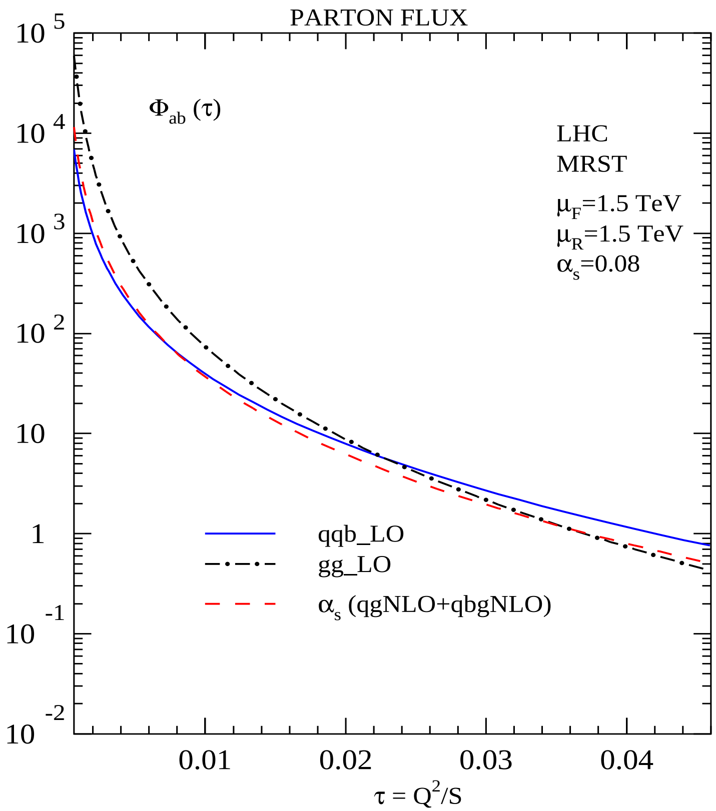

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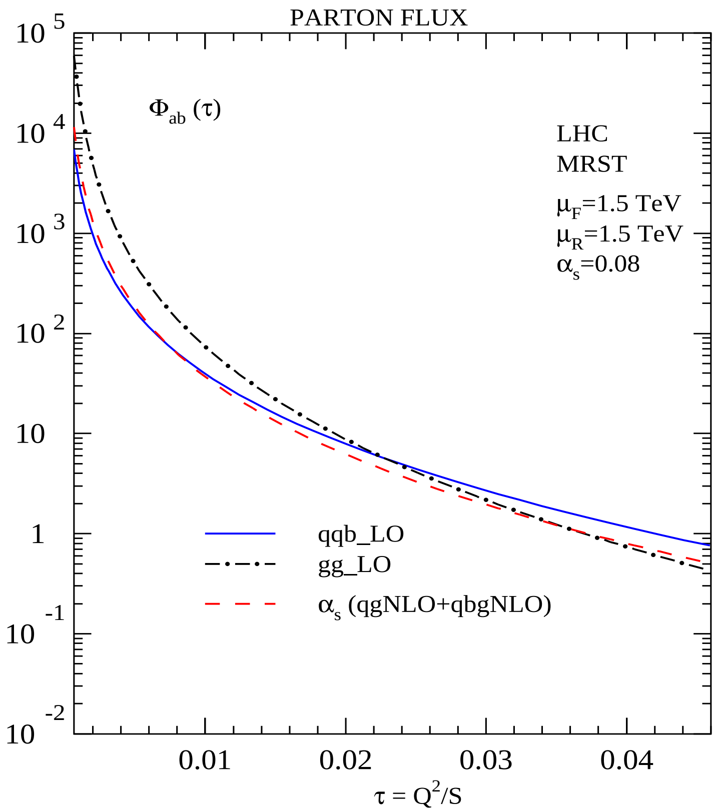

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- Expand the partonic cross section around $x = \tau$.
- Dominantly come from virtual and soft gluon emission processes (SV)

Soft plus Virtual

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- Soft Collinear Effective theory

  Becher, Neubert

A master formula for Soft plus Virtual

VR

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Following Sterman et al, Catani et al, namely using "factorisation" of UV, Soft and Collinear:

$$\Delta^{sv}_{I,P}(z,q^2,\mu_R^2,\mu_F^2) = \mathcal{C}\exp\left(\Psi^I(z,q^2,\mu_R^2,\mu_F^2,\varepsilon)\right)\Big|_{\varepsilon=0} \qquad I=q,g \qquad n=4+\varepsilon$$

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$$q^2 \frac{d}{dq^2}\Phi^I\left(\hat{a}_s, q^2, \mu^2, z, \varepsilon\right) = \frac{1}{2}\left[\overline{K}^I\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right) + \overline{G}^I\left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, z, \varepsilon\right)\right]$$

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Ansatz for the solution:

$$\Phi^I(\hat a_s, q^2, \mu^2, z, \varepsilon) = \sum_{i=1}^{\infty} \hat a_s^i \left(\frac{q^2(1-z)^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\,\varepsilon}{1-z}\right) \hat\phi^{\,I,(i)}(\varepsilon)$$

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- That is, the missing $\delta(1-z)$ contribution to Φ^g to third order is nown known from Higgs production at N^3LO.

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- This fixes the missing $\delta(1-z)$ part of third order Φ^q as well due to the Casimir scaling.
- Knowledge of Φ^q completes the computation of N^{3}LO threshold corrections to Drell-Yan.

Φ^g from Φ^q

VR

Φ^I is maximally non-abelian (known only up to three loops).

$$\boxed{\Phi^q = \frac{C_F}{C_A} \Phi^g}$$

- At every order in a_s, all the terms in Φ^I expect $\delta(1-z)$ can be fixed from the lower order terms thanks to RG, Sudakov $K+G$ equations and finiteness of the cross section.
- $\delta(1-z)$ component has been the missing term for both Φ^I for $I = g, q$ till recently.
- The most recent computation on Threshold N^3LO to Higgs production by Anastasiou et al. provides the missing piece.
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Soft plus Virtual part at N^3LO_{SV} for DY Production

Moch, Vogt, ;Ahmed, Mahakhud, Rana, VR

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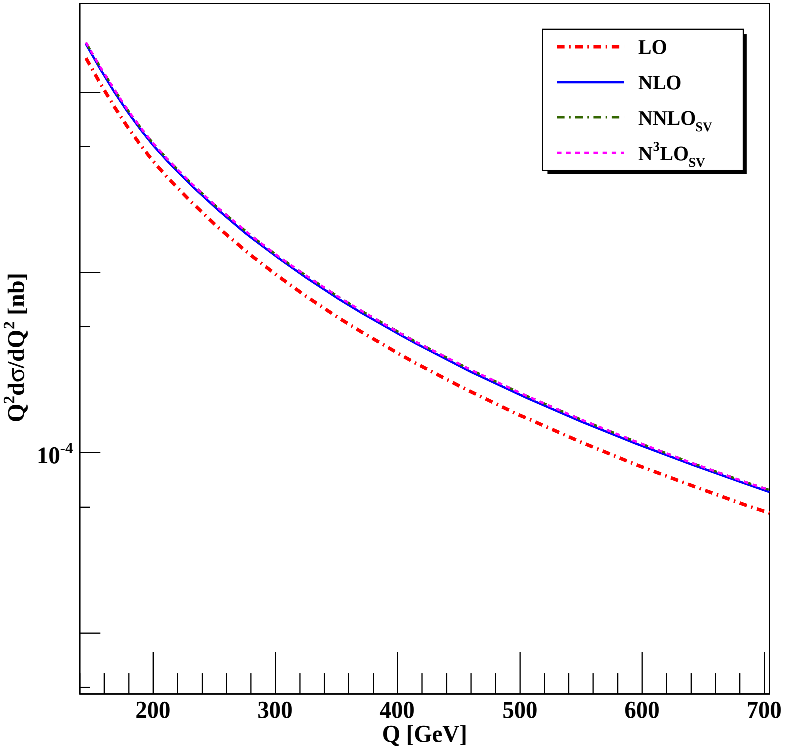
$$2S\, d\sigma^{P_1P_2}(\tau, m_h) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}(x)\, 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h\right) \qquad \tau = \frac{m_h^2}{S}$$



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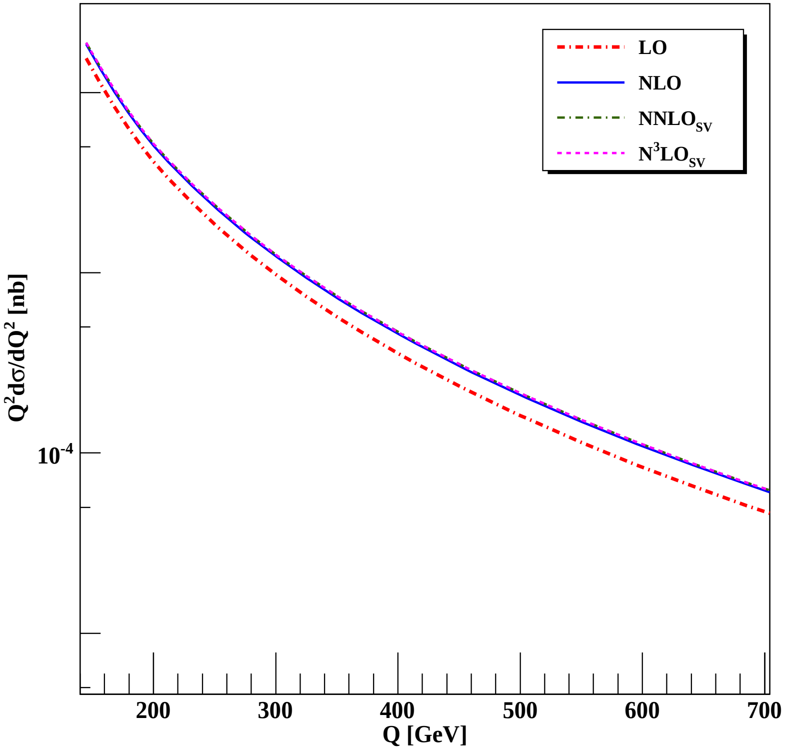

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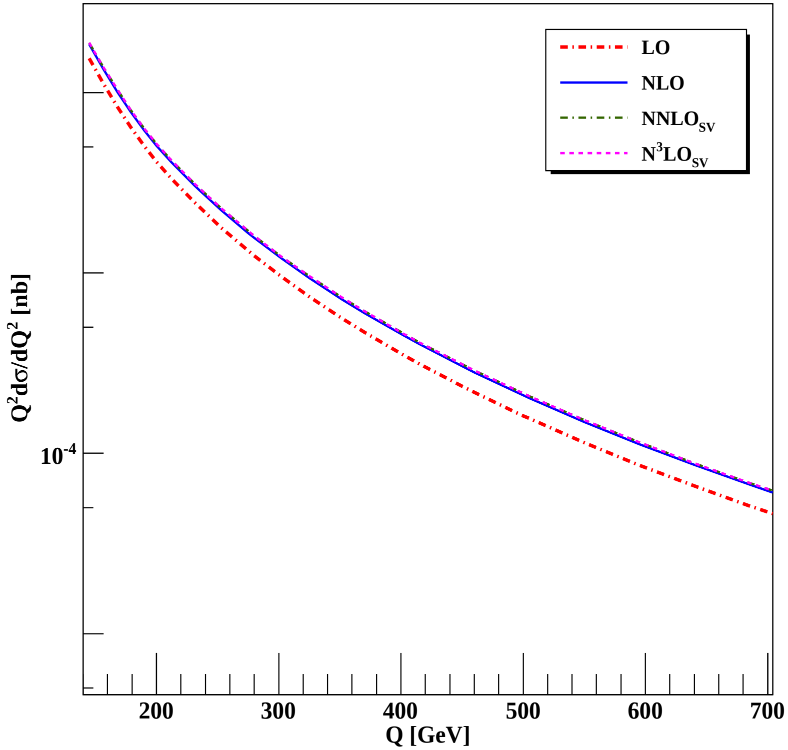

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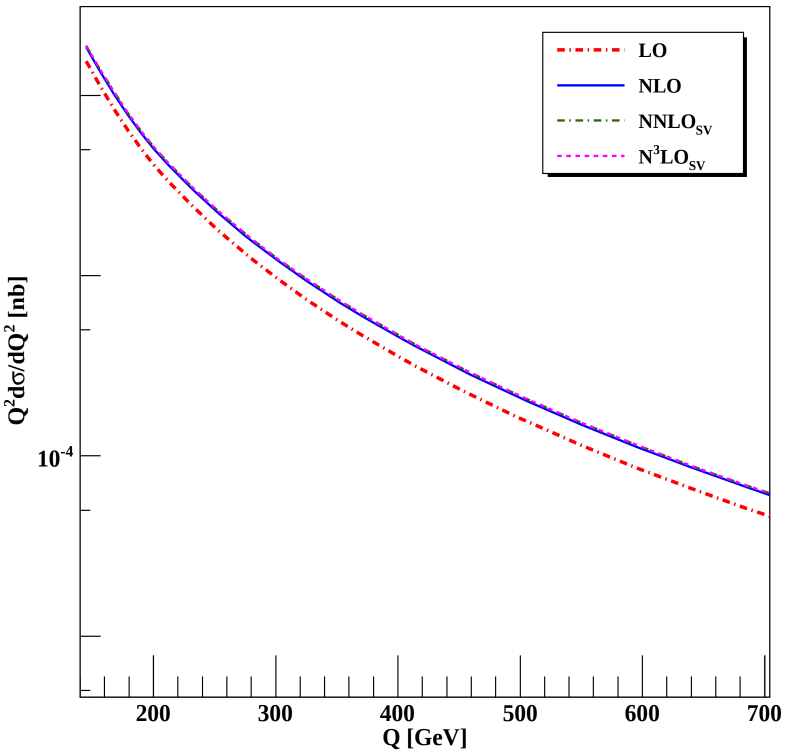

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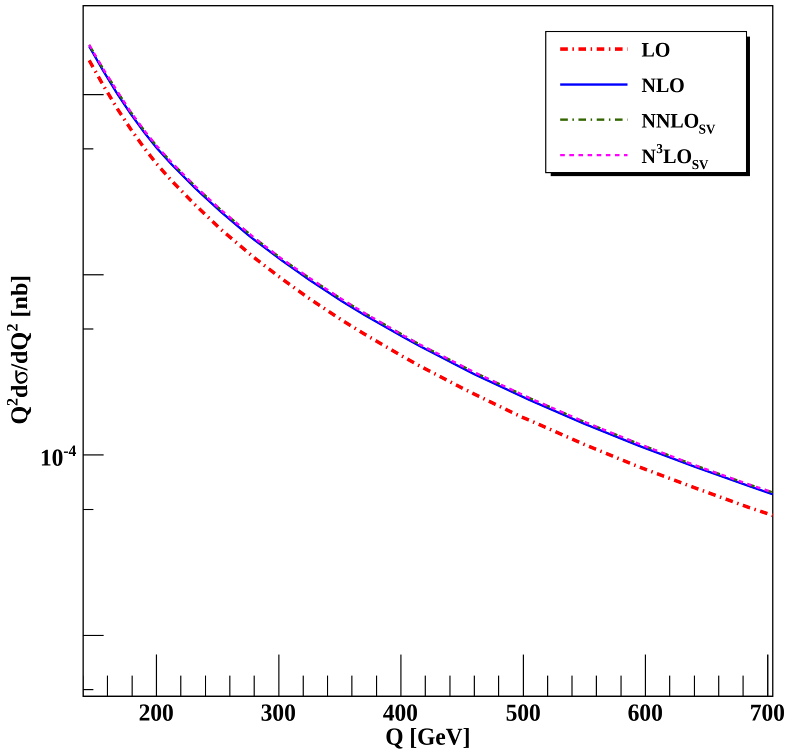

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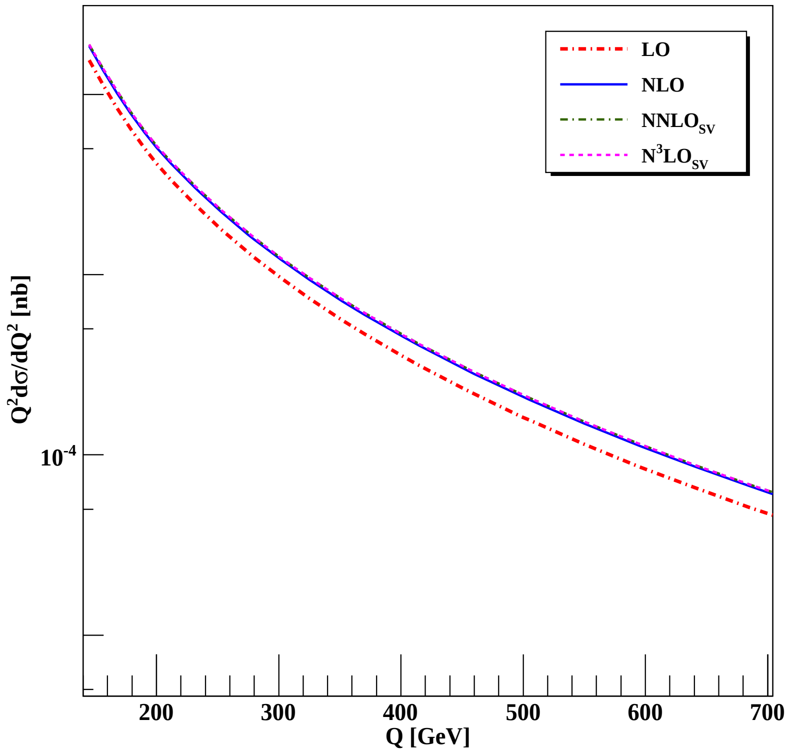

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- At 4-loop:
$$\mathcal{D}_j \qquad j = 7, 6, 5, 4, 3, 2, 1$$
- Most of the contributions from $\mathcal{D}_i$s is cancelled by $\delta(1-z)$ making N^3LO further subleading

$\delta(1-z)$ and Scale dependence for N^3LO DY production

Ahmed, Mahakhud, Rana, VR

$$R = \frac{\sigma_{N^iLO}(\mu)}{\sigma_{N^iLO}(\mu_0)}$$

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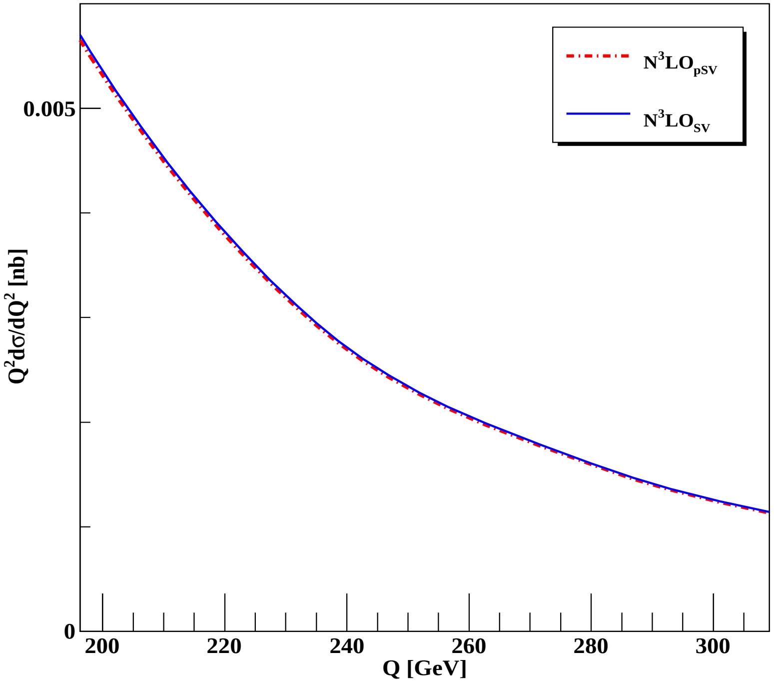

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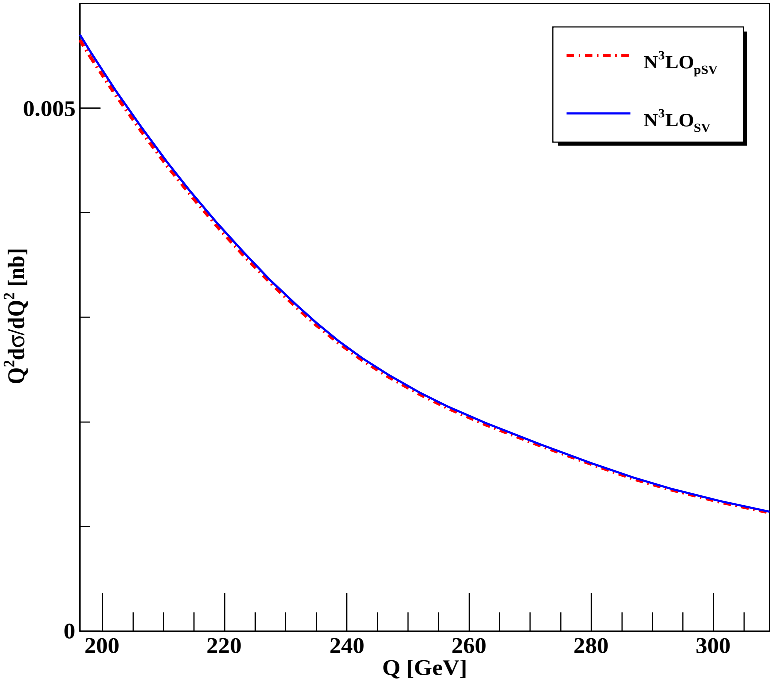

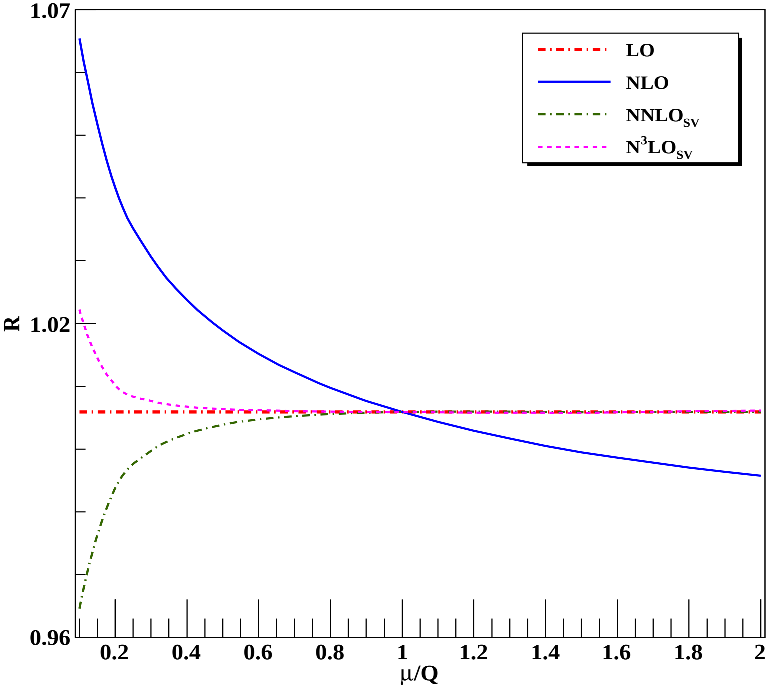

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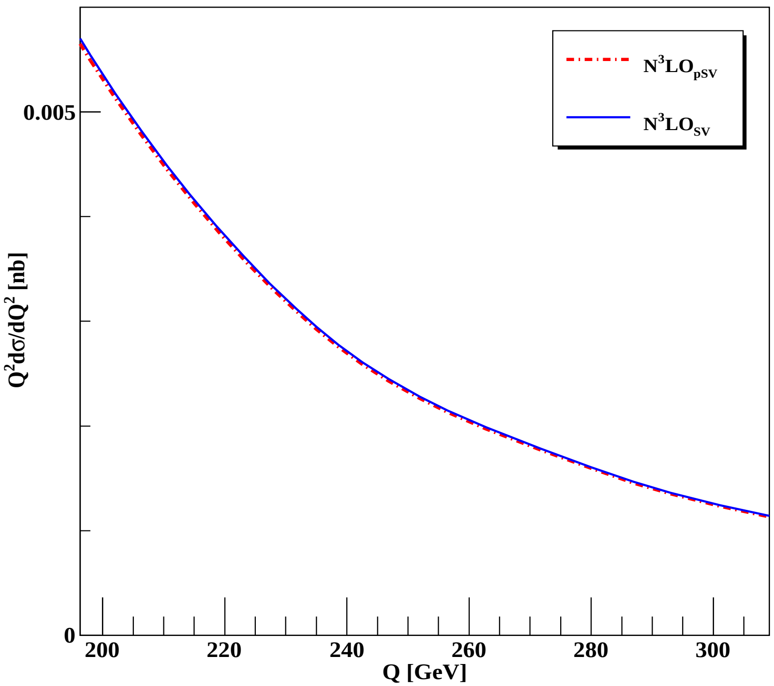

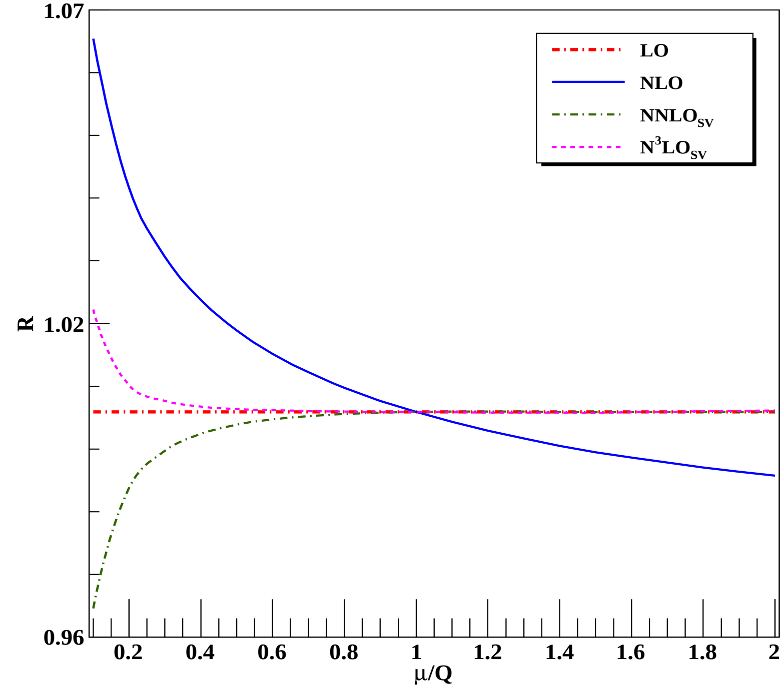

- Scale uncertainity improves a lot
- Perturbative QCD works at LHC

Rapidity distributions

V. Ravindran, J. Smith and W. van Neerven

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$$\frac{d\sigma^I}{dY} = \sigma^I_{\rm Born}(x_1^0, x_2^0, q^2) W^I(x_1^0, x_2^0, q^2) \ , \qquad I = q, b, g \quad ,$$

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$$W^I(x_1^0, x_2^0, q^2) = \sum_{ab=q,\overline{q},g} \int_0^1 dx_1 \int_0^1 dx_2\, \mathcal{H}^I_{ab}(x_1, x_2, \mu_F^2)$$

$$\times \int_0^1 dz_1 \int_0^1 dz_2\, \delta(x_1^0 - x_1 z_1)\, \delta(x_2^0 - x_2 z_2)\, \Delta^I_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2)\ .$$

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$$\Delta^I_{d,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) = \Delta^{\rm hard}_{I,ab}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) + \delta_{a\overline{b}} \Delta^{\rm sv}_{d,I}(z_1, z_2, q^2, \mu_F^2, \mu_R^2),$$

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Threshold region corresspond to $z_1, z_2 \rightarrow 1$, that $Y \rightarrow 0$.

$$\left\{ Z^I(\mu_F^2), \quad F^I(Q^2), \quad \Gamma_{II}(\mu_F^2), \Phi^I(Q^2) \right\} \Rightarrow \Delta^{sv}_{d,I}(z_1, z_2, q^2, \mu_F^2, \mu_R^2) \,:$$

Soft Sudakov Equation

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$$q^2 \frac{d}{dq^2} \Phi_d^I \left(\hat{a}_s, q^2, \mu^2, \vec{z}, \varepsilon\right) = \frac{1}{2}\left[\overline{K}_d^I \left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \vec{z}, \varepsilon\right) + \overline{G}_d^I \left(\hat{a}_s, \frac{q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \vec{z}, \varepsilon\right)\right]$$

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- $\vec{z} = (z_1, z_2)$
- $\overline{K}^I_d$ contains all the infra-red poles
- $\overline{G}^I_d$ is regular as $\varepsilon \to 0$

$$\begin{aligned}\Phi^I_d(\hat{a}_s, q^2, \mu^2, z, \varepsilon) &= \Phi^I_d\left(\hat{a}_s, q^2 \prod_i (1 - z_i)^2, \mu^2, \varepsilon\right) \\ &= \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{q^2 \prod_i (1 - z_i)^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \left(\frac{i\,\varepsilon}{\prod_i (1 - z_i)}\right) \hat{\phi}_d^{I,(i)}(\varepsilon)\end{aligned}$$

where

$$\hat{\phi}_d^{I,(i)}(\varepsilon) = \hat{\mathcal{L}}_F^{I,(i)}(\varepsilon)\left(A^I \to -A^I, G^I(\varepsilon) \to \overline{\mathcal{G}}^I_d(\varepsilon)\right)$$

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results

Ahmed, Mandal, Rana, VR

The exclusive $\overline{\mathcal{G}}^{I,k}_{d,i}$ can be expressed in terms of inclusive $\overline{\mathcal{G}}^{I,k}_{i}$:

$$\int_0^1 dx_1^0 \int_0^1 dx_2^0 \, (x_1^0 x_2^0)^{N-1} \frac{d\sigma^I}{dY} = \int_0^1 d\tau \, \tau^{N-1} \, \sigma^I \, ,$$

where the inclusive σ^I are now known for both inclusive Drell-Yan and Higgs boson productions up to N^{3}LO level in threshold limit

In the threshold limit, $N \to \infty$:

$$\hat{\phi}_d^{I,(i)}(\varepsilon) = \frac{\Gamma(1 + i\,\varepsilon)}{\Gamma^2\left(1 + i\frac{\varepsilon}{2}\right)} \hat{\phi}^{I,(i)}(\varepsilon).$$

$$\begin{aligned} \Delta^{sv}_{d,I}(\vec{z}) \quad = \quad & \sum_{i=0}^{\infty} a_s^i(\mu_R^2) \Big[\Delta^{sv}_{d,I,i}|_{\delta\delta} \delta(\overline{z}_1)\delta(\overline{z}_2) \\ & + \sum_{j=0}^{2i-1} \Delta^{sv}_{d,I,i}|_{\delta\mathcal{D}_j} \delta(\overline{z}_2)\mathcal{D}_j + \sum_{j=0}^{2i-1} \Delta^{sv}_{d,I,i}|_{\delta\overline{\mathcal{D}}_j} \delta(\overline{z}_1)\overline{\mathcal{D}}_j \\ & + \sum_{j \circledS k} \Delta^{sv}_{d,I,i}|_{\mathcal{D}_j\overline{\mathcal{D}}_k} \mathcal{D}_j \overline{\mathcal{D}}_k \, , \qquad \overline{z}_i = 1 - z_i \end{aligned}$$

The symbol $j \circledS k$ implies $j, k \geq 0$ and $j + k \leq (2i - 2)$.

N^3LO_{SV} results for Drell-Yan rapidity

Ahmed, Mandal, Rana, VR

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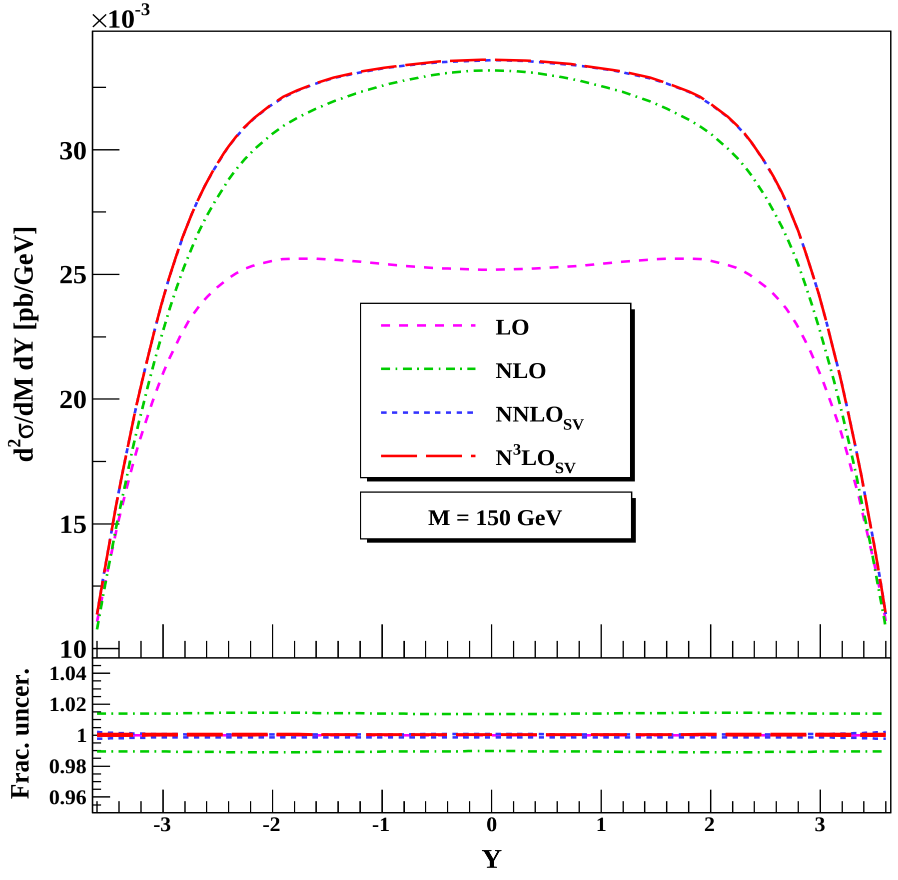

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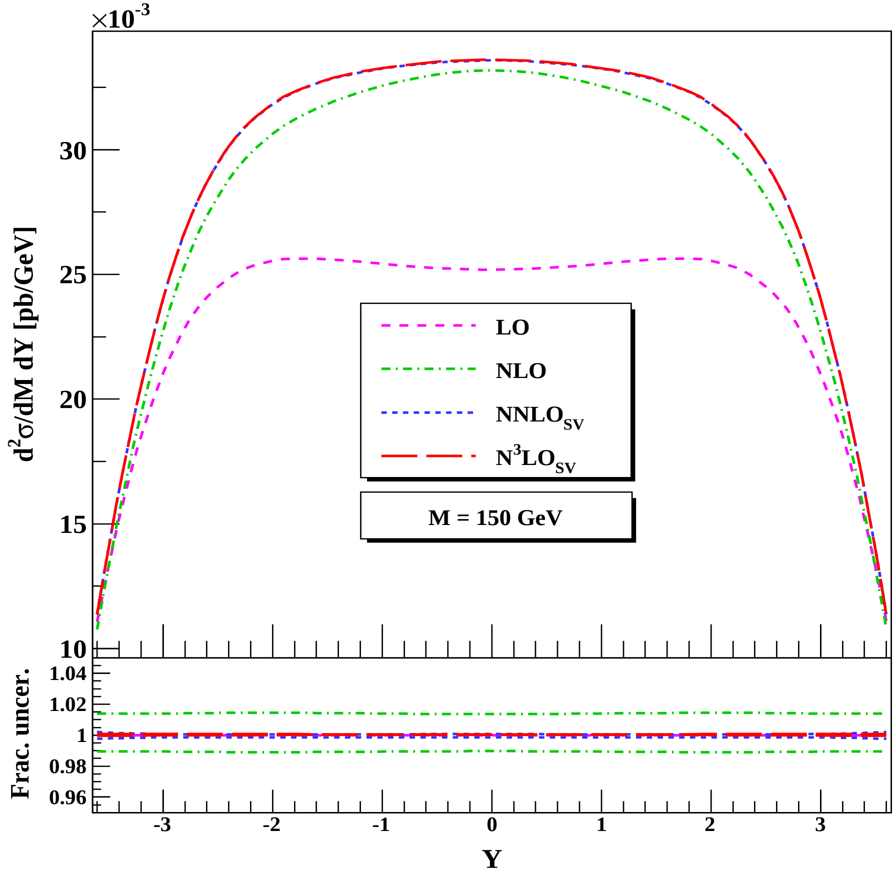

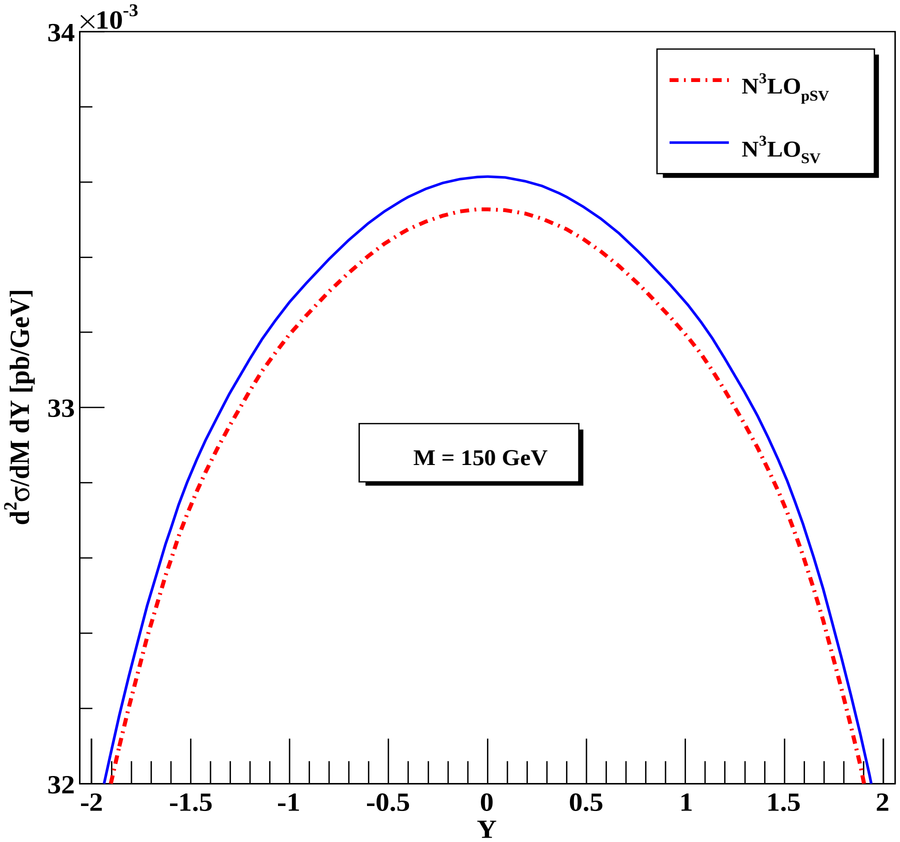

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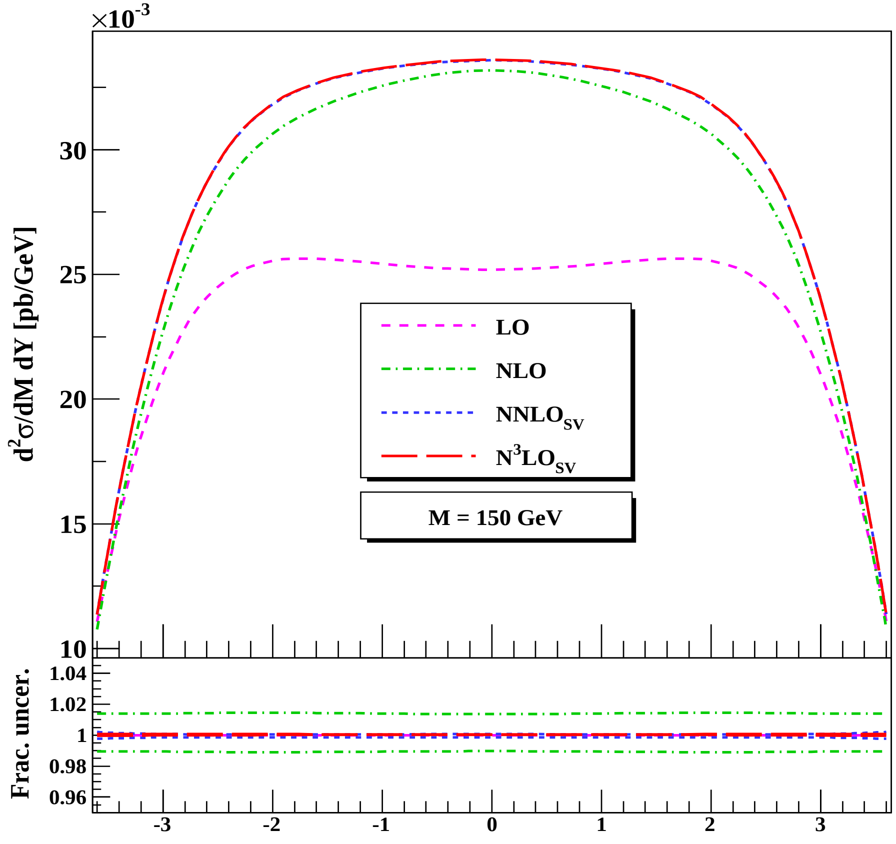

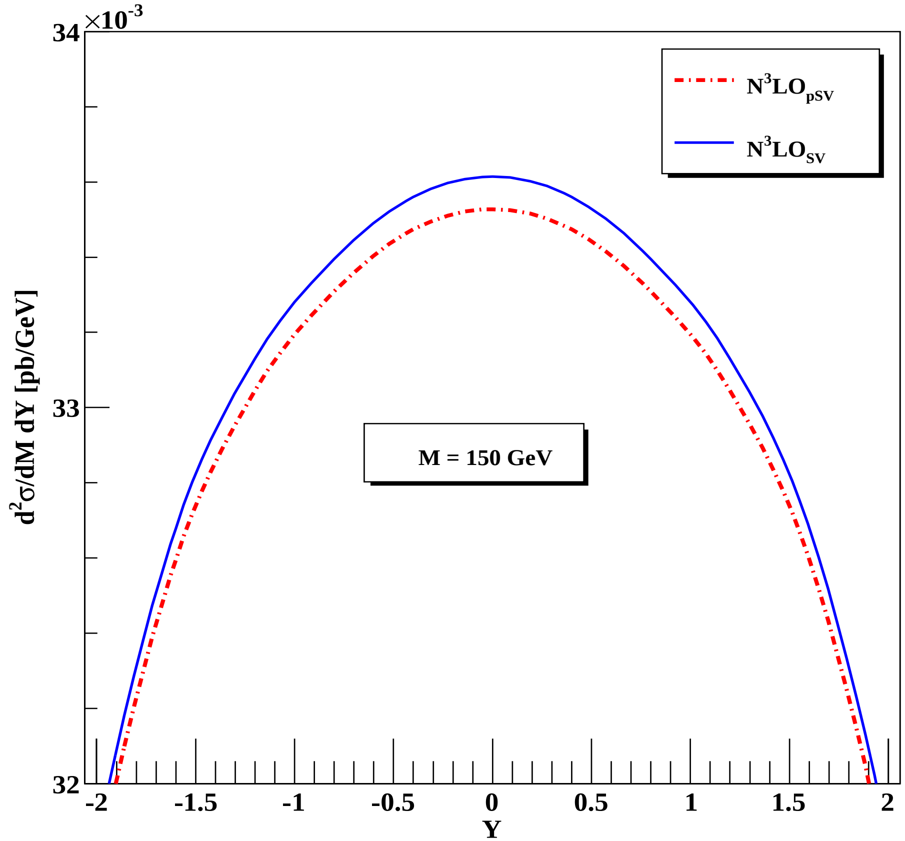

- Compared against Dixon,Anastasiou,Melnikov,Petriello NNLO results for Drell-Yan,$\boldsymbol{Higgs}, \boldsymbol{Z}, \boldsymbol{W^{\pm}}$ productions.

N^3LO_{SV} results for Higgs rapidity

Ahmed, Mandal, Rana, VR

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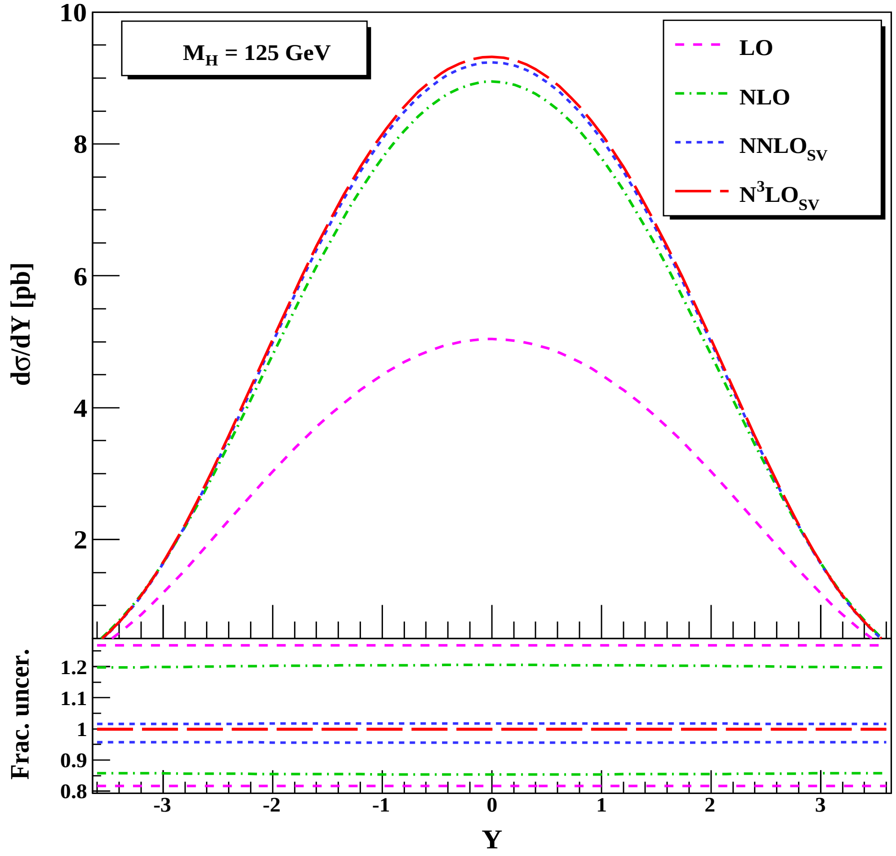

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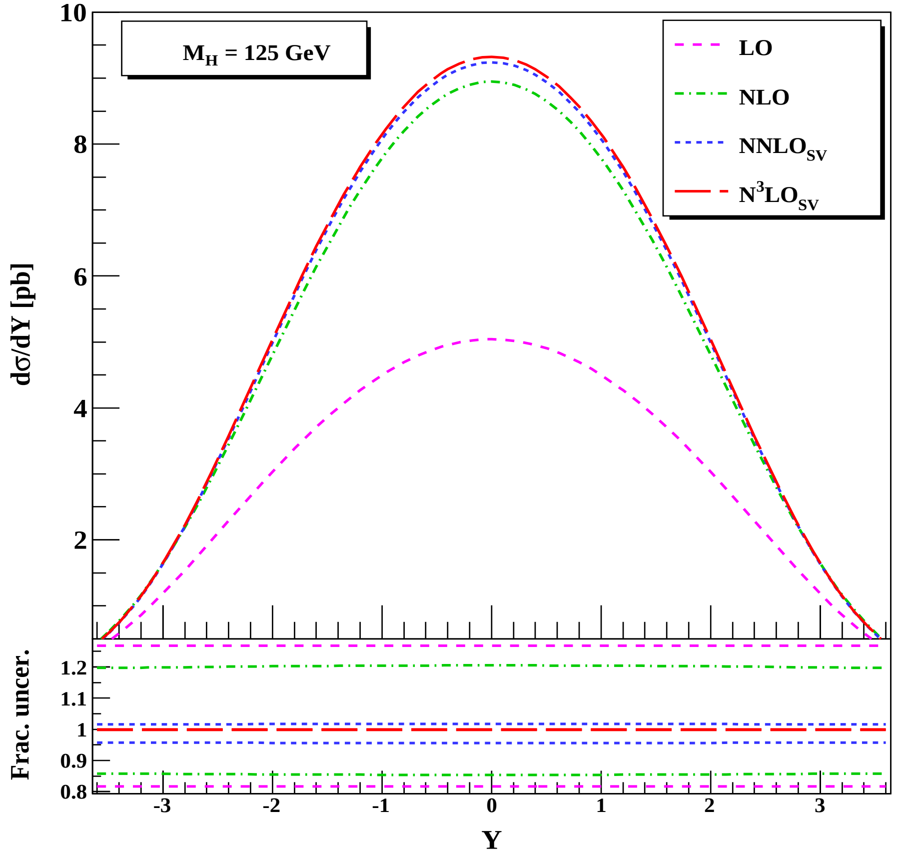

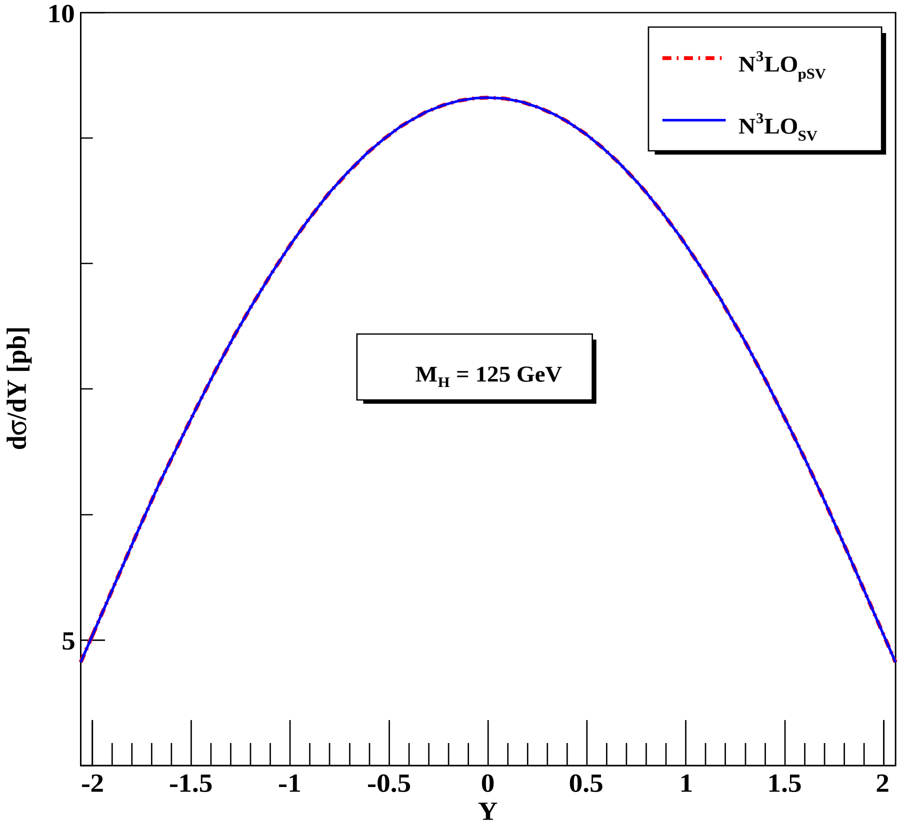

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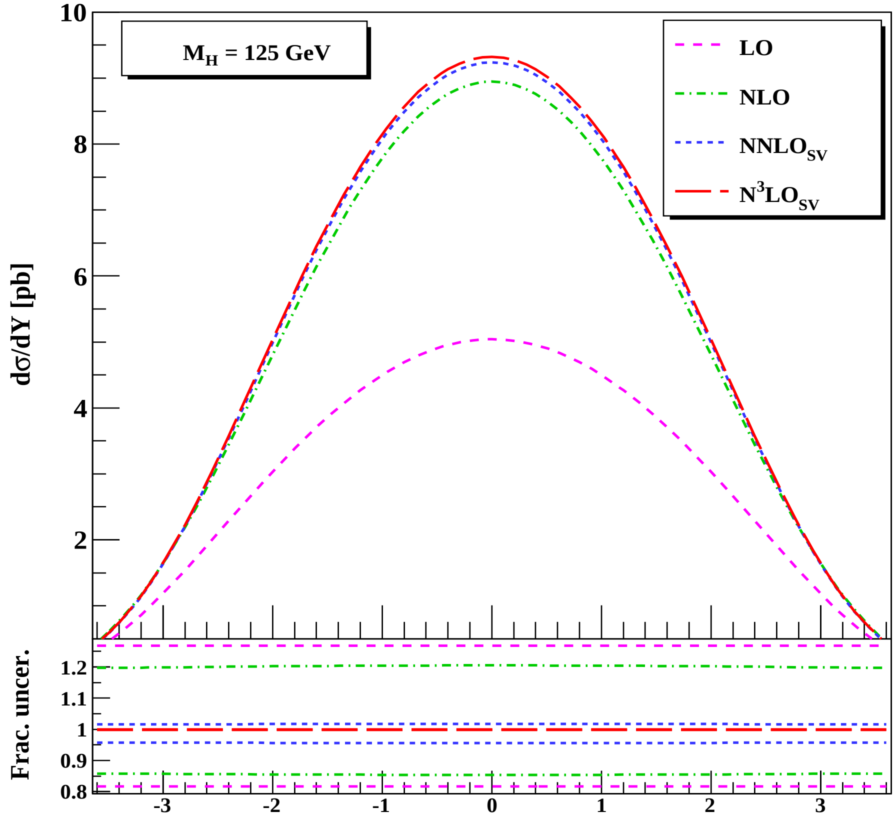
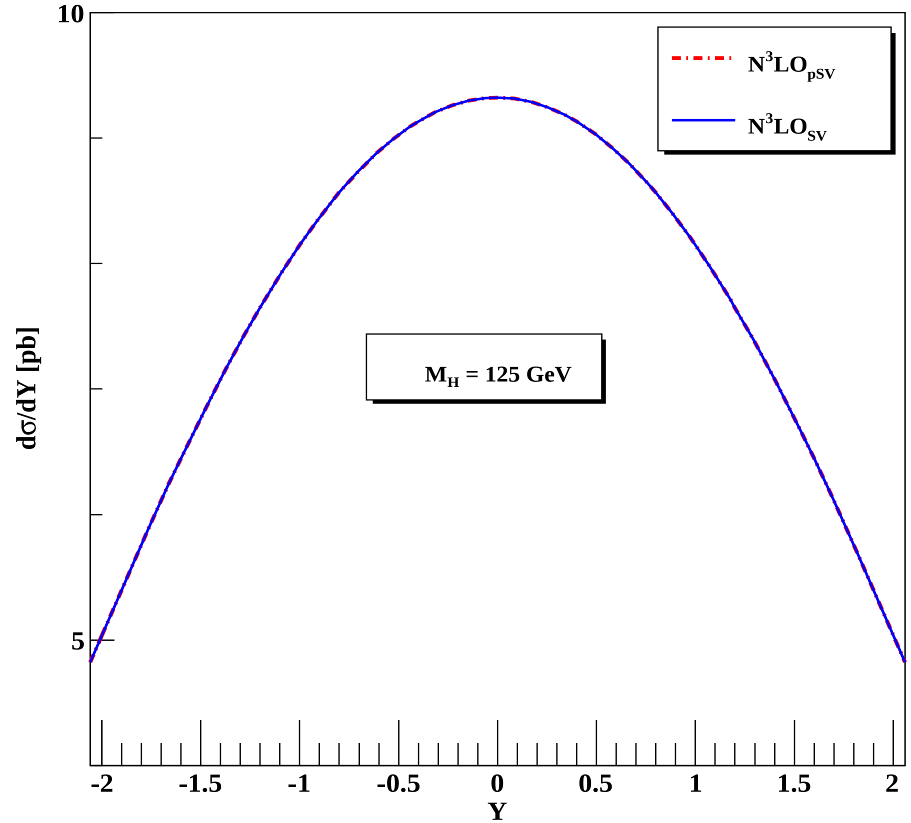




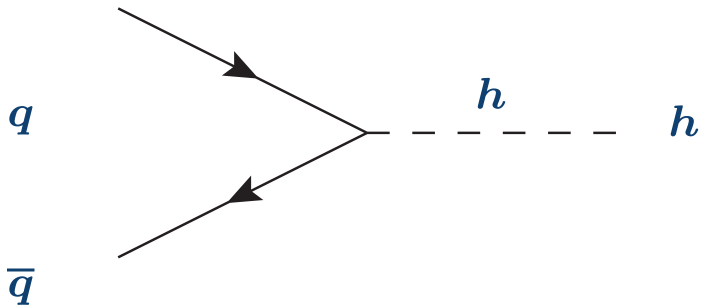
- Scale uncertainity improves a lot

	$\delta\delta$	$\delta\bar{\mathcal{D}}_0$	$\delta\bar{\mathcal{D}}_1$	$\delta\bar{\mathcal{D}}_2$	$\delta\bar{\mathcal{D}}_3$	$\delta\bar{\mathcal{D}}_4$	$\delta\bar{\mathcal{D}}_5$	$\mathcal{D}_0\bar{\mathcal{D}}_0$	$\mathcal{D}_0\bar{\mathcal{D}}_1$	$\mathcal{D}_0\bar{\mathcal{D}}_2$	$\mathcal{D}_0\bar{\mathcal{D}}_3$	$\mathcal{D}_0\bar{\mathcal{D}}_4$	$\mathcal{D}_1\bar{\mathcal{D}}_1$	$\mathcal{D}_1\bar{\mathcal{D}}_2$	$\mathcal{D}_1\bar{\mathcal{D}}_3$	$\mathcal{D}_2\bar{\mathcal{D}}_2$
%	73.3	16.0	9.1	31.4	1.0	-9.9	-23.1	-13.7	-10.7	-0.3	3.1	7.3	-0.2	3.8	8.6	4.2

Relative contributions of pure N^3LO terms

$b + \overline{b} \rightarrow Higgs$ at Threshold N^3LO

$b + \overline{b} \rightarrow Higgs$ at Threshold N^3LO



N^3LO_{SV} results for $b + \overline{b} \rightarrow H$

Ahmed, Mandal, Rana, VR

N^3LO_{SV} results for $b + \overline{b} \rightarrow H$

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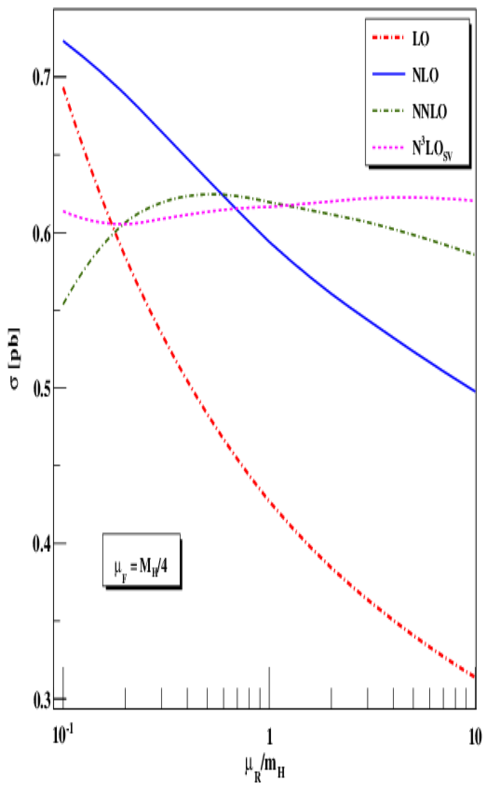

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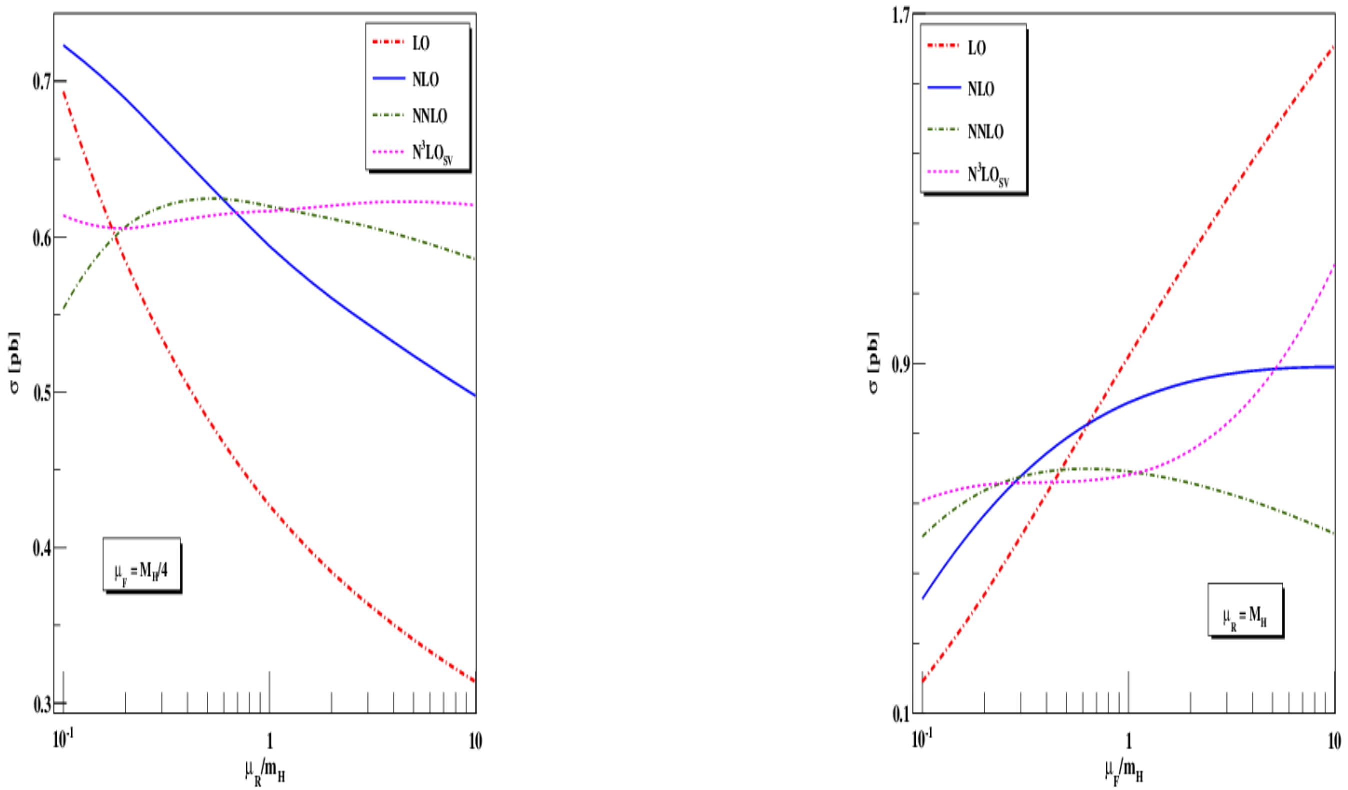
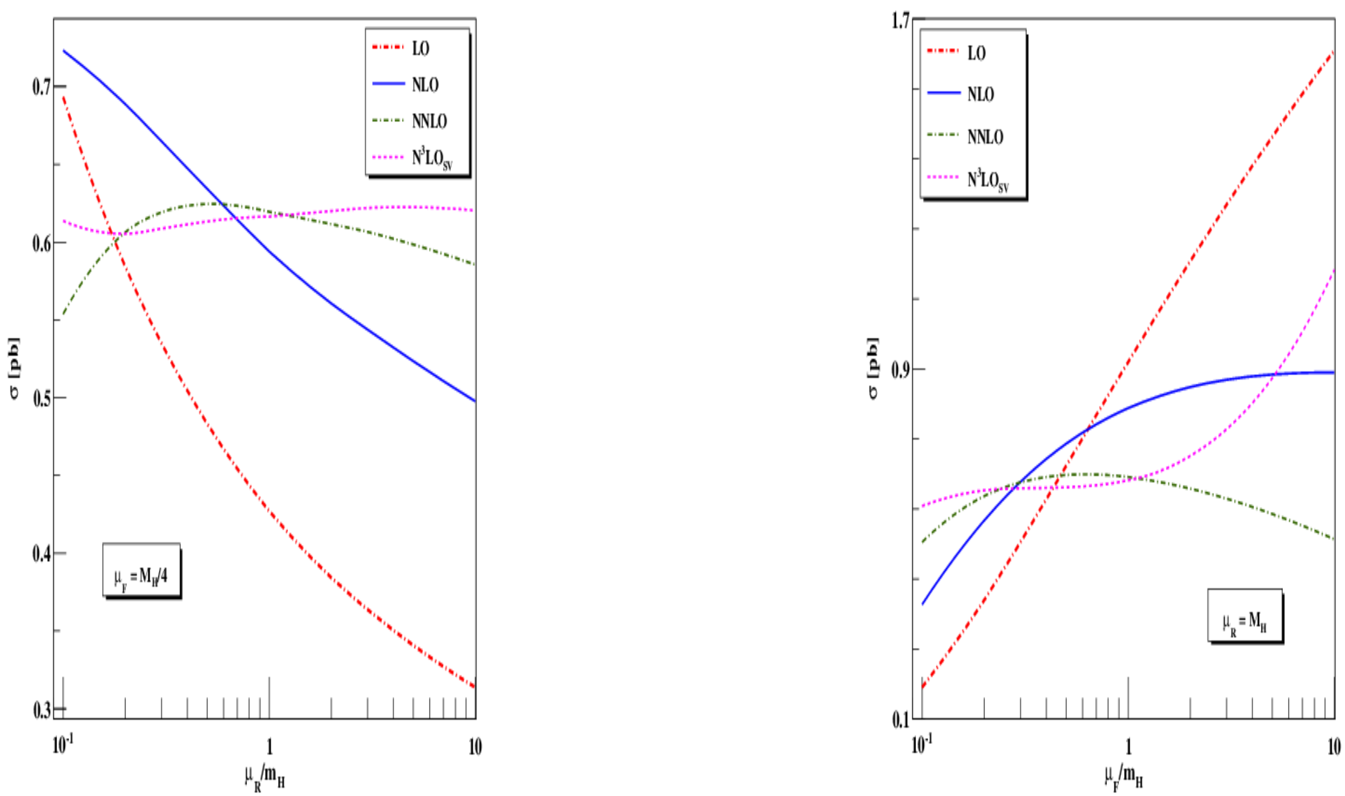


- Total cross section for Higgs production in $b\bar{b}$ annihilation at various orders in a_s as a function of μ_R/m_H (left panel) and of μ_F/m_H (right panel) at the LHC with $\sqrt{s} = 14$ TeV.

N^3LO_{SV} results for $b + \overline{b} \rightarrow H$

Ahmed, Mandal, Rana, VR



- Total cross section for Higgs production in $b\bar{b}$ annihilation at various orders in a_s as a function of μ_R/m_H (left panel) and of μ_F/m_H (right panel) at the LHC with $\sqrt{s} = 14$ TeV.

N^3LO_{SV} results for $d\sigma/dY$ in $b + \overline{b} \to H$

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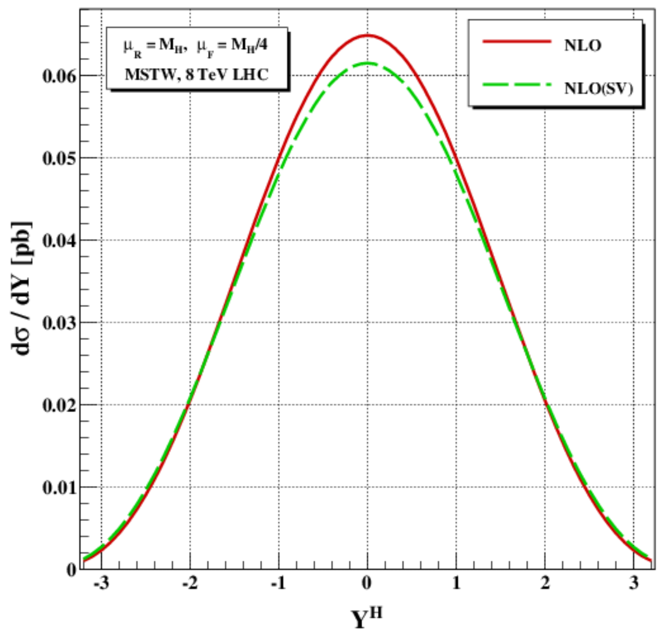

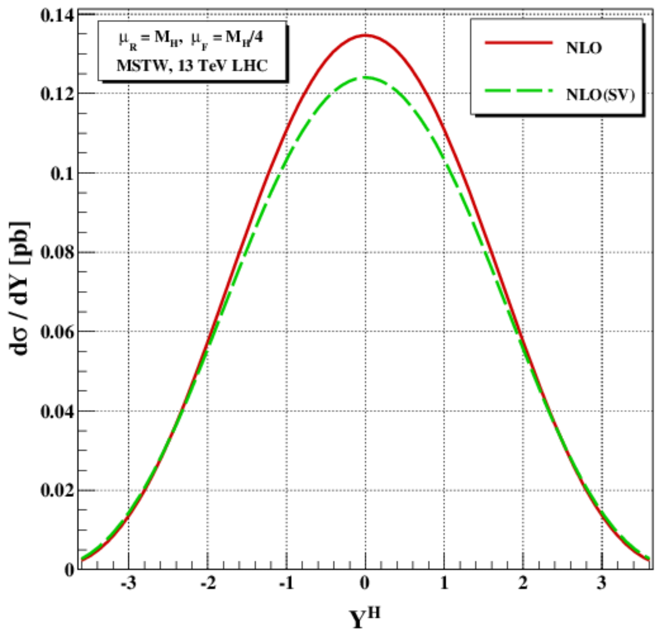

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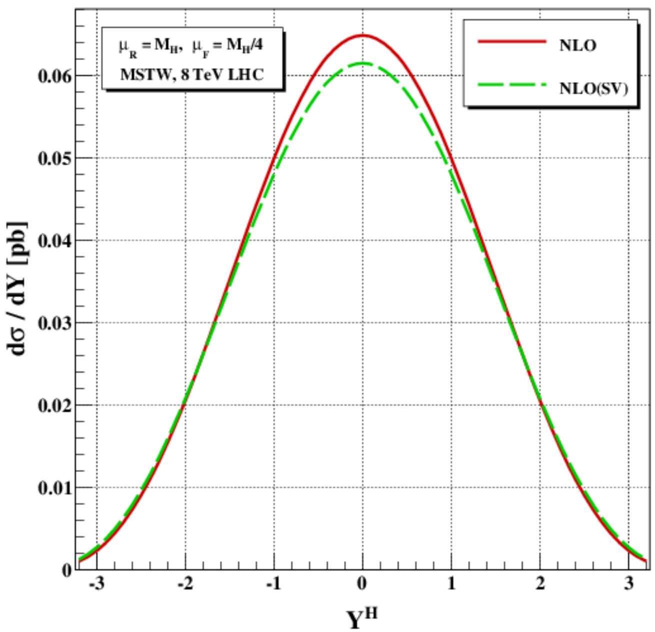

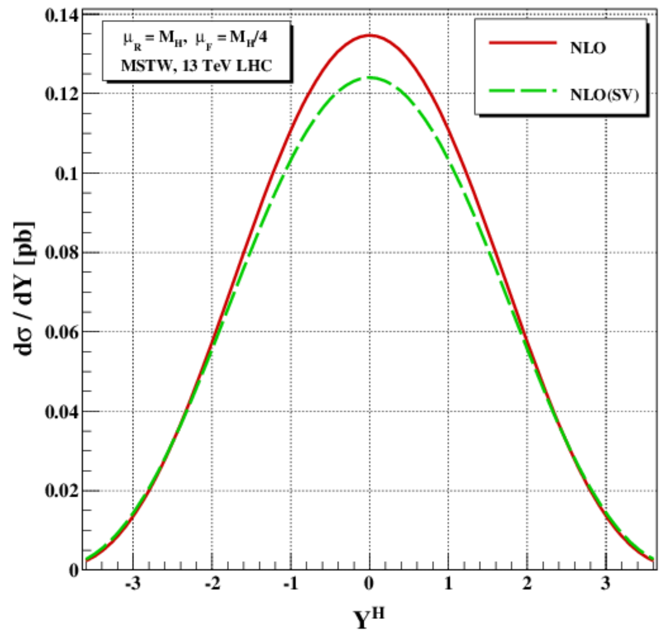

- The comparison between NLO and $\text{NLO}_{\mathbf{SV}}$ with the renormalization scale $\mu_R = m_H$ and factorization scale $\mu_F = m_H/4$ at 8 TeV(left panel) and 13 TeV (right panel) LHC.

N^3LO_{SV} results for $d\sigma/dY$ in $b + \overline{b} \rightarrow H$

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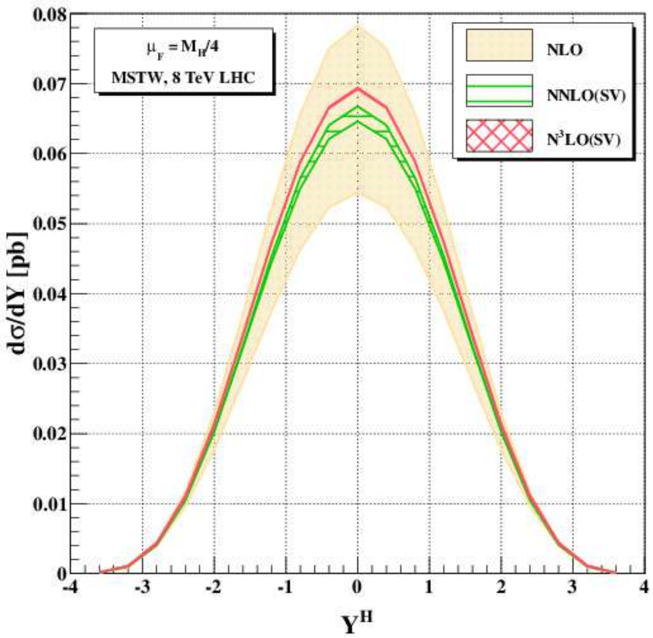

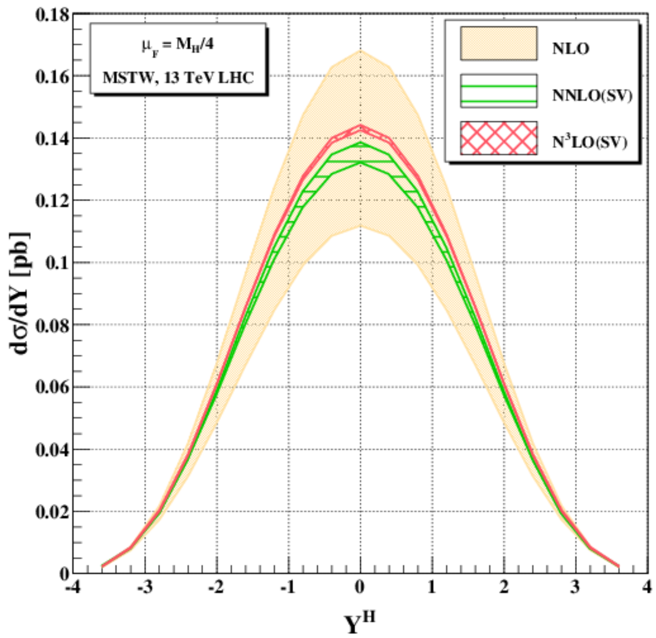

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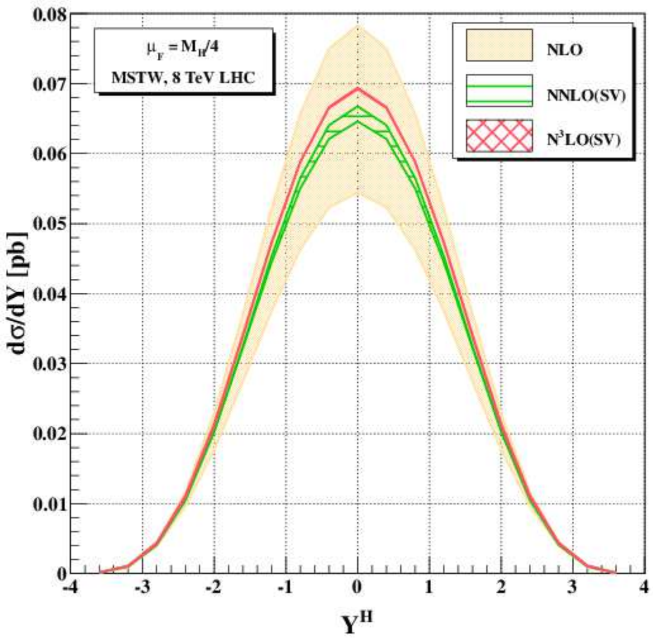
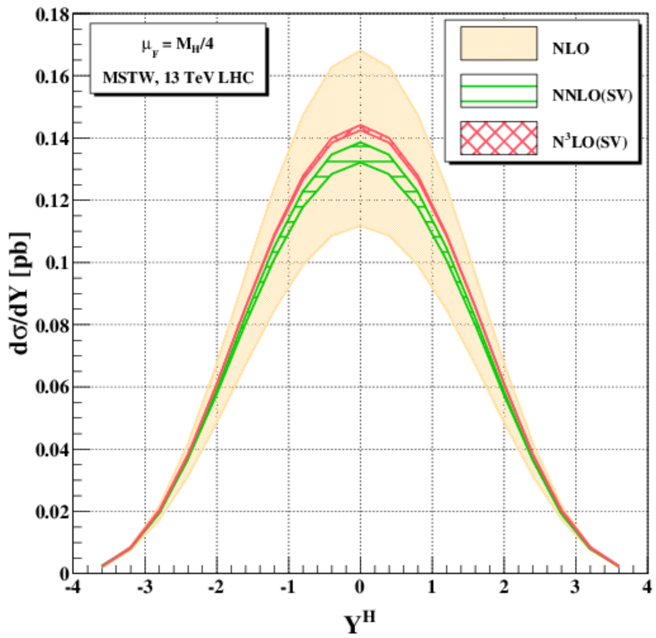
Ahmed, Mandal, Rana, VR





- The rapidity distribution of the Higgs boson at NLO, NNLO(SV) and N^3LO(SV) at 8 TeV(left panel) and 13 TeV (right panel) LHC. The band indicates the uncertainty due to renormalization scale.

N^3LO_{SV} results for $d\sigma/dY$ in $b+\overline{b}\rightarrow H$

Ahmed, Mandal, Rana, VR

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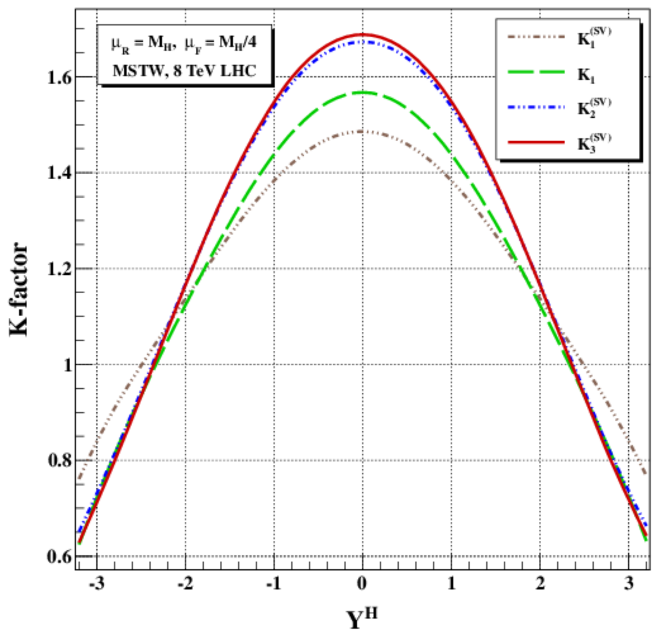
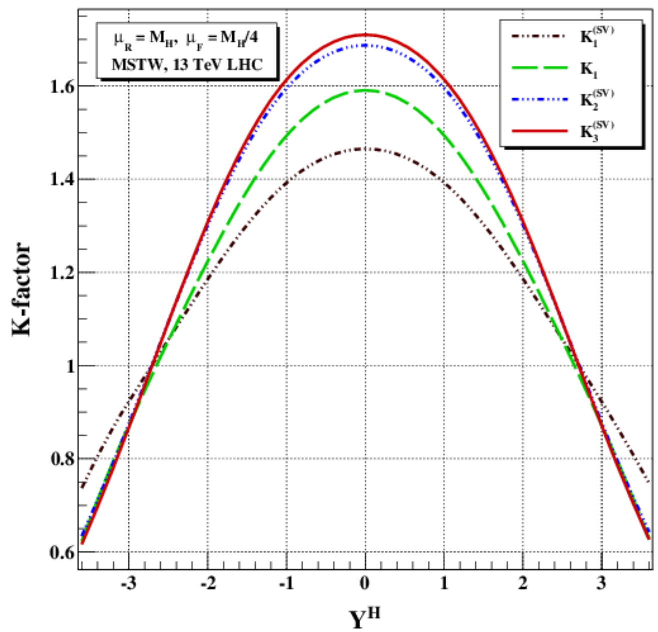
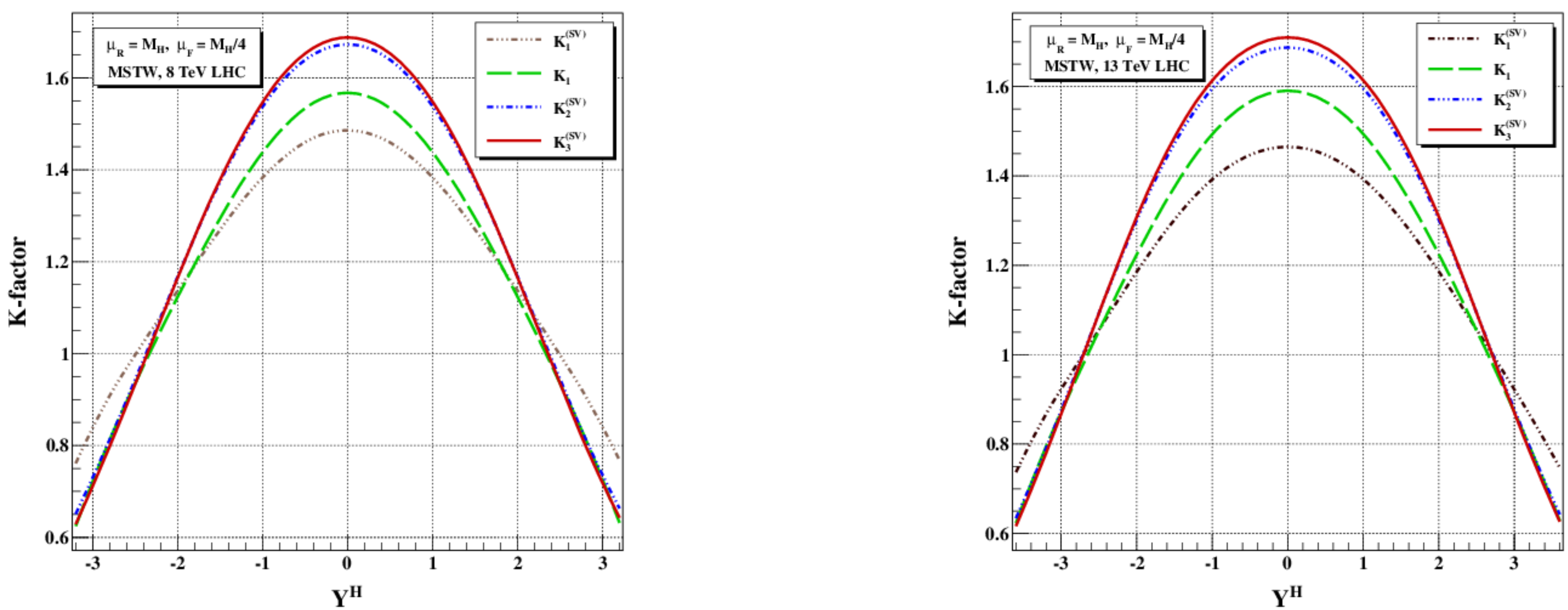




N^3LO_{SV} results for $d\sigma/dY$ in $b+\bar{b}\to H$

Ahmed, Mandal, Rana, VR



- The distribution of $\boldsymbol{K_1^{(SV)}}$, $\boldsymbol{K_1}$, $\boldsymbol{K_2^{(SV)}}$ and $\boldsymbol{K_3^{(SV)}}$ at different perturbative order at 8 TeV(left panel) and 13 TeV (right panel) LHC.

N^3LO_{SV} results for $d\sigma/dY$ in $b + \overline{b} \to H$

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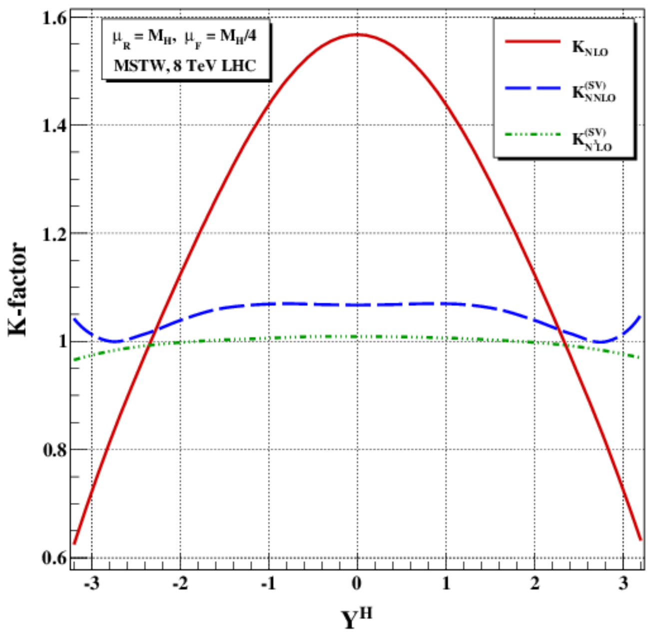
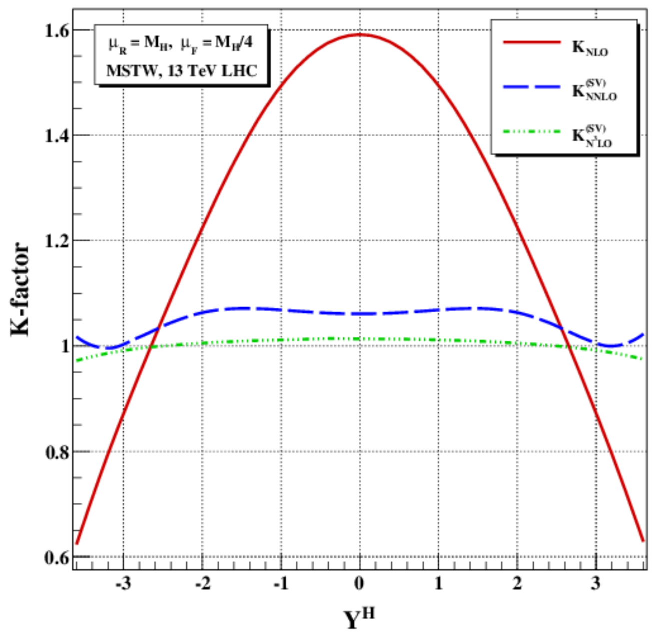
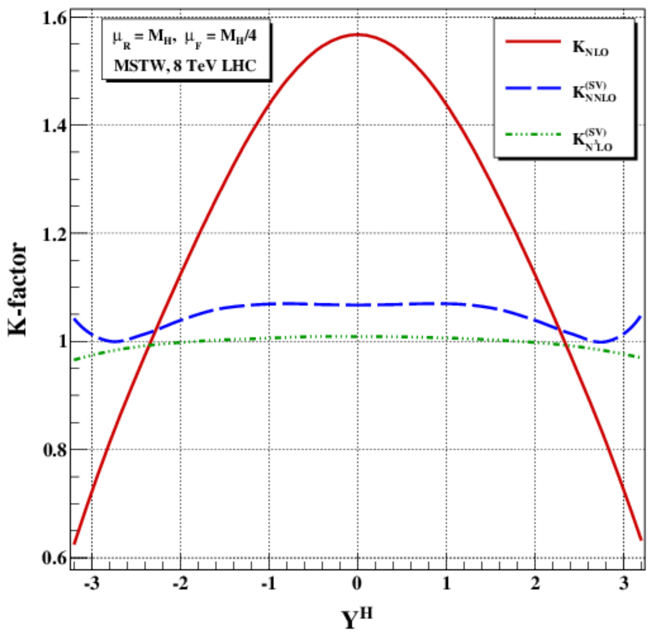
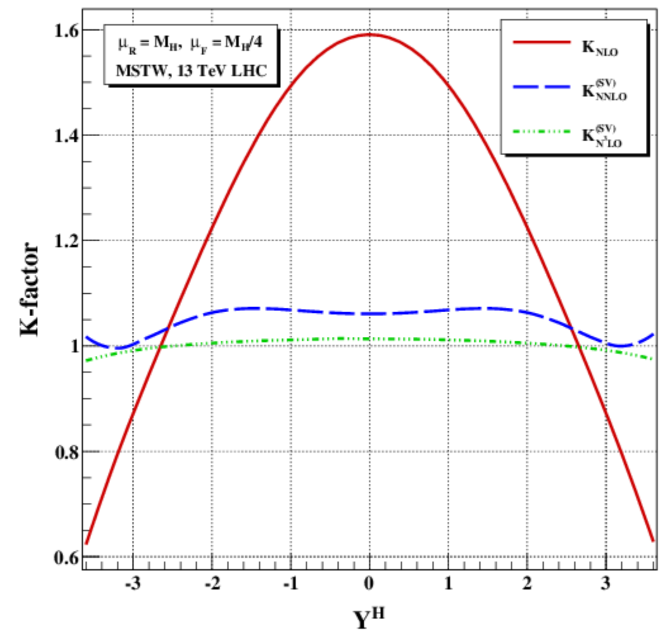




N^3LO_{SV} results for $d\sigma/dY$ in $b + \bar{b} \rightarrow H$

Ahmed, Mandal, Rana, VR

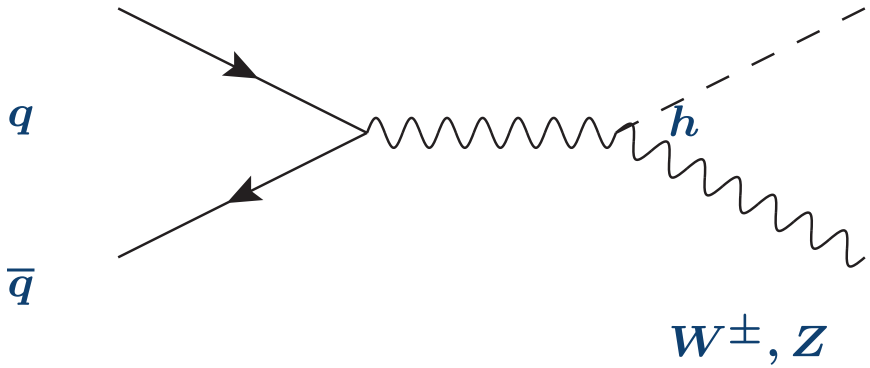




- The distribution of K_{NLO}, $K^{(\text{SV})}_{\text{NNLO}}$ and $K^{(\text{SV})}_{\text{N}^3\text{LO}}$ at different perturbative order at 8 TeV(left panel) and 13 TeV (right panel) LHC.

Higgs in association with W/Z

HIGGS IN ASSOCIATION WITH W/Z



N^3LO_{SV} results for Higgsstralung with Z/W

Mandal, Kumar, VR

N^3LO_{SV} results for Higgsstralung with Z/W

Mandal, Kumar, VR

$\mathbf{E_{CM}}$	LO	$\mathbf{NLO_{SV}}$	NLO	$\mathbf{NNLO_{SV}}$	NNLO	$\mathbf{N^3LO_{SV}}$
7	0.2415	0.2987	0.3183	0.3203	0.3257	0.3254
8	0.2977	0.3667	0.3901	0.3932	0.3993	0.3991
13	0.6120	0.7363	0.7788	0.7900	0.7975	0.7970
14	0.6801	0.8150	0.8604	0.8730	0.8808	0.8807

- DY like contributions (in pb) for different center of mass energies (TeV) at LHC with MSTW2008 PDFs. The factorization and renormalization scales are set to $\mu_F = \mu_R = Q$.

N^3LO_{SV} results for Higgsstralung with Z/W

Mandal, Kumar, VR

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7	0.2415	0.2987	0.3183	0.3203	0.3257	0.3254
8	0.2977	0.3667	0.3901	0.3932	0.3993	0.3991
13	0.6120	0.7363	0.7788	0.7900	0.7975	0.7970
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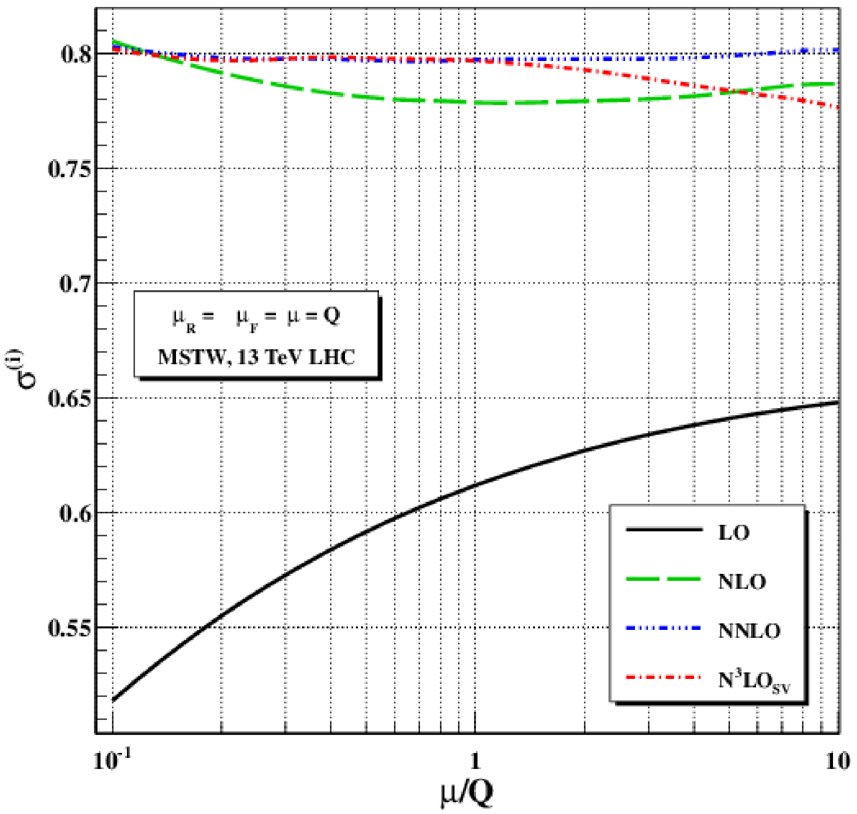
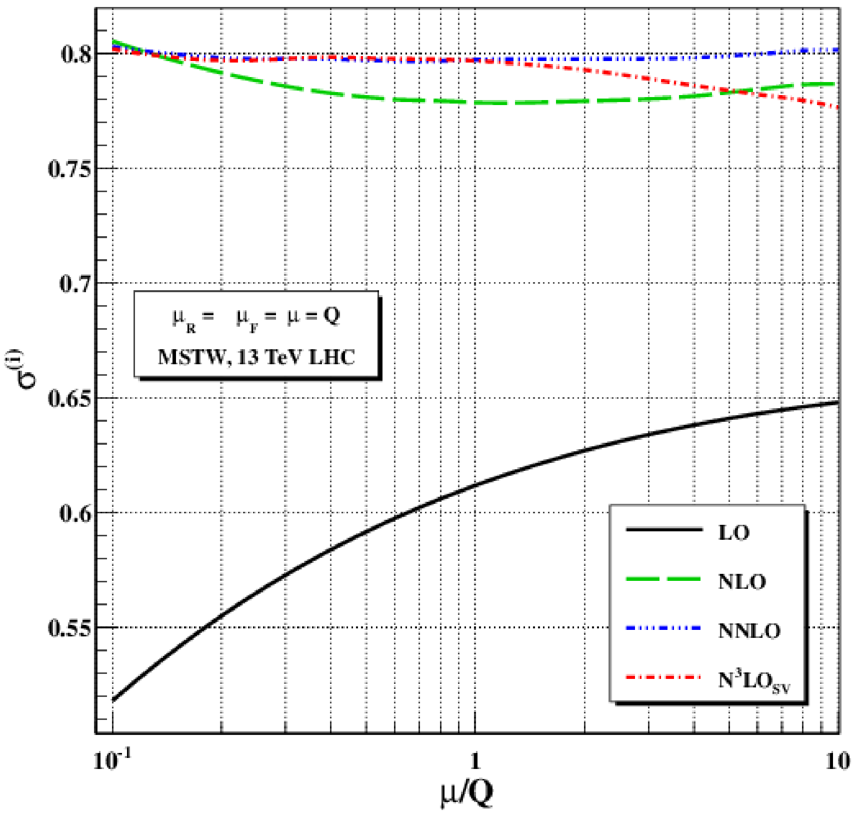


N^3LO_{SV} results for Higgsstralung with Z/W

Mandal, Kumar, VR



- Scale uncertainties of DY type cross sections for LHC13 by varying the factorization and renormalization scales in the range $0.1 < \mu/Q < 10.0$, where $\mu = \mu_F = \mu_R$.

N^3LO_{SV} results for Higgsstralung with Z/W

Mandal, Kumar, VR

N^3LO_{SV} results for Higgsstralung with Z/W

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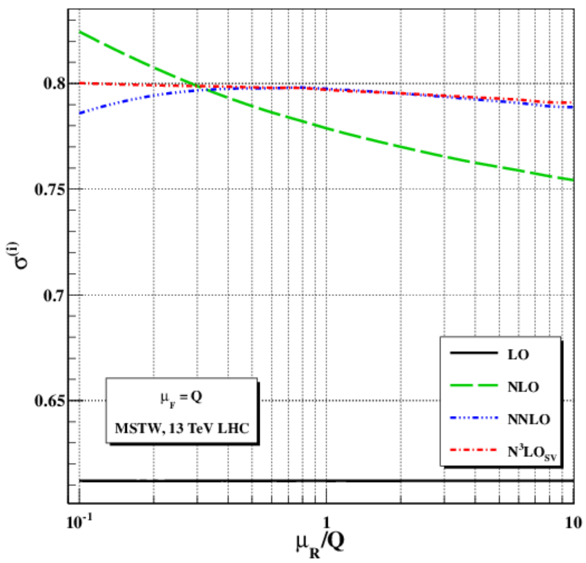
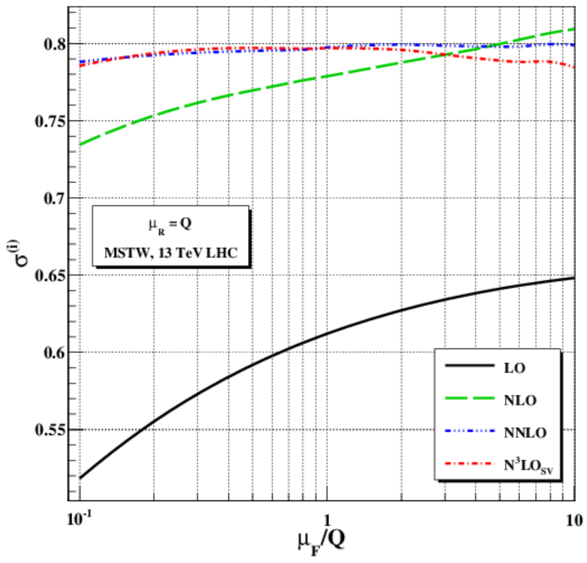
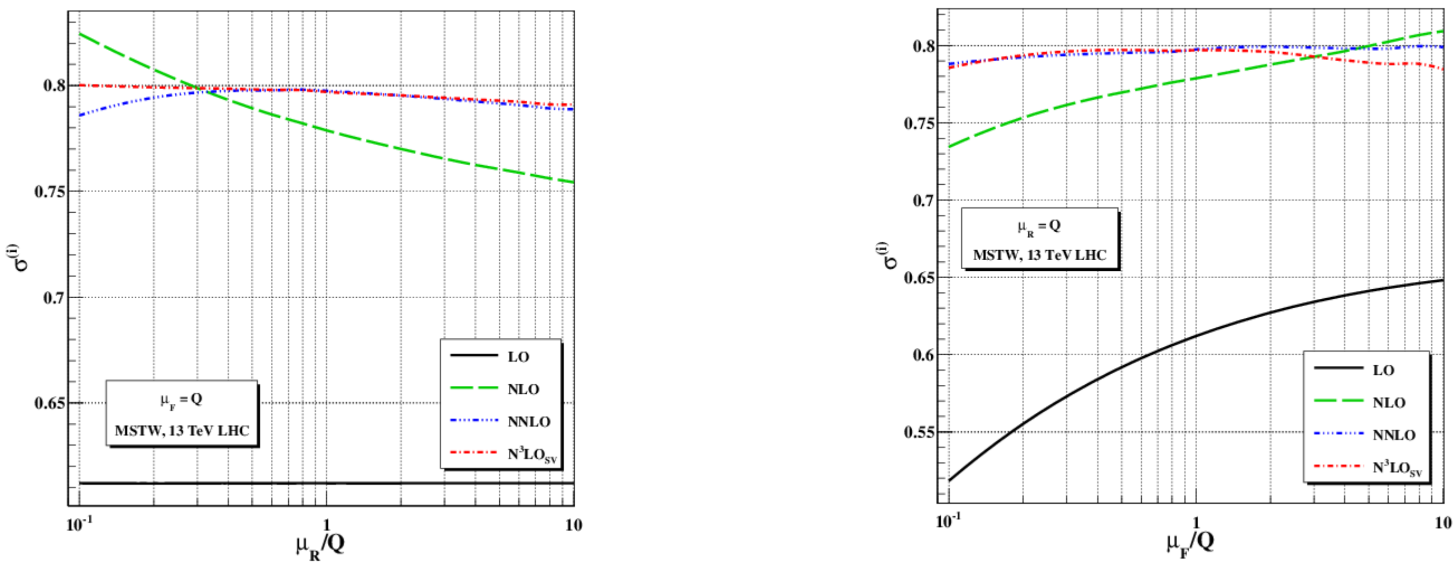




N^3LO_{SV} results for Higgsstralung with Z/W

Mandal, Kumar, VR



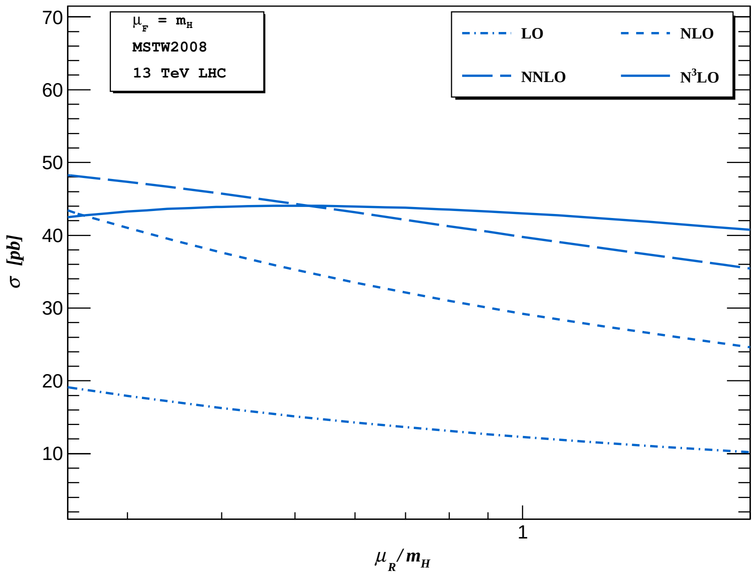
- Scale uncertainties of DY type cross sections for LHC13. In the left panel, we show the renormalization scale uncertainty for $0.1 < \mu_R/Q < 10.0$ keeping $\mu_F = Q$ fixed. In the right panel, we show the factorization scale uncertainty for the similar range variation as μ_R.

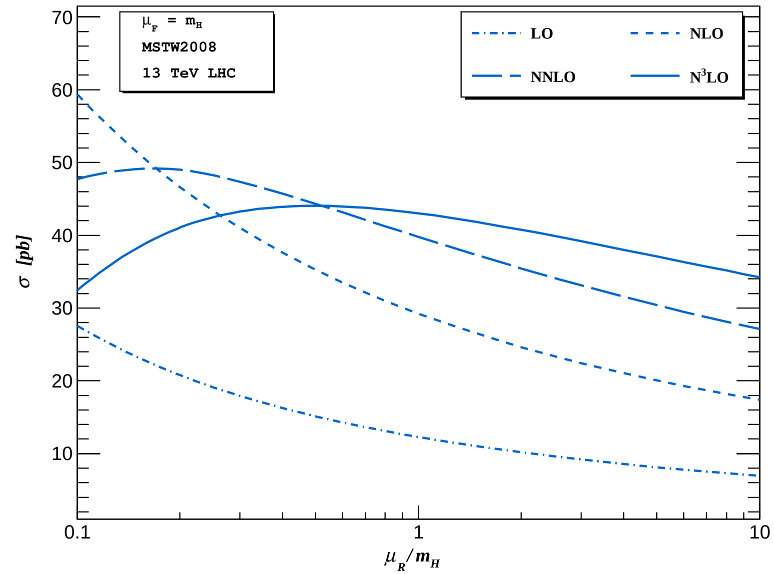
Renormalisation scale dependence at N^3LO in gluon fusion

$Kumar, Ahmed, \ Rana, Goutam, VR$

Renormalisation scale dependence at N^3LO in gluon fusion

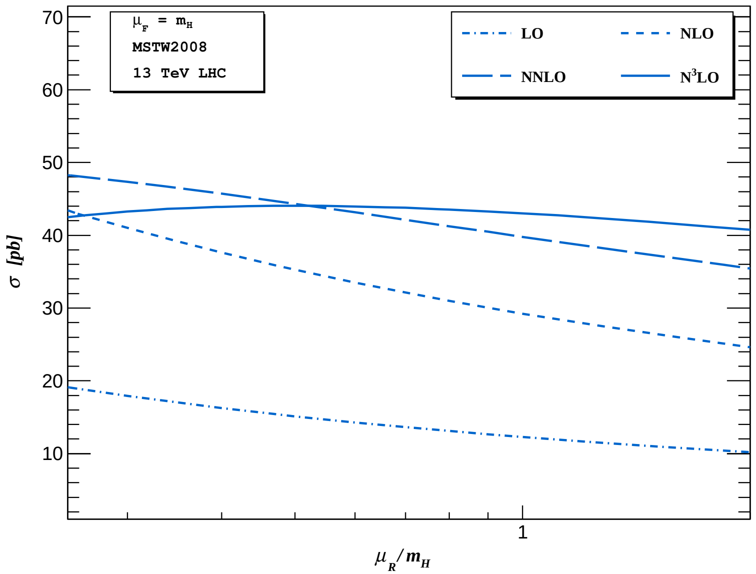
Kumar, Ahmed, Rana, Goutam, VR

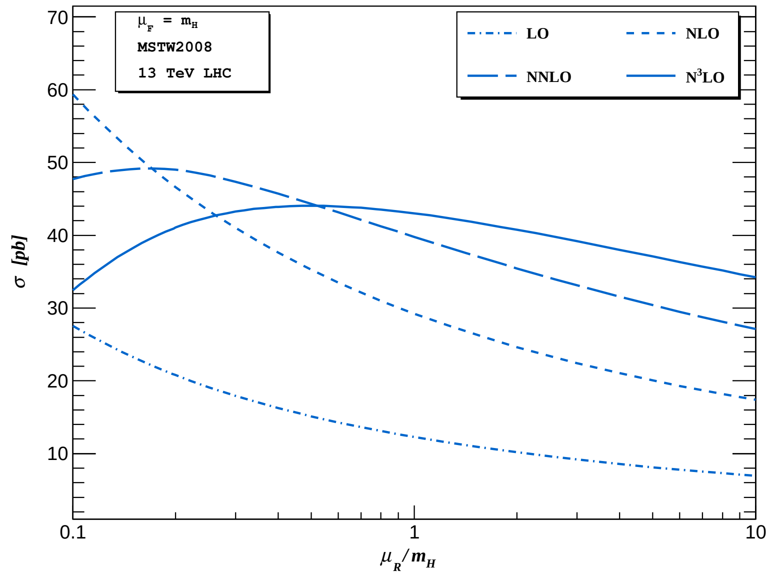




Renormalisation scale dependence at N^3LO in gluon fusion

Kumar, Ahmed, Rana, Goutam, VR





- Scale uncertainties of Higgs cross section for LHC13 by varying the renormalization scale μ_R in the
  - canonical range $1/4 < \mu_R/m_H < 2.0$
  - wider range $1/10 < \mu_R/m_H < 10.$
- Falls of like NLO and NNLO cross sections.
- Scale improvement is visible only when mu_R is closer to m_H.

Renormalisation Group Invariance (RGI)

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$$\begin{aligned}\sigma^{\mathrm{H}}(s, m_{\mathrm{H}}^2) =& \sigma^0 a_s^2(\mu_R^2) \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \\ & \times \mathcal{C}_{\mathrm{H}}^2 \left(a_s(\mu_R^2)\right) \Delta_{ab}^{\mathrm{H}} \left(\frac{\tau}{x_1 x_2}, m_{\mathrm{H}}^2, \mu_R^2, \mu_F^2\right) \\ =& a_s^2(\mu_R^2) \overline{\sigma}(\mu_R^2)\end{aligned}$$

Renormalisation Group Invariance (RGI)

$$
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& \times \mathcal{C}_{\rm H}^2 \left(a_s(\mu_R^2)\right) \Delta_{ab}^{\rm H} \left(\frac{\tau}{x_1 x_2}, m_{\rm H}^2, \mu_R^2, \mu_F^2\right) \\
=& a_s^2(\mu_R^2) \overline{\sigma}(\mu_R^2)
\end{aligned}
$$

Th RG invariance of $\sigma^{\rm H}$

$$
\mu_R^2 \frac{d}{d\mu_R^2} \left[a_s^2(\mu_R^2) \overline{\sigma}(\mu_R^2)\right] = 0
$$

- The solution is given by

$$
\overline{\sigma}(\mu_R^2) = \sum_{n=0}^{\infty} \sum_{k=0}^{n} a_s^n(\mu_R^2)\, \mathcal{R}_{n,k}\, L_R^k
$$

where, $L_R \equiv \ln\left(\frac{\mu_R^2}{m_H^2}\right)$.

- The RG invariance dictates

$$
\mathcal{R}_{n,n-m} = \frac{1}{(n-m)} \sum_{i=0}^{m} (n-i+1) \beta_i \mathcal{R}_{n-i-1,n-m-1}
$$

Resummation of large $\log(\mu_R^2)$ using RG equation

Resummation of large $\log(\mu_R^2)$ using RG equation

$$\overline{\sigma}(\mu_R^2) = \sum_{m=0}^{\infty} a_s^m(\mu_R^2) \sum_{n=m}^{\infty} \mathcal{R}_{n,n-m}(a_s L_R)^{n-m}$$

$$= \sum_{m=0}^{\infty} a_s^m(\mu_R^2)\, \overline{\sigma}_{\Sigma}^{(m)}$$

Resummation of large $\log(\mu_R^2)$ using RG equation

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$$= \sum_{m=0}^{\infty} a_s^m(\mu_R^2)\, \overline{\sigma}_{\Sigma}^{(m)}$$

To compute the $\overline{\sigma}_{\Sigma}^{(m)}$s, we multiply $\left(a_s(\mu_R^2)L_R\right)^{n-m-1}$ with the recursion relation and sum from $n = m+1$ to ∞

$$\sum_{n=m+1}^{\infty} (a_s L_R)^{n-m-1}\left[(n-m)\mathcal{R}_{n,n-m} - \sum_{i=0}^{m}(n-i+1)\beta_i \mathcal{R}_{n-i-1,n-m-1}\right] = 0$$

which leads to the following first-order differential equation

$$\left[\omega\frac{d}{d\omega} + (m+2)\right]\overline{\sigma}_{\Sigma}^{(m)} = \Theta_{m-1}\sum_{i=1}^{m}\eta_i\left[(1-\omega)\frac{d}{d\omega} - (m-i+2)\right]\overline{\sigma}_{\Sigma}^{(m-i)}$$

Θ_{m-1} is Heaviside Theta function, $\omega = (1-\beta_0 a_s L_R)$ and $\eta_i = \frac{\beta_i}{\beta_0}$

Resummed cross sections

Resummed cross sections

$$\overline{\sigma}_{\Sigma}^{(0)} = \frac{1}{\omega^2}\Big\{\mathcal{R}_{0,0}\Big\}\,, \quad \overline{\sigma}_{\Sigma}^{(1)} = \frac{1}{\omega^3}\Big\{\mathcal{R}_{1,0} - 2\eta_1\mathcal{R}_{0,0}\ln(\omega)\Big\}\,,$$

$$\overline{\sigma}_{\Sigma}^{(2)} = \frac{1}{\omega^3}\Big\{2\mathcal{R}_{0,0}\left(\eta_1{}^2 - \eta_2\right)\Big\} + \frac{1}{\omega^4}\Big\{\mathcal{R}_{2,0} + 2\mathcal{R}_{0,0}\left(\eta_2 - \eta_1{}^2\right) + \ln(\omega)\Big(-2\eta_1{}^2\mathcal{R}_{0,0} - 3\eta_1\mathcal{R}_{1,0}\Big) + 3\eta_1{}^2\mathcal{R}_{0,0}\ln^2(\omega)\Big\}\,,$$

$$\overline{\sigma}_{\Sigma}^{(3)} = \frac{1}{\omega^3}\Big\{\mathcal{R}_{0,0}\left(-\eta_1{}^3 + 2\eta_1\eta_2 - \eta_3\right)\Big\} + \frac{1}{\omega^4}\Big\{\mathcal{R}_{0,0}\left(2\eta_1{}^3 - 2\eta_1\eta_2\right) + \mathcal{R}_{1,0}\left(3\eta_1{}^2 - 3\eta_2\right) + \mathcal{R}_{0,0}\left(6\eta_1\eta_2 - 6\eta_1{}^3\right)\ln(\omega)\Big\} + \frac{1}{\omega^5}\Big\{\mathcal{R}_{3,0} + \mathcal{R}_{0,0}\left(\eta_3 - \eta_1{}^3\right) + \mathcal{R}_{1,0}\left(3\eta_2 - 3\eta_1{}^2\right) + \ln(\omega)\Big(\mathcal{R}_{0,0}\left(6\eta_1{}^3 - 8\eta_1\eta_2\right) - 3\eta_1{}^2\mathcal{R}_{1,0} - 4\eta_1\mathcal{R}_{2,0}\Big) + \ln^2(\omega)\Big(7\eta_1{}^3\mathcal{R}_{0,0} + 6\eta_1{}^2\mathcal{R}_{1,0}\Big) - 4\eta_1{}^3\mathcal{R}_{0,0}\ln^3(\omega)\Big\}\,,$$

$$\overline{\sigma}_{\Sigma}^{(4)} = \textit{Big expression}$$

- We have resummed only μ_R dependent logarithms.
- μ_F has been chosen to some specific value m_H
  $\rightsquigarrow$ μ_F dependence remains unchanged.
- Conclusions are independent of the choice of μ_F.

Numerical Implications

Numerical Implications

(a) Result is almost μ_R independent for a wide range of $\mu_R \in [0.1 m_{\rm H}, 10 m_{\rm H}]$

	LO	NLO	NNLO	N^{3}LO
FO (%)	167.26	143.40	54.99	27.01
RESUM (%)	6.11	5.47	3.39	1.23

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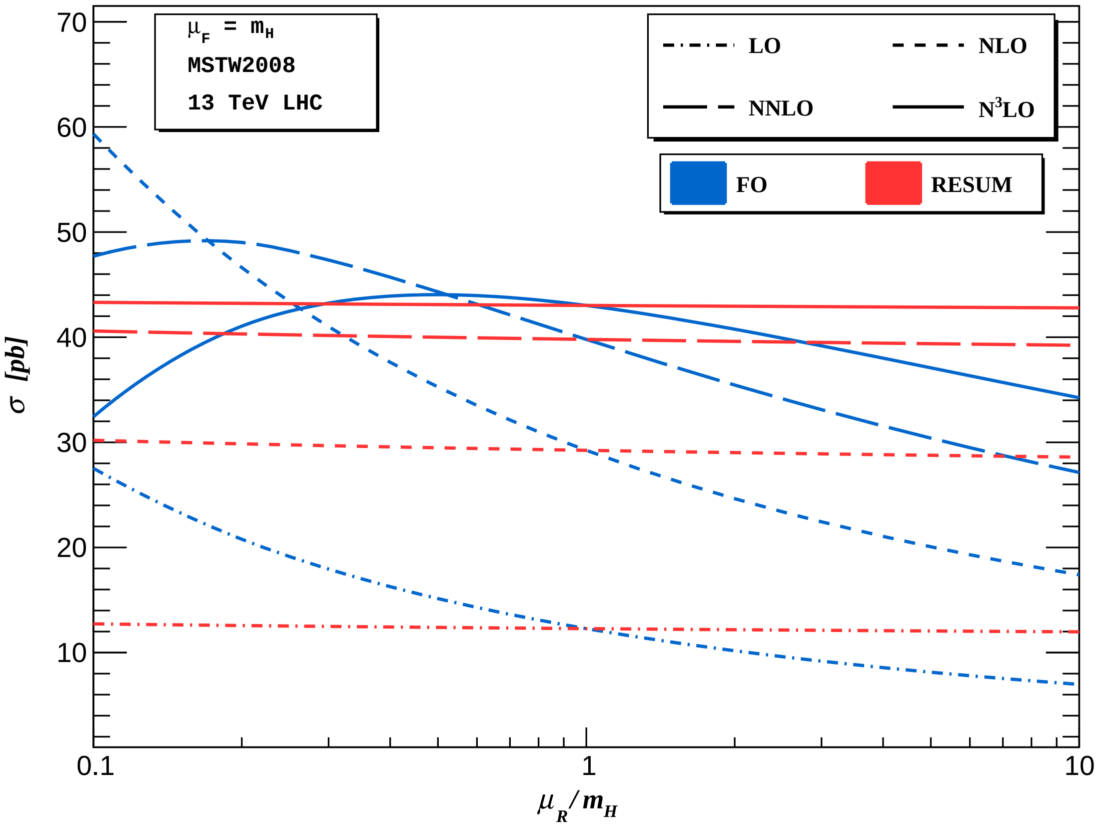

RG improvements through N^{3}LO

$$\sigma^{\mathrm{H}}(s, m_{\mathrm{H}}^2) = a_s^2(\mu_R^2)\left(\overline{\sigma}_{\Sigma}^{(0)} + a_s(\mu_R^2)\overline{\sigma}_{\Sigma}^{(1)} + a_s^2(\mu_R^2)\overline{\sigma}_{\Sigma}^{(2)} + a_s^3(\mu_R^2)\overline{\sigma}_{\Sigma}^{(3)} + \ldots\right)$$

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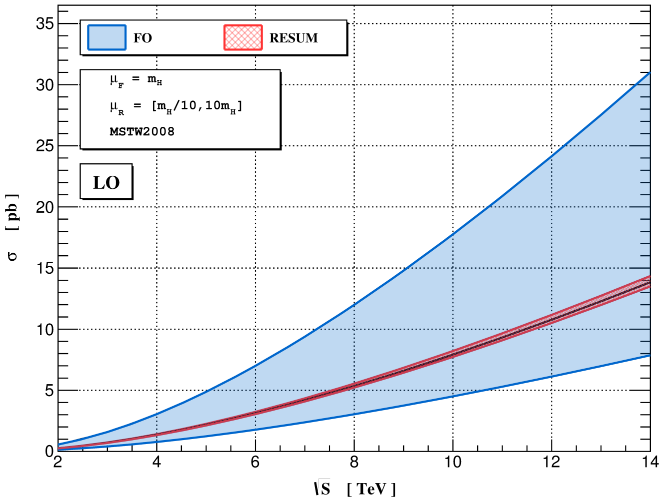
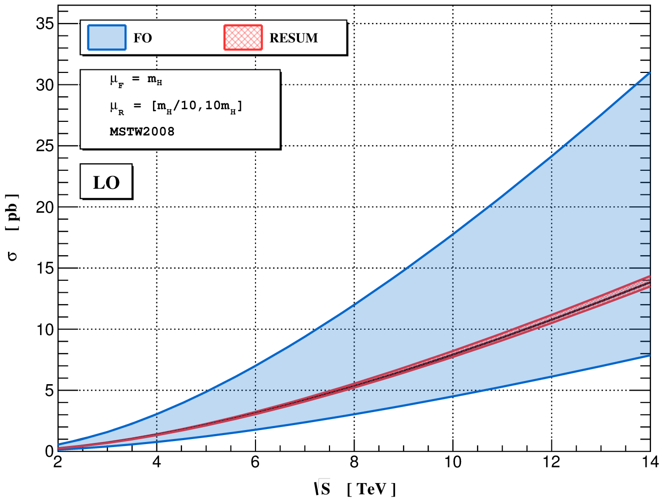
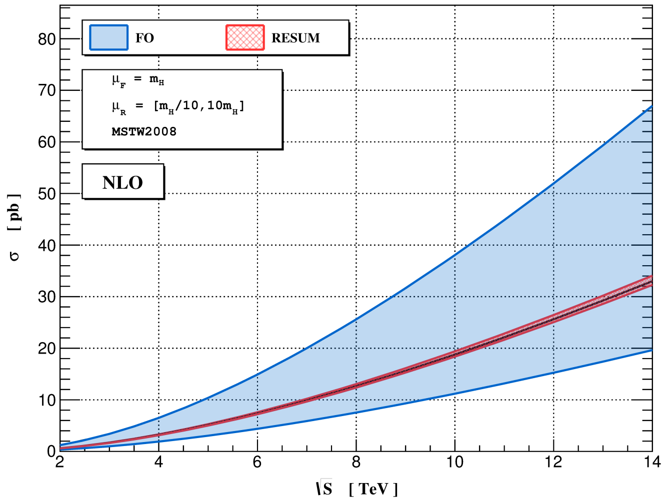


RG improvements through N^3LO

$$\sigma^{H}(s, m_H^2) = a_s^2(\mu_R^2)\left(\overline{\sigma}_{\Sigma}^{(0)} + a_s(\mu_R^2)\overline{\sigma}_{\Sigma}^{(1)} + a_s^2(\mu_R^2)\overline{\sigma}_{\Sigma}^{(2)} + a_s^3(\mu_R^2)\overline{\sigma}_{\Sigma}^{(3)} + \ldots\right)$$





RG improvements through N^{3}LO

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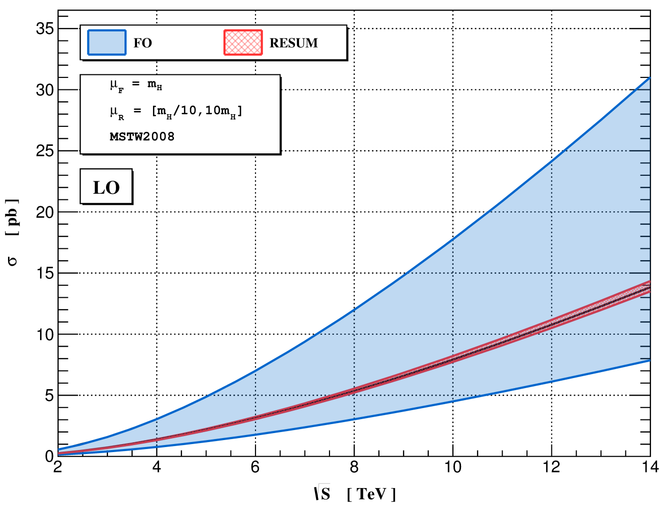

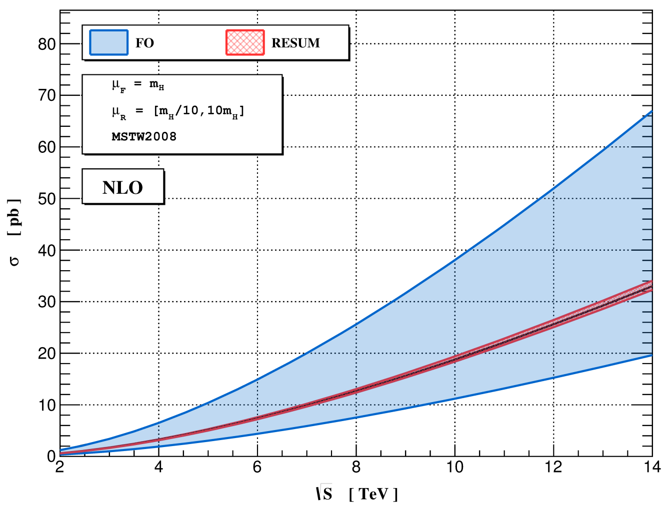

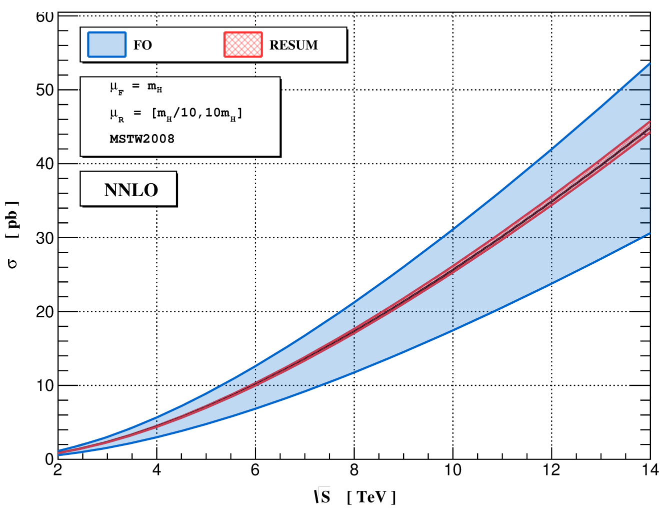

RG improvements through N^{3}LO

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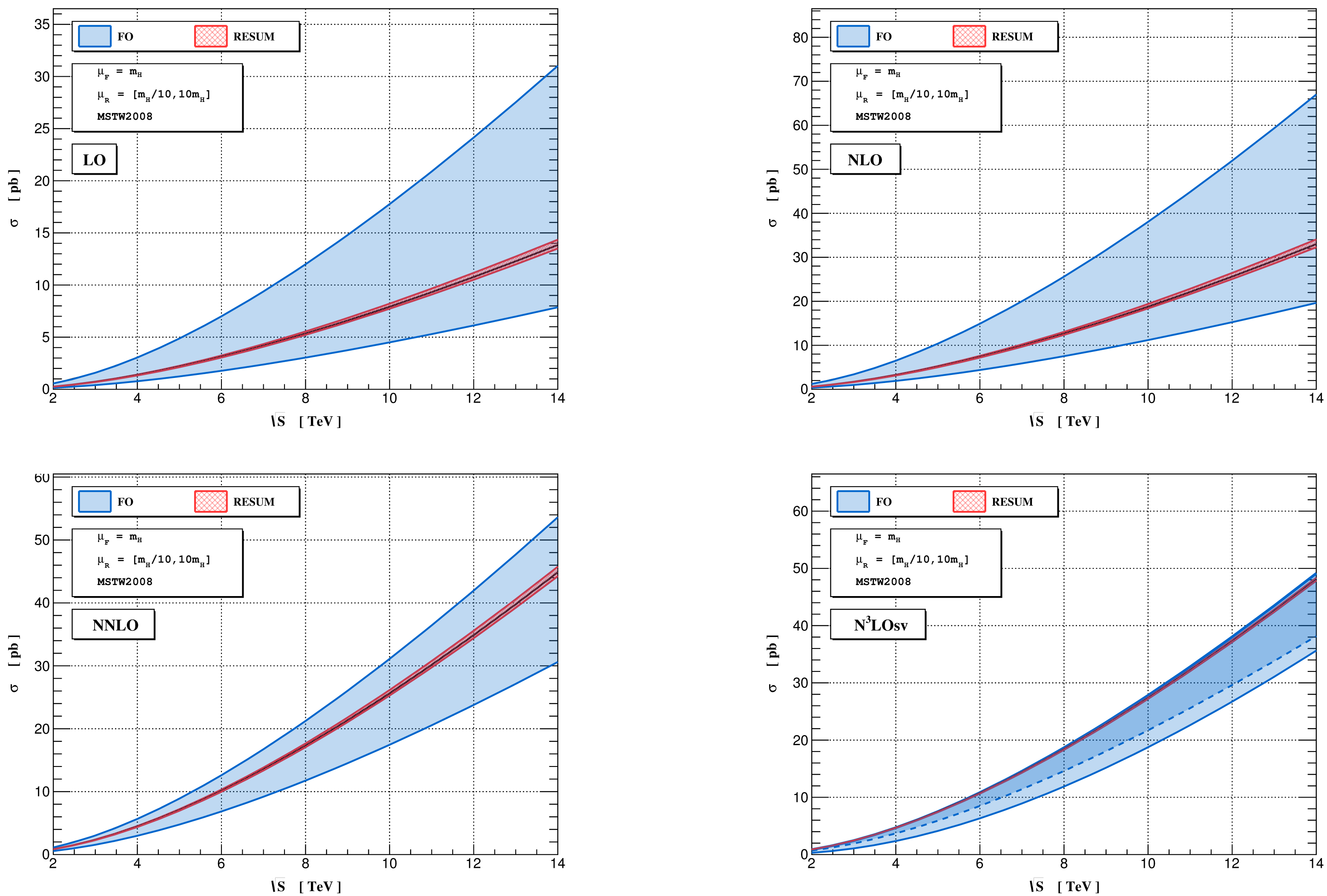
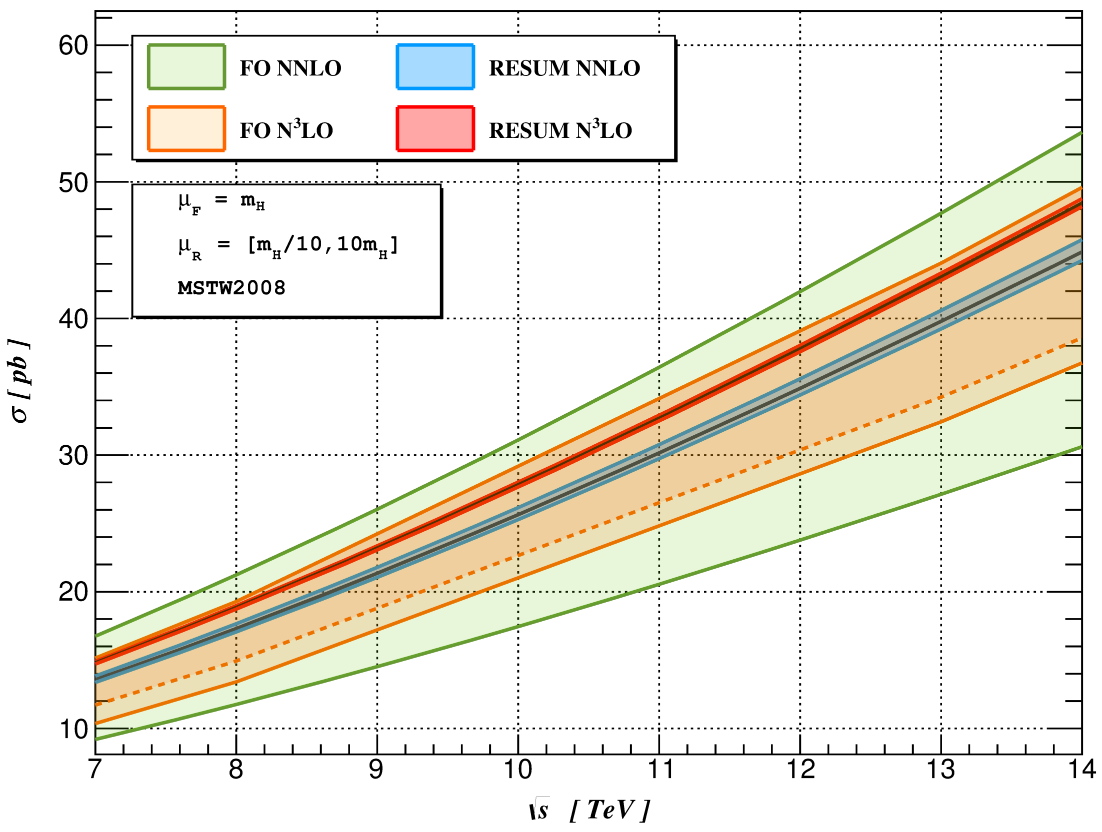


RG improved Higgs production at N^{3}LO

RG improved Higgs production at N³LO



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- Renormalisation group improved expression at N^{3}LO behaves better over wide range of μ_R.