Integrand-Reduction Techniques for NLO Calculations and Beyond

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Based on work with GoSam Collaboration
G. Cullen, H. van Deurzen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, E. Mirabella, T. Peraro, J. Schlenk, J.F. von Soden-Fraunhofen, F. Tramontano

and P. Mastrolia, E. Mirabella, T. Peraro

Work in progress with: R. Frederix, V. Hirschi, U. Schubert, Z. Zhang

Outline

Outline of this talk:

1. NLO Automation
   - ⇝ Recent developments in GoSam 2.0
   - ⇝ Progress on Integrand-Reduction Techniques for one-loop multi-leg
   - ⇝ Ninja, a.k.a. Integrand Reduction via Laurent Expansion

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   - $\rightsquigarrow$ Ninja, a.k.a. Integrand Reduction via Laurent Expansion
2. Physical applications at NLO precision
   - $\rightsquigarrow$ Higgs Boson production in conjunction with jets and top-quark pairs
   - $\rightsquigarrow$ Interfacing with Monte Carlo tools

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   - ⇝ Higgs Boson production in conjunction with jets and top-quark pairs
   - ⇝ Interfacing with Monte Carlo tools
3. From NLO towards NNLO (and higher orders)
   - ⇝ A glance beyond GoSam 2.0
   - ⇝ Integrand Reduction via Multivariate Polynomial Division
   - ⇝ The Maximum-Cut theorem

NLO Automation

$$\sigma_{NLO} = \int_n \left(d\sigma^B + d\sigma^V + \int_1 d\sigma^A \right) + \int_{n+1} \left(d\sigma^R - d\sigma^A \right)$$

Monte Carlo Tools (MC) $\rightsquigarrow$

- Tree-level Contributions
- Subtraction Terms
- Integration over phase-space

One-Loop Programs (OLP) $\rightsquigarrow$

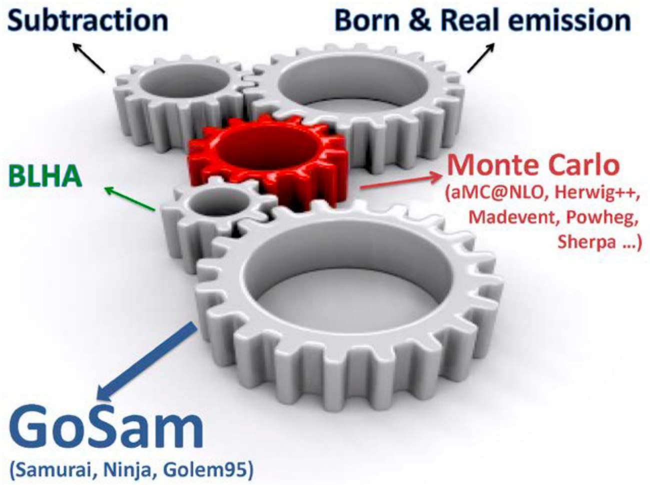
- The values of the Virtual Contributions (at each given phase-space point)

Strategies to full NLO automation:

$\rightsquigarrow$ **MC** controls the **OLP** via Binoth Les Houches Accord interface (BLHA)

$\rightsquigarrow$ **OLP** is fully incorporated within the **MC**

The GoSam Project



The GoSam Project

http://gosam.hepforge.org/

GoSam 1.0

"Automated One-Loop Calculations with GoSam"
Cullen, Greiner, Heinrich, Luisoni, Mastrolia, G.O., Reiter, Tramontano
Eur.Phys.J. C72 (2012) 1889 [arXiv:1111.2034]

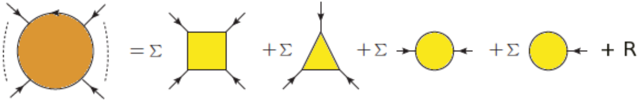
GoSam 2.0

"GOSAM-2.0: a tool for automated one-loop calculations within the Standard Model and beyond"
Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, G.O., Peraro, Schlenk, von Soden-Fraunhofen, Tramontano
Eur.Phys.J. C74 (2014) 3001 [arXiv:1404.7096]

Virtual NLO: One-Loop "master" formula

$$\int d^d\bar{q}\,\frac{\mathcal{N}(\bar{q})}{\bar{D}_{i_0}\bar{D}_{i_1}\ldots\bar{D}_{n-1}} = \sum_i d_i\,\mathrm{Box}_i + \sum_i c_i\,\mathrm{Triangle}_i + \sum_i b_i\,\mathrm{Bubble}_i + \sum_i a_i\,\mathrm{Tadpole}_i + \mathrm{R}\,,$$

1) **Generation**: Compute the unintegrated amplitudes for all diagrams
2) **Reduction**: Extract all coefficients and rational terms
3) **Master Integrals**: Calculate the Master Integrals (scalar integrals) and combine with the coefficients



There are several techniques available for Generation+Reduction and available codes to compute the one-loop Scalar Integrals

Virtual corrections with GoSam

1. Algebraic Generation
   - Amplitudes generated with **Feynman diagrams**
   - Optimization: **grouping of diagrams**, smart **caching**
   - Algebra in **dimension** d, different schemes
   - Suited for QCD, EW, effective Higgs coupling and BSM models$^{(*)}$

GoSam employs a Python "wrapper" which:

- ⇝ generates analytic integrands from **Feynman diagrams** using QGRAF
- ⇝ manipulates and simplifies them with FORM
- ⇝ writes them into FORTRAN95 code for the reduction

Virtual corrections with GoSam

1. Algebraic Generation
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2. Flexibility in the Reduction
   - Different reduction algorithms available at run-time
   - **Integrand-Level** and/or **Tensorial** Reduction available

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3. Selection of codes for the evaluation of the Master Integrals

OneLoop (**van Hameren**); QCDLoop (**Ellis, Zanderighi**)
Golem95C (**Binoth et al.**); LoopTools (**Hahn et al.**)

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   Golem95C (**Binoth et al.**); LoopTools (**Hahn et al.**)
4. Fully interfaced within several Monte Carlo frameworks

Modular Structure → Ability to incorporate **new ideas** and **techniques**
The GoSam framework is in continuous evolution

Reduction algorithms within GoSam

Ninja

$\rightsquigarrow$ **Default in GoSam 2.0**, more **Stable** and **Fast**

Integrand-Level Reduction + Laurent Expansion

van Deurzen, Luisoni, Mastrolia, Mirabella, G.O., Peraro

Golem95

$\rightsquigarrow$ **Default Rescue System**

Tensorial Reduction

Binoth, Guillet, Heinrich, Pilon, Reiter, von Soden-Fraunhofen

Samurai

$\rightsquigarrow$ **Default in GoSam 1.0**

$d-$dimensional Integrand-Level Reduction

Mastrolia, G.O., Reiter, Tramontano

One-loop Integrand Decomposition

$$\mathcal{N}(\bar{q}) = \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\ + \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h$$

#1 What is the Functional Form of all the Residues $\Delta_{ij\ldots k\ell}$?

- ⇝ They are polynomials in the components of $\bar{q}$
- ⇝ They have a universal, process-independent form . . .
- ⇝ . . . parametrised by process-dependent coefficients

#2 Extract all coefficients by sampling on the kinematic cuts

- ⇝ Polynomial fitting allows to find all coefficients (“fit-on-the-cut” approach)
- ⇝ we only need Numerator Function evaluated on the cuts

GO, Papadopoulos, Pittau (2007); Ellis, Giele, Kunszt, Melnikov (2008)
Mastrolia, GO, Reiter, Tramontano (2010); Mastrolia, Mirabella, GO, Peraro (2012)

ONE-LOOP INTEGRAND DECOMPOSITION $\rightsquigarrow$ NINJA-STYLE

$$
\begin{aligned}
\mathcal{N}(\bar{q}) \;=\; & \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h + \\
& + \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h
\end{aligned}
$$

#1 What is the Functional Form of all the Residues $\Delta_{ij\dots k\ell}$?

- $\rightsquigarrow$ They are polynomials in the components of $\bar{q}$
- $\rightsquigarrow$ They have a universal, process-independent form . . .
- $\rightsquigarrow$ . . . parametrised by process-dependent coefficients

#2 Extraction of coefficients via Laurent Expansion

$\rightsquigarrow$ If the numerator functions are known analytically, all coefficients can be extracted by performing a Laurent expansion with respect to the free parameters which appear in the solutions of the cuts $\rightsquigarrow$ NINJA

Mastrolia, Mirabella, Peraro (2012)

INTEGRAND REDUCTION NINJA-STYLE

Mastrolia, Mirabella, Peraro (2012)

$\rightsquigarrow$ The coefficients of the integrand can be extracted by performing a Laurent expansion with respect to one of the free parameters which appear in the solutions of the cut

Advantages:

- Corrections at the coefficient level replace subtractions at the integrand level
- Lighter reduction algorithm where fewer coefficients are computed
- Quadruple-cut decoupled from triple-, double-, and single-cut.
- No more "sampling on the cuts" (no need of finding solutions for the cuts)

INTEGRAND REDUCTION NINJA-STYLE

Mastrolia, Mirabella, Peraro (2012)
van Deurzen, Luisoni, Mastrolia, Mirabella, G.O., Peraro (2014)

$\rightsquigarrow$ This method has been implemented in the library NINJA and interfaced with GOSAM providing an improvement in the computational performance

Series of selected processes involving **massive particles**, from six to eight legs

Timings refer to **full color-summed** and **helicity-summed** amplitudes

```
Intel Core i7 CPU @ 3.40 GHz
        compiled with ifort

(*) Intel(R) Xeon(R) CPU E5-2650 @ 2.00 GHz
                     compiled with gfortran
```

Benchmarks: GOSAM + NINJA

Process		# NLO diagrams	ms/event
$W+3j$	$d\bar{u} \to \bar{\nu}_e e^- ggg$	1 411	226
$Z+3j$	$d\bar{d} \to e^+e^- ggg$	2 928	1 911
$ZZZ+1j$	$u\bar{u} \to ZZZg$	1 305	*12 000
$WWZ+1j$	$u\bar{u} \to W^+W^-Zg$	972	*7 050
$WZZ+1j$	$u\bar{d} \to W^+ZZg$	836	*3 300
$WWW+1j$	$u\bar{d} \to W^+W^-W^+g$	729	*1 800
$ZZZZ$	$u\bar{u} \to ZZZZ$	741	*1 070
$t\bar{t}b\bar{b}\,(m_b \neq 0)$	$d\bar{d} \to t\bar{t}b\bar{b}$	275	178
	$gg \to t\bar{t}b\bar{b}$	1 530	5 685
$t\bar{t}+2j$	$gg \to t\bar{t}gg$	4 700	13 827
$Zb\bar{b}+1j\,(m_b \neq 0)$	$dug \to ue^+e^-b\bar{b}$	708	*1 070
$Wb\bar{b}+1j\,(m_b \neq 0)$	$u\bar{d} \to e^+\nu_e b\bar{b}g$	312	67
$Wb\bar{b}+2j\,(m_b \neq 0)$	$u\bar{d} \to e^+\nu_e b\bar{b}s\bar{s}$	648	181
	$u\bar{d} \to e^+\nu_e b\bar{b}d\bar{d}$	1 220	895
	$u\bar{d} \to e^+\nu_e b\bar{b}gg$	3 923	5387
$WWb\bar{b}\,(m_b \neq 0)$	$d\bar{d} \to \nu_e e^+\bar{\nu}_\mu\mu^- b\bar{b}$	292	115
	$gg \to \nu_e e^+\bar{\nu}_\mu\mu^- b\bar{b}$	1 068	*5 300
$WWb\bar{b}+1j\,(m_b = 0)$	$u\bar{u} \to \nu_e e^+\bar{\nu}_\mu\mu^- b\bar{b}g$	3 612	*2 000
$H+3j$ in GF	$gg \to Hggg$	9 325	8 961
$t\bar{t}H+1j$	$gg \to t\bar{t}Hg$	1 517	1 505
$H+3j$ in VBF	$u\bar{u} \to Hgu\bar{u}$	432	101
$H+4j$ in VBF	$u\bar{u} \to Hggu\bar{u}$	1 176	669
$H+5j$ in VBF	$u\bar{u} \to Hgggu\bar{u}$	15 036	29 200

INTEGRAND REDUCTION NINJA-STYLE

Mastrolia, Mirabella, Peraro (2012)
van Deurzen, Luisoni, Mastrolia, Mirabella, G.O., Peraro (2014)

⇝ The coefficients of the integrand can be extracted by performing a Laurent expansion with respect to one of the free parameters which appear in the solutions of the cut

⇝ This method has been implemented in the library NINJA and interfaced with GOSAM providing an improvement in the computational performance

⇝ **Standalone** and **public** version of the NINJA library Peraro (2014)

⇝ Interfaced within FORMCALC 8.4: FORMCALC generates all numerator expansions needed by NINJA as an independent subroutine

Hahn *et al.* (2014)

Example: One-loop Bubbles via Laurent Expansion

- The residue of a bubble

$$\Delta_{ij}(q) = b_0 + b_1\,(q\cdot e_2) + b_2\,(q\cdot e_2)^2 + b_3\,(q\cdot e_3) + b_4\,(q\cdot e_3)^2 + b_5\,(q\cdot e_4) + b_6\,(q\cdot e_4)^2 + b_7\,(q\cdot e_2)(q\cdot e_3) + b_8\,(q\cdot e_2)(q\cdot e_4) + b_9\,\mu^2$$

- solutions of a double cut $D_i = D_j = 0$, parametrized by the free variables t, x and μ^2

$$q_+ = x\,e_1 + (\alpha_0 + x\,\alpha_1)e_2 + t\,e_3 + \frac{\beta_0 + \beta_1 x + \beta_2 x^2 + \mu^2}{2\,t}\,e_4$$

$$q_- = x\,e_1 + (\alpha_0 + x\,\alpha_1)e_2 + \frac{\beta_0 + \beta_1 x + \beta_2 x^2 + \mu^2}{2\,t}\,e_3 + t\,e_4$$

- in the limit $t \to \infty$

$$\left.\frac{\mathcal{N}(q_\pm)}{\prod_{m\neq i,j} D_m}\right|_{\text{cut}} = \Delta_{ij} + \sum_k \frac{\Delta_{ijk}}{D_k} + \sum_{kl} \frac{\Delta_{ijkl}}{D_k D_l} + \sum_{klm} \frac{\Delta_{ijklm}}{D_k D_l D_m}$$

$$= \Delta_{ij} + \sum_k \frac{\Delta_{ijk}}{D_k} + \mathcal{O}(1/t)$$

Example: One-loop Bubbles via Laurent Expansion

- In the asymptotic limit $t \to \infty$
  - the integrand

$$\left.\frac{\mathcal{N}(q_\pm)}{\prod_{m\neq i,j,k} D_m}\right|_{\text{cut}} = n_0^\pm + n_6^\pm\,\mu^2 + n_1^\pm\,x + n_2^\pm\,x^2 + \left(n_3^\pm + n_4^\pm x\right)t + n_5^\pm\,t^2 + \mathcal{O}(1/t)$$

  - the subtraction term

$$\frac{\Delta_{ijk}(q_\pm)}{D_k} = \tilde{b}_0^{k,\pm} + \tilde{b}_6^{k,\pm}\,\mu^2 + \tilde{b}_1^{k,\pm}\,x + \tilde{b}_2^{k,\pm}\,x^2 + \left(\tilde{b}_3^{k,\pm} + \tilde{b}_4^{k,\pm}x\right)t + \tilde{b}_5^{k,\pm}\,t^2 + \mathcal{O}(1/t)$$

    - $\tilde{b}_i^{k,\pm}$ are known functions of the triangle coefficients
  - the residue

$$\Delta_{ij}(q_+) = b_0 + b_9\,\mu^2 + b_1\,x + b_2 x^2 - \left(b_5 + b_8 x\right)t + b_6\,t^2 + \mathcal{O}(1/t)$$

$$\Delta_{ij}(q_-) = b_0 + b_9\,\mu^2 + b_1\,x + b_2 x^2 - \left(b_3 + b_7 x\right)t + b_4\,t^2 + \mathcal{O}(1/t)$$

  - by comparison, applying subtractions at the coefficient level

$$b_0 = n_0^\pm - \sum_k \tilde{b}_0^{k,\pm}, \quad b_1 = n_1^\pm - \sum_k \tilde{b}_1^{k,\pm}, \quad b_3 = -n_3^- + \sum_k \tilde{b}_3^{k,-}, \quad \ldots$$

GoSam – NLO Pheno Results

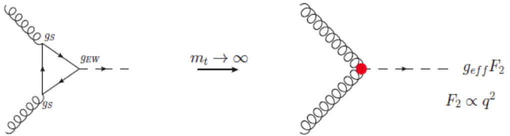
- N. Greiner, S. Hoeche, G. Luisoni, M. Schoenherr, V. Yundin, J. Winter, "Phenomenological analysis of Higgs boson production through gluon fusion in association with jets", arXiv:1506.01016
- G. Luisoni, C. Oleari, F. Tramontano, "Wbbj production at NLO with POWHEG+MiNLO", JHEP 1504 (2015) 161
- M. J. Dolan, C. Englert, N. Greiner, M. Spannowsky, "Further on up the road: hhjj production at the LHC", Phys.Rev.Lett. 112 (2014) 101802
- G. Heinrich, A. Maier, R. Nisius, J. Schlenk, J. Winter, "NLO QCD corrections to WWbb production with leptonic decays in the light of top quark mass and asymmetry measurements", JHEP 1406 (2014) 158
- T. Gehrmann, N. Greiner, and G. Heinrich, "Precise QCD predictions for the production of a photon pair in association with two jets", Phys.Rev.Lett. 111 (2013) 222002
- N. Greiner, G. Heinrich, J. Reichel, and J. F. von Soden-Fraunhofen, "NLO QCD corrections to diphoton plus jet production through graviton exchange", JHEP 1311 (2013) 028
- H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G.O. and T. Peraro, "NLO QCD corrections to Higgs boson production in association with a top quark pair and a jet", Phys.Rev.Lett. 111 (2013) 171801
- G. Cullen, H. van Deurzen, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G.O., T. Peraro, and F. Tramontano, "NLO QCD corrections to Higgs boson production plus three jets in gluon fusion," Phys.Rev.Lett. 111 (2013) 131801
- S. Hoeche, J. Huang, G. Luisoni, M. Schoenherr and J. Winter, "Zero and one jet combined NLO analysis of the top quark forward-backward asymmetry," Phys.Rev. D88 (2013) 014040
- G. Luisoni, P. Nason, C. Oleari and F. Tramontano,"HW/HZ + 0 and 1 jet at NLO with the POWHEG BOX interfaced to GoSam and their merging within MiNLO", JHEP 1310 (2013) 083
- M. Chiesa, G. Montagna, L. Barze', M. Moretti, O. Nicrosini, F. Piccinini and F. Tramontano, "Electroweak Sudakov Corrections to New Physics Searches at the CERN LHC," Phys.Rev.Lett. 111 (2013) 121801
- T. Gehrmann, N. Greiner, and G. Heinrich, "Photon isolation effects at NLO in gamma gamma + jet final states in hadronic collisions," JHEP 1306, 058 (2013)
- H. van Deurzen, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G.O., T. Peraro, J. F. von Soden-Fraunhofen, and F. Tramontano, "NLO QCD corrections to the production of Higgs plus two jets at the LHC," Phys. Lett. B 721, 74 (2013)
- G. Cullen, N. Greiner, and G. Heinrich, "Susy-QCD corrections to neutralino pair production in association with a jet," Eur. Phys. J. C 73, 2388 (2013)
- N. Greiner, G. Heinrich, P. Mastrolia, G. O., T. Reiter and F. Tramontano, "NLO QCD corrections to the production of W+ W- plus two jets at the LHC," Phys. Lett. B 713, 277 (2012)

Interfaces with external MC

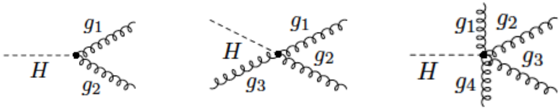
- GoSam + MadGraph+MadDipole+MadEvent
  → ad-hoc interface [Greiner]
- GoSam + Sherpa
  → fully automated [Luisoni, Schönherr, Tramontano]
- GoSam + Powheg
  → fully automated apart from phase space generation [Luisoni, Nason, Oleari, Tramontano]
- GoSam + Herwig++/Matchbox
  → fully automated [Bellm, Gieseke, Greiner, Heinrich, Plätzer, Reuschle, von Soden-Fraunhofen]
- GoSam + mg5_aMC@NLO
  → fully automated [van Deurzen, Frederix, Hirschi, Luisoni, Mastrolia, G.O.]

$pp \to H$+Jets in Gluon Fusion (GF)

Large Top-Mass Approximation ($m_t \to \infty$) : the Higgs coupling to gluons, mediated by a top-quark loop, can be described by an effective operator



→ **new Feynman rules**: vertices involving the Higgs field and up to four gluons



Higher Rank Extension → XSamurai

Theoretical Challenge: Effective Hgg coupling leads to **numerators with rank** r **larger than the number** n **of the denominators**, i.e. $r \leq n+1$

- The form of the integrand-level identity for the numerator was extended

Mastrolia, Mirabella, Peraro (2012)

- The decomposition of any one-loop n-point amplitude in terms of master integrals (MIs) acquires new contributions

$$\mathcal{M}_n^{\text{one-loop}} = \mathcal{A}_n + \delta\mathcal{A}_n$$

$\mathcal{A}_n$ corresponds to the standard decomposition ($r \leq n$)
$\delta\mathcal{A}_n$ enters only if $r \leq n+1$

- The **extended** integrand decomposition has been implemented in the SAMURAI library

Mastrolia, GO, Reiter, Tramontano (2010)
van Deurzen et al. (2012)

Higher Rank Extension → XSamurai

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- The **extended** integrand decomposition has been implemented in the SAMURAI library

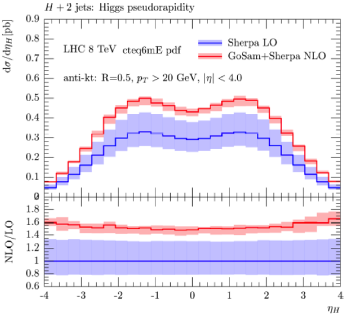
  Mastrolia, GO, Reiter, Tramontano (2010)
  van Deurzen et al. (2012)

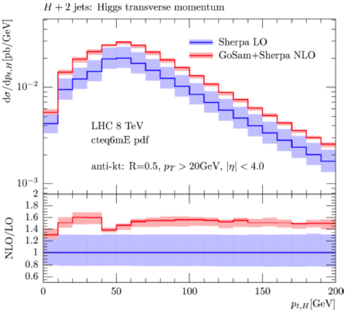
- Also the tensorial reduction of Golem95C was extended to support higher ranks $\rightsquigarrow$ applied in BSM physics (i.e. spin two particles)

  Guillet, Heinrich, von Soden-Fraunhofen (2014)

Warm-up: Higgs + 2 jets in GF @ NLO

- Results obtained with GoSam+Sherpa
- Agreement with MCFM v6.4 [Campbell, Ellis, Williams]





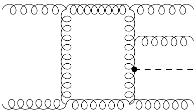
van Deurzen, Greiner, Luisoni, Mastrolia, Mirabella, G.O., Peraro, von Soden-Fraunhofen, Tramontano (2013)

(also appeared in *Handbook of LHC Higgs Cross Sections: 3. Higgs Properties*)

from H+2 to H+3 jets

Huge Theoretical and Computational Challenges

- More than 10,000 diagrams
- Higher-Rank terms
- Rank-7 hexagons



		Diagrams
H+0 jets	$g+g \longrightarrow H$	1
H+1 jets	$q+\bar{q} \longrightarrow H+g$	14
	$g+g \longrightarrow H+g$	48
		62
H+2 jets	$q+\bar{q} \longrightarrow H+q'+\bar{q}'$	32
	$q+\bar{q} \longrightarrow H+q+\bar{q}$	64
	$q+\bar{q} \longrightarrow H+g+g$	179
	$g+g \longrightarrow H+g+g$	651
		926
H+3 jets	$q+\bar{q} \longrightarrow H+q'+\bar{q}'+g$	467
	$q+\bar{q} \longrightarrow H+q+\bar{q}+g$	868
	$q+\bar{q} \longrightarrow H+g+g+g$	2519
	$g+g \longrightarrow H+g+g+g$	9325
		13179

$\rightsquigarrow$ The calculation of $H+3$ jets required lots of work and theoretical progress

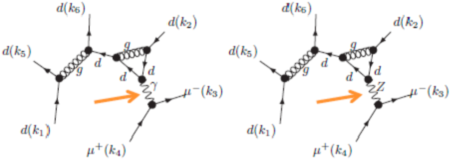
from H+2 to H+3 jets

$\rightsquigarrow$ **Improvements in the reduction**

- Upgrade Reduction Algorithms to include Higher-Rank Terms (effective couplings)
- Implementation of the corresponding Higher-Rank Master Integrals
- Ninja reduction based on Integrand reduction via Laurent expansion
- Interface for Ninja implemented in GoSam

FROM H+2 TO H+3 JETS

- ⇝ **Improvements in the reduction**
- ⇝ **Improvements in the code generation**
  - Diagrams with identical sets of denominators are summed algebraically during generation



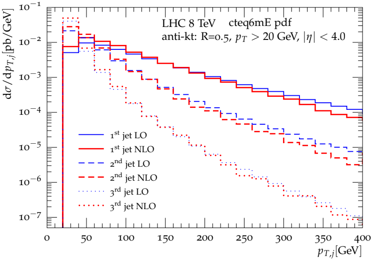
  - New optimization strategy via FORM 4.0
  - Numerical polarization vectors → Reduced code size
  - Parallelization of diagram generation → Reduction of generation time

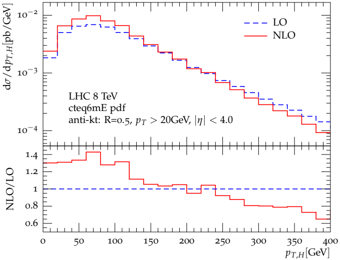
from H+2 to H+3 jets

- ⇝ **Improvements in the reduction**
- ⇝ **Improvements in the code generation**
- ⇝ **Improvements in the Monte Carlo** ⇝ see talk of **Nicolas Greiner**
  - In the original paper, all cross sections obtained with a hybrid setup
    ⇝ GoSam+ Sherpa for Born and of the Virtual contributions
    ⇝ `MadGraph+MadDipole+MadEvent` for reals/subtraction/integrated dipoles

FROM H+2 TO H+3 JETS

- **Improvements in the reduction**
- **Improvements in the code generation**
- **Improvements in the Monte Carlo** ⇝ see talk of **Nicolas Greiner**
- **Finally... a first result for Higgs + 3 jets in GF**

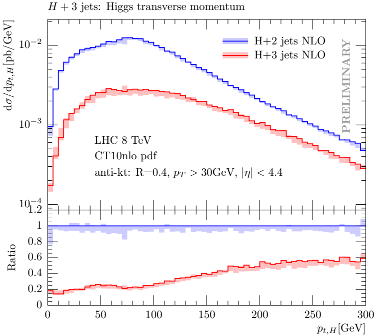


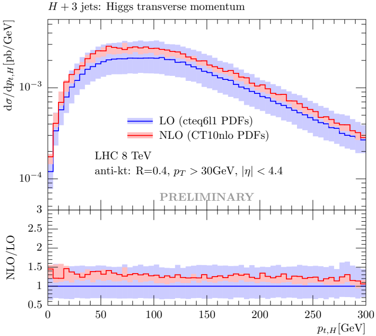


Cullen, van Deurzen, Greiner, Luisoni, Mastrolia,
Mirabella, G.O., Peraro, Tramontano (2013)

Higgs + 3 jets GF @ NLO: MC analyses

An updated analysis appeared in **Physics at TeV Colliders: Standard Model Working Group Report** [arXiv:1405.1067]





Cullen, van Deurzen, Greiner, Huston, Luisoni, Mastrolia, Mirabella, G.O., Peraro,Tramontano, Yundin, Winter (2014)

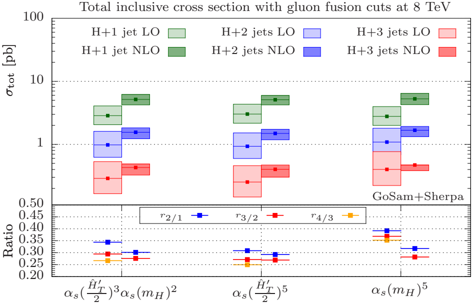
Higgs + 3 jets GF @ NLO: MC analyses

New results with GoSam + Sherpa !!

Greiner, Hoeche, Luisoni, Schoenherr, Yundin, Winter, "Phenomenological analysis of Higgs boson production through gluon fusion in association with jets",

arXiv:1506.01016

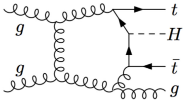
⇝ **Talk of Nicolas Greiner** on Monday

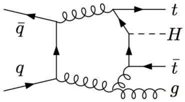


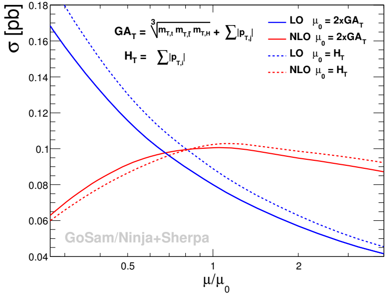
$pp \to Ht\bar{t}$ + 1 jet @ NLO

⇝ First Application of GoSam/Ninja + Sherpa

		Diagrams
H t t +1 jets	$q + \bar{q} \longrightarrow H + t + \bar{t} + g$	320
	$g + g \longrightarrow H + t + \bar{t} + g$	1575
		1895







- Two different mass scales: Higgs and Top
- 51 hexagons in the gluon-gluon channel
- Timing/PS-point: $qq \to 0.2$ sec
  $gg \to 2.5$ sec

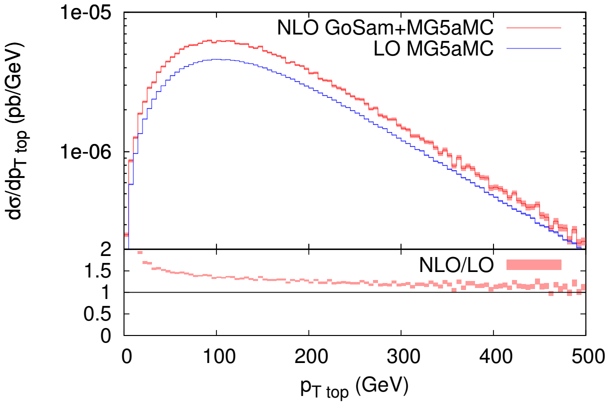
Central Scale	σ_{LO} [fb]	σ_{NLO} [fb]
$2{\times}GA_T$	$80.03^{+35.64}_{-23.02}$	$100.6^{+0.00}_{-9.43}$
H_T	$88.93^{+41.41}_{-26.13}$	$102.3^{+0.00}_{-15.82}$

van Deurzen, Luisoni, Mastrolia, Mirabella, G.O., Peraro (2013)

⋆ Updated and improved analysis under way

$pp \to t\bar{t}\gamma\gamma$ @ NLO

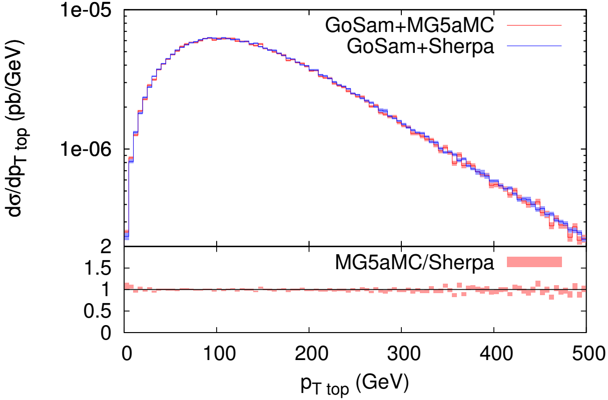
⇝ First Application of GoSam + mg5_aMC@NLO



van Deurzen, Frederix, Hirschi, Luisoni, Mastrolia, G.O. (work in progress)

$pp \to t\bar{t}\gamma\gamma$ @ NLO

⇝ First Application of GoSam + mg5_aMC@NLO



van Deurzen, Frederix, Hirschi, Luisoni, Mastrolia, G.O. (work in progress)

WHAT ABOUT HIGHER LOOPS?

Problem: at two loops (and higher), it is not available a **Standard Complete Basis of Master Integrals**

Gluza, Kosower, Kajda; Feng, Zeng, Huang, Zhou; Søgaard, Zhang; von Manteuffel, Schabinger; [. . .]

The most common (and successful) approach to NNLO relies on:

- Amplitude generation via **Feynman diagrams**
- Reduction to a minimal set of MIs using **Integration-by-Parts Identities**
- Computation of the MIs (analytic or numerical) $\rightsquigarrow$ **Differential Equations**

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Plenty of **progress in the field** (several talks at **UCLARadCor-LoopFest**)

This is the "Time of NNLO calculations"

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The **path to full automation** is still **long** and difficult

A glimpse at the use of GoSam beyond NLO

1. GoSam 2.0 has been already used within NNLO calculations, for the evaluation of the real-virtual contributions:
   - V. Del Duca, C. Duhr, G. Somogyi, F. Tramontano, Z. Trocsanyi, "Higgs boson decay into b-quarks at NNLO accuracy", JHEP 1504 (2015) 036
   - J. Gao, H.X. Zhu, "Top-quark forward-backward asymmetry in e+e- annihilation at NNLO in QCD", Phys.Rev.Lett. 113 (2014) 262001.
   - J. Gao, H.X. Zhu, "Electroweak prodution of top-quark pairs in e+e- annihilation at NNLO in QCD: the vector contributions", Phys.Rev. D90 (2014) 114022.
2. Towards "GoSam 2-Loops"

   Greiner, Heinrich, Jones, Kerner, Luisoni, Mastrolia, Schlenk, Zirke

   - Extension of the GoSam **generator** to produce all two-loop Feynman diagrams for any proceess: algebra done automatically (FORM), uses projectors on tensor structures, depicts all contributing diagrams as output on file.
   - Interfaces to Reduze, LiteRed, FIRE in progress.

Approximate NNLO: Hard Functions at NLO with GoSam

Broggio, Ferroglia, Pecjak, Yang

- The partonic cross section for **top pair + Higgs production** receives potentially large corrections from soft gluon emission
- The resummation of soft emission corrections can be carried out by means of effective field theory methods $\rightsquigarrow$ hard functions and soft functions
- The calculation of the NLO hard function requires the evaluation of one loop amplitudes obtained separating out the various color components

$$H_{IJ}^{(1)} = \frac{1}{4}\frac{1}{\langle c_I|c_I\rangle\,\langle c_J|c_J\rangle}\left[\left\langle c_I \middle| \mathcal{M}_{\rm ren}^{(0)}\right\rangle\left\langle \mathcal{M}_{\rm ren}^{(1)} \middle| c_J\right\rangle + \left\langle c_I \middle| \mathcal{M}_{\rm ren}^{(1)}\right\rangle\left\langle \mathcal{M}_{\rm ren}^{(0)} \middle| c_J\right\rangle\right]$$

- By default GoSam provides squared amplitudes summed over colors $\rightsquigarrow$ to build the hard functions we need to combine color decomposed (complex) amplitudes $\rightsquigarrow$ This required ad-hoc modifications of the GoSam code

[see talk of A. Ferroglia at SCET 2015]

Integrand-Level Reduction at Higher Orders

Can we build **alternative approaches** for **multi-loop reduction** by exploiting the multi-pole structure of scattering amplitudes ?

To pursue this challenging task, we need a **better understanding** of integrand decomposition and unitarity beyond one loop

- **Multiloop Integrand Reduction**
  Mastrolia, G.O. (2011); Badger, Frellersvig, Kleiss, Malamos, Papadopoulos, Verheyen; Zhang; Mastrolia, G.O., Mirabella, Peraro; Feng, Huang.
- see also **Maximal Unitarity** $\rightsquigarrow$ Talk of **K. Larsen**
  Kosower, Larsen (2011); Johansson, Kosower, Larsen; Søgaard, Zhang; Johansson, Kosower, Larsen, Søgaard.

Integrand-Level Reduction at Higher Orders

Let's consider a two-loop integral with n denominators:

$$\int dq\ dk\ \frac{N(q,k)}{D_1 D_2 \dots D_n}$$

As in the one-loop case, we want to construct an identity for the integrands:

$$N(q,k) = \sum_{i_1<<i_8}^{n} \Delta_{i_1,\dots,i_8}(q,k) \prod_{h\neq i_1,\dots,i_8}^{n} D_h + \dots + \sum_{i_1<<i_2}^{n} \Delta_{i_1,i_2}(q,k) \prod_{h\neq i_1,i_2}^{n} D_h$$

$$A(q,k) = \sum_{i_1<<i_8}^{n} \frac{\Delta_{i_1,\dots,i_8}(q,k)}{D_{i_1} D_{i_2} \dots D_{i_8}} + \sum_{i_1<<i_7}^{n} \frac{\Delta_{i_1,\dots,i_7}(q,k)}{D_{i_1} D_{i_2} \dots D_{i_7}} + \dots + \sum_{i_1<<i_2}^{n} \frac{\Delta_{i_1,i_2}(q,k)}{D_{i_1} D_{i_2}}$$

- Which terms appear in the above expressions?
- What is the general form of the residues $\Delta_{i_1,\dots,i_m}$?
- Can we detect the Master Integrals at the Integrand Level?

Work in collaboration with P. Mastrolia, E. Mirabella, T. Peraro, U. Schubert

Using the language of Algebraic Geometry...

- Set of Multivariate Polynomials $\{\omega_i(z)\}$ where $z = (z_1, z_2, \ldots)$
- **Ideal** $\mathcal{J} = \langle \omega_1(z), \ldots, \omega_s(z) \rangle \rightsquigarrow \mathcal{J} = \left\{ \sum_i \ h_i(z)\ \omega_i(z) \right\}$
- **Multivariate Polynomial Division** of a function $F(\mathbf{z})$ modulo $\{\omega_1, \ldots, \omega_s\}$
  - $\rightsquigarrow$ $F(z) = \sum_i \ h_i(z)\ \omega_i(z) + \mathcal{R}(z)$
  - $\rightsquigarrow$ $h_i(z)$ and $\mathcal{R}(z)$ are **not** unique
- **Gröbner Basis** $\{g_1(z), \ldots, g_r(z)\}$
  - $\rightsquigarrow$ It exists (Buchberger's algorithm) and generates the ideal $\mathcal{J}$
  - $\rightsquigarrow$ Provides a **unique** $\mathcal{R}(z)$
- Hilberts Nullstellensatz
  - $\rightsquigarrow$ $V(\mathcal{J}) \rightarrow$ set of common zeros of $\mathcal{J}$
  - $\rightsquigarrow$ Weak Nullstellensatz: $V(\mathcal{J}) = \mathcal{O} \Leftrightarrow 1 \in \mathcal{J}$

...Integrand Reduction via Multivariate Polynomial Division

$$\mathcal{I}_{i_1\cdots i_n} = \frac{\mathcal{N}_{i_1\cdots i_n}(z)}{D_{i_1}(z)\cdots D_{i_n}(z)}$$

- Ideal: $\mathcal{J}_{i_1\cdots i_n} = \langle D_{i_1}, \ldots, D_{i_n} \rangle$
- Gröbner basis $\mathcal{G}_{i_1\cdots i_n}$: same zero as the denominators
- Multivariate division of $\mathcal{N}_{i_1\cdots i_n}$ modulo $\mathcal{G}_{i_1\cdots i_n}$

$$\mathcal{N}_{i_1\cdots i_n}(z) = \Gamma_{i_1\cdots i_n} + \Delta_{i_1\cdots i_n}(z)$$

- The quotient $\Gamma_{i_1\cdots i_n}$ can be expressed in terms of denominators

$$\Gamma_{i_1\cdots i_n} = \sum_{\kappa=1}^{n} \mathcal{N}_{i_1\cdots i_{\kappa-1} i_{\kappa+1}\cdots i_n}(z) D_{i_\kappa}(z)$$

- Which provides the Recursive Formula

$$\mathcal{I}_{i_1\cdots i_n} = \sum_{\kappa=1}^{n} \mathcal{I}_{i_1\cdots i_{\kappa-1} i_{\kappa+1} i_n} + \frac{\Delta_{i_1\cdots i_n}}{D_{i_1}\cdots D_{i_n}}$$

Integrand Reduction at Arbitrary Loops: a Theorem

Let's look at the on-shell conditions, and impose

$$D_1 = D_2 = \ldots = D_n = 0$$

1) There are no solutions $\rightarrow$ the diagram is **reducible**

- $\rightsquigarrow$ The integrand with n denominators can be expressed in terms of integrands with $(n-1)$ denominators
- $\rightsquigarrow$ The diagram is fully reducible in terms of lower point functions
  $\rightsquigarrow$ i.e. *six-point functions at one-loop*

Integrand Reduction at Arbitrary Loops: a Theorem

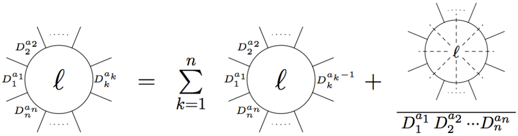
Let's look at the on-shell conditions, and impose

$$D_1 = D_2 = \ldots = D_n = 0$$

1) There are no solutions → **reducible**
2) The cut has solutions → there is a **residue** Δ

Divide the numerator modulo the Gröbner basis of the n-ple cut (a set of polynomials vanishing on the same on-shell cuts of the denominators)

⇝ The **remainder** of the division is the **residue** Δ of the n-ple cut.
⇝ The **quotients** generate integrands with $(n-1)$ denominators.



Integrand Reduction at Arbitrary Loops: a Theorem

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1) There are no solutions $\rightarrow$ **reducible**
2) The cut has solutions $\rightarrow$ **residue Δ**
3) Finite number of solutions $n_s \rightarrow$ **Maximum Cut**
   - $\rightsquigarrow$ First term in the integrand decomposition i.e. four-point function at one-loop in 4-dim
   - $\rightsquigarrow$ its residue is a **univariate polynomial** parametrized by n_s coefficients
   - $\rightsquigarrow$ the corresponding residue **can always be reconstructed at the cut**
   - $\rightsquigarrow$ the **residue** is determined as in the previous case, via MPD

Integrand Reduction at Arbitrary Loops: a Theorem

Let's look at the on-shell conditions, and impose

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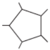
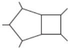
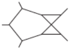
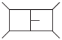
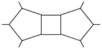
diagram	Δ	n_s	diagram	Δ	n_s
	c_0	1		$c_0 + c_1 z$	2
	$\sum_{i=0}^{3} c_i z^i$	4		$\sum_{i=0}^{3} c_i z^{\,i}$	4
	$\sum_{i=0}^{7} c_i z^{\,i}$	8		$\sum_{i=0}^{7} c_i z^{\,i}$	8

Mastrolia, G.O., Mirabella, Peraro

Examples of Applications of MPD

1. Reconstruct **all residues** in the OPP and d-dimensional One-Loop Integrand Decomposition

$$\mathcal{A} \equiv \frac{\mathcal{N}}{D_0 \ldots D_{n-1}} = \sum_{k=1}^{5} \sum_{\{i_1,\cdots,i_k\}} \frac{\Delta_{i_1\cdots i_k}}{D_{i_1}\cdots D_{i_k}}$$

$\rightsquigarrow$ Take a rank-k polynomial in $z \equiv (x_1, x_2, x_3, x_4, \mu^2)$ as $\mathcal{N}(z)$

$$\mathcal{N}(z) = \sum_{\vec{j}} \alpha_{\vec{j}}\, z_1^{j_1}\, z_2^{j_2}\, z_3^{j_3}\, z_4^{j_4}\, z_5^{j_5}$$

$\rightsquigarrow$ Apply the recursion formula:

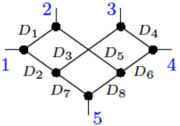
$$\mathcal{I}_{i_1\cdots i_n} = \sum_{\kappa=1}^{n} \mathcal{I}_{i_1\cdots i_{\kappa-1} i_{\kappa+1} i_n} + \frac{\Delta_{i_1\cdots i_n}}{D_{i_1}\cdots D_{i_n}}$$

$\rightsquigarrow$ Read the residues:

$$\Delta_{i_1\cdots i_5} = c_0\,,\;\; \Delta_{i_1\cdots i_4} = c_0 + c_1 x_4 + \mu^2(c_2 + c_3 x_4 + \mu^2 c_4)\,,\;\ldots$$

Examples of Applications of MPD

1. Reconstruct **all residues** in the OPP and d-dimensional One-Loop Integrand Decomposition
2. Examples of Two-loop topologies in $\mathcal{N} = 4$ SYM
   - Example: five-point $\mathcal{N} = 4$ SYM topology



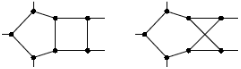
$$\mathcal{I} = \frac{c_0 + c_1(\mathrm{k}_1 \cdot p_5) + c_1(\mathrm{k}_1 \cdot p_1)}{D_1 \cdots D_8} + \sum_{i=1}^{8} \frac{\hat{c}_i}{\prod_{k \neq i} D_k}$$

$$\mathcal{N} = \alpha + v_1 \cdot \mathrm{k}_1 + v_2 \cdot \mathrm{k}_2$$ [Carrasco, Johansson '11]

Step 1. Reducing the integrand $\mathcal{I} = \frac{\mathcal{N}}{D_1 \cdots D_8}$

Step 2. Reducing the integrand $\mathcal{I}_{i_1 \cdots i_7} = \frac{\mathcal{N}_{i_1 \cdot i_7}}{D_{i_1} \cdots D_{i_7}}$

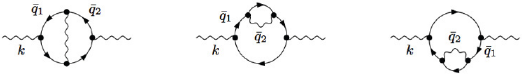
- reduction completed after two steps ($\mathcal{N} = 4$) (Checked via the $N = N$ test)
- other topologies & $\mathcal{N} = 8$ SUGRA reduced



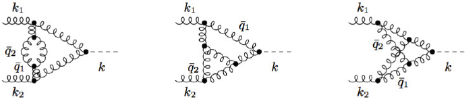
Examples of Applications of MPD

1. Reconstruct **all residues** in the OPP and d-dimensional One-Loop Integrand Decomposition
2. Examples of Two-loop topologies in $\mathcal{N} = 4$ SYM
3. Two-loop diagrams in QED and QCD processes

$\rightsquigarrow$ Two-loop QED corrections to the photon self energy



$\rightsquigarrow$ Two-loop diagrams entering the QCD corrections to $gg \to H$ in the heavy top mass approximation



Mastrolia, Mirabella, GO, Peraro

Conclusions/Outlook

- At present, there is a **great variety of methods** available for NLO scattering amplitudes $\rightsquigarrow$ Multi-Purpose NLO Computational Tools
- Generalized Unitarity, Integrand Reduction methods, Recursion Relations, Improved Tensorial techniques **revolutionized** the calculation of **NLO multi-leg** amplitudes

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- **OLP** have been incorporated **in the MC tools** to generate theoretical prediction for a large variety of processes: high multiplicities, several scales, effective vertices . . .
- **NLO automation has been fully achieved** $\rightsquigarrow$ Several **NLO analyses** under way

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⇝ In this talk, I described some **applications of Integrand Reduction methods**

- At **NLO**, Integrand reduction via Laurent (available with NINJA) provided a stable and reliable reduction for GoSam
- GoSam is in the process of being **upgraded to be used in NNLO calculations** (generation of NNLO virtual, numerical evaluation of real-virtual, . . .)
- Integrand Reduction can be extended at Arbitrary Loops (**Maximum-Cut Theorem**). Not ready yet to be competitive beyond NLO.

⇝ Will we ever achieve at NNLO the same degree of automation of the NLO? Which **new ideas** will play a role in the process?