Radiative μ and τ leptonic decays at NLO

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Work in collaboration with Lorenzo Mercolli, arXiv:1506.03416

BaBar's branching ratio measurements

RAPID COMMUNICATIONS

PHYSICAL REVIEW D **91**, 051103(R) (2015)

Measurement of the branching fractions of the radiative leptonic τ decays $\tau \to e\gamma\nu\bar{\nu}$ and $\tau \to \mu\gamma\nu\bar{\nu}$ at *BABAR*

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

BABAR coll., PRD 91 (2015) 051103

- Babar experimental precision around 3%.
- More precise than CLEO results: T. Bergfeld et al., PRL 84 (2000) 830
  $1.75\,(6)_{\rm st}(17)_{\rm sy} \times 10^{-2}\,(\tau \to e\gamma\nu\bar{\nu})$,
  $3.61\,(16)_{\rm st}(35)_{\rm sy} \times 10^{-3}\,(\tau \to \mu\gamma\nu\bar{\nu})$.
- To be compared with SM branching ratio at NLO of order $(\alpha/\pi)\ln(m_l/m_\tau)\ln(\omega_0/m_\tau)$, $\sim 10\%$ for $l = e$, $\sim 3\%$ for $l = \mu$.

Radiative μ and τ leptonic decays

- Very clean, can be predicted with very high precision.
- TH formulation in terms of Bouchiat-Michel-Kinoshita-Sirlin parameters allows to test couplings beyond the SM V-A; additional BMKS-like param. accessible in radiative decays.
- SM background for μ and τ flavour violating decays like

$$\mu \to e\,\gamma$$

$$\tau \to l\,\gamma$$

$$\mu^+ \to e^+ e^- e^+$$

- Precise data on radiative τ decays may allow to determine its anomalous magnetic moment.

The τ g-2: SM prediction

- The Standard Model prediction of the tau g-2 is:

a_τ^{SM} =	117324 (2)	x 10^{-8}	QED
+	47.4 (0.5)	x 10^{-8}	EW
+	337.5 (3.7)	x 10^{-8}	HLO
+	7.6 (0.2)	x 10^{-8}	HHO (vac)
+	5 (3)	x 10^{-8}	HHO (lbl)

$$a_\tau^{SM} = 117721\ (5) \times 10^{-8}$$

Eidelman & MP 2007

- $(m_\tau/m_\mu)^2 \sim 280$: great opportunity to look for New Physics, and a "clean" NP test too…

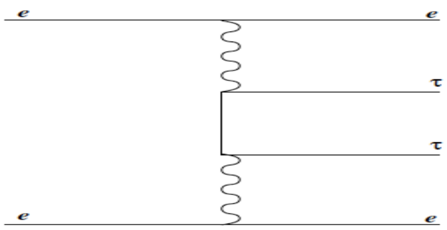
	Muon	Tau
a_{EW}/a_H	1/45	1/7
$a_{EW}/\ \delta a_H$	3	10

... if only we could measure it!!

The τ g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.

- DELPHI's result, from $e^+e^- \to e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):



$$a_\tau = -0.018\ (17)$$

PDG 2014

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

$$-0.004 < a_\tau^{NP} < 0.006 \quad (95\%\ CL)$$

Escribano & Massó 1997

$$-0.007 < a_\tau^{NP} < 0.005 \quad (95\%\ CL)$$

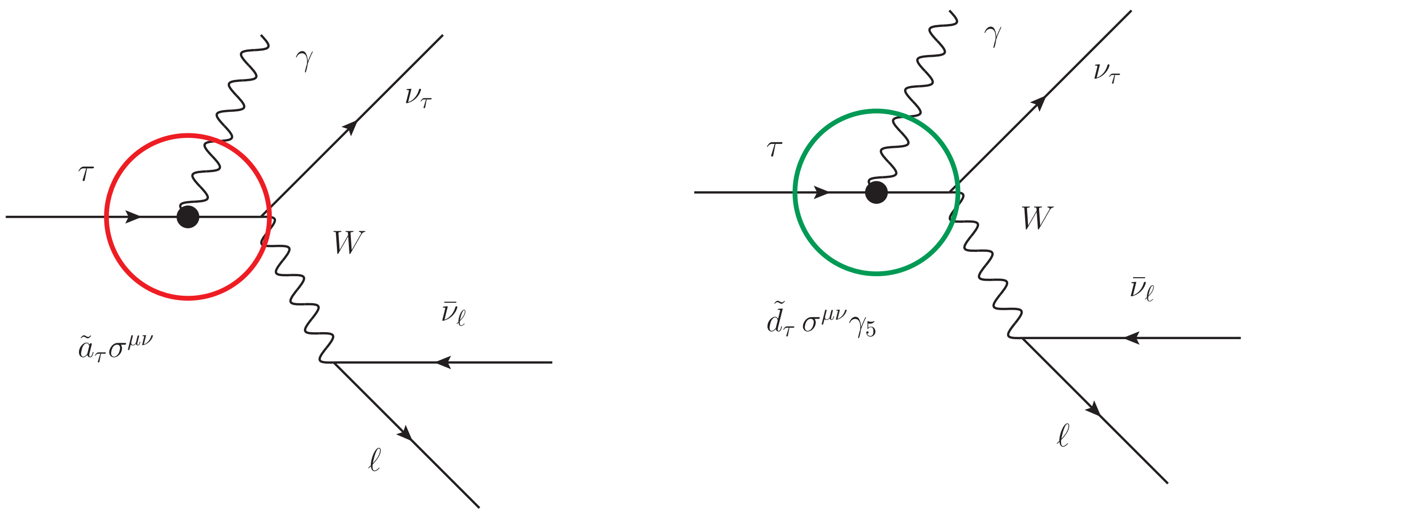
Gonzáles-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2=M_\Upsilon^2)$ from $e^+e^- \to \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160

The τ g-2 via τ radiative leptonic decays: a proposal

- Must employ the polarized differential decay rate.
- Study the full phase space.

$$d\Gamma = d\Gamma_0 + \left(\frac{m_\tau}{M_W}\right)^2 d\Gamma_W + \frac{\alpha}{\pi} d\Gamma_{\rm NLO} + \tilde{a}_\tau \, d\Gamma_{\rm a} + \tilde{d}_\tau \, d\Gamma_{\rm d}$$

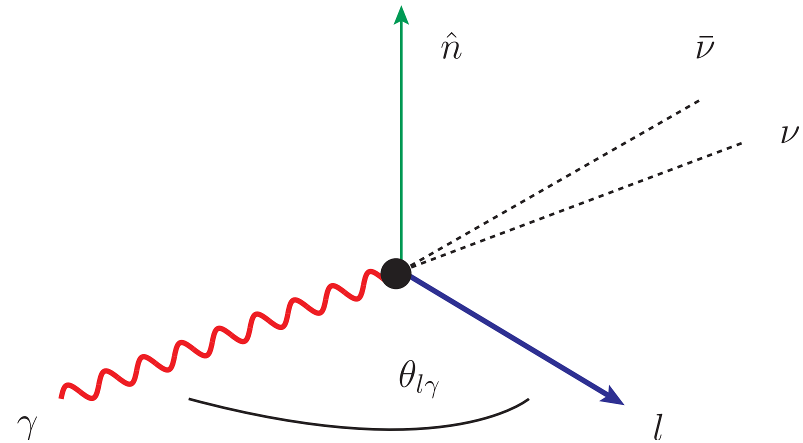


- Under investigation for Belle-II analysis. Should at least improve the Delphi bound. Work in progress with S. Eidelman & D. Epifanov

Differential decay rates

Radiative μ and τ leptonic decays: definitions

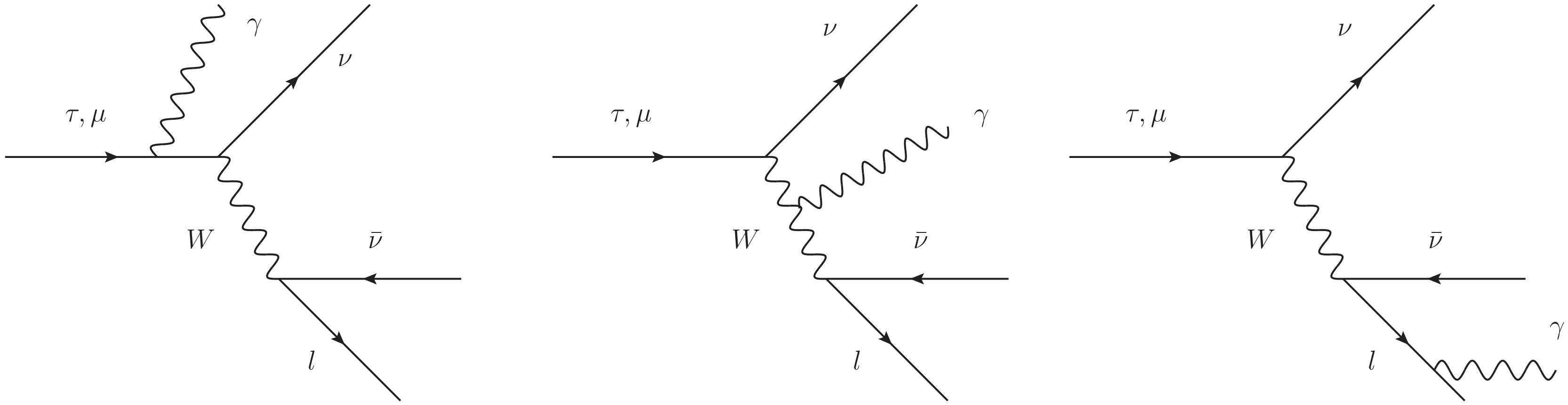
$$\mu^- \to e^- \gamma \nu_\mu \bar{\nu}_e,$$
$$\tau^- \to l^- \gamma \nu_\tau \bar{\nu}_l, \ (l = e, \mu)$$



- M: mass of the μ or τ.
- m_l: final charged lepton mass.
- $\hat{n}$: polarization vector of the initial μ or τ.
- Mass ratio $r = m_l/M$.
- Minimum photon energy ω_0.

Differential decay rates ($d\Gamma_0$)

$$d\Gamma = d\Gamma_0 + \frac{m^2_{\mu,\tau}}{M^2_W}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\mathrm{virt}} + \int_0^{\omega'_0} d\omega'\, d\Gamma_{\gamma\gamma}\right]$$



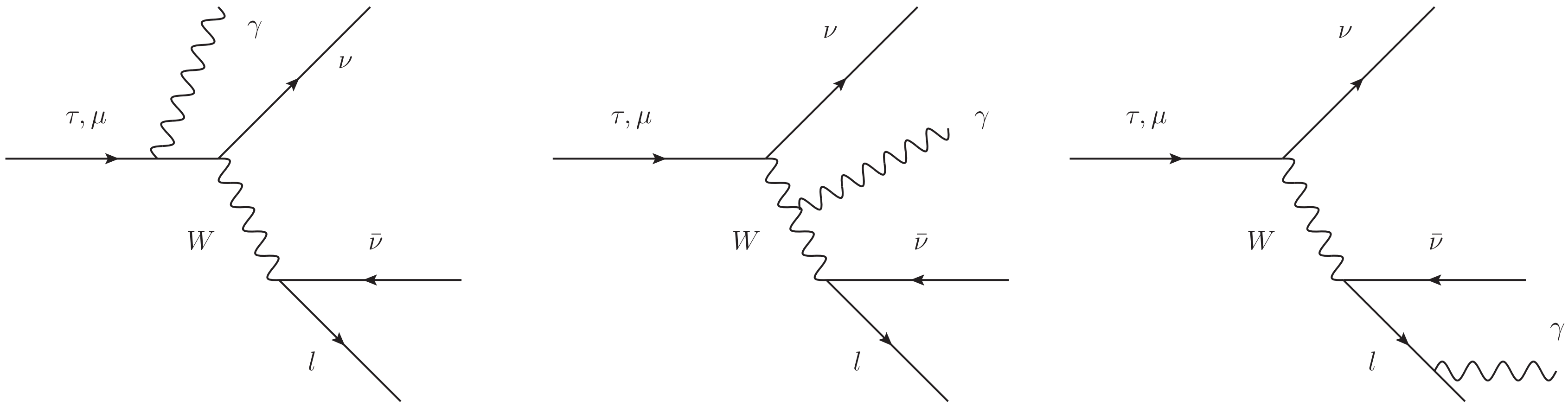
Kinoshita, Sirlin, PRL 2 (1959) 177; Fronsdal, Uberall, PR 133 (1959) 654

Eckstein, Pratt, Ann. Phys. 8 (1959) 297; Kuno, Okada, RMP 73 (2001) 151

Differential decay rates (dΓ_W)

$m_\mu^2/M_W^2 \sim 2\times 10^{-6}$ $\qquad$ $m_\tau^2/M_W^2 \sim 5\times 10^{-4}$

$$d\Gamma = d\Gamma_0 + \frac{m_{\mu,\tau}^2}{M_W^2}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\rm virt} + \int_0^{\omega_0'} d\omega'\, d\Gamma_{\gamma\gamma} \right]$$

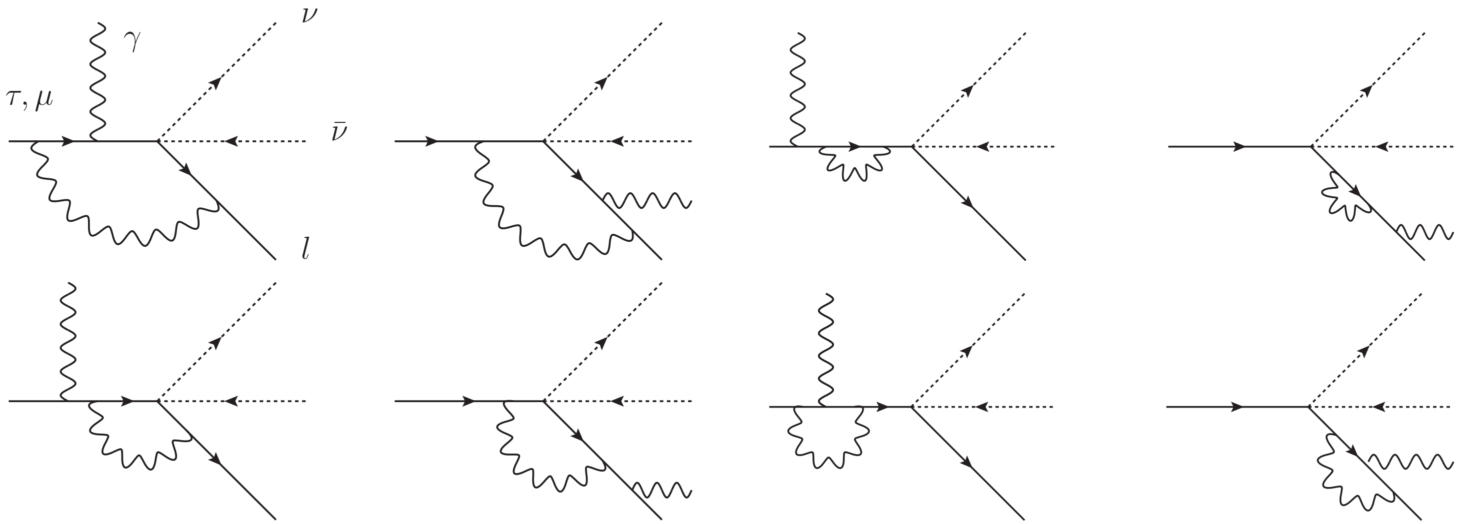


M. Fael, L. Mercolli, MP, PRD 88 (2013) 093011

Differential decay rates ($d\Gamma_V$)

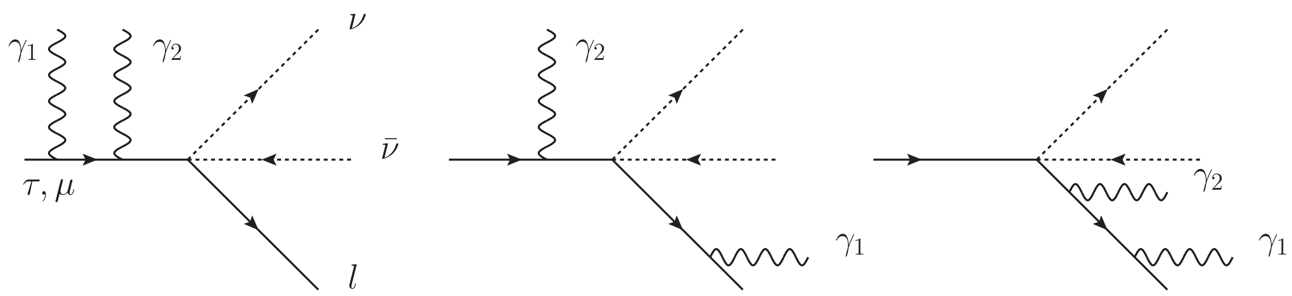
$$d\Gamma = d\Gamma_0 + \frac{m_{\mu,\tau}^2}{M_W^2}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\rm virt} + \int_0^{\omega_0'} d\omega'\, d\Gamma_{\gamma\gamma} \right]$$

- NLO corrections computed in the effective Fermi Lagrangian.



Fischer et al., PRD 49 (1994) 3426; Arbuzov, Scherbakova, PLB 597 (2004) 285; also as part of NNLO corrections to μ decay: T. van Ritbergen, R. G. Stuart, PRL 82 (1999) 488, C. Anastasiou, K. Melnikov, F. Petriello, JHEP 0709 (2007) 014, F. Caola, A. Czarnecki, Y. Liang, K. Melnikov, R. Szafron, PRD 90 (2014) 5.

$$d\Gamma = d\Gamma_0 + \frac{m^2_{\mu,\tau}}{M^2_W}\, d\Gamma_W + \frac{\alpha}{\pi}\left[d\Gamma_{\text{virt}} + \int_0^{\omega'_0} d\omega'\, d\Gamma_{\gamma\gamma}\right]$$

$2 \times$ 

- Extra soft photon emission $\omega' < \omega'_0$ integrated analytically:

$$d\Gamma^{\text{soft}}_{\gamma\gamma}(\omega'_0) = \int_0^{\omega'_0} d\omega'\, d\Gamma_{\gamma\gamma} \quad (\omega'_0 \ll M/2)$$

- IR divs of soft bremsstrahlung and virtual corrections cancel.
- Double "hard" bremss integrated numerically ($\omega' > \omega'_0$).

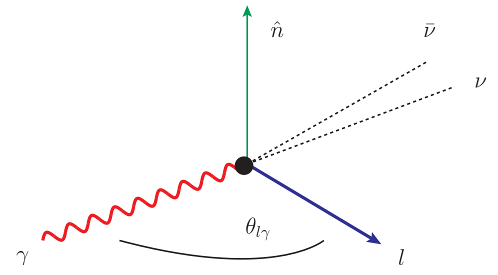
Differential decay rates (full $d\Gamma_{NLO}$)

$$\frac{d^6\Gamma}{dE_l\, d\omega\, d\Omega_l \Omega_\gamma} \propto G + \hat{n}\cdot\left[\hat{p}_l J + \hat{p}_\gamma K + (\hat{p}_l \times \hat{p}_\gamma) L\right]$$

$$G(\omega, E_l, \cos\theta_{le}) = G_0 + \frac{m_l^2}{M_W^2} G_W + \frac{\alpha}{\pi} G_{\rm NLO},\ \text{same for } J \text{ and } K$$

$$L(\omega, E_l, \cos\theta_{le}) = \frac{\alpha}{\pi} L_{\rm NLO}$$ NEW @ NLO!

- Analytically integrated over neutrinos.
- Visible particle kinematics $\omega, E_l, \Omega_l, \Omega_\gamma$.
- G: isotropic function.
- J, K, L: spin dependent parts.



Differential decay rates - comparison with previous results

$$\frac{d^6\Gamma}{dE_l\, d\omega\, d\Omega_l \Omega_\gamma} \propto G + \hat{n}\cdot\left[\hat{p}_l J + \hat{p}_\gamma K + (\hat{p}_l \times \hat{p}_\gamma) L\right]$$

One-loop corrections to $\mu \to e\gamma\nu\bar{\nu}$ differential decay rate previously computed in:

- A. Fischer et al., PRD 49 (1994) 3426
  Only the isotropic part G. Files with results unavailable.
- A. Arbuzov, E. S. Scherbakova, PLB 597 (2004) 285
  Full spin dependence, but calculation performed in the $m_l \to 0$ limit.
  - Perfect agreement with our isotropic function G (in that limit).
  - The anisotropic functions J, K differ from ours.
  - L has been overlooked.

Branching Ratios

Branching ratios at LO

B.R. for a minimum photon energy ω_0

$$\Gamma_{\rm LO}(y_0) = \frac{G_F^2 M^5}{192\pi^3}\frac{\alpha}{3\pi}\Big[3{\rm Li}_2(y_0) - \frac{\pi^2}{2} - \frac{1}{2}\left(6+\bar{y}_0^3\right)\bar{y}_0\ln\bar{y}_0$$

$$+\left(\ln r + \frac{17}{12}\right)\left(6\ln y_0 + 6\bar{y}_0 + \bar{y}_0^4\right) + \frac{1}{48}\left(125 + 45 y_0 - 33 y_0^2 + 7 y_0^3\right)\bar{y}_0\Big]$$

where $y_0 = 2\omega_0/M$, $\bar{y}_0 = 1 - y_0$, $r = m_l/M$

Kinoshita & Sirlin, PRL 2 (1959) 177; Eckstein & Pratt, Ann. Phys. 8 (1959) 297

- In $\Gamma_{\rm LO}(y_0)$ only $\ln(r)$ are kept, terms $\mathcal{O}(r)$ are neglected.
- However terms $\propto r^2$ were not, and cannot, be neglected in the integrand $d\Gamma$ (be careful also at NLO!).

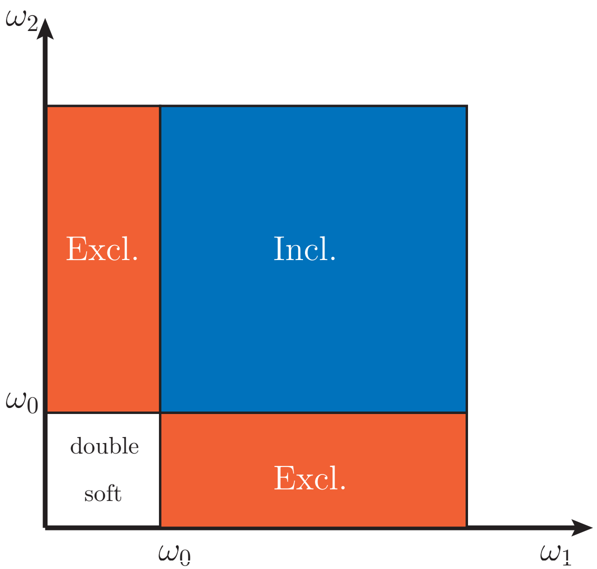
When r is small, $\Gamma_{\rm LO}(y_0)$ gives a good prediction for the branching ratios $\mathcal{B}_{\rm LO}$, compared with result integrated numerically. For $\omega_0 = 10$ MeV:

	$\mathcal{B}_{\rm LO}$	$\mathcal{B}_{\rm LO}^{\rm num}$	
$\mu \to e\gamma\nu\bar{\nu}$	1.31×10^{-2}	1.308×10^{-2}	
$\tau \to e\gamma\nu\bar{\nu}$	1.83×10^{-2}	1.834×10^{-2}	
$\tau \to \mu\gamma\nu\bar{\nu}$	3.58×10^{-3}	3.663×10^{-3}	OFF by ~2%

Inclusive and exclusive branching ratios at NLO

The branching ratio of radiative μ and τ leptonic decays for a minimum photon energy ω_0:

$$\mathcal{B}(\omega_0) \propto \int d\Phi_4 \left(d\Gamma_{\rm LO} + d\Gamma_{\rm virt} \right) + \int d\Phi_5 \, d\Gamma_{\gamma\gamma}$$



- $\mathcal{B}^{\rm Exc}(\omega_0)$: only one γ of energy $\omega > \omega_0$, additional second soft photon $\omega' < \omega_0$.

$$\mathcal{B}^{\rm Exc}(\omega_0) = \blacksquare$$

- $\mathcal{B}^{\rm Inc}(\omega_0)$: at least one γ of energy $\omega > \omega_0$.

$$\mathcal{B}^{\rm Inc}(\omega_0) = \blacksquare + \blacksquare$$

Branching ratios: τ results

B.R. of radiative τ leptonic decays ($\omega_0 = 10$ MeV)		
	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\mathcal{B}_{\rm LO}$	1.834×10^{-2}	3.663×10^{-3}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.06\,(1)_n(10)_N \times 10^{-3}$	$-5.8\,(1)_n(2)_N \times 10^{-5}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-1.89\,(1)_n(19)_N \times 10^{-3}$	$-9.1\,(1)_n(3)_N \times 10^{-5}$
$\mathcal{B}^{\rm Inc}$	$1.728\,(10)_{\rm th}(3)_\tau \times 10^{-2}$	$3.605\,(2)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\rm Exc}$	$1.645\,(19)_{\rm th}(3)_\tau \times 10^{-2}$	$3.572\,(3)_{\rm th}(6)_\tau \times 10^{-3}$
$\mathcal{B}^{\dagger}_{\rm EXP}$	$1.847\,(15)_{\rm st}(52)_{\rm sy} \times 10^{-2}$	$3.69\,(3)_{\rm st}(10)_{\rm sy} \times 10^{-3}$

(n): numerical errors

(N): uncomputed NNLO corr.

$\sim (\alpha/\pi) \ln r \ln(\omega_0/M) \times \mathcal{B}^{\rm Exc/Inc}_{\rm NLO}$

†BABAR - PRD 91 (2015) 051103

(th): combined $(n) \oplus (N)$

(τ): experimental error of τ lifetime: $\tau_\tau = 2.903(5) \times 10^{-13}$ s

	$\tau \to e\bar{\nu}\nu\gamma$	$\tau \to \mu\bar{\nu}\nu\gamma$
$\Delta^{\rm Exc}$	$2.02\,(57) \times 10^{-3} \to 3.5\sigma$	$1.2\,(1.0) \times 10^{-4} \to 1.1\sigma$

Branching ratios: μ results

B.R. of radiative μ leptonic decays ($\omega_0 = 10$ MeV)	
$\mathcal{B}_{\rm LO}$	1.308×10^{-2}
$\mathcal{B}^{\rm Inc}_{\rm NLO}$	$-1.91\,(5)_n(6)_N \times 10^{-4}$
$\mathcal{B}^{\rm Exc}_{\rm NLO}$	$-2.25\,(5)_n(7)_N \times 10^{-4}$
$\mathcal{B}^{\rm Inc}$	$1.289\,(1)_{\rm th} \times 10^{-2}$
$\mathcal{B}^{\rm Exc}$	$1.286\,(1)_{\rm th} \times 10^{-2}$
$\mathcal{B}^{\dagger}_{\rm EXP}$	$1.4\,(4) \times 10^{-2}$

† Crittenden *et al.* PR 1961 (1961) 1823

⋆ J. Adam *et al.* arXiv:1312.3217 [hep-ex]

‡ D. Počanić *et al.* 1403.7416 [nucl-ex].

Preliminary new measurements		
	cuts	$\mu \to e\gamma\nu\bar{\nu}$
MEG⋆	$E_e > 45$ MeV, $\omega_0 > 40$ MeV	$6.03\,(14)_{\rm st}\,(53)_{\rm sy} \times 10^{-8}$
PIBETA‡	$\omega_0 > 10$ MeV, $\theta_{e\gamma} > 30^\circ$	$4.365\,(9)_{\rm st}\,(42)_{\rm sy} \times 10^{-3}$

Conclusions

- We studied the differential rates and BRs of the radiative decays $\tau \to l\gamma\nu\bar{\nu}$ $(l = \mu, e)$ and $\mu \to e\gamma\nu\bar{\nu}$ in the SM at NLO in α.
- QED RC were computed taking into account the full mass dependence $r = m_l/M$, needed for the correct determination of the BRs.
- We found agreement with an earlier calculation of the isotropic function G in the $r \to 0$ limit, while our anisotropic functions J and K differ. Also, the function L has been previously overlooked.
- BRs: our results agree with the experimental value for $\mathcal{B}(\mu \to e\gamma\nu\bar{\nu})$ and BaBar's recent measurement of $\mathcal{B}(\tau \to \mu\gamma\nu\bar{\nu})$, both for a minimum photon energy $\omega_0 = 10$ MeV.
- On the contrary, BaBar's recent precise measurement of $\mathcal{B}(\tau \to e\gamma\nu\bar{\nu})$, for the same ω_0, differs from our prediction by 3.5σ.

Backup Slides

Differential decay rates (details)

The total differential decay for a polarized μ or τ lepton in the tau r.f. for $y > y_0'$ is

$$\frac{d^6\Gamma(y_0')}{dx\, dy\, d\Omega_l\, d\Omega_\gamma} = \frac{\alpha\, G_F^2 m_\tau^5}{(4\pi)^6} \frac{x\beta}{1+\delta_{\rm W}(m_\mu, m_e)} \Big[G \; + $$

$$+ \; x\beta\, \hat{n}\cdot\hat{p}_l\, J \; + \; y\, \hat{n}\cdot\hat{p}_\gamma\, K \; + \; y\, x\beta\, \hat{n}\cdot(\hat{p}_l \times \hat{p}_\gamma)\; L \Big]$$

where $x = 2E_l/m_\tau$, $y = 2E_\gamma/m_\tau$, $c = \cos\theta_{l\gamma}$. The polarization vector $n = (0, \hat{n})$ satisfies $n^2 = -1$ and $n\cdot p_\tau = 0$. The function $G(x, y, c; y_0')$, and similarly for J and K, is given by

$$G(x, y, c; y_0') = \frac{4}{3yz^2} \left[g_0(x, y, z) + \frac{\alpha}{\pi}\, g_{\rm NLO}(x, y, z; y_0') + \left(\frac{m_\tau}{M_W}\right)^2 g_{\rm W}(x, y, z) \right]$$

Inclusive and Exclusive Branching Ratios (2)

$$\mathcal{B}(\omega_0) \propto \lim_{\omega_0' \to 0} \left\{ \int d\Phi_4 \left(d\Gamma_{\mathrm{LO}} + d\Gamma_{\mathrm{virt}} + d\Gamma_{\gamma\gamma}^{\mathrm{soft}}(\omega_0') \right) + \int_{\omega_0'} d\Phi_5 \, d\Gamma_{\gamma\gamma} \right\}$$

- We use a soft threshold $\omega_0' \ll \omega_0$ to cancel IR.
- Soft photon radiation $d\Gamma_{\gamma\gamma}^{\mathrm{soft}}(\omega_0')$ computed thanks to the factorization of the amplitude.
- Integration over the phase space performed numerically.
- Double "hard" emission also checked with MadGraph5.