High-Energy Limit of QED beyond Sudakov Approximation

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Topics discussed

- Introduction
  - *high energy limit, power corrections and renormalization group*

- QED vector form factor at high energy to $\mathcal{O}(m_e^2/s)$
  - *Sudakov and non-Sudakov double logarithms*
  - *soft electron pair emission and eikonal charge nonconservation*
  - *two-loop result, all order resummation and asymptotic formula*
  - *generalizations: QCD, scattering amplitudes, etc.*

- Based on:

  A.A. Penin, Phys.Lett. **B745** *(2015) 69, arXiv:1412.0671 [hep-ph]*

Sudakov limit

- Sudakov limit
  - *on-shell*
  - *exclusive*
  - *high energy*
  - *fixed angle*
- Sudakov logarithms *(leading power in* $1/Q^2$*)*
  - *each external line gets* $e^{-\frac{\alpha}{4\pi}\frac{C_R}{2}\ln^2(Q^2)}$

    Sudakov (1956); Frenkel, Taylor (1976)
  - *subleading logs exponentiate as well*

    Mueller (1979); Collins (1980); Sen (1981); Sterman (1987); ...
- What about power suppressed logs?

High energy limit beyond Sudakov approximation

- Logarithmically enhanced power corrections
  - *can be phenomenologically important at moderate energies*
  - *can become asymptotically dominant*
  - *intriguing from QFT point of view*

- Power corrections and renormalization group
  - *OPE, large mass expansion* ⇨ *local composite operators*
  - *threshold, nonrelativistic limit* ⇨ *spartially nonlocal potentials*
  - *ultrarelativistic limit* ⇨ ?

QED form factor

- $$\mathcal{F} = \bar{\psi}(p_1)\left(\gamma_\mu F_1 + \frac{i\sigma_{\mu\nu}q^\nu}{2m_e}F_2\right)\psi(p_2)$$

  kinematics $\quad p_1^2 = p_2^2 = m_e^2, \quad Q^2 = -(p_2 - p_1)^2 \gg m_e^2 \gg \lambda^2$

- High-energy expansion

$$F_1 = e^{-\frac{\alpha}{2\pi}\ln(\lambda^2/m_e^2)\ln\rho}\sum_{n=0}^{\infty}\rho^n F_1^{(n)} \qquad (\rho = m_e^2/Q^2)$$

- Sudakov double logarithmic result

$$F_1^{(0)} = e^{-x} \qquad \left(x = \frac{\alpha}{4\pi}\ln^2\rho\right)$$

Dirac form factor to $\mathcal{O}(\rho)$

- Main idea
  - *expansion by regions* ➪ *homogeneous integrals*
  - *leading singularity of a given region* ➪ *double log*
- Sudakov logs ➪ photon momentum $l \to 0$

soft photons | *eikonal electrons*

$$D = \frac{-g_{\mu\nu}}{l^2 - \lambda^2}$$

$$S \approx -\frac{\not{p}_i + m_e}{2p_i l}$$

↪ *either* $\mathcal{O}(\rho^2)$ *or subleading log*

- Cusp anomalous dimension

Korchemsky, Radyushkin (1987)

$$\Gamma_{cusp} = -\frac{\alpha}{\pi} \ln\rho \left(1 + \mathcal{O}(\rho^2)\right)$$

Dirac form factor to $\mathcal{O}(\rho)$

- Non-Sudakov logs ⇨ $l' = p_i - l \to 0$

  eikonal photons — *soft electrons*

  $$D = \frac{g_{\mu\nu}}{2p_i l' - m_e^2 + \lambda^2}$$

  $$S \approx \frac{m_e}{l'^2 - m_e^2}$$

  Gorshkov, Gribov, Lipatov, Frolov (1968)

- Does not contribute at one-loop

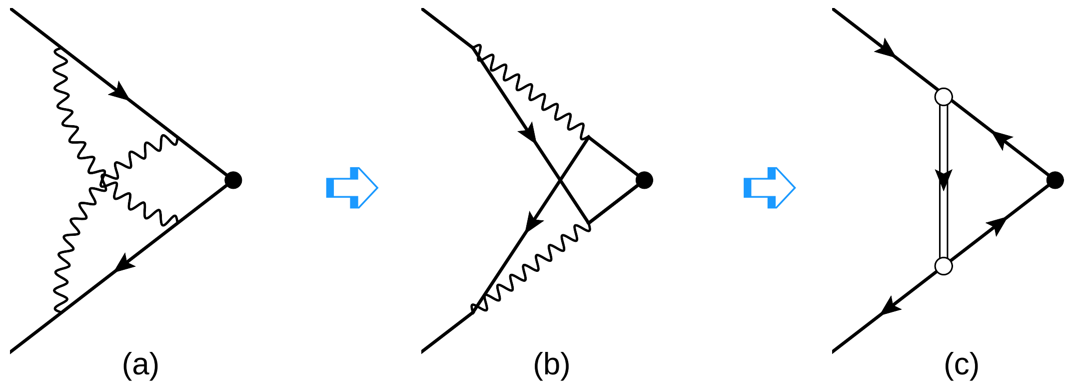
- Shows up in two-loop non-planar diagram

  - $l_1' = p_1 - l_1 \to 0, \qquad l_2' = p_2 - l_2 \to 0$

  ➥ *does not destroy eikonal structure*

Soft pair emission

- "Twist" the diagram



- *soft electon pair exchange*
- *eikonal charge nonconservation*

Evaluation of double logs

- *New variables*
  - *Sudakov parameters* $l = up_1 + vp_2 + l_\perp$
  - *hypercube coordinates* $\eta = \ln v / \ln\rho,\ \xi = \ln u/\ln\rho,\ 0 < \eta,\ \xi < 1$
- *Double log coefficient*
  - *volume under hypersurface*

$$F_1^{(1)}\Big|_{2-loop} = -4x^2 \int_{0<\eta_i,\ \xi_i<1} K(\eta_1, \eta_2, \xi_1, \xi_2) \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2$$

  - *kernel* $K(\eta_1, \eta_2, \xi_1, \xi_2) = \theta(1-\eta_1-\xi_1)\theta(1-\eta_2-\xi_2)\theta(\eta_2-\eta_1)\theta(\xi_1-\xi_2)$
  - *result* $F_1^{(1)}\Big|_{2-loop} = -x^2/3$

Mastrolia, Remiddi (2003)

Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi (2005)

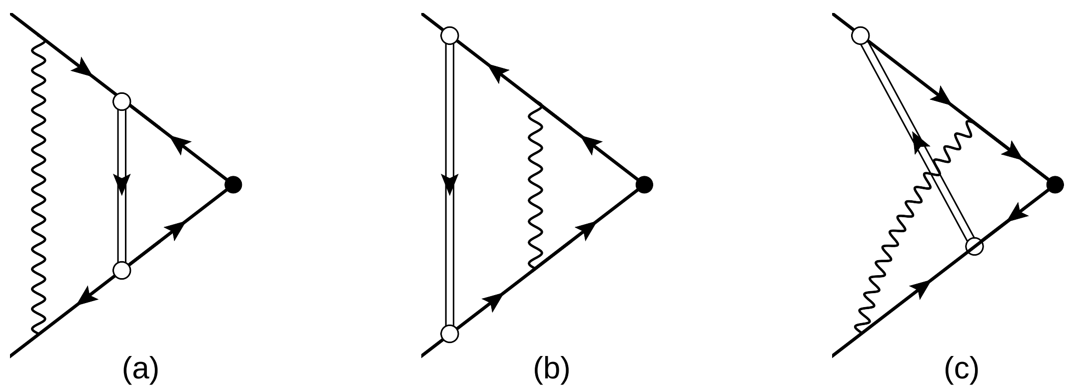
Universality

- *Quasi-universal character*
  - *leading power corrections are due to soft pair exchange*
  - *the coefficient depends on scattering details*
  - ⮱ *not determined by external particle charge only*

- *Two-loop form factors*
  - *Dirac, Pauli*
  - *scalar, axial*
  - ⮱ *agree with known results*

Resummation

- *Dressing with Sudakov photons*



➥ *no coupling to scalar quarks*

Resummation

- *Multiple Sudakov exchange*

$$F_1^{(1)} = -4x^2 \int \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2 \, \phi^b(\eta_1, \xi_2)\phi^c(\eta_1, \xi_1)\phi^c(\xi_2, \eta_2) \times \Big[\phi^a(\eta_2, \xi_1) K_1(\eta_1, \eta_2, \xi_1, \xi_2) + K_2(\eta_1, \eta_2, \xi_1, \xi_2)\Big]$$

  - *form factors:*

$$\phi^a(\eta, \xi) = \exp\left[-x\,(1-\eta-\xi)^2\right],$$
$$\phi^b(\eta, \xi) = \exp\left[-2x\eta\xi\right],$$
$$\phi^c(\eta, \xi) = \exp\left[x\eta\,(\eta+2\xi-2)\right],$$

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- *kernels:*

$$K_1(\eta_1, \eta_2, \xi_1, \xi_2) = \theta(1 - \eta_2 - \xi_1)\theta(1 - \eta_2 - \xi_2) \times\theta(\eta_2 - \eta_1)\theta(\xi_1 - \xi_2),$$
$$K_2(\eta_1, \eta_2, \xi_1, \xi_2) = \theta(1 - \eta_1 - \xi_1)\theta(1 - \eta_2 - \xi_2) \times\theta(\eta_2 - \eta_1)\theta(\xi_1 + \eta_2 - 1),$$

Resummation

- *The series*

$$F_1^{(1)} = -\frac{x^2}{3}\left(1 + \sum_{n=1}^{\infty} c_n x^n\right)$$

- *The coefficients*

n	1	2	3	4	5	6	7
$n!\,n^3 c_n$	$\frac{7}{30}$	$\frac{68}{105}$	$\frac{509}{350}$	$\frac{992}{945}$	$\frac{32225}{29106}$	$\frac{208044}{175175}$	$\frac{3946313}{3281850}$

- *The asymptotics*

$$c_n = \frac{C}{n^3 n!}\left[1 + \mathcal{O}\left(\frac{1}{n^{1/2}}\right)\right] \qquad C = 1.1994\ldots$$

High energy limit

- Large-x behaviour

$$F_1^{(1)} \sim e^{x - \ln x} \left[-\frac{C}{3} + \mathcal{O}\left(\frac{1}{x^{1/2}} \right) \right]$$

➥ *exponential enhancement*

- Leading term *vs* Power corrections

$$F_1^{(0)} \sim \rho^{-\frac{\alpha}{4\pi} \ln \rho}$$

$$\rho F_1^{(1)} \sim \rho^{1 + \frac{\alpha}{4\pi} \ln \rho}$$

➥ *power correction determine the asymptotic for* $|\ln \rho| > \frac{2\pi}{\alpha}$
below Landau pole, weak coupling $\alpha(Q^2) \approx 3\alpha$

Summary

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- New universal source of double logs
  - *soft pair exchange*

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- New universal source of double logs
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- Drastically different from Sudakov logs
  - *eikonal charge nonconservation*
  - ↪ *DL enhancement*

Summary

- The first resummation beyond Sudakov
- New universal source of double logs
  - *soft pair exchange*
- Drastically different from Sudakov logs
  - *eikonal charge nonconservation*
  - ↪ *DL enhancement*
- To be done
  - *amplitudes beyond two legs*
  - *QCD beyond two loops*
  - *subleading logs*