UCLA Physics|Amplitudes

Subtleties in Nonsupersymmetric Gravity at Two Loops

Josh Nohle, UCLA

[Zvi Bern, Clifford Cheung, Huan-Hang Chi, Scott Davies, Lance Dixon, JN – to appear]
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18 June, 2015

Radcor/Loopfest

Introduction

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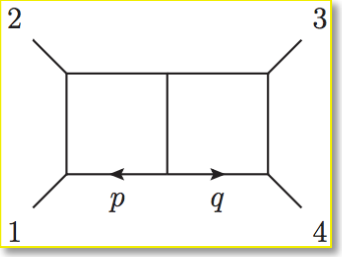

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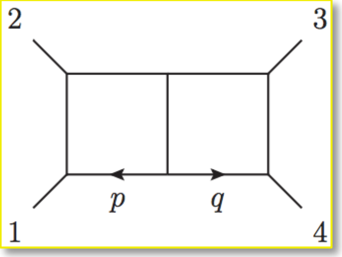

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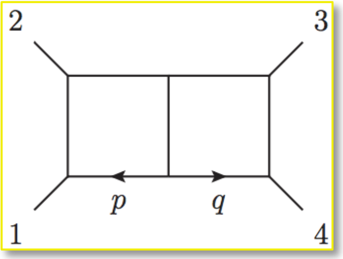

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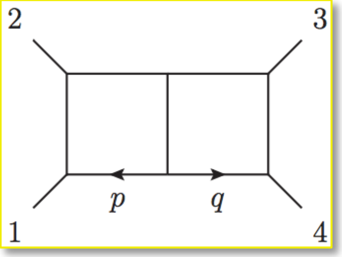

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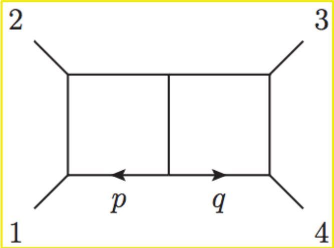

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  - Quantum (in)equivalence of dilaton and anti-symmetric tensor fields
  - Contribution of three-form field – zero degrees of freedom in D=4
  - [Color-kinematics duality constraints only on spanning set of cuts]

Outline

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1. Layout of Pertinent NonSUSY Gravity Theories
   - Graviton, dilaton, antisymmetric tensor, three-form

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- Unitarity, spinor-helicity, [color-kinematics duality]

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⑤ [Color-Kinematics Duality at Two-Loops]

Nonsupersymmetric Gravity Theories

NonSUSY Gravity Theories

- Pure Gravity Lagrangian:

$$\mathcal{L}_{\rm G} = \sqrt{-g}\left(-\frac{2}{\kappa^2}R\right)$$

- Graviton, Three-Form Lagrangian:

$$\mathcal{L}_{\rm G3} = \sqrt{-g}\left(-\frac{2}{\kappa^2}R + \frac{1}{8}H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}\right)$$

$H_{\mu\nu\rho\sigma} \equiv \partial_\mu A_{\nu\rho\sigma} - \partial_\nu A_{\rho\sigma\mu} + \partial_\rho A_{\sigma\mu\nu} - \partial_\sigma A_{\mu\nu\rho}$: Field strength of three-form field, $A_{\mu\nu\rho}$

- Note the D=4 duality transformation

$$\sqrt{\Lambda} \leftrightarrow \epsilon_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}$$

NonSUSY Gravity Theories

- Graviton, Dilaton Lagrangian:

$$\mathcal{L}_{\mathrm{GD}} = \sqrt{-g}\left(-\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\right)$$

- Graviton, Antisymmetric Tensor Lagrangian:

$$\mathcal{L}_{\mathrm{GA}} = \sqrt{-g}\left(-\frac{2}{\kappa^2}R + \frac{1}{6}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

$H_{\mu\nu\rho} \equiv \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu}$: Field strength of antisymmetric tensor field, $A_{\mu\nu}$

- Note the D=4 duality transformation between GD and GA

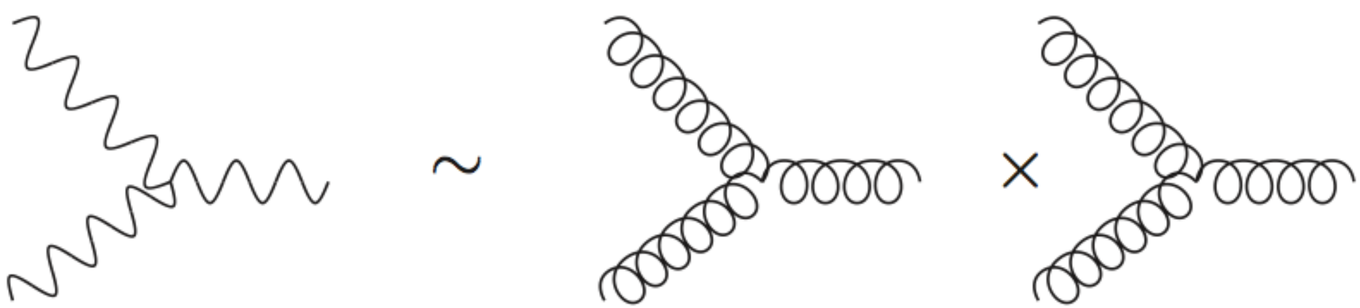
$$\partial_\mu\phi \leftrightarrow \epsilon_{\mu\nu\rho\sigma}H^{\nu\rho\sigma}$$

NonSUSY Gravity Theories

- Graviton, Dilaton, Antisymmetric Tensor Lagrangian:

$$\mathcal{L}_{\text{GDA}} = \sqrt{-g}\left(-\frac{2}{\kappa^2}R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{6}e^{-2\kappa\phi/\sqrt{D_s-2}}H_{\mu\nu\rho}H^{\mu\nu\rho}\right)$$

- Low-energy limit of bosonic string theory
- This theory emerges from KLT or as a double-copy of nonsupersymmetric Yang-Mills theory



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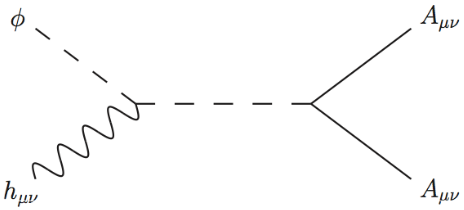

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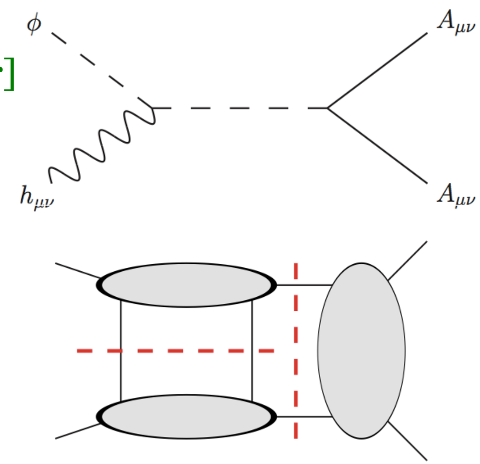

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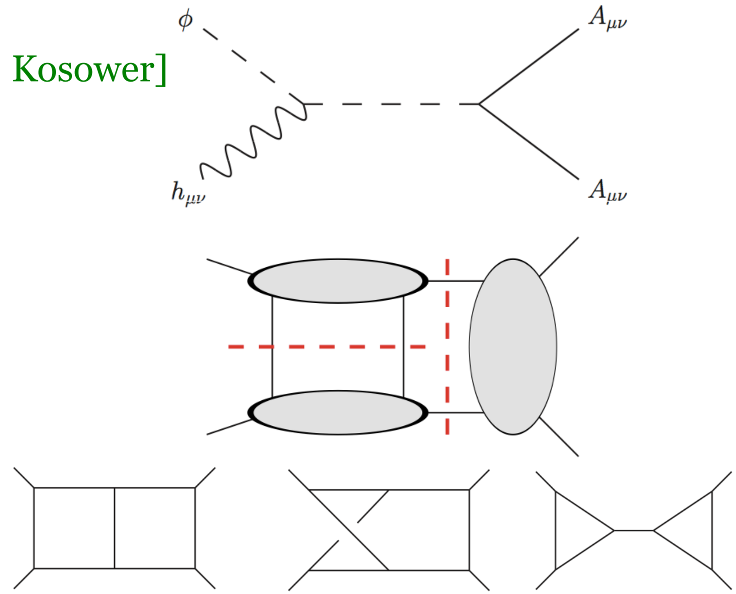

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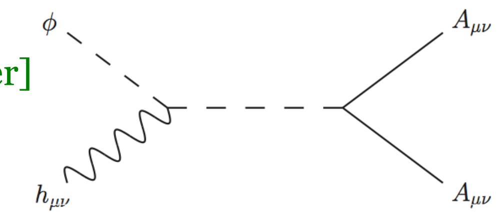

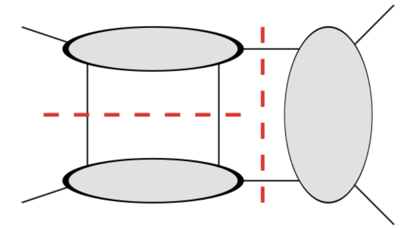

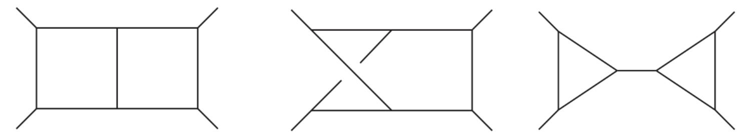

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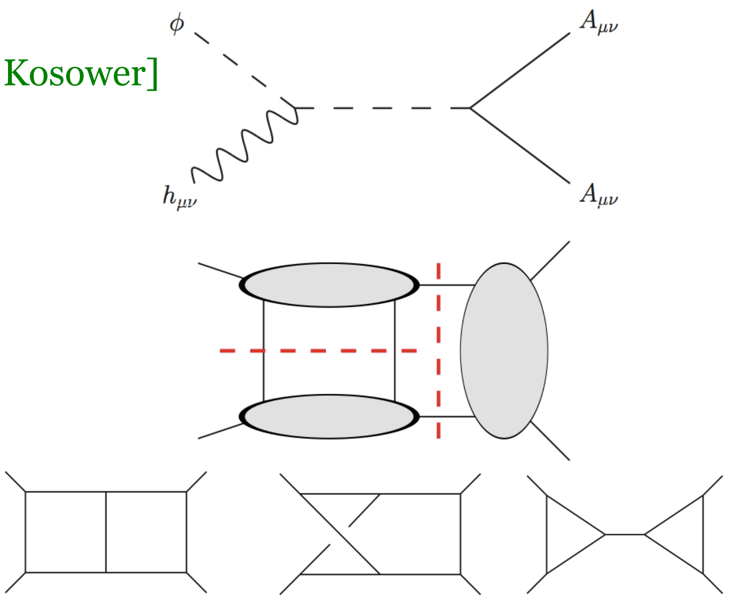
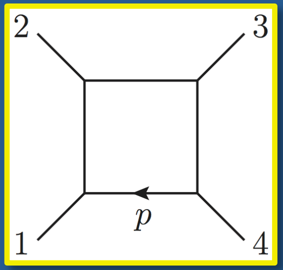


- Spinor-Helicity Formalism
  - Used for two-loop “bare” integrands
- Color-Kinematics Duality [Bern, Carrasco, Johansson]
  - Useful for GDA theory

[See talk by Scott Davies]

One-Loop Gravity



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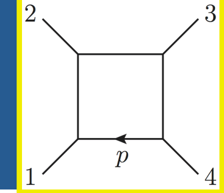

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  - Possible external graviton counterterms in D=4:

$$R^2, R_{\mu\nu}^2, R_{\mu\nu\rho\sigma}^2$$

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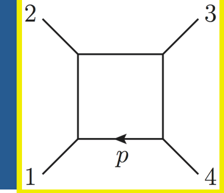

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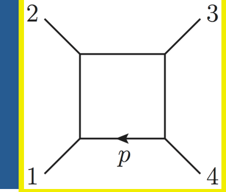

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  - In D=4, Gauss-Bonnet Theorem states

$$\int d^4x\sqrt{-g}\left(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right) = 32\pi^2\chi$$

  - In asymptotically flat space, χ=0 → Riemann curvature vanishes

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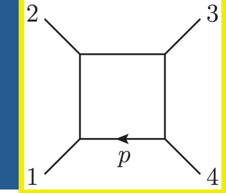

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- With nontrivial topology, pure gravity counterterm is

$$\mathcal{L}^{(\mathrm{GB})} = -\frac{1}{\epsilon}\frac{1}{(4\pi)^2}\frac{53}{90}\sqrt{-g}\left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2\right)$$

[Capper and Duff (1974); Tsao (1977); Critchley (1978); Gibbons, Hawking, Perry (1978); Goroff and Sagnotti (1986); Bornsen and van de Ven (2009)]

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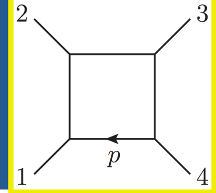

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  - Computed long ago for many different fields in the loop (ext. gravitons)

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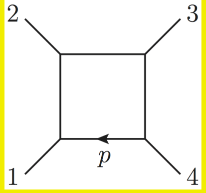

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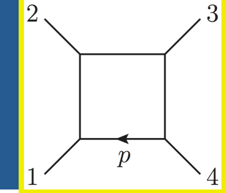

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  - For the theories we discussed,

$$\mathcal{L}^{(\mathrm{GB})} = -\frac{1}{\epsilon} \frac{1}{(4\pi)^2} \frac{1}{360} (4 \cdot 53 + 1 + 91 - 180) \sqrt{-g} \left(R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}\right)$$

graviton dilaton antisym. 3-form

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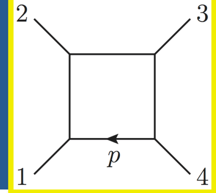

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graviton dilaton antisym. 3-form

- Notice that the dilaton and antisymmetric tensor contribute differently in spite of the D=4 duality transformation,

$$\partial_\mu \phi \;\leftrightarrow\; \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}$$

- Quantum (in)equivalence under duality transformations?

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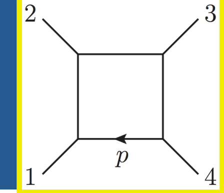

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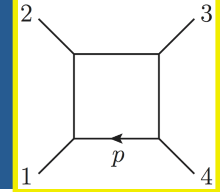

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[Capper and Kimber (1980)]

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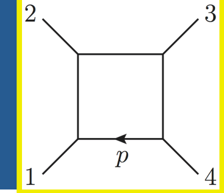

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- Notice that the three-form contributes despite having zero degrees of freedom in D=4.
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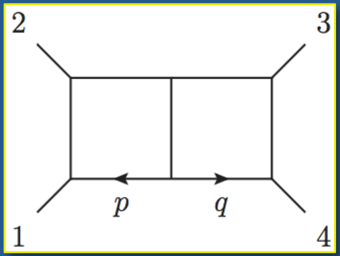
- Gauss-Bonnet operator is evanescent, but could affect 2 loops

Two-Loop Gravity



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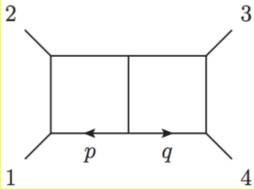
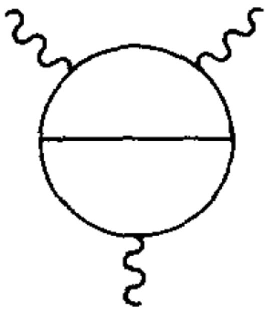


- There is a valid R^3 counterterm and corresponding divergence [Goroff and Sagnotti (1986); van de Ven (1992)]



$$\mathcal{L}^{(R^3)} = -\frac{1}{\epsilon}\frac{1}{(4\pi)^4}\frac{209}{5760}\sqrt{-g}\,R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$$

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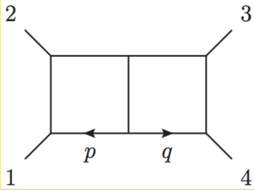

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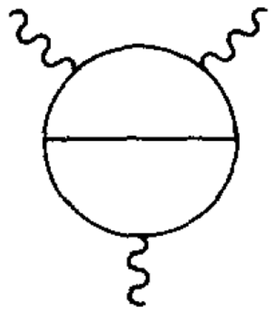

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- For four plus-helicity gravitons, this corresponds to

$$\mathcal{M}_4^{(2)}(1^+,2^+,3^+,4^+)\Big|_{\text{UV div.}} = \frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4}\frac{209}{24}stu\,\mathcal{T}^2 \quad \text{where} \quad \mathcal{T} \equiv \frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}$$

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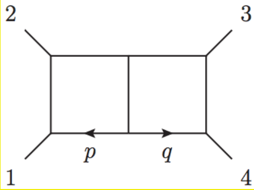

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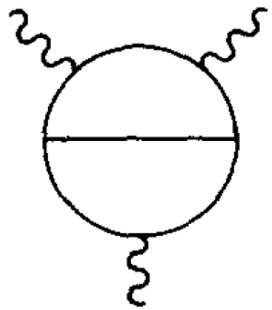


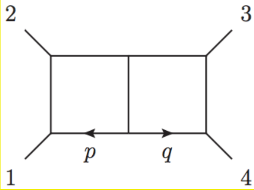
$$\mathcal{L}^{(R^3)} = -\frac{1}{\epsilon}\frac{1}{(4\pi)^4}\frac{209}{5760}\sqrt{-g}\,R^{\alpha\beta}{}_{\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$$

- For four plus-helicity gravitons, this corresponds to

$$\mathcal{M}_4^{(2)}(1^+,2^+,3^+,4^+)\Big|_{\text{UV div.}} = \frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^6\frac{i}{(4\pi)^4}\frac{209}{24}stu\,\mathcal{T}^2 \quad \text{where} \quad \mathcal{T} \equiv \frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}$$

- They performed the bare integration and subtracted any subdivergences, which can appear even if artificial

Two-Loop Gravity



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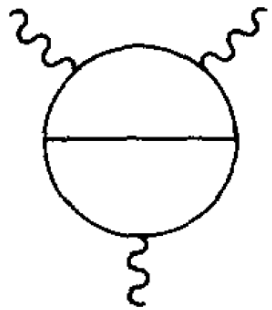

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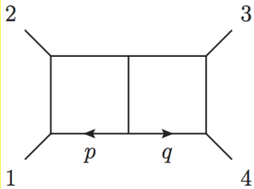
- They performed the bare integration and subtracted any subdivergences, which can appear even if artificial
  - Are there meaningful subdivergences even though there is no one-loop divergence in D=4? I.e., does the evanescent Gauss-Bonnet operator (trace anomaly) feed into the two-loop divergence?

Two-Loop Gravity



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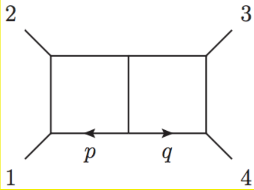
Two-Loop Gravity



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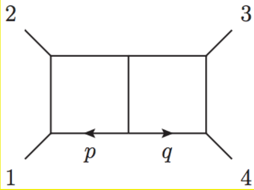
Two-Loop Gravity



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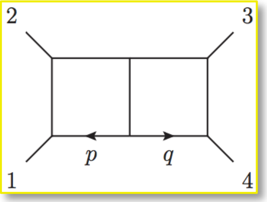
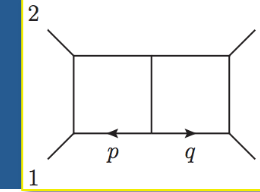


$$= \frac{D_s(D_s-3)}{2}(\lambda_p^2\lambda_q^2+\lambda_p^2\lambda_{p+q}^2+\lambda_q^2\lambda_{p+q}^2)^2 - \frac{D_s(D_s-6)}{2}\lambda_p^2\lambda_q^2\lambda_{p+q}^2(\lambda_p^2+\lambda_q^2+\lambda_{p+q}^2)$$
$$+ 12D_s((\lambda_p\cdot\lambda_q)^2-\lambda_p^2\lambda_q^2)(\lambda_p^2\lambda_q^2+\lambda_p^2\lambda_{p+q}^2+\lambda_q^2\lambda_{p+q}^2) + 144((\lambda_p\cdot\lambda_q)^2-\lambda_p^2\lambda_q^2)^2\,,$$

where the λ's represent the -2ε-components of the loop momenta

Two-Loop Gravity



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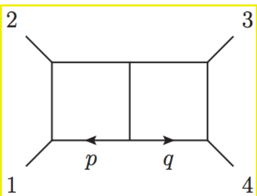

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- Integrating, we find the "bare" UV divergence

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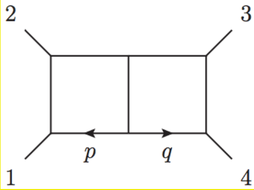
Two-Loop Gravity



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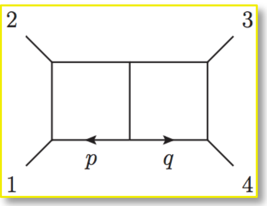

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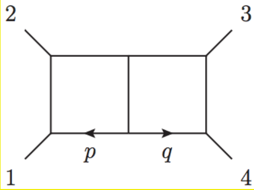
Two-Loop Gravity



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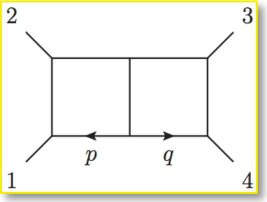

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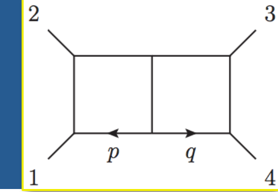
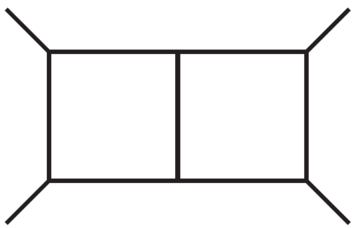
- Does not match Goroff & Sagnotti, but if we look closely, we will see meaningful subdivergences!
- We will deal with the subtractions using counterterms

Two-Loop Gravity

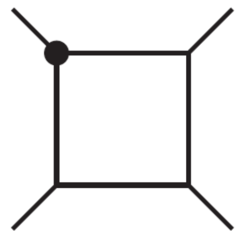


[Bern, Cheung, Chi, Davies, Dixon, JN (to appear)]

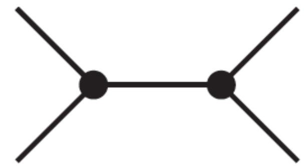
- We now consider the bare result with the single- and double- Gauss-Bonnet counterterm insertions in D=4-2ε



Bare

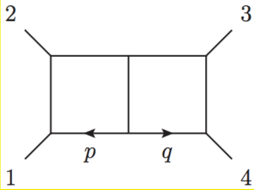


Single GB



Double GB

Two-Loop Gravity



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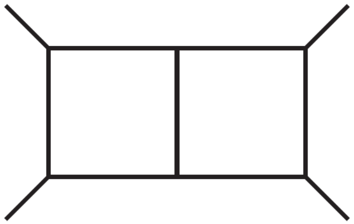
- We now consider the **bare** result with the **single-** and **double-Gauss-Bonnet** counterterm insertions in $D=4-2\varepsilon$



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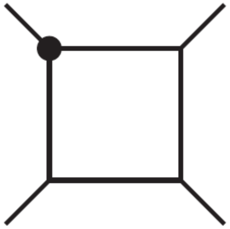

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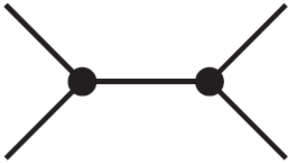


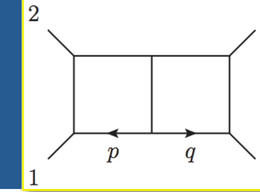
Double GB

$$\mathcal{M}_4^{(2)}\Big|^{\text{bare}}_{\text{UV div.}} = \left(-\frac{3431}{5400}\right)\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} stu\,\mathcal{T}^2$$

$$\mathcal{M}_4^{(1)}\Big|^{\text{GB}}_{\text{UV div.}} = \left(+\frac{689}{675}\right)\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} stu\,\mathcal{T}^2$$

$$\mathcal{M}_4^{(0)}\Big|^{\text{GB}^2}_{\text{UV div.}} = \left(+\frac{5618}{675}\right)\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} stu\,\mathcal{T}^2$$

Two-Loop Gravity



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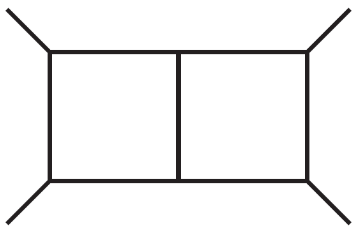

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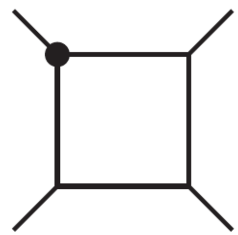

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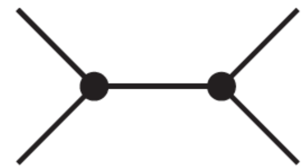

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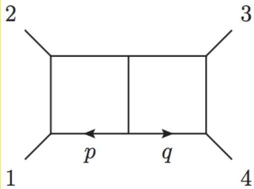
$$\boxed{\mathcal{M}_4^{(2)}\Big|^{\text{full}}_{\text{UV div.}} = \left(+\frac{209}{24}\right)\frac{1}{\epsilon}\left(\frac{\kappa}{2}\right)^6 \frac{i}{(4\pi)^4} stu\,\mathcal{T}^2}$$

Two-Loop Gravity



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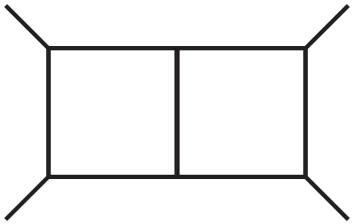

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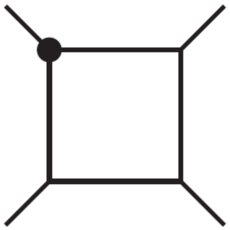
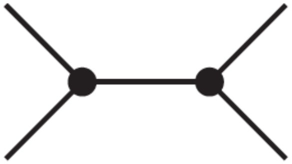


Single GB



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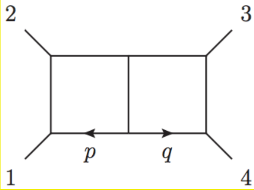
- Matches Goroff & Sagnotti after meaningful subtractions!

Two-Loop Gravity



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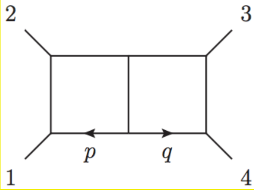
- For other theories, the coefficient of $1/\varepsilon$ is given by

theory	g	gd	ga	gda
bare divergence	$-\frac{3431}{5400}$	$-\frac{793}{1200}$	$\frac{2027}{1200}$	$\frac{83}{2700}$
GB c.t. in 1 loop	$\frac{4\cdot 53}{360}\cdot\frac{2\cdot 13}{15}$	$\frac{4\cdot 53+1}{360}\cdot\frac{2\cdot(13-1)}{15}$	$\frac{4\cdot 53+91}{360}\cdot\frac{2\cdot(13-91)}{15}$	$\frac{4\cdot 53+91+1}{360}\cdot\frac{2\cdot(13-91-1)}{15}$
GB^2 c.t. in tree	$24\left(\frac{4\cdot 53}{360}\right)^2$	$24\left(\frac{4\cdot 53+1}{360}\right)^2$	$24\left(\frac{4\cdot 53+91}{360}\right)^2$	$24\left(\frac{4\cdot 53+91+1}{360}\right)^2$
RHH c.t. in 1 loop	0	0	$\frac{1}{4}\cdot 20$	$\frac{5}{12}\cdot 20$
total	$\frac{209}{24}$	$\frac{139}{16}$	$\frac{239}{16}$	$\frac{199}{12}$

- [Not shown is graviton + n_3 3-forms: total = $209/24 - n_3\,(15/2)$]

Two-Loop Gravity



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GB^2 c.t. in tree	$24\left(\frac{4\cdot 53}{360}\right)^2$	$24\left(\frac{4\cdot 53+1}{360}\right)^2$	$24\left(\frac{4\cdot 53+91}{360}\right)^2$	$24\left(\frac{4\cdot 53+91+1}{360}\right)^2$
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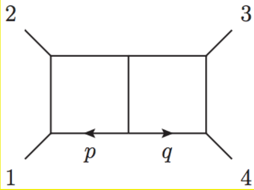
- Again, it appears that dilatons and antisymmetric tensors are quantum-mechanically inequivalent

Two-Loop Gravity



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GB^2 c.t. in tree	$24\left(\frac{4\cdot 53}{360}\right)^2$	$24\left(\frac{4\cdot 53+1}{360}\right)^2$	$24\left(\frac{4\cdot 53+91}{360}\right)^2$	$24\left(\frac{4\cdot 53+91+1}{360}\right)^2$
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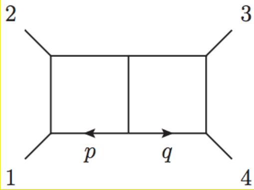
Two-Loop Gravity



[Bern, Cheung, Chi, Davies, Dixon, JN (to appear)]

- For other theories, the coefficient of $1/\varepsilon$ is given by

theory	g	gd	ga	gda
bare divergence	$-\frac{3431}{5400}$	$-\frac{793}{1200}$	$\frac{2027}{1200}$	$\frac{83}{2700}$
GB c.t. in 1 loop	$\frac{4\cdot53}{360}\cdot\frac{2\cdot13}{15}$	$\frac{4\cdot53+1}{360}\cdot\frac{2\cdot(13-1)}{15}$	$\frac{4\cdot53+91}{360}\cdot\frac{2\cdot(13-91)}{15}$	$\frac{4\cdot53+91+1}{360}\cdot\frac{2\cdot(13-91-1)}{15}$
GB^2 c.t. in tree	$24\left(\frac{4\cdot53}{360}\right)^2$	$24\left(\frac{4\cdot53+1}{360}\right)^2$	$24\left(\frac{4\cdot53+91}{360}\right)^2$	$24\left(\frac{4\cdot53+91+1}{360}\right)^2$
RHH c.t. in 1 loop	0	0	$\frac{1}{4}\cdot20$	$\frac{5}{12}\cdot20$
total	$\frac{209}{24}$	$\boxed{\frac{139}{16}}$	$\boxed{\frac{239}{16}}$	$\frac{199}{12}$

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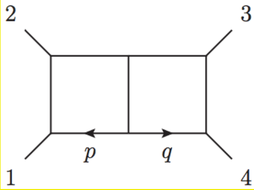
Two-Loop Gravity



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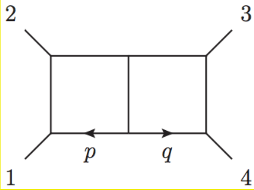
- …look at the finite log pieces as well
  - Pure Gravity:
  - Gravity + 3-Form

Two-Loop Gravity



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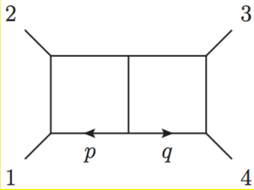

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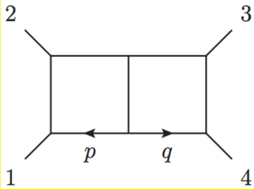

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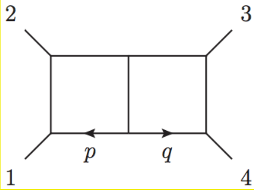

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- Similar story for GD vs. GA theories – same logarithms
  - Quantum equivalence under duality

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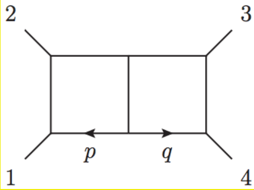

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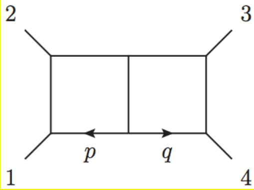

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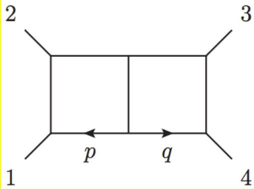

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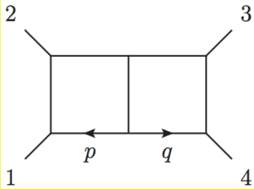

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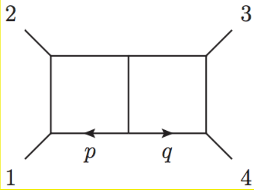

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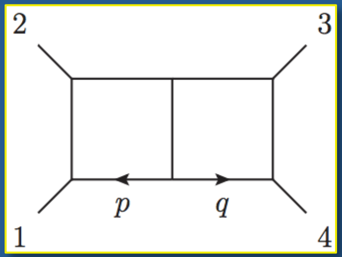
- Another angle of attack:
  - Two-loop integrands in *D*-dimensions with formal polarization

BCJ at Two Loops



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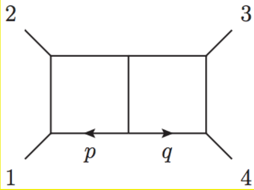
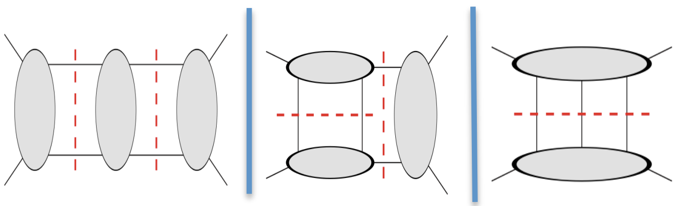
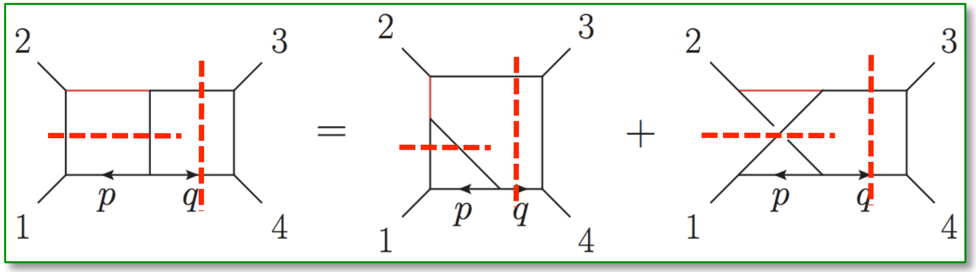


[Bern, Davies, JN (to appear)]

- Constructed Yang-Mills numerators at two loops that obey kinematic Jacobi identities on the spanning set of cuts





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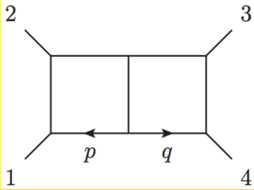

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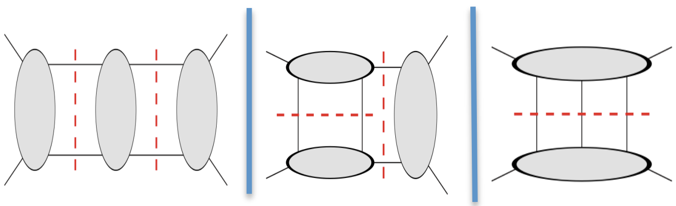

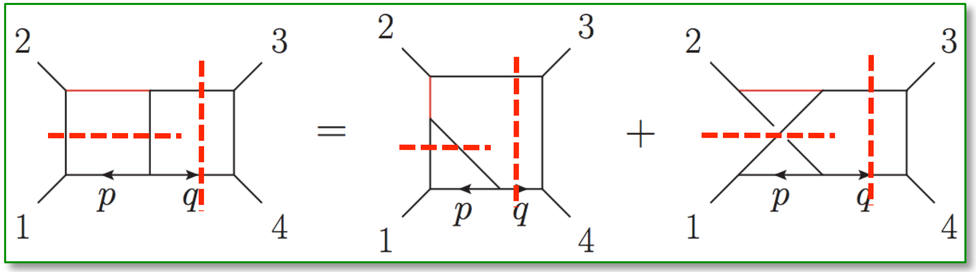

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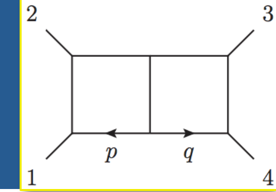

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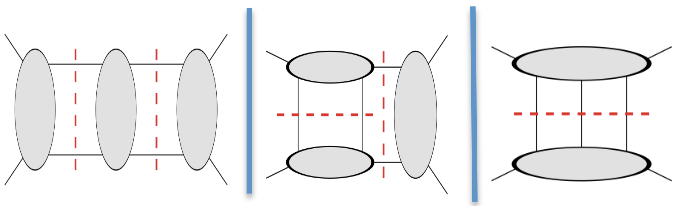

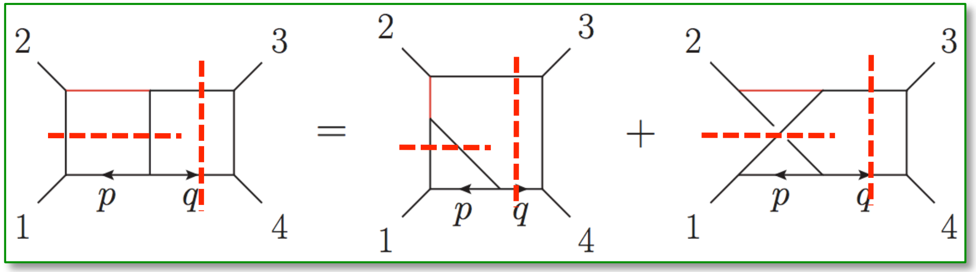

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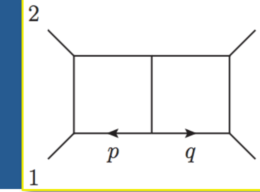

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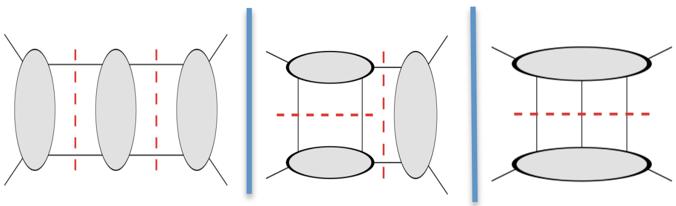

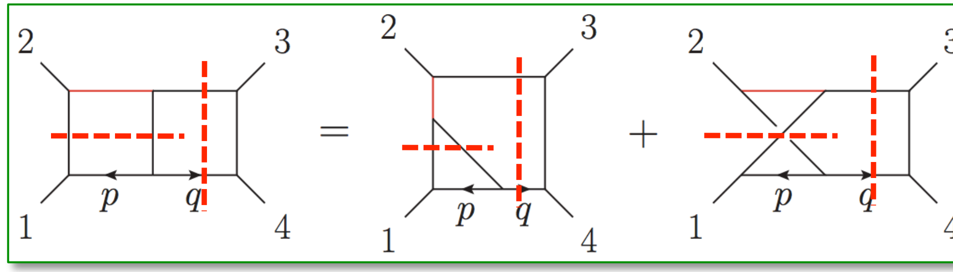

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- Square numerators to get GDA theory numerators

$$\mathcal{A}_m^{L\text{-loop}} = i^L g^{m-2+2L} \sum_{\mathcal{S}_m} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{c_j n_j}{\prod_{\alpha_j} p_{\alpha_j}^2} \quad \Rightarrow \quad \mathcal{M}_m^{L\text{-loop}} = i^{L+1} \left(\frac{\kappa}{2}\right)^{m-2+2L} \sum_{\mathcal{S}_m} \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{\tilde{n}_j n_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

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- Future work:
  - Perform two-loop calculation using formal polarization vectors
    - Closer to Goroff & Sagnotti, avoids FDH scheme and more helicity configurations
    - Already found BCJ numerators for GDA theory