Z+jet at next-to-next-to-leading order

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Motivation - Why $Z+$jet?

- An important background for beyond the standard model searches.
- Very precise measurements can be obtained.
  - Provides a fantastic testing ground for precision QCD and electroweak corrections
  - Useful for detector calibration, jet energy scale can be determined from the recoil of the jet against the $Z$ boson.
- Useful process for PDF determination (particularly gluon distributions).

from Matthias Weber’s talk on Monday
Overview of current calculations

- Calculation of Z+jet including Next-to-Leading Order (NLO) known for a long time - Giele, Glover, Kosower [1993] & Campbell, Ellis [2002]
- Electroweak radiative corrections calculated - Denner, Dittmaier, Tasprzik, Mück [2010]
- NLO calculation matched to parton shower - Alioli, Nason, Oleri, Re [2010]
**Theoretical Challenges**

$Z+\text{jet}$ has a very rich set of partonic channels that contribute to it. At NLO for a given set of cuts and scale choice,

<table>
<thead>
<tr>
<th>Initial state</th>
<th>$\sigma$ (pb)</th>
<th>% contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qg$</td>
<td>80.2</td>
<td>55.6%</td>
</tr>
<tr>
<td>$q\bar{q}$</td>
<td>33.1</td>
<td>22.9%</td>
</tr>
<tr>
<td>$\bar{q}g$</td>
<td>33.1</td>
<td>22.9%</td>
</tr>
<tr>
<td>$gg$</td>
<td>-4.0</td>
<td>-2.7%</td>
</tr>
<tr>
<td>$qq$</td>
<td>1.8</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\bar{q}\bar{q}$</td>
<td>0.1</td>
<td>0.1%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>144.3</strong></td>
<td><strong>100.0%</strong></td>
</tr>
</tbody>
</table>

- We present preliminary results for channels in blue calculated to NNLO accuracy.
- Work is on going for all the subdominant channels.
Multi-channel NNLO calculations

For the calculation to be used in phenomenology we need to calculate many channels at the same time, preferably full channel. This poses several problems,

- code must be **numerically efficient**
- book-keeping numerous channels.

Several options are now on the market for calculations involving jet observables at proton colliders.

- Antenna subtraction
- Sector improved decomposition
- N-jettiness subtraction

We present a calculation of Z+jet at NNLO using **antenna subtraction**.
Antenna subtraction in a nutshell

Exploit the universal factorisation of QCD in Infrared (IR) limits. We pick simple processes and recycle their pole structures for use in more complicated processes.

\[ M^0_3(1_q, i_g, 2_{\bar{q}}) \overset{i_g}{\rightarrow} \text{Antenna function} \]

\[ A^0_3(1_q, i_g, 2_{\bar{q}}) \equiv \frac{M^0_2(\hat{1}_q, \hat{i}_{2\bar{q}})}{M^0_2((1\bar{q}), (i2_{\bar{q}}))}. \]

- We need to define a phase space mapping from the \( n + 1 \rightarrow n \) phase space.
- The processes need to be suitably simple so that they can be integrated analytically.
Antenna subtraction in a nutshell

Three separate integrals,

\[ d\hat{\sigma}^{\text{NNLO}} = \int d\sigma_{n+2} (d\hat{\sigma}^{RR,\text{NNLO}} - d\hat{\sigma}^{S,\text{NNLO}}) \]

\[ + \int d\sigma_{n+1} (d\hat{\sigma}^{RV,\text{NNLO}} - d\hat{\sigma}^{T,\text{NNLO}}) \]

\[ + \int d\sigma_n (d\hat{\sigma}^{VV,\text{NNLO}} - d\hat{\sigma}^{U,\text{NNLO}}) \]

Each bracket is IR-finite.
Types of Antenna at NNLO

single unresolved tree level emission

$$X_3^0(1, 2, 3)$$

Real-Real double unresolved emission

$$X_4^0(1, 2, 3, 4)$$

Real-Virtual single unresolved emission at one loop

$$X_3^1(1, 2, 3)$$

- Each antenna contain many limits
- The antenna used are dependent on the flavour of the partons within the process and the initial state configuration.
- All antenna are integrated analytically.
Testing our subtraction

- Define a ratio of the matrix element against the subtraction term

\[ R = \frac{d\sigma^M}{d\sigma^S} \]

- In all unresolved limits \( R \to 1 \)

- Feed in all possible unresolved limits into the matrix element and subtraction term.
Cuts and physical setup

- Included $qg$, $q\bar{q}$ and $\bar{q}g$ processes including most subleading colour contributions.
- Comparisons to LO and NLO data are all full channel at all colour levels.

- Computation for 8TeV LHC using NNPDF2.3 set, $\alpha_s(M_Z) = 0.118$
- Anti-$k_T$ jet clustering algorithm with $R=0.5$, $p_T^{\text{jet}} > 30$ GeV and $|\eta^{\text{jet}}| < 3$
- $80$ GeV $< m_{ll} < 100$ GeV and $|\eta^{l}| < 5$
- Central scale $\mu_R = \mu_F = M_Z$ with scale variation between $0.5M_Z$ and $2M_Z$
Calculation time

<table>
<thead>
<tr>
<th></th>
<th>No. jobs</th>
<th>CPU time per job</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real-Real</td>
<td>1000</td>
<td>12 hours</td>
</tr>
<tr>
<td>Real-Virtual</td>
<td>300</td>
<td>15 hours</td>
</tr>
<tr>
<td>Virtual-Virtual</td>
<td>1</td>
<td>24 hours</td>
</tr>
</tbody>
</table>

- Warmup is now performed using a multi-threaded setup using openMP, significantly reducing the calculation time.
- All calculations done using double precision.
- Multiple scale choices calculated simultaneously.
Total cross section breakdown

**PRELIMINARY**

\[ \sigma_{\text{LO}} = 103.6^{+7.7}_{-7.5} \text{ pb} \]

\[ \sigma_{\text{NLO}} = 147.6^{+9.5}_{-7.6} \text{ pb} \]

\[ \sigma_{\text{NNLO}}(qg + \bar{q}g + q\bar{q}) = 157.6^{+3.4}_{-1.7} \text{ pb} \]
Excellent convergence of NNLO in the high \( p_T \) tail of the distribution.

Significant reduction in the scale uncertainty.
NNLO corrections uniform in rapidity, approximately 7%.
Significant reduction in scale uncertainty.
A qualitative comparison of the $P_T^Z$ distribution against the $P_T^W$ distribution from the $W+$jet calculation at NNLO.

Observe a similar shape to the corrections.
We see a separation between the NNLO and NLO results in the central $\eta$ region.
We see a separation between the NNLO and NLO results in the central $\eta$ region.

Zoom in on this transition region.
• Scale uncertainty drops in the central region
Comparing the full NLO result to a partial NLO result where we have removed the $gg$, $qq$ and $\bar{q}\bar{q}$ channels.

- Distribution is distorted across the whole central region
- $gg$ is a new channel at NLO, scale uncertainty is therefore larger than existing channels
Overshoot at low $p_T$ - missing $gg$ channel
Small undershoot at very high $p_T$ - missing $qq$ channel
Moral of the story

- Even channels that give a small contribution to the total cross section can have a huge impact on the differential distributions.
- Only by doing the full calculation can we determine the importance of channels at a given order.
Conclusions and Outlook

- Antenna subtraction is a powerful subtraction scheme that extracts the infrared singularities **analytically** and enables the pole cancellation to be verified **analytically**.

- All of the relevant antenna are available in unintegrated and integrated form, enabling the systematic evaluation of NNLO corrections to a variety of processes relevant to the LHC.

- We present preliminary results for Z+jet production at NNLO including the most dominant channels ($qg$, $\bar{q}g$ and $q\bar{q}$).

- We get an excellent scale reduction compared to NLO calculations both in the total cross section and in differential distributions.

- Subdominant channels will be calculated in due course.

- We expect the $gg$ channel to have an impact on the $\eta^Z$ distribution in the central region. Exactly how much of an impact requires a full calculation.