NNLO matching and parton shower developments

Stefan Prestel

SLAC NATIONAL ACCELERATOR LABORATORY

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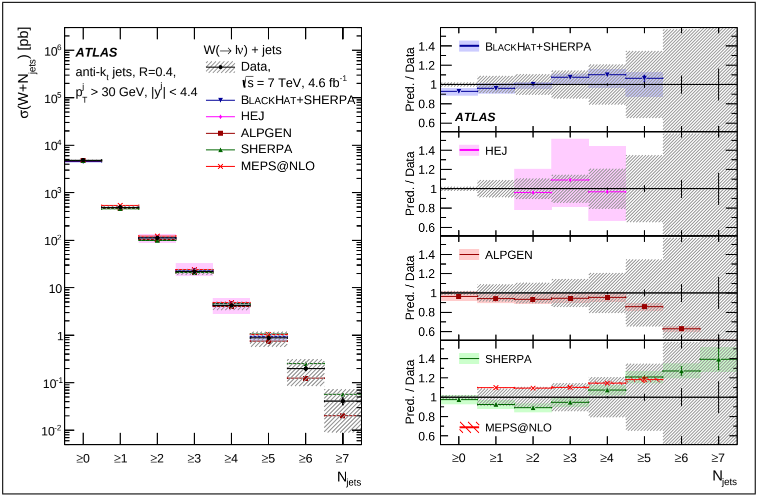
(in collaboration with Stefan Höche and Ye Li)

Outline

- Motivation / introduction:
  - A pessimist's view on NLO matching, LO merging, NLO merging and NNLO matching (e.g. everything that I usually turn to)
- New parton showers for PYTHIA and SHERPA
- Summary

Event generators should describe multijet observables



(Figures taken from EPJC 75 (2015) 2 82 and CMS summary of https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsCombined)

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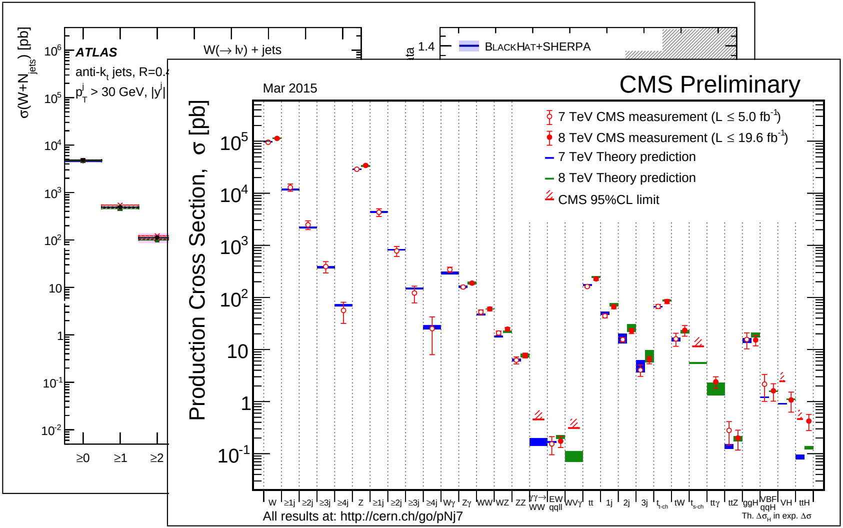

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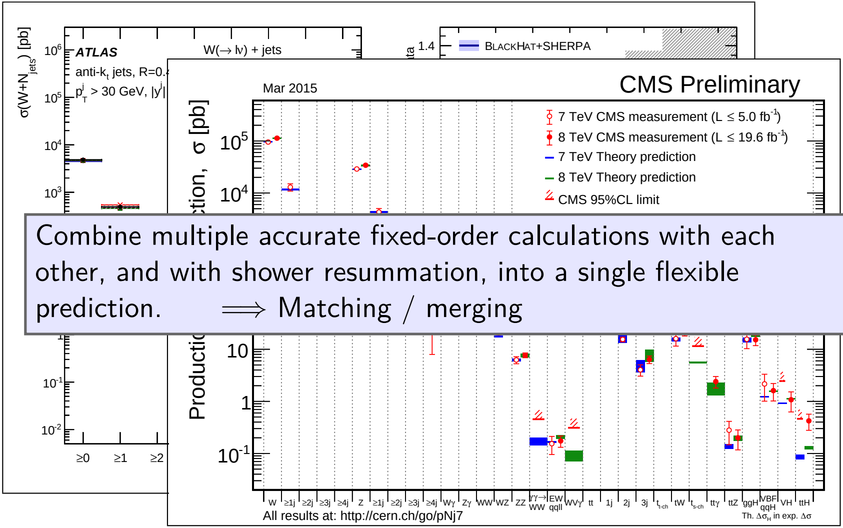

Combine multiple accurate fixed-order calculations with each other, and with shower resummation, into a single flexible prediction. $\Longrightarrow$ Matching / merging

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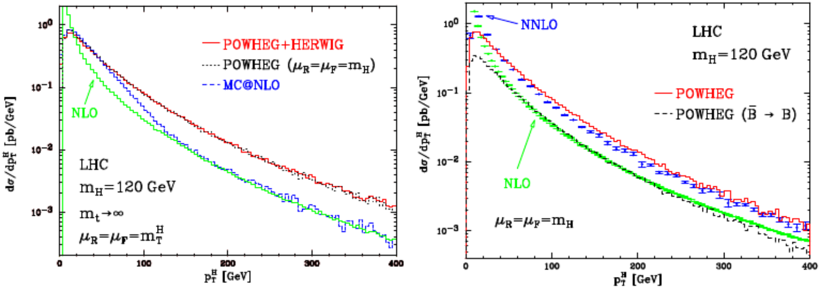
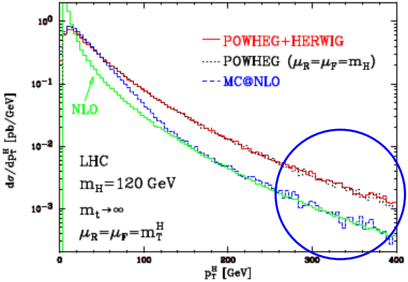
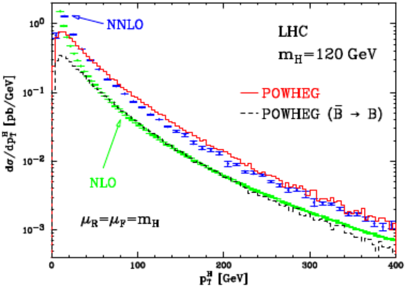
NLO+PS matched predictions



NLO+PS methods mostly behave well, but may sometimes show large differences





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◇ in the real-emission dominated region, since (smearing of NLO "seed" cross section into) real emission treated differently;
⇒ Merging with further accurate multi-jet calculations helps.

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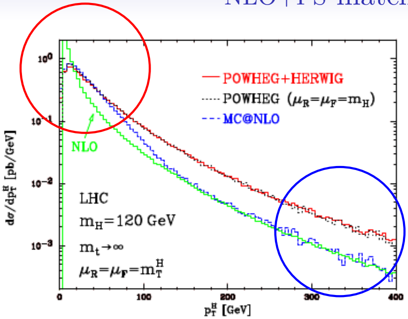

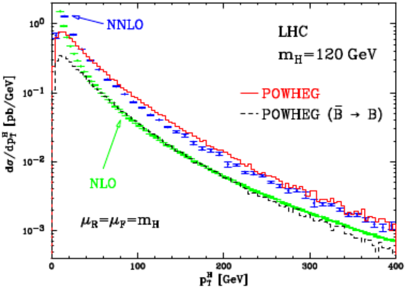

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◇ in the real-emission dominated region, since (smearing of NLO "seed" cross section into) real emission treated differently;
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◇ in the resummation region, since exponentiation different.
⇒ Compare to inclusive resummation, improve shower accuracy.

Leading-order merging

Start with seed cross sections and no-emission probabilities

$$\mathrm{B}_n F_n(\Phi_n) \qquad , \qquad \Delta(t_0, t) = \exp\left(- \int_t^{t_0} d\Phi_{\mathrm{rad}} \alpha_s(\Phi_R) P(\Phi_R) \right)$$

$$\Delta^u(t_n, t_{\mathrm{MS}}) = 1 - \int_{t_{\mathrm{MS}}}^{t_n} \mathrm{B}_{n+1} F_{n+1}(\Phi_{n+1}) \Delta(t_n, t) \qquad \text{("unitarised Sudakov")}$$

where $F_m(\Phi_m) = \Theta(t(\Phi_m) - t_{\mathrm{MS}})$, $t_0 \sim Q$. Two multiplicities merged by

$$\begin{aligned}
& \mathrm{B}_n \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) \mathcal{O}_0(S_{+0}) \\
+ & \int^{t_0} d\Phi_R \left[\mathrm{B}_n \alpha_s(\Phi_R) P(\Phi_R)(1 - F_{n+1}) \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) \; + \; \mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1) \Delta(t_1, t_2) \right] \mathcal{O}_1(S_{+1}) \\
+ & \int^{t_0} d\Phi_{RR} \; \mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1) \Delta(t_1, t_2) \alpha_s(\Phi_R) P(\Phi_R) \mathcal{O}_2(S_{+2}) \; + \; \text{further emissions in } (t_2, t_{cut}]
\end{aligned}$$

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where $F_m(\Phi_m) = \Theta(t(\Phi_m) - t_{\mathrm{MS}})$, $t_0 \sim Q$. Three multiplicities merged by

$$\mathrm{B}_n \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) \mathcal{O}_0(S_{+0})$$
$$+ \int^{t_0} d\Phi_R \left[\mathrm{B}_n \alpha_s(\Phi_R) P(\Phi_R)(1 - F_{n+1}) \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) + \mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1) \Delta(t_1, t_2)\right] \mathcal{O}_1(S_{+1})$$
$$+ \int^{t_0} d\Phi_{RR} \Big[\mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1)(1 - F_{n+2}) \Delta^u(t_1, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) \alpha_s(\Phi_R) P(\Phi_R)$$
$$+ \mathrm{B}_{n+2} F_{n+2} \Delta(t_0, t_1) \Delta(t_1, t_2) \Delta(t_2, t_3)\Big] \mathcal{O}_2(S_{+2}) + \text{further emissions in } (t_3, t_{cut}]$$

Leading-order merging

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Merging methods valid for any multiplicity.
Merging methods use shower Sudakov factors $\Rightarrow$ Improve when showers improve.
NB: Usual CKKW approximation $\Delta^u(t_n, t_{\mathrm{MS}}) \to \Delta(t_n, t_{\mathrm{MS}})$ calls for $t_{\mathrm{MS}} \gg t_{cut}$ or $\mathrm{B}_n P(\Phi) \to \mathrm{B}_{n+1}$. We use $t_{\mathrm{MS}} = \mathcal{O}(1\mathrm{GeV})$ later.

$$+ \int^{t_0} d\Phi_R \left[\mathrm{B}_n \alpha_s(\Phi_R) P(\Phi_R)(1 - F_{n+1}) \Delta^u(t_0, t_{\mathrm{MS}}) \Delta(t_{\mathrm{MS}}, t_{cut}) + \mathrm{B}_{n+1} F_{n+1} \Delta(t_0, t_1) \Delta(t_1, t_2) \right] \mathcal{O}_1(S_{+1})$$

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$$\left. + \mathrm{B}_{n+2} F_{n+2} \Delta(t_0, t_1) \Delta(t_1, t_2) \Delta(t_2, t_3) \right] \mathcal{O}_2(S_{+2}) + \text{further emissions in } (t_3, t_{cut}]$$

NLO merging

Any NLO matching method contains only approximate multi-jet kinematics (given by PS).
Any LO merging method **X** only ever contains approximate virtual corrections.

Use the full NLO results for any multi-jet state!

NLO multi-jet merging from LO scheme **X**:

- Subtract approximate **X** $\mathcal{O}(\alpha_s)$-terms, add multiple NLO calculations.
- Make sure fixed-order calculations do not overlap careful by cutting, vetoing events and/or vetoing emissions (e.g. remove real contribution to n-jet in favour of $n + 1$-jet NLO).
- Adjust higher orders to suit other needs (e.g. to preserve the inclusive cross section).

$\Rightarrow$ **X**@NLO

NLO-merged predictions for Higgs + jets

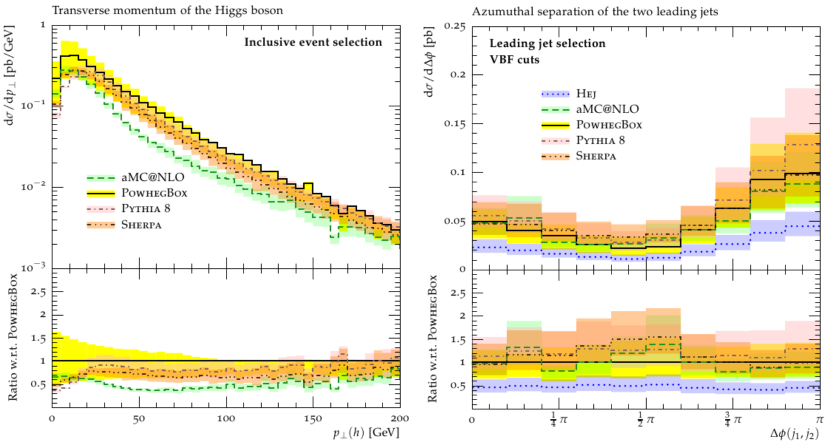
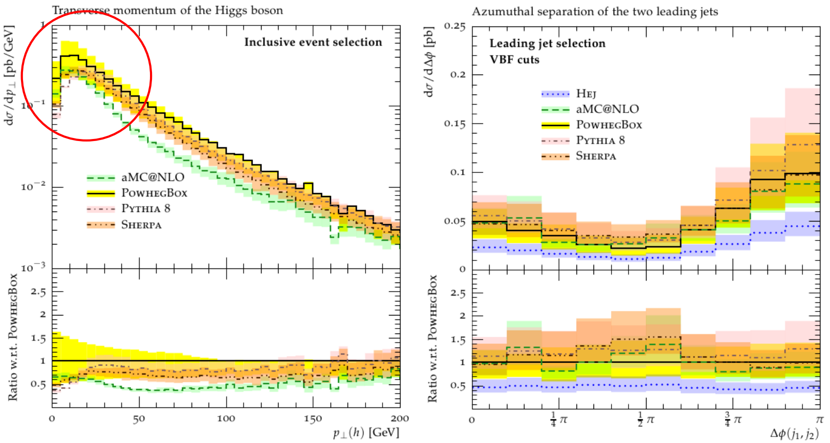


Figure: $p_{\perp,H}$ and $\Delta\phi_{12}$ for gg$\to$H after merging (H+0)@NLO, (H+1)@NLO, (H+2)@NLO, (H+3)@LO, compared to other generators.

$\Rightarrow$ LH study: The generators come closer together if NLO merging is employed.

NLO-merged predictions for Higgs + jets



Still $\mathcal{O}(50\%)$ differences in resummation region – important uncertainty for analyses that use jet binning!

⇒ LH study: The generators come closer together if NLO merging is employed.

The next step(s): Matching @ NNLO

Aim: For important processes – lumi monitors like Drell-Yan, precision studies (ggH, ZH, WBF,...) – reduce uncertainties and remove personal bias. But make sure all other improvements stay intact!

Observation: If an NLO merged calculation leads to a well-defined zero-jet inclusive cross section, it is easy to upgrade this cross section to NNLO.

$\Longrightarrow$ Fulfilled by two NLO merging schemes: MiNLO and UNLOPS

Ways to matching @ NNLO

$$\mathrm{B}_0\left[\mathcal{O}_0\Pi_{S_{+0}} + \int_1 \frac{\mathrm{B}_1}{\mathrm{B}_0}\mathcal{O}_1\Pi_{S_{+0}}\right] \sim \mathrm{B}_0\mathcal{O}_0 - \int_1 \mathrm{B}_1\mathcal{O}_0\Pi_{S_{+0}} + \int_1 \mathrm{B}_1\mathcal{O}_1\Pi_{S_{+0}}$$

NNLOPS:

Start from MiNLO, upgrade analytic CKKW Sudakov to match integral of $q_\perp$ onto zero-jet NLO cross section (LO+NLL calculation of $q_\perp$), then include differential NNLO K-factor.

Benefits: Analytic control over 1st emission, improved resummation.

Limitations: Process-dependent, uses pre-tabulated K-factors.

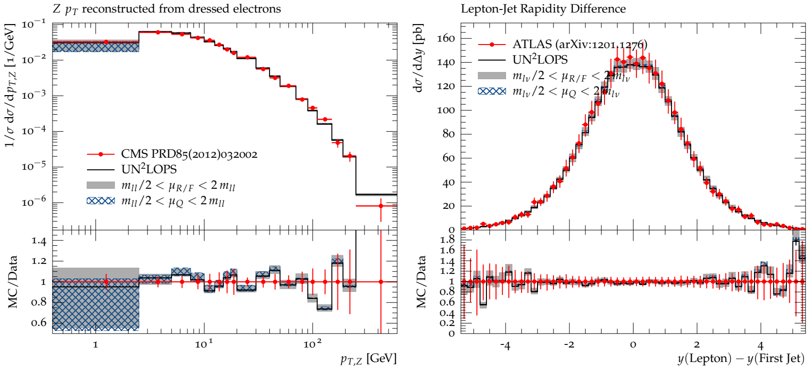
UN^{2}LOPS

Start from UNLOPS, refine unitarised Sudakov factor so that only one-jet NLO cross section survives at $\mathcal{O}(\alpha_s^2)$, then include NNLO jet-vetoed cross section for NNLO accuracy.

Benefits: Easy, process- and shower-independent.

Limitations: Does does not shower $\alpha_s^2\delta(p_\perp)$ terms, or only a subset – i.e. has bin edges.

UN^{2}LOPS (Drell-Yan)



CMS data for Z-boson $p_\perp$. UN^{2}LOPS does quite well. Large band at low $p_\perp$ reflects log scale and shower modelling. ATLAS data for charged current well described.

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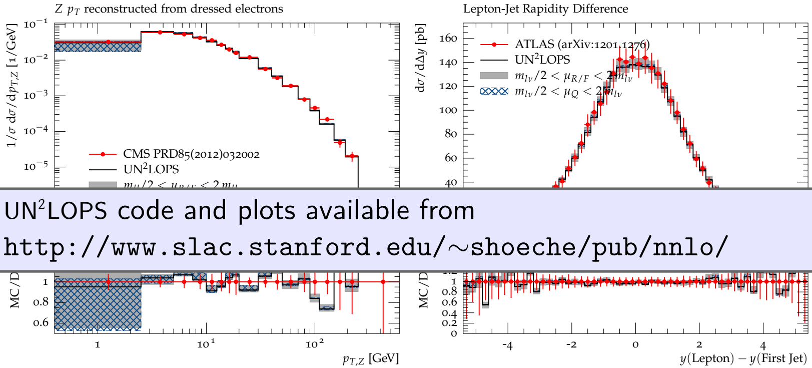

UN²LOPS code and plots available from

`http://www.slac.stanford.edu/~shoeche/pub/nnlo/`

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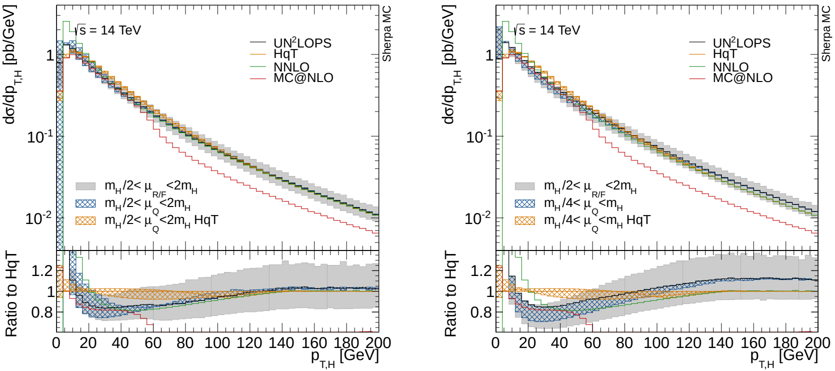
UN^{2}LOPS (Higgs production)



$p_\perp$ of the Higgs-boson for two different matching schemes in UN^{2}LOPS – mimicing the philosophical differences between common NLO matching schemes.

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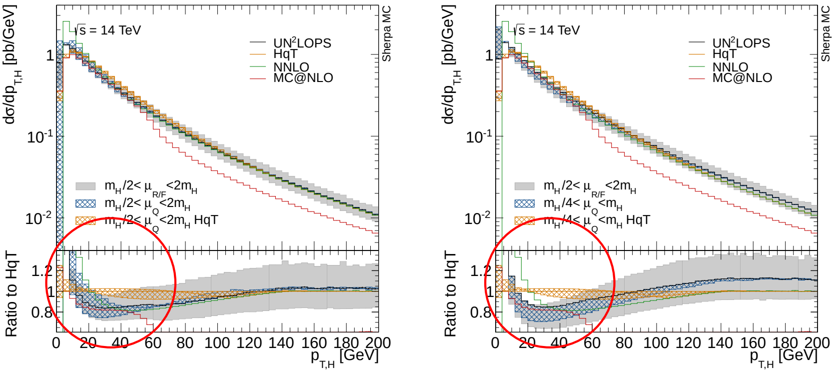

We have NLO matching, NLO merging and NNLO matching. Still, predictions differ appreciably because of limited shower accuracy.

$\Longrightarrow$ Back to Parton Shower Resummation

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The PS prediction for an observable O is

$$\mathcal{F}_{\vec{a}}(\Phi_n, t_c, t_0; O) = \mathcal{F}_{\vec{a}}(\Phi_n, t_c, t_0)\, O(\Phi_n) + \int_{t_c}^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d\ln \mathcal{F}_{\vec{a}}(\Phi_n, \bar{t}, t_0)}{d\ln \bar{t}} \mathcal{F}_{\vec{a}'}(\Phi'_{n+1}, t_c, \bar{t}; O)$$

with no-emission probabilities and Sudakov factors defined by

$$\mathcal{F}_a(x, t, \mu^2) = f_a(x, t)\Delta_a(t, \mu^2) = f_a(x, \mu^2)\, \Pi_a(x, t, \mu^2)\ .$$

Now if $t_c \to 0$, then we find

$$\mathcal{F}_{\vec{a}}(\Phi_n, t_c, t_0; O) \to \int^{t_0} \frac{d\bar{t}}{\bar{t}} \frac{d\ln \mathcal{F}_{\vec{a}}(\Phi_n, \bar{t}, t_0)}{d\ln \bar{t}} \mathcal{F}_{\vec{a}'}(\Phi'_{n+1}, t, \bar{t}; O)$$

which allows a comparison to analytic resummation formulae.

Parton Shower Sudakovs

The main object on which showers are built are Sudakov form factors. To improve showers, we should improve the accuracy of the Sudakov factors by comparing

$$\Delta^{\mathrm{AN}}(\rho_0,\rho_1) = \exp\left(-\int_{\rho_0}^{\rho_1}\frac{d\rho}{\rho}\left[\ln\left(\frac{Q^2}{\rho}\right)\sum_i\left(\frac{\alpha_s}{2\pi}\right)^i A_i - \sum_i\left(\frac{\alpha_s}{2\pi}\right)^i B_i\right]\right)$$

$$\Delta^{\mathrm{PS}}(\rho_0,\rho_1) = \exp\left(-\int_{\rho_1}^{\rho_0}\frac{d\rho}{\rho}\int_{z_{min}}^{z_{max}}dz\frac{\alpha_s(\rho)}{2\pi}\mathcal{K}(z,\rho)\right)$$

where A and B are related to γ_{cusp} and γ_i (e.g. $A_1 = C_a \times 1$, $A_2 = C_a \times \gamma_{cusp}^{(2)}$)

"Good" parton shower should yield a sensible result for the perturbative exponent ($A's$, $B's$). Higher-order normalisation ($C's$, $H's$) may be included through matching.

Reliable Event Generator predictions?

We have hints that current parton showers (e.g. PYTHIA, SHERPA) are not ideal in the Sudakov region. This can mean

- Problems in the (matching of shower and) fixed-order parts
- Problems in the parton showers
- Problems in the non-perturbative modelling
- Bugs

$\Rightarrow$ We want a simple[1], theoretically clean[2], extendable parton shower, to so that comparison to observable-based resummation becomes possible.

[1] Simply splitting functions, simple phase space boundaries.
[2] Eikonal in soft limit, AP kernels in collinear limit, collinear anomalous dimensions unchanged, flavour/momentum sum rules, no choices introducing iffy sub-leading logs.

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We have defined a new dipole-parton shower, and implemented independently in the PYTHIA and SHERPA to minimise bug contamination (arXiv:1506.05057). Codes will become PYTHIA and SHERPA plugins.

Deriving a DIRE shower

Catani-Seymour dipoles are a nice starting point because

- A capture the correct divergence structure (including coherence)
- B correct single log for $q_\perp$ (even with trivial phase space boundaries).
- C come with exact phase space factorisation

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3 not immediately applicable for evolution, because not normalized by momentum / flavour sum rules $\leftarrow$ demand sum rules exactly.

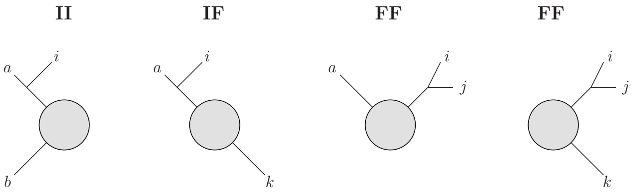
Addressing these points completely fixes *all* (II, IF, FI, FF) dipoles to

$$P_{qq}(z,\kappa^2) = 2\,C_F\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ - \frac{1+z}{2}\right] + \frac{3}{2}\,C_F\,\delta(1-z)$$

$$P_{gg}(z,\kappa^2) = 2\,C_A\left[\left(\frac{1-z}{(1-z)^2+\kappa^2}\right)_+ + \frac{z}{z^2+\kappa^2} - 2 + z(1-z)\right] + \delta(1-z)\left(\frac{11}{6}C_A - \frac{2}{3}n_f T_R\right)$$

$$P_{gq}(z,\kappa^2) = 2\,C_F\left[\frac{z}{z^2+\kappa^2} - \frac{2-z}{2}\right] \qquad P_{qg}(z,\kappa^2) = T_R\left[z^2+(1-z)^2\right]$$

DIRE ordering variables



More concretely, we use the phase space variables

$$\rho_{ii} = \frac{s_{ai}s_{bi}}{s_{ab}}\frac{s_{aib}}{s_{ab}} \qquad z_{ii} = 1 - \frac{s_{bi}}{s_{ab}}$$

$$\rho_{if} = \frac{s_{ai}s_{ik}}{s_{ai}+s_{ak}} \qquad z_{if} = 1 - \frac{s_{ik}}{s_{ai}+s_{ak}}$$

$$\rho_{fi} = \frac{s_{aj}s_{ij}}{s_{ai}+s_{aj}} \qquad z_{fi} = \frac{s_{ai}}{s_{ai}+s_{aj}}$$

$$\rho_{ff} = \frac{s_{ij}s_{jk}}{s_{ij}+s_{ik}+s_{jk}} \qquad z_{ff} = \frac{s_{ij}+s_{ik}}{s_{ij}+s_{ik}+s_{jk}}$$

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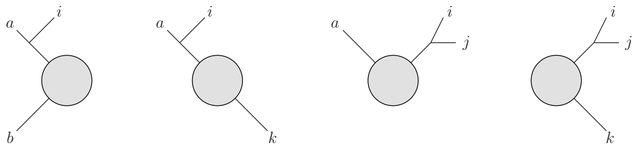

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Kinematical $p_\perp$ for small $p_\perp$, but covers full $p_\perp$ range even for small ρ_{start}

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Ariandne $p_\perp$

Improvements

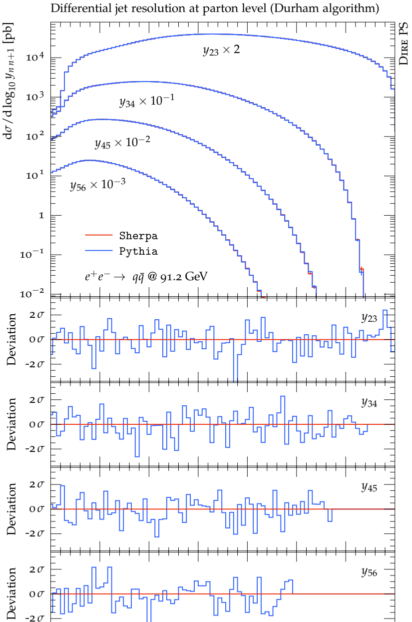
Ordering variables were chosen to obtain very simple phase space boundaries and splitting functions, so that a comparison with analytic results is possible.

$\Longrightarrow$ The soft terms can directly be rescaled with higher-order cusp anomalous dimensions to get A_2 and A_3:

$$\frac{2(1-z)}{(1-z)^2+\kappa^2} \longrightarrow \frac{2(1-z)}{(1-z)^2+\kappa^2}\left(1+\frac{\alpha_s}{2\pi}\gamma_{cusp}^{(1)}+\frac{\alpha_s^2}{4\pi^2}\gamma_{cusp}^{(2)}\right)$$

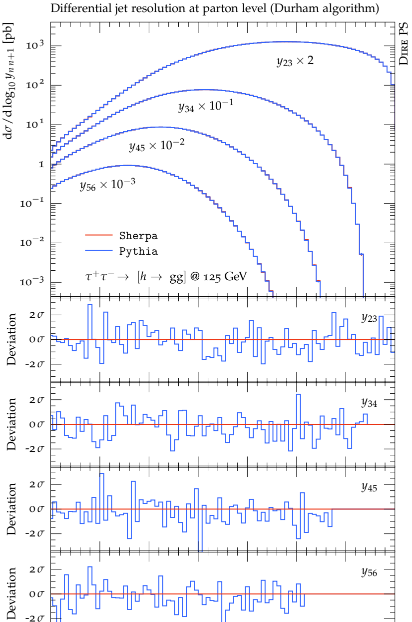
Note: α_s not in the CMW scheme (two-loop cusp is absorbed via redefinition of Λ_{QCD}), as this can introduce problematic collinear anomalous dimensions beyond NLL (depending on shower details).

Cross-validation at the sub-permille level: Durham jet rates

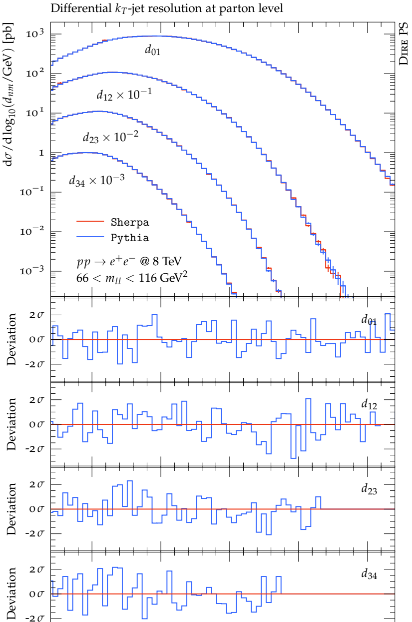


Cross-validation at the sub-permille level: Durham jet rates

Lepton colliders ✓



Cross-validation at the sub-permille level: $k_\perp$ clustering scales



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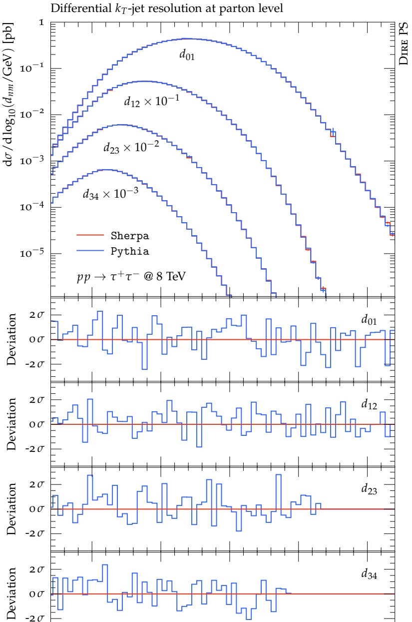
Hadron colliders ✓



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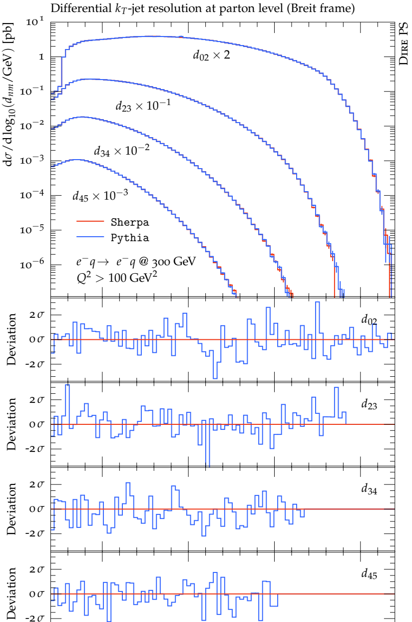
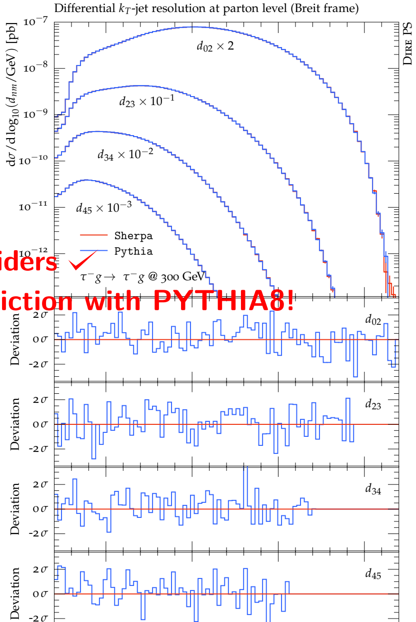


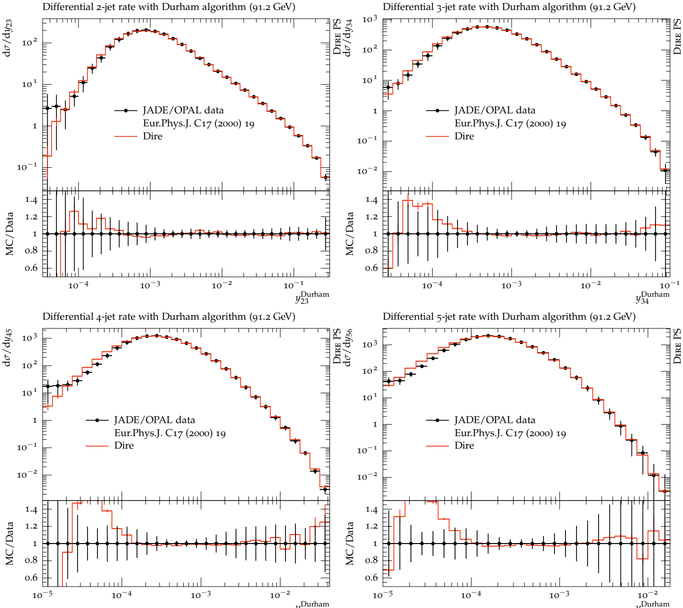
Cross-validation at the sub-permille level: Jet scales



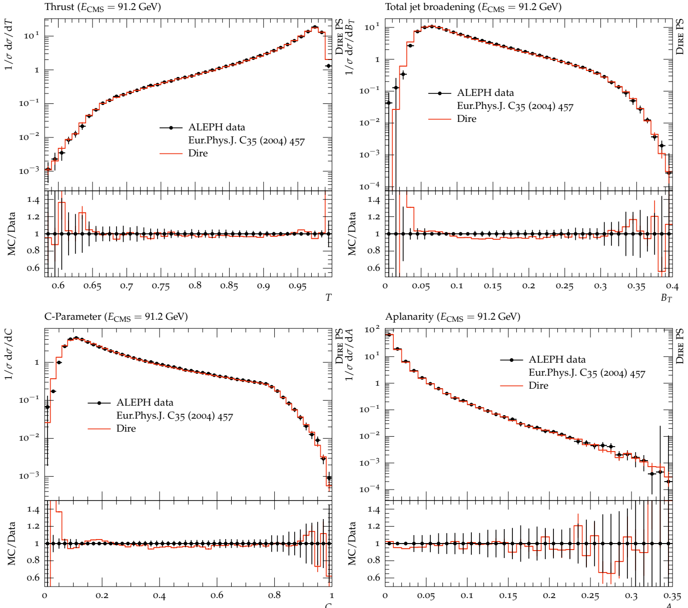
Lepton-hadron colliders ✓

First ever DIS prediction with PYTHIA8!

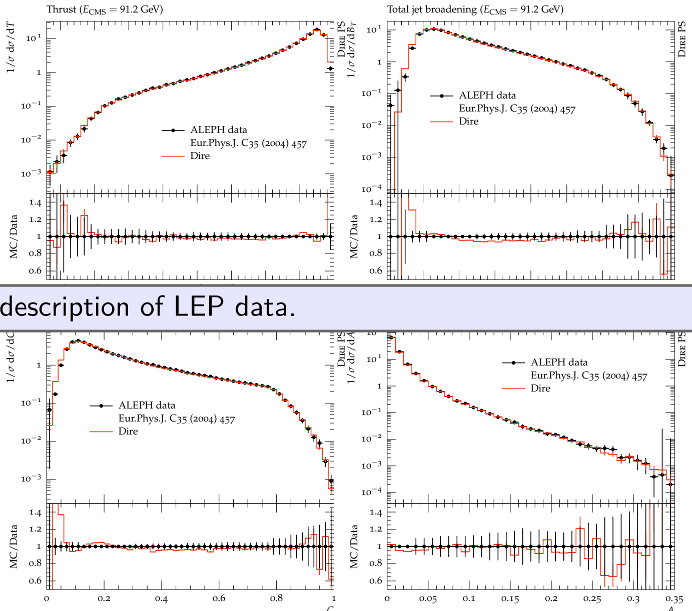
LEP data comparisons (plain showering)



LEP data comparisons (plain showering)

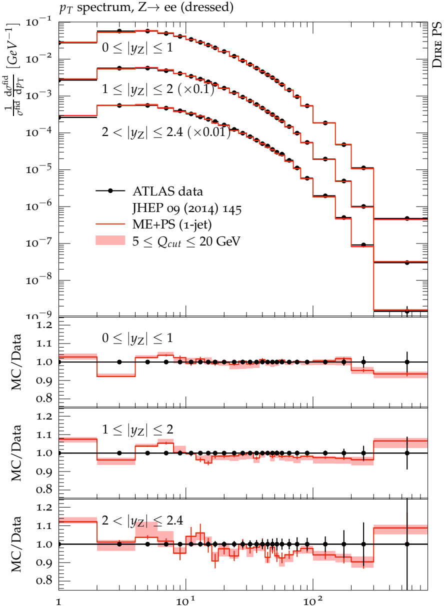


LEP data comparisons (plain showering)

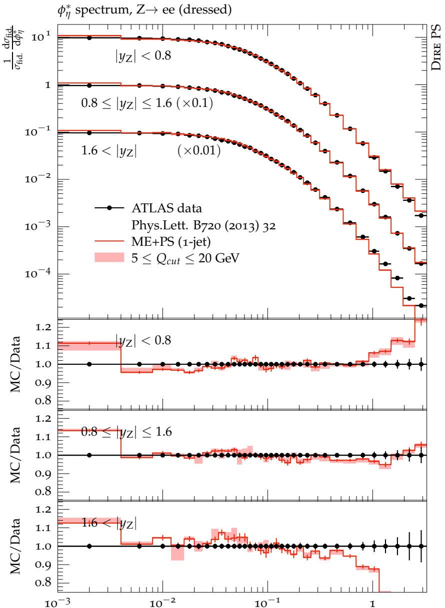


Nice description of LEP data.

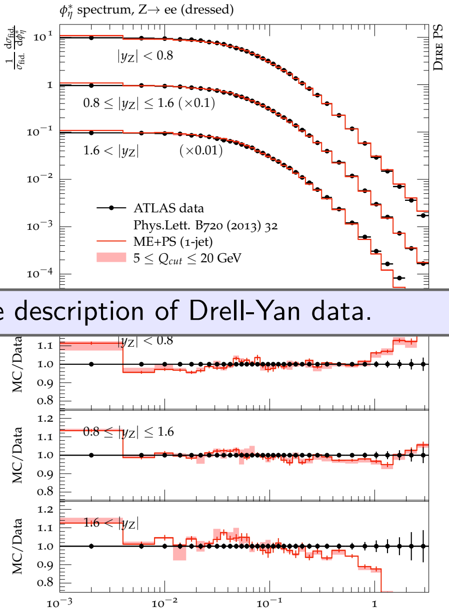
LHC data comparisons (1-jet CKKW-L merged)



LHC data comparisons (1-jet CKKW-L merged)



LHC data comparisons (1-jet CKKW-L merged)

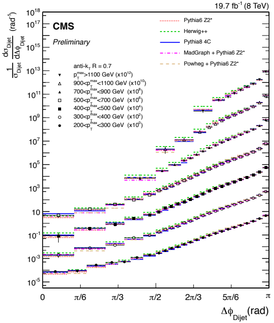


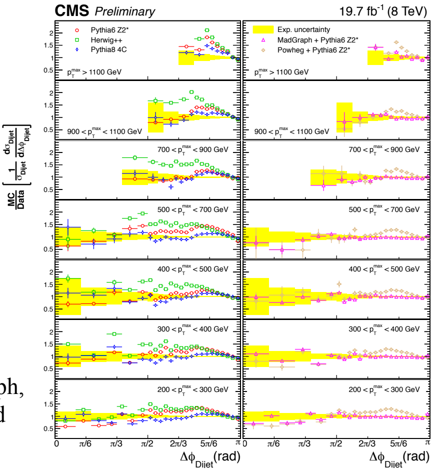
Not too terrible description of Drell-Yan data.

LHC data comparisons (plain showering)

Slide taken from Monday's CMS summary by Matthias Weber

Dijet azimuthal decorrelation at 8 TeV



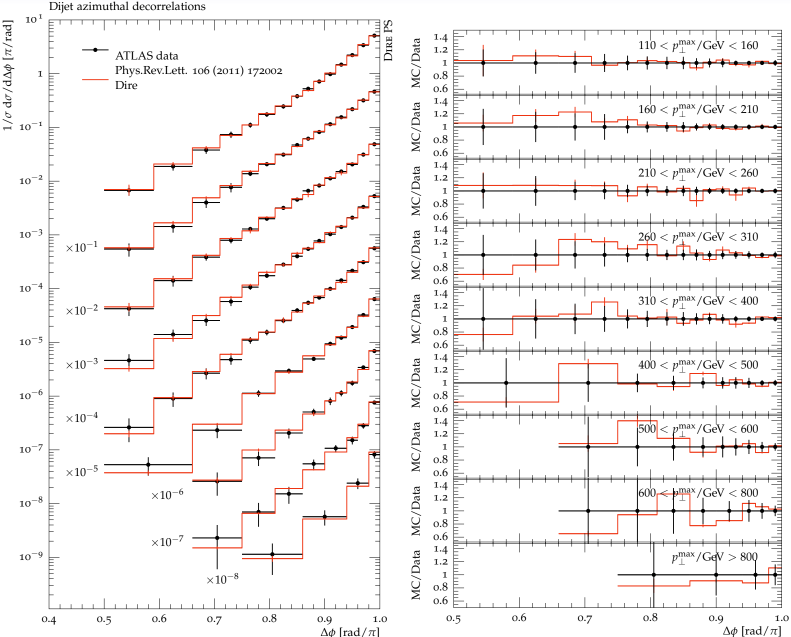


Compared to several MC generators

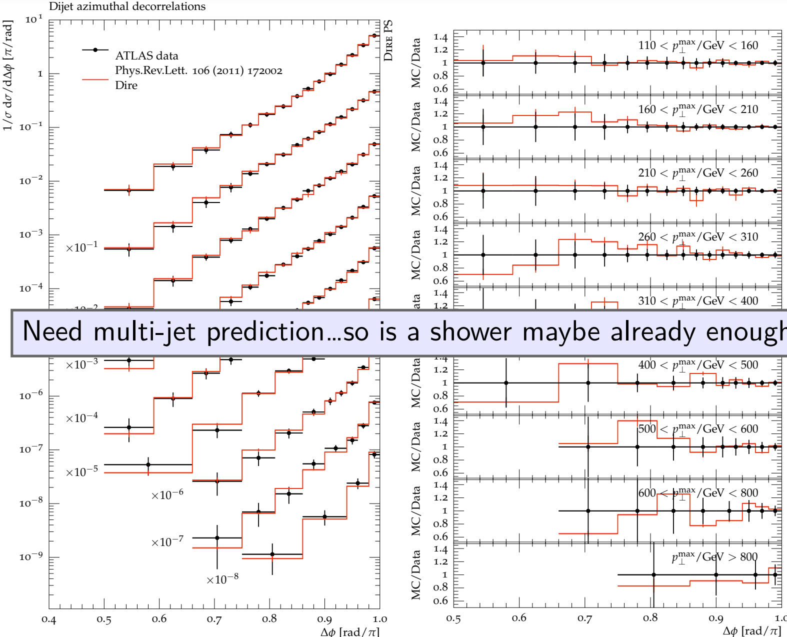
➡ Good agreement only for MadGraph, need multi-leg calculation for good description

CMS-PAS-SMP-14-015

LHC data comparisons (plain showering)



LHC data comparisons (plain showering)



Need multi-jet prediction...so is a shower maybe already enough?

Summary

- Fixed-order + parton shower continues to be an active field, but we need to also improve showers.
  We can combine multiple LO calculations, or multiple NLO calculations, or match $pp \to colour\ singlett$ at NNLO +PS.
- Now the main issue is the accuracy of the shower.
- For simple processes, we can in principle match terms in the shower with analytic results.
  ...needed a new, cleanly defined shower for that though.
- We introduced a new shower combining the simplicity of parton showers and the symmetry of dipole showers.
- Results of the new DIRE shower in PYTHIA and SHERPA. look promising. We hope to improve this shower in the future.

Back-up supplement

References

POWHEG
JHEP 0411 (2004) 040 JHEP 0711 (2007) 070 POWHEG-BOX (JHEP 1006 (2010) 043)

MC@NLO
Original (JHEP 0206 (2002) 029) Herwig++ (Eur.Phys.J. C72 (2012) 2187)
Sherpa (JHEP 1209 (2012) 049) aMC@NLO (arXiv:1405.0301)

NLO matching results and comparisons
Plots taken from Ann.Rev.Nucl.Part.Sci. 62 (2012) 187 Plots taken from JHEP 0904 (2009) 002

Tree-level merging MLM (Mangano, http://www-cpd.fnal.gov/personal/mrenna/tuning/nov2002/mlm.pdf. Talk presented at the Fermilab ME/MC Tuning Workshop, Oct 4, 2002, Mangano et al. JHEP 0701 (2007) 013)
Pseudoshower (JHEP 0405 (2004) 040)
CKKW (JHEP 0111 (2001) 063, JHEP 0208 (2002) 015)
CKKW-L (JHEP 0205 (2002) 046, JHEP 0507 (2005) 054, JHEP 1203 (2012) 019)
METS (JHEP 0911 (2009) 038, JHEP 0905 (2009) 053)

Unitarised merging
Pythia (JHEP 1302 (2013) 094) Herwig (JHEP 1308 (2013) 114) Sherpa (arXiv:1405.3607)

FxFx: Jet matching @ NLO: JHEP 1212 (2012) 061

MEPS@NLO
JHEP 1304 (2013) 027 JHEP 1301 (2013) 144 Plots taken from arXiv:1401.7971

UNLOPS JHEP 1303 (2013) 166 arXiv:1405.1067

MiNLO: Original (JHEP 1210 (2012) 155) Improved (JHEP 1305 (2013) 082)

MiNLO-NNLOPS: JHEP 1310 (2013) 222 arXiv:1407.2940
arXiv:1501.04637

UN^{2}LOPS: arXiv:1405.3607 arXiv:1407.3773

Parton showers are **unitary** all-order operators:

$$\mathbf{PS}\left[\sigma^{\mathsf{ME}}_{+0}\right]$$

Parton shower basics

Parton showers are **unitary** all-order operators:

$$\begin{aligned}\mathbf{PS}\left[\sigma^{\mathsf{ME}}_{+0}\right] &= \sigma^{\mathsf{PS}}_{+0} + \\ &= \sigma^{\mathsf{ME}}_{+0}\Pi_{S_{+0}}\left(\rho_0, \rho_{\mathsf{min}}\right) \\ &+\end{aligned}$$

$\longleftarrow$ 0 emissions in $[\rho_0, \rho_{\mathsf{min}}]$

Parton shower basics

Parton showers are **unitary** all-order operators:

$$\mathbf{PS}\left[\sigma_{+0}^{\mathsf{ME}}\right] = \sigma_{+0}^{\mathsf{PS}} + \sigma_{+1}^{\mathsf{PS}} +$$

$$= \sigma_{+0}^{\mathsf{ME}}\Pi_{S_{+0}}\left(\rho_0, \rho_{\mathsf{min}}\right) \longleftarrow \text{0 emissions in } \left[\rho_0, \rho_{\mathsf{min}}\right]$$

$$+ \sigma_{+0}^{\mathsf{ME}}\Pi_{S_{+0}}\left(\rho_0, \rho_1\right)\alpha_s w_f^0 P_0 \Pi_{S_{+1}}\left(\rho_1, \rho_{\mathsf{min}}\right) \longleftarrow \text{1 emission in } \left[\rho_0, \rho_{\mathsf{min}}\right]$$

$$+$$

Parton showers are **unitary** all-order operators:

$$
\begin{aligned}
\mathbf{PS}\left[\sigma_{+0}^{\mathrm{ME}}\right] &= \sigma_{+0}^{\mathrm{PS}} + \sigma_{+1}^{\mathrm{PS}} + \sigma_{+\geq 2} \\
&= \sigma_{+0}^{\mathrm{ME}}\Pi_{S_{+0}}\left(\rho_0,\rho_{\mathrm{min}}\right) \qquad \longleftarrow \boxed{0 \text{ emissions in } [\rho_0,\rho_{\mathrm{min}}]} \\
&+ \sigma_{+0}^{\mathrm{ME}}\Pi_{S_{+0}}\left(\rho_0,\rho_1\right)\alpha_s w_f^0 P_0 \Pi_{S_{+1}}\left(\rho_1,\rho_{\mathrm{min}}\right) \longleftarrow \boxed{1 \text{ emission in } [\rho_0,\rho_{\mathrm{min}}]} \\
&+ \sigma_{+0}^{\mathrm{ME}}\Pi_{S_{+0}}\left(\rho_0,\rho_1\right)\alpha_s w_f^0 P_0 \Pi_{S_{+1}}\left(\rho_1,\rho_2\right)\alpha_s w_f^1 P_1 \left[\Pi_{S_{+2}}\left(\rho_2,\rho_{\mathrm{min}}\right) + \ldots\right] \\
&\qquad \uparrow \qquad \uparrow \\
&\qquad \boxed{2 \text{ or more emissions in } [\rho_0,\rho_{\mathrm{min}}]} \\
&\stackrel{!}{=} \sigma_{+0}^{\mathrm{ME}}
\end{aligned}
$$

Parton shower basics

Parton showers are **unitary** all-order operators:

$$\mathbf{PS}\left[\sigma^{\mathsf{ME}}_{+0}\right] = \sigma^{\mathsf{PS}}_{+0} + \sigma^{\mathsf{PS}}_{+1} + \sigma_{+\geq 2}$$

$$= \sigma^{\mathsf{ME}}_{+0}\Pi_{S_{+0}}(\rho_0,\rho_{\mathsf{min}}) \longleftarrow \boxed{\text{0 emissions in } [\rho_0,\rho_{\mathsf{min}}]}$$

$$+ \sigma^{\mathsf{ME}}_{+0}\Pi_{S_{+0}}(\rho_0,\rho_1)\,\alpha_s w_f^0 P_0 \Pi_{S_{+1}}(\rho_1,\rho_{\mathsf{min}}) \longleftarrow \boxed{\text{1 emission in } [\rho_0,\rho_{\mathsf{min}}]}$$

$$+ \sigma^{\mathsf{ME}}_{+0}\Pi_{S_{+0}}(\rho_0,\rho_1)\,\alpha_s w_f^0 P_0 \Pi_{S_{+1}}(\rho_1,\rho_2)\,\alpha_s w_f^1 P_1\left[\Pi_{S_{+2}}(\rho_2,\rho_{\mathsf{min}}) + \ldots\right]$$

2 or more emissions in $[\rho_0, \rho_{\mathsf{min}}]$

$$\stackrel{!}{=} \sigma^{\mathsf{ME}}_{+0}$$

The no-emission probabilities

$$\Pi_{S_{+i}}(\rho_1,\rho_2) = \exp\left\{-\int_{\rho_2}^{\rho_1} d\rho\,\alpha_s w_f^i P_i\right\}$$

define *exclusive* cross sections and remove the overlap between samples!

Next-to-leading order calculations

Pen-and-paper: Add Born + Virtual + Real.

$$\langle \mathcal{O} \rangle^{\mathsf{NLO}} = \int \mathrm{B}_n \mathcal{O}(\Phi_n) d\Phi_n + \int \mathrm{V}_n \mathcal{O}_n(\Phi_n) d\Phi_n + \int \mathrm{B}_{n+1} \mathcal{O}(\Phi_n) d\Phi_{n+1}$$

Next-to-leading order calculations

Pen-and-paper: Add Born + Virtual + Real.

$$\langle\mathcal{O}\rangle^{\text{NLO}} = \int \mathrm{B}_n\mathcal{O}(\Phi_n)d\Phi_n + \int \mathrm{V}_n\mathcal{O}_n(\Phi_n)d\Phi_n + \int \mathrm{B}_{n+1}\mathcal{O}(\Phi_n)d\Phi_{n+1}$$

Reality: Phase space integral separately divergent $\Rightarrow$ Add zero!

$$\langle\mathcal{O}\rangle^{\text{NLO}} = \int \left[\mathrm{B}_n + \mathrm{V}_n + \int \mathrm{D}_{n+1}\right]\mathcal{O}(\Phi_n)d\Phi_n + \int \left[\mathrm{B}_{n+1}\mathcal{O}(\Phi_{n+1}) - \mathrm{D}_{n+1}\mathcal{O}(\Phi'_n)\right]d\Phi_{n+1}$$

Next-to-leading order calculations

Pen-and-paper: Add Born + Virtual + Real.

$$\langle\mathcal{O}\rangle^{\text{NLO}} = \int \mathrm{B}_n \mathcal{O}(\Phi_n) d\Phi_n + \int \mathrm{V}_n \mathcal{O}_n(\Phi_n) d\Phi_n + \int \mathrm{B}_{n+1} \mathcal{O}(\Phi_n) d\Phi_{n+1}$$

Reality: Phase space integral separately divergent $\Rightarrow$ Add zero!

$$\langle\mathcal{O}\rangle^{\text{NLO}} = \int \left[\mathrm{B}_n + \mathrm{V}_n + \int \mathrm{D}_{n+1}\right] \mathcal{O}(\Phi_n) d\Phi_n + \int \left[\mathrm{B}_{n+1}\mathcal{O}(\Phi_{n+1}) - \mathrm{D}_{n+1}\mathcal{O}(\Phi_n')\right] d\Phi_{n+1}$$

Real reality: States Φ_{n+1} and Φ_n' are correlated. $\Rightarrow$ Problematic, since further manipulations (e.g. hadronisation) can spoil the cancellations

$\Rightarrow$ Add more zeros!

$$\begin{aligned}\langle\mathcal{O}\rangle^{\text{NLO}} &= \int \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\text{rad}} \left(\mathrm{B}_{n+1}' - \mathrm{D}_{n+1}\right)\right] \mathcal{O}(\Phi_n) d\Phi_n \\ &+ \int \left(\mathrm{B}_{n+1} - \mathrm{B}_{n+1}'\right) \mathcal{O}(\Phi_{n+1}) \\ &+ \int \left(\mathrm{B}_{n+1}'\mathcal{O}(\Phi_{n+1}) - \mathrm{B}_{n+1}'\mathcal{O}(\Phi_n)\right) \longleftarrow \text{That's the } \mathcal{O}(\alpha_s) \text{ of a PS step!}\end{aligned}$$

CKKW(-L)

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.

$$
\begin{aligned}
\langle \mathcal{O} \rangle \quad &= \mathrm{B}_0 \mathcal{O}(S_{+0j}) \\
&\qquad\qquad - \int d\rho\ \mathrm{B}_0 P_0(\rho) \Theta_>^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1 \Theta_>^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+1j}) \\
&\qquad\qquad - \int d\rho\ \mathrm{B}_1 P_1(\rho) \Theta_>^{(2)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2 \Theta_>^{(2)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+2j})
\end{aligned}
$$

Changes inclusive cross sections

$\Longrightarrow$ Can contain numerically large (sub-leading) logs.

$\Longrightarrow$ Needs fixing!

Bug vs. Feature in CKKW(-L)

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).

These are the improvements that we need to describe multiple hard jets!

If we simply add samples, the "improvements" will degrade the inclusive cross section: σ_{inc} will contain $\ln(t_{\mathrm{MS}})$ terms.

THE INCLUSIVE CROSS SECTION DOES NOT CONTAIN LOGS RELATED TO CUTS ON HIGHER MULTIPLICITIES.

Traditional approach: Don't use a too small merging scale.

$\rightarrow$ Uncancelled terms numerically not important.

Unitary approach[1]:

Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on t_{MS}.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

Unitarised ME+PS

Aim: If you add too much, then subtract what you add!

$$
\begin{aligned}
\langle \mathcal{O} \rangle \quad &= \mathrm{B}_0 \mathcal{O}(S_{+0j}) \\
&\quad - \int d\rho \; \mathrm{B}_1 \Theta_{>}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+0j}) \; - \int d\rho \; \mathrm{B}_2 \Theta_{>}^{(2)} \Theta_{<}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1 \Theta_{>}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+1j}) \\
&\quad - \int d\rho \; \mathrm{B}_2 \Theta_{>}^{(2)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2 \Theta_{>}^{(2)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+2j}) \; + \int \mathrm{B}_2 \Theta_{>}^{(2)} \Theta_{<}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+}
\end{aligned}
$$

Inclusive cross sections preserved by construction.
Cancellation between different "jet bins".
$\Rightarrow$ Statistics needs fixing.

NLO matching with MC@NLO

Aim: Achieve NLO for inclusive +0-jet, and LO for inclusive +1-jet observables and attach PS resummation.

To get there, remember that the (regularised) NLO cross section is

$$\begin{aligned}
\mathrm{B}_{\mathrm{NLO}} &= \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1}\mathcal{O}_1 - \mathrm{D}_{n+1}\mathcal{O}_0\right) \\
&= \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{S}_{n+1}\mathcal{O}_0 - \mathrm{D}_{n+1}\mathcal{O}_0\right) \\
&\quad + \int d\Phi_{\mathrm{rad}}\left(\mathrm{S}_{n+1}\mathcal{O}_1 - \mathrm{S}_{n+1}\mathcal{O}_0\right) + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1}\mathcal{O}_1 - \mathrm{S}_{n+1}\mathcal{O}_1\right)
\end{aligned}$$

where S_{n+1} are some additional "transfer functions", e.g. the PS kernels.

Red term is the $\mathcal{O}(\alpha_s)$ part of a shower from B_n. $\Rightarrow$ Discard from $\mathrm{B}_{\mathrm{NLO}}$.

Thus, we have the seed cross section

$$\widehat{\mathrm{B}}_{\mathrm{NLO}} = \left[\mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_n + \int d\Phi_{\mathrm{rad}}\left(\mathrm{S}_{n+1} - \mathrm{D}_{n+1}\right)\right]\mathcal{O}_0 + \int d\Phi_{\mathrm{rad}}\left(\mathrm{B}_{n+1} - \mathrm{S}_{n+1}\right)\mathcal{O}_1$$

This is not the NLO result...but showering the $\mathcal{O}_0$-part will restore this!

UMEPS, MC@NLO-style (Plätzer JHEP 1308 (2013) 114)

Aim: Combine multiple tree-level calculations with each other and (PS) resummation. Fill in soft and collinear regions with parton shower.

$$
\begin{aligned}
\langle\mathcal{O}\rangle \quad &= \mathrm{B}_0 \Pi_{S_{+0}}(\rho_0, \rho_{\mathrm{MS}}) \mathcal{O}(S_{+0j}) \\
&\quad - \int d\rho \;\; [\mathrm{B}_1 - \mathrm{B}_0 P_0(\rho)]\, \Theta_{>}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \mathcal{O}(S_{+0j}) \\
&+ \int \mathrm{B}_1 \Theta_{>}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho) \Pi_{S_{+1}}(\rho, \rho_{\mathrm{MS}}) \mathcal{O}(S_{+1j}) \\
&\quad - \int d\rho \;\; [\mathrm{B}_2 - \mathrm{B}_1 P_1(\rho)]\, \Theta_{>}^{(2)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+1j}) \\
&+ \int \mathrm{B}_2 \Theta_{>}^{(2)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \Pi_{S_{+1}}(\rho_1, \rho) \mathcal{O}(S_{+2j}) \; + \int \mathrm{B}_2 \Theta_{>}^{(2)} \Theta_{<}^{(1)} w_f w_{\alpha_s} \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+2
\end{aligned}
$$

Inclusive cross sections preserved by construction.
Less cancellation between different "jet bins" fixed.
$\Longrightarrow$ Statistics okay.

Comparison of NLO merging schemes

FxFx: Restricts the range of merging scales. Cross section changes thus numerically small.
Probably fewest counter events.

MEPS@NLO: Improved, colour-correct Sudakov of MC@NLO for the first emission. Larger t_{MS} range.
Smaller cross section changes.
Improved resummation in process-independent way.

UNLOPS: Inclusive observables strictly NLO correct. Further shower improvements also directly improve the results.
Many counter events if done naively.

MiNLO: applies analytical (N)NLL Sudakov factors, which cancel problematic logs, only merging two multiplicities.
Was moulded into an NNLO matching.

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \qquad - \int \widehat{\mathrm{B}}_{1\to 0} \qquad - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\qquad \widehat{\mathrm{B}}_1 \qquad - \int \widehat{\mathrm{B}}_{2\to 1} \qquad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_s^n)$- and $\mathcal{O}(\alpha_s^{n+1})$-terms:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(- \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

Add full NLO results:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \left[\int \widehat{\mathrm{B}}_{1\to 0}\right]_{-1,2} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg) + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1\right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1}\right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j})\widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Unitarise:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

Deriving an UN²LOPS matching

We basically follow a "merging strategy":

- Pick calculations to combine (two MC@NLOs) with each other *and* with the PS resummation.
- Remove kinematic overlaps between the two MC@NLOs by dividing the one-jet phase space.
- Reweight one-jet MC@NLO (*to make it exclusive* $\leftrightarrow$ *want to describe hardest jet with this*),
  remove all undesired terms at $\mathcal{O}(\alpha_s^{1+1})$
  and make sure that the whole thing is numerically stable.
  Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with virtuals.
- Add and subtract reweighted one-jet MC@NLO, ($\rightarrow$ unitarise) to ensure inclusive zero-jet cross section is unchanged w.r.t. NLO.
- Remove all terms up to $\mathcal{O}(\alpha_s^2)$ in the zero-jet contribution, replace by NNLO jet-vetoed cross section.

UN^{2}LOPS matching

Aim: Combine just two NLO calculations, then upgrade to NNLO directly.

Work with Stefan Höche and Ye Li.

UN^{2}LOPS matching

Aim: Combine just two NLO calculations, then upgrade to NNLO directly.

Start over again, now combining MC@NLO's because those are resonably stable. Thus:

- ◇ Use 0-jet matched (MC@NLO $_0$) and 1-jet matched calculation (MC@NLO $_1$).
- ◇ Remove hard ($q_T > \rho_{\mathrm{MS}}$) reals in MC@NLO $_0$.
- ◇ Reweight B_1 of MC@NLO $_1$ with "zero-jet Sudakov" factor $\Pi_{S_{+0}}/\alpha_s$ running.
- ◇ Reweight NLO part $\widetilde{\mathrm{B}}_1^{\mathrm{R}}$ of MC@NLO $_1$ with "zero-jet Sudakov" factor.
- ◇ Subtract erroneous $\mathcal{O}(\alpha_s^{+1})$ terms multiplying B_1.
- ◇ Reweight subtractions with $\Pi_{S_{+0}}$ to be able to group them with $\widetilde{\mathrm{B}}_1^{\mathrm{R}}$.
- ◇ Put $\rho_{\mathrm{MS}} \to \rho_c < 1\mathrm{GeV}$. ($\to$ MC@NLO $_0$ becomes exclusive NLO)
- ◇ Unitarise by subtracting the processed MC@NLO $_1'$ from the "zero-q_T bin".
- ◇ Remove all terms up to α_s^2 from the "zero-q_T bin" and add the q_T-vetoed NNLO cross section.

$\Rightarrow$ $\sigma_{inclusive}$ @ NNLO, resummation as accurate as Sudakov, stats fine.
NNLO logarithmic parts from q_T-vetoed TMDs (EFT calculation),
hard coefficients from q_T-subtraction (i.e. DYNNLO, HNNLO),
power corrections from MC@NLO $_1$.

Work with Stefan Höche and Ye Li.

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} =\; & \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

Note that this is just an extention of the old Sudakov veto algorithm:
Run trial shower on the reconstructed zero-jet state,
If trial shower produces an emission, keep zero-jet kinematics and stop;
else start PS off one-jet state.

UN^{2}LOPS matching

$$
\begin{aligned}
\mathcal{O}^{(\mathrm{UN^2LOPS})} = & \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2,O) \\
& + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2,O)
\end{aligned}
$$

Note: $\left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}$ etc. comes from using q_T-vetoed cross sections.

UN^{2}LOPS matching

$$\mathcal{O}^{(\mathrm{UN^2LOPS})} = \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$

$$\bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}} + \tilde{\mathrm{B}}_1^{\mathrm{R}} + \mathrm{H}_1^{\mathrm{R}} + \mathrm{H}_1^{\mathrm{E}} = \mathrm{B}_{\mathrm{NNLO}}$$

Other terms drop out in inclusive observables.

UN^{2}LOPS matching

$$\mathcal{O}^{(\mathrm{UN^2LOPS})} = \int d\Phi_0\, \bar{\bar{\mathrm{B}}}_0^{q_{T,\mathrm{cut}}}(\Phi_0)\, O(\Phi_0)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right)\right] \mathrm{B}_1(\Phi_1)\, O(\Phi_0)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1\, \Pi_0(t_1,\mu_Q^2)\left(w_1(\Phi_1) + w_1^{(1)}(\Phi_1) + \Pi_0^{(1)}(t_1,\mu_Q^2)\right) \mathrm{B}_1(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_1 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_1 \Pi_0(t_1,\mu_Q^2)\, \tilde{\mathrm{B}}_1^{\mathrm{R}}(\Phi_1)\, \bar{\mathcal{F}}_1(t_1, O)$$

$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2 \left[1 - \Pi_0(t_1,\mu_Q^2)\right] \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, O(\Phi_0) + \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \Pi_0(t_1,\mu_Q^2)\, \mathrm{H}_1^{\mathrm{R}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$

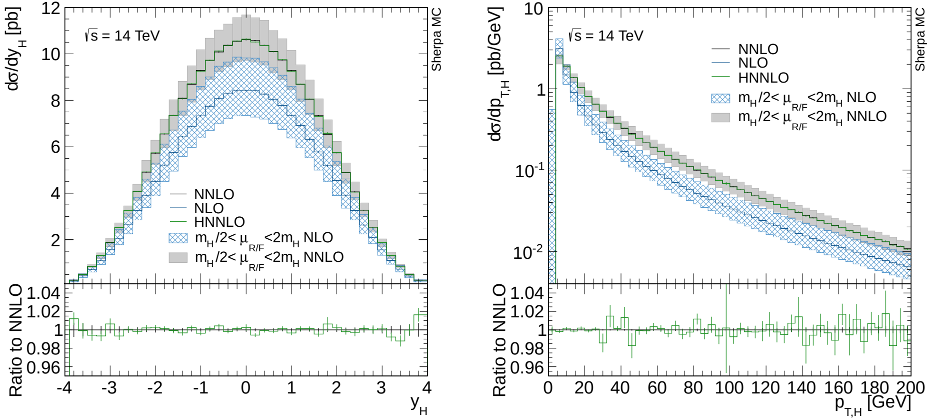
$$+ \int_{q_{T,\mathrm{cut}}} d\Phi_2\, \mathrm{H}_1^{\mathrm{E}}(\Phi_2)\, \mathcal{F}_2(t_2, O)$$

Orange terms do not contain any universal α_s corrections present in the PS.

H_1 do not contribute in the soft/collinear limit.

$\Longrightarrow$ PS accuracy is preserved.

UN^{2}LOPS (Higgs production)



Rapidity and $p_\perp$ of the Higgs-boson, comparing SHERPA-NNLO and HNNLO. Our independent NNLO calculation works nicely.

Shower tricks: Weighted shower (JHEP 1209 (2012) 049)

For frequently negative contributions, or if we do not want to find an ideal overestimate, we can use a weighted shower instead:

- For each phase space point, store the full splitting probability f, the differential overestimate g, and an auxiliary weight $h = \mathrm{sgn}(f) \cdot g$.
- Pick/discard splitting as usual according to $\mathrm{abs}\left(\frac{f}{g}\right)$. If the emission...

  ...was picked, multiply $w_{picked} = \frac{h}{g}\frac{g-f}{h-f}$ to event weight

  ...was discarded, multiply $w_{discarded} = \frac{h}{g}$ to event weight

If the differential overestimate is not ideal, i.e. $\frac{f}{g} = a > 1$, then use

$$\frac{f}{g} \to \frac{f}{(ka) \cdot g} \quad \text{and} \quad h \to (ka) \cdot h$$

with an additional factor $k \gtrsim 1$, and continue as before.

The anomalous dimensions

$$\gamma_{ab}(N,\kappa^2) = \int_0^1 z^N P_{ab}(z,\kappa^2) \qquad \text{are given by} \tag{1}$$

$$\begin{aligned}
\gamma_{qq}(N,\kappa^2) &= 2C_F\,\Gamma(N,\kappa^2) - \frac{C_F\,(2N+3)}{(N+1)(N+2)} + \gamma_q \\
\gamma_{gq}(N,\kappa^2) &= 2C_F\,\mathrm{K}(N,\kappa^2) - \frac{C_F\,(N+3)}{(N+1)(N+2)} \\
\gamma_{gg}(N,\kappa^2) &= 2C_A\,\Gamma(N,\kappa^2) + 2C_A\,\mathrm{K}(N,\kappa^2) - \frac{2C_A\,(N+3)}{(N+1)(N+2)} - \frac{2C_A}{N+3} + \gamma_g \\
\gamma_{qg}(N,\kappa^2) &= \ -\frac{T_R\,N}{(N+1)(N+2)} + \frac{2T_R}{N+3}
\end{aligned} \tag{2}$$

where

$$\begin{aligned}
2\Gamma(N,\kappa^2) &= \frac{2\,{}_3F_2\left(1,1,\frac{3}{2};\frac{N+3}{2},\frac{N+4}{2};-\kappa^{-2}\right)}{(N+1)(N+2)\,\kappa^2} - \ln\frac{1+\kappa^2}{\kappa^2}\ , \\
2\mathrm{K}(N,\kappa^2) &= \frac{2\,{}_2F_1\left(1,\frac{N+2}{2};\frac{N+4}{2};-\kappa^{-2}\right)}{(N+2)\,\kappa^2}\ .
\end{aligned} \tag{3}$$