Generalised Resummation for QCD Jet Observables

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Based on work done with:
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and ongoing work

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Outline

- All-order perturbation theory in the Sudakov regime
- Practical relevance of resummation for QCD jet observables
- Factorisation theorems and resummation
- Devising a general technique at NNLL without a factorisation theorem
    - Observable's properties
    - Amplitudes
    - Relevant phase space regions
- Automation for processes with two Born coloured legs
- Conclusions and perspectives

Fixed order QCD and resummation

- High energy strong interaction can be very well described by perturbative QCD (PT) through a power series in the (small) coupling constant (fixed order approximation)

- Each radiative emission is associated with an extra power of the coupling, evaluated at a scale of the order of the emission's transverse momentum

$$\alpha_s(k_t^2) \sim \frac{\alpha_s(Q^2)}{1 - \alpha_s(Q^2)\beta_0 \ln \frac{Q^2}{k_t^2}}$$

- If the transverse momentum of the QCD radiation is constrained to be small, an arbitrary amount of QCD emissions become equally important - need for a all-orders description (large logs)

- Additional (double and single) large logarithms L of kinematical origin appear as a left-over of the real-virtual cancellation of IRC divergences

Fixed order QCD and resummation

- In the perturbative regime these logarithms can become as large as (breakdown of the PT below this limit)

$$L \sim \frac{1}{\alpha_s}$$

- This makes "higher order" corrections as large as leading order ones, i.e. $(\alpha_s L)^n L \sim \alpha_s L^2$

- The PT series breaks down and the probability of the reaction diverges logarithmically in the large L limit instead of being suppressed

- The resummation of the large logarithms to all perturbative orders restores the correct physical (Sudakov) suppression and rescues the predictive power of perturbation theory

Logarithmic accuracy

- Double logarithms due radiation kinematics commonly happen to exponentiate exactly (see later)
    - non-exponentiating observables are avoided because of issues with the simulation in event generators, e.g. JADE algorithm
- For such observables we can define a new perturbative order by expressing the cross section as an exponential function

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\rm Born}} \frac{d\sigma}{dv'} dv' \sim e^{\overbrace{\alpha_s^n L^{n+1}}^{\rm LL} + \overbrace{\alpha_s^n L^n}^{\rm NLL} + \overbrace{\alpha_s^n L^{n-1}}^{\rm NNLL} + \dots}$$

- In the region where $L \sim 1/\alpha_s(Q)$, LL are enhanced w.r.t. the Born, NLL are as large as the Born cross section itself, NNLL count as NLO corrections, and so on

Are these regimes of interest ?

- The large-logarithms (Sudakov) regime is probed in several situations whenever tight phase space cuts are applied in the definition of the physical observables (e.g. event-shapes, jet rates, definition of fiducial/control regions for signal or background).

- Phenomenological interests:
  - fit of the strong coupling constant
  - precise simulation of background/signal
  - tuning/developing Monte Carlo event generators
  - design of better-behaved observables (e.g. substructure)

- Theoretical interests:
  - properties of the QCD radiation to all-orders
  - understanding of IRC singular structure (subtraction)
  - unveiling perturbative scalings in the deep IRC region
  - probing the boundary with the non-perturbative regime, and study of non-perturbative dynamics

Glimpses of automation

- Several NLL resummations exist for a number of observables at lepton-lepton (hadron) and hadron-hadron colliders (long literature)

- Automation at NLL for rIRC safe observables in e+e- and hadronic collisions (CAESAR)

[Banfi, Salam, Zanderighi (2001-2010)]

- NNLL corrections are generally sizeable (count as NLO) and important for precision physics - few NNLL results exist for 2 scale observables in e+e- collisions and even fewer for hadronic collisions

- Automation for two hard Born legs at NNLL (ARES)

[Banfi, McAslan, Monni, Zanderighi (2014)]

- Leading jet's transverse momentum distribution in colour singlet production at the LHC automated in MadGraph5_MC@NLO (general radiation structure initially established in Higgs production, one-loop virtual and luminosity required as an input)

[Becher, Frederix, Neubert, Rothen (2014)]

Finding a general pattern

- A generic observable in a hadronic process has a complex logarithmic structure (e.g. non-global, multi-legs, multiple sources of large logs)
- Can we understand the logarithmic structure of a big class of observables at once ?
- So far NNLL resummation relies on our ability to factorise (see later) the observable in some (smartly defined) conjugate space - resummation often leads to very tedious calculations (~ 14-16 yrs to go from NLL to the next order)
- GOAL: devise a (numerical) resummation approach that:

  [Banfi, McAslan, Monni, Zanderighi]

  - does not rely on factorisation properties of the observable
  - is NNLL accurate and extendable to higher orders
  - is fully general for a very broad category of observables (~all that we can currently resum to NNLL)
  - it is suitable for automation (only input: observable's routine)

Finding a general pattern

- Describe the all-orders QCD radiation to a given logarithmic accuracy in a (as much as) general way

- To achieve that, we need to study the behaviour of both the squared amplitude and the observable in the presence of an arbitrary number of emissions

- We divide the problem (and its solution) in three parts

    - Observable’s properties

    - Amplitudes in the logarithmic regime

    - Relevant phase space regions

Observable's properties to all orders

- We consider an Infrared and Collinear (IRC) safe observable in the Born kinematics (e.g. two-jet event)

- In this limit the radiative corrections are described exclusively by virtual corrections, and collinear and/or soft real emissions - amplitudes must factorise at all orders in these regimes w.r.t. the Born up to regular terms

$$|\mathcal{M}(\{\tilde{p}\}, k_1, ..., k_n)|^2 \simeq |M_{\rm Born}(\{\tilde{p}\})|^2 |M(k_1, ..., k_n)|^2 + \dots$$

Breaking of factorisation found at high orders for specific processes [J. R. Forshaw, A. Kyrieleis, and M. Seymour 2006-2009] [S. Catani, D. de Florian, and G. Rodrigo 2012] [J. R. Forshaw, M. H. Seymour, and A. Siodmok 2012]

- Conventional approach to resummation: set up a factorisation theorem (i.e. scales separation) by factorising the observable's measurement function. Resum large logarithms by means of evolution equations

Observable's properties to all orders

- The observable does not trivially factorise in a product of terms arising from each kinematic mode contributing to the factorised amplitude - it often requires to transform into a conjugate space where the factorisation is explicit (e.g. Mellin - Laplace, Fourier)

  - OK for simple additive cases: e.g. thrust in e+e-

$$1 - T \simeq \sum_{i=1}^{n} \frac{k_{ti}}{Q} e^{-\eta_i} \qquad \rightarrow \qquad \Theta(1 - T < \tau) = \int \frac{d\nu}{2\pi i \nu} e^{\nu\tau} \prod_{i=1}^{n} e^{-\nu \frac{k_{ti}}{Q} e^{-\eta_i}}$$

  - more difficult for involved observables: e.g. jet broadening in e+e- or inclusive vector-boson kt in hadron collisions

  - tough/impossible for observables which mix various kinematic modes or require iterative optimisations: e.g. jet rates, thrust major

- Main idea: factorisation of the measurement function is an unnecessary requirement for resummation. All one needs is some scaling properties of the observable

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Observable's properties to all orders

- The standard requirement of IRC safety implies that the value of the observable does not change in the presence of one or more unresolved emissions (i.e. very soft and/or collinear)

- In addition, we require recursive IRC (rIRC) safety (see backup slides), i.e. [Banfi, Salam, Zanderighi 2004]

  - that in the presence of multiple emissions the observable scales in the same fashion as for a single emission (IRC divergences have an exponential form)

    - this property ensures the exponentiation of leading logarithms and a cancellation of divergences to all orders;

  - that for sufficiently small $\bar{v}$ there exists some ϵ that can be chosen *independently* of $\bar{v}$ such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$

$$V(\{\tilde{p}\}, k_1, \ldots, k_n, \ldots, k_m) \simeq V(\{\tilde{p}\}, k_1, \ldots, k_n) + \epsilon^p v$$

    - this property allows one to associate a logarithmic order to each real n-emission probability, establishing a logarithmic hierarchy between correlated branchings

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We limit ourselves to *continuously* global observables*, i.e. constrain the radiation equally everywhere in the phase space
(it ensures the absence of non-global logarithms)

[Dasgupta, Salam 2001; Banfi, Marchesini, Smye 2002]

*Not a real limitation, however currently the full NNLL structure of non-global logarithms is still unknown

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Virtual and (unresolved) real emissions

- RGE evolution for virtual corrections leads to a complete exponentiation of IRC singularities. To perform the cancellation of IRC poles to all orders, some sort of factorisation of the real corrections is required!

[Parisi (1980); Magnea, Sterman (1990)]

- Solution: consider primary emissions off the Born legs *inclusive* in secondary (gluon) branchings. We introduce a resolution scale ϵ and define a subset of (*unresolved*) emissions, such that $V_{\rm sc}(\{\tilde{p}\}, k_i) < \epsilon v$

[In progress]

- The observable for unresolved emissions takes the simple (and factorising) form

$$\Theta\left(\epsilon v - \max\{V_{\rm sc}(\{\tilde{p}\}, k_1), \ldots, V_{\rm sc}(\{\tilde{p}\}, k_n)\}\right)$$

  - rIRC ensures that these emissions do not contribute significantly to the observable, their contribution factorises at all-orders. Use virtual RGE to cancel IRC singularities

  - the soft-collinear approximation of the observable for unresolved emissions is enough to ensure the cancellation of the IRC poles arising from the virtual corrections (a different choice can be adopted - final result is scheme-invariant)

  - details of the actual observable's scaling for several emissions away from the soft-collinear region and correct treatment of the gluon branchings are introduced at a later stage (next slides)

Virtual and (unresolved) real emissions

- The combination of unresolved real and virtual corrections gives rise to an exponential factor that defines the no-emission probability at scales larger than ϵv

$$P(\mathrm{no-emissions}) \sim e^{-R(\epsilon v)}$$

- Since we're interested in vetoing emissions above v, this can be further expanded as

$$P(\mathrm{no-emissions}) \sim e^{-R(v)-R'(v)\ln\frac{1}{\epsilon}+\frac{1}{2!}R''(v)\ln^2\frac{1}{\epsilon}+\dots}$$

- The factor $R(v)$ is called the *radiator*, and defines the physical region where no radiation is allowed

- The logarithmic dependence on the resolution parameter cancels when all-order resolved real emissions are taken into account (rIRC safety)

- Owing to the above definition of unresolved emissions, the radiator is universal for all observables with the same soft-collinear scaling in the presence of a single emission

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Virtual and (unresolved) real emissions

- The resulting cumulative resummed cross section takes the form

$$\Sigma(v) = \int_0^v \frac{1}{\sigma_{\text{Born}}} \frac{d\sigma}{dv'} dv' \sim e^{-R(v)} \mathcal{F}(v)$$

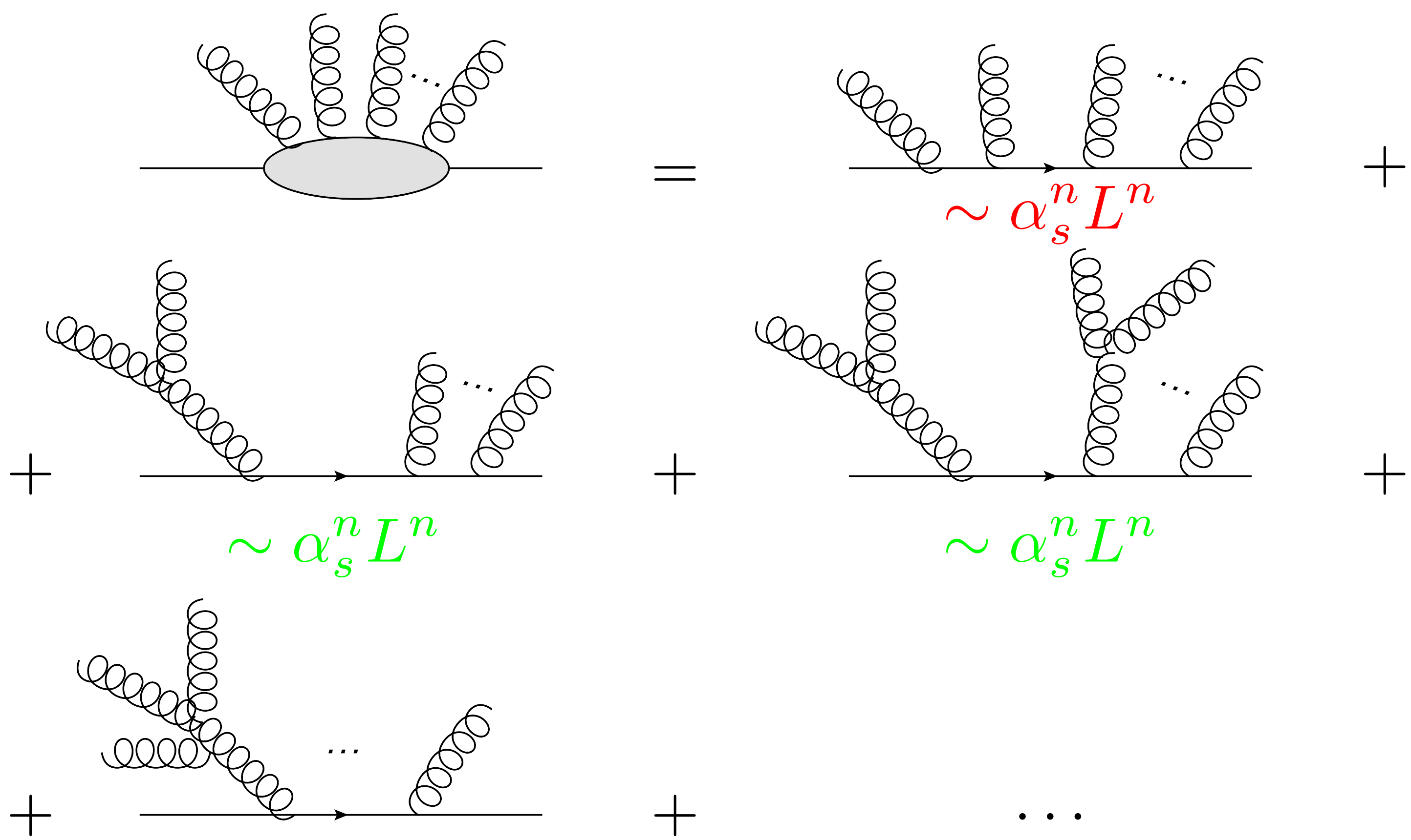
Multiple emissions function

- rIRC safety ensures that all double logarithms are contained in the radiator, and that the multiple emissions function is non trivial (at most) at NLL level

- Because of the lower cutoff defined by the resolution parameter, each *resolved real* emission "loses" one logarithmic power

e.g. soft (resolved) matrix elements

- It is useful to decompose the matrix element for n soft emissions (w.r.t. the Born) as a sum of terms with an increasing number of colour-correlated emissions (i.e. non-abelian contributions)



- Which diagrams do we need to achieve NNLL, i.e. neglect terms of order $\sim \alpha_s^n L^{n-2}$?

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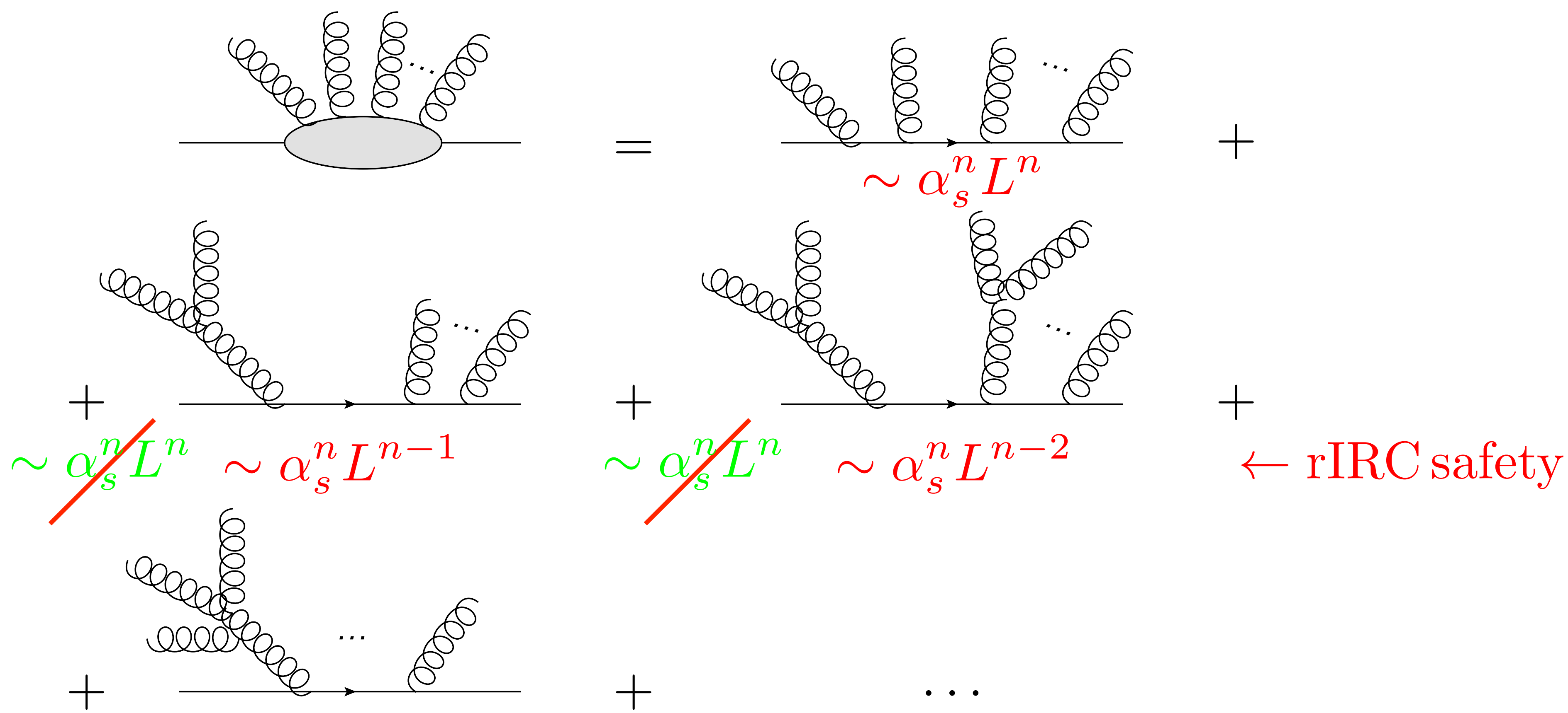

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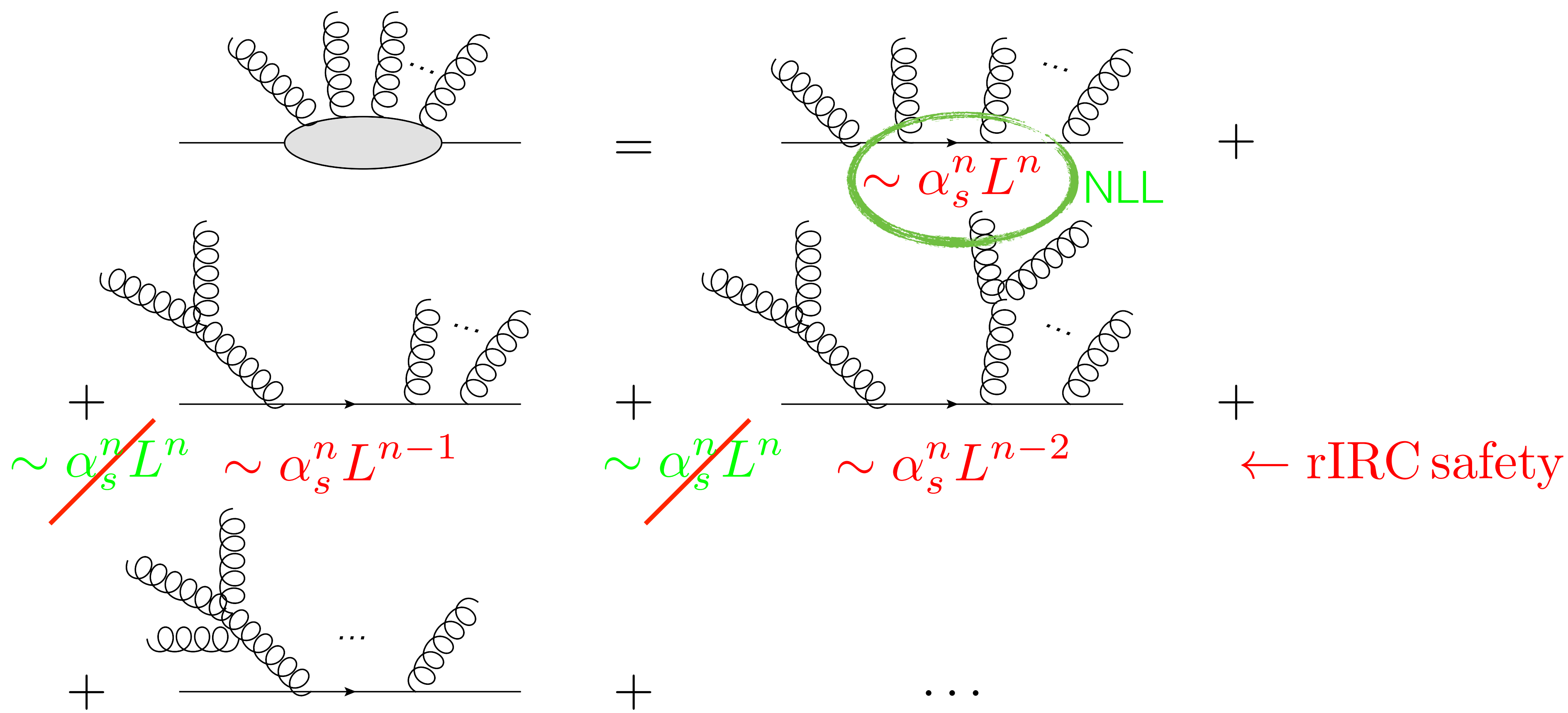

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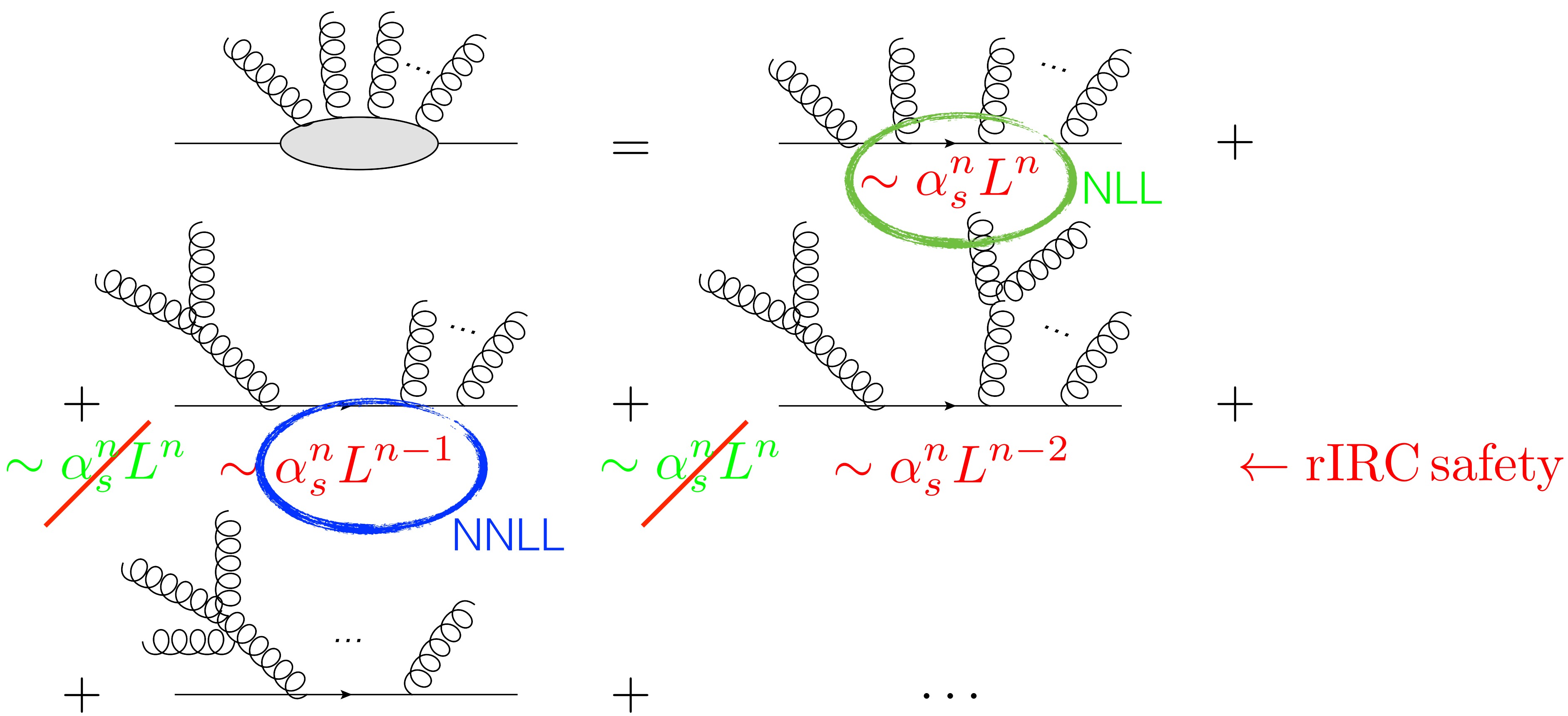

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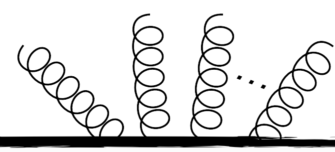
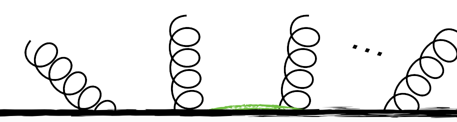




- At NNLL keep allow at most for a single soft gluon branching in the real corrections (correct for the inclusive approximation in the radiator)
- Analogously, allow for one single emission to be emitted in non-soft-collinear (i.e. NLL) regions of the phase space (i.e. wide angle, hard collinear)
- This corresponds to a NLO correction to the NLL cross section
- Subleading terms in the expansion can be included systematically

- Which diagrams do we need to achieve NNLL, i.e. neglect terms of order $\sim \alpha_s^n L^{n-2}$?

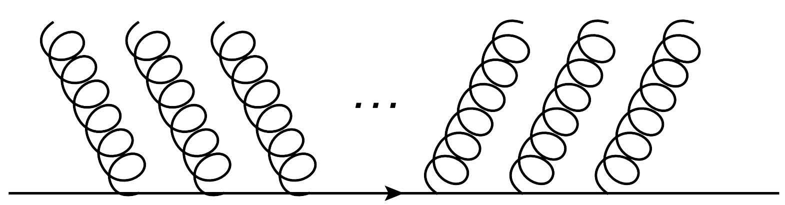
Phase space at NLL

[Banfi, Salam, Zanderighi]

- At NLL the multiple emission function is given by an ensemble of soft and collinear independent (abelian) emissions widely separated in rapidity (coherence)

- Non-abelian effects are completely accounted for in the radiator (i.e. inclusive gluon branching treatment -> CMW scheme)

Measure defined by the soft-collinear ensemble



$$= \int d\mathcal{Z}[\{R'_{\mathrm{NLL},\ell_i}, k_i\}]\,\Theta\left(1 - \lim_{v\to 0} \frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v}\right)$$

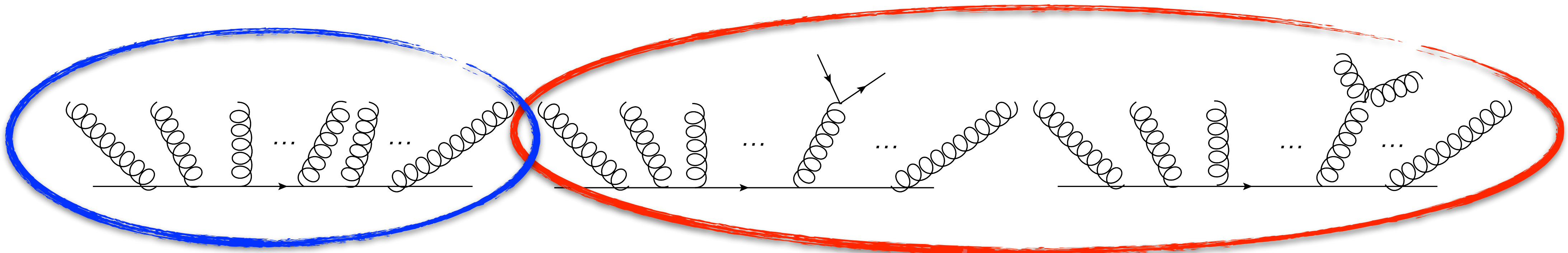
$$\mathcal{F}_{\mathrm{NLL}}(v) = \langle \Theta(1 - \lim_{v\to 0} \frac{V_{\mathrm{sc}}(\{\tilde{p}\},\{k_i\})}{v}) \rangle$$

- Differences in rapidity bounds of different emissions contribute at NNLL (can be neglected at this order)

Phase space at NNLL

[Banfi, McAslan, Monni, Zanderighi]

- Extension to NNLL involves additional kinematic configurations:
  - (at most) two soft-collinear emissions get close in rapidity



Clustering correction
(jet algorithms only)

Correlated corrections

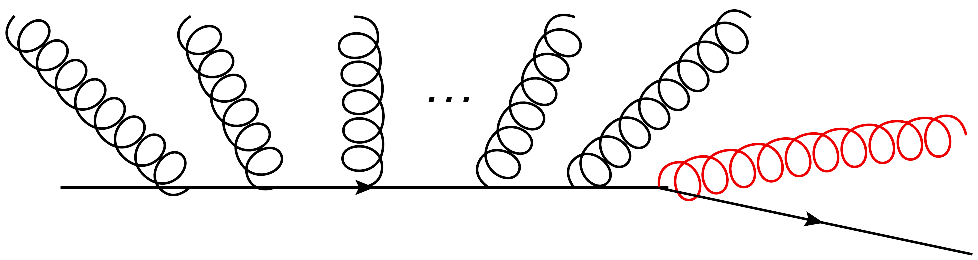
$$\delta\mathcal{F}_{\rm correl}(\lambda) = \int_0^\infty \frac{d\zeta_a}{\zeta_a}\int_0^{2\pi}\frac{d\phi_a}{2\pi}\sum_{\ell_a=1,2}\left(\frac{2C_{\ell_a}\lambda}{\beta_0}\frac{R''_{\ell_a}(v)}{\alpha_s(Q)}\right)\int_0^\infty\frac{d\kappa}{\kappa}\int_{-\infty}^{\infty}d\eta\int_0^{2\pi}\frac{d\phi}{2\pi}\frac{1}{2!}C_{ab}(\kappa,\eta,\phi)\times$$
$$\times\int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i},k_i\}]\left[\Theta\left(v-V_{\rm sc}(\{\tilde{p}\},k_a,k_b,\{k_i\})\right)-\Theta\left(v-V_{\rm sc}(\{\tilde{p}\},k_a+k_b,\{k_i\})\right)\right]$$

$$C_{ab}(\kappa,\eta,\phi) = \frac{\tilde{M}^2(k_a,k_b)}{M^2_{\rm sc}(k_a)M^2_{\rm sc}(k_b)}$$

All corrections in terms of four-dimensional integrals

Phase space at NNLL

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one collinear emission can carry a significant fraction of the energy of the hard emitter (which recoils against it)



  - Corrections affect both matrix element (hc) and observable (rec)

$$\delta\mathcal{F}_{\rm hc}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}] \times$$
$$\times \int_0^1 \frac{dz}{z} \left(z p_\ell(z) - 2C_\ell\right) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, \{k_i\})}{v}\right)\Theta(1-\zeta)\right]$$

$$\delta\mathcal{F}_{\rm rec}(\lambda) = \sum_{\ell=1,2} \frac{\alpha_s(v^{1/(a+b_\ell)}Q)}{\alpha_s(Q)(a+b_\ell)} \int_0^\infty \frac{d\zeta}{\zeta} \int_0^{2\pi} \frac{d\phi}{2\pi} \int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i}, k_i\}] \times$$
$$\times \int_0^1 dz\, p_\ell(z) \left[\Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm hc}^{(k')}(\{\tilde{p}\}, k', \{k_i\})}{v}\right) - \Theta\left(1 - \lim_{v\to 0} \frac{V_{\rm sc}(\{\tilde{p}\}, k, \{k_i\})}{v}\right)\right]$$

Phase space at NNLL**

- Extension to NNLL involves additional kinematic configurations:
  - (at most) one soft-collinear emission has the correct rapidity bounds (approximated in the NLL ensemble)

$$\delta\mathcal{F}_{\rm sc}(\lambda) = \frac{\pi}{\alpha_s(Q)}\int_0^\infty \frac{d\zeta}{\zeta}\int_0^{2\pi}\frac{d\phi}{2\pi}\sum_{\ell=1,2}\left(\delta R'_{\rm NNLL,\ell} + R''_{\ell_i}\ln\frac{d_\ell g_\ell(\phi)}{\zeta}\right)\int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i},k_i\}]\times$$
$$\times\left[\Theta\left(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde p\},k,\{k_i\})}{v}\right)-\Theta(1-\zeta)\Theta\left(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde p\},\{k_i\})}{v}\right)\right],$$

  - (at most) one soft emission can have very small rapidity (wide angle)

$$\delta\mathcal{F}_{\rm wa}(\lambda) = \frac{2C_F}{a}\frac{\alpha_s(v^{1/a}Q)}{\alpha_s(Q)}\int_0^\infty\frac{d\zeta}{\zeta}\int_{-\infty}^{\infty}d\eta\int_0^{2\pi}\frac{d\phi}{2\pi}\int d\mathcal{Z}[\{R'_{{\rm NLL},\ell_i},k_i\}]$$
$$\times\left[\Theta\left(1-\lim_{v\to 0}\frac{V_{\rm wa}^{(k)}(\{\tilde p\},k,\{k_i\})}{v}\right)-\Theta\left(1-\lim_{v\to 0}\frac{V_{\rm sc}(\{\tilde p\},k,\{k_i\})}{v}\right)\right]$$

**With more than two coloured Born legs there are additional (NLL) contributions due to the coherent interference between hard legs

Application to processes with 2 Born legs

[Banfi, McAslan, Monni, Zanderighi]

- Observables with very different logarithmic structure can be modelled with the same method

NEW

correction type	$p_{t,\text{veto}}$	$1-T$	B_T	B_W	C	ρ_H	T_M	O	$y_3^{\text{Dur.}}$	$y_3^{\text{Cam.}}$
$\mathcal{F}_{\text{NLL}}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\delta\mathcal{F}_{\text{rap}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x
$\delta\mathcal{F}_{\text{wa}}$	x	x	x	x	✓	x	x	x	✓	✓
$\delta\mathcal{F}_{\text{hc}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x
$\delta\mathcal{F}_{\text{rec}}$	x	✓	✓	✓	✓	✓	✓	✓	✓	x
$\delta\mathcal{F}_{\text{clust}}$	✓	x	x	x	x	x	x	x	✓	✓
$\delta\mathcal{F}_{\text{correl}}$	✓	x	✓	✓	x	x	✓	✓	✓	✓

- Reproduce existing results in the literature up to NNLL:

[Banfi, Monni, Salam, Zanderighi] [Becher, Schwartz] [Becher, Bell] [Chien, Schwartz] [Becher, Neubert, Rothen] [Gehrmann, Luisoni, Monni] [Hoang, Kolodrubetz, Mateu, Stewart] [Stewart, Tackmann, Walsh, Zuberi]

- Obtain new results for involved observables (no factorisation theorem)

e.g. Thrust major: $T_M \equiv \max_{\vec{n}\cdot\vec{n}_T=0} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q}$

The two-jet rate in e+e-

- The three-jet resolution parameter defines the maximum value of $y_{\rm cut}$ that leads to two QCD jets in the final state

- Clustering is performed according to a jet algorithm defined by a ordering variable v_{ij} and a test variable y_{ij}

  - Durham kt clustering:

$$y_{ij}^{(D)} = v_{ij}^{(D)} = 2\frac{\min\{E_i, E_j\}^2}{Q^2}(1-\cos\theta_{ij})$$

  - Cambridge kt clustering:

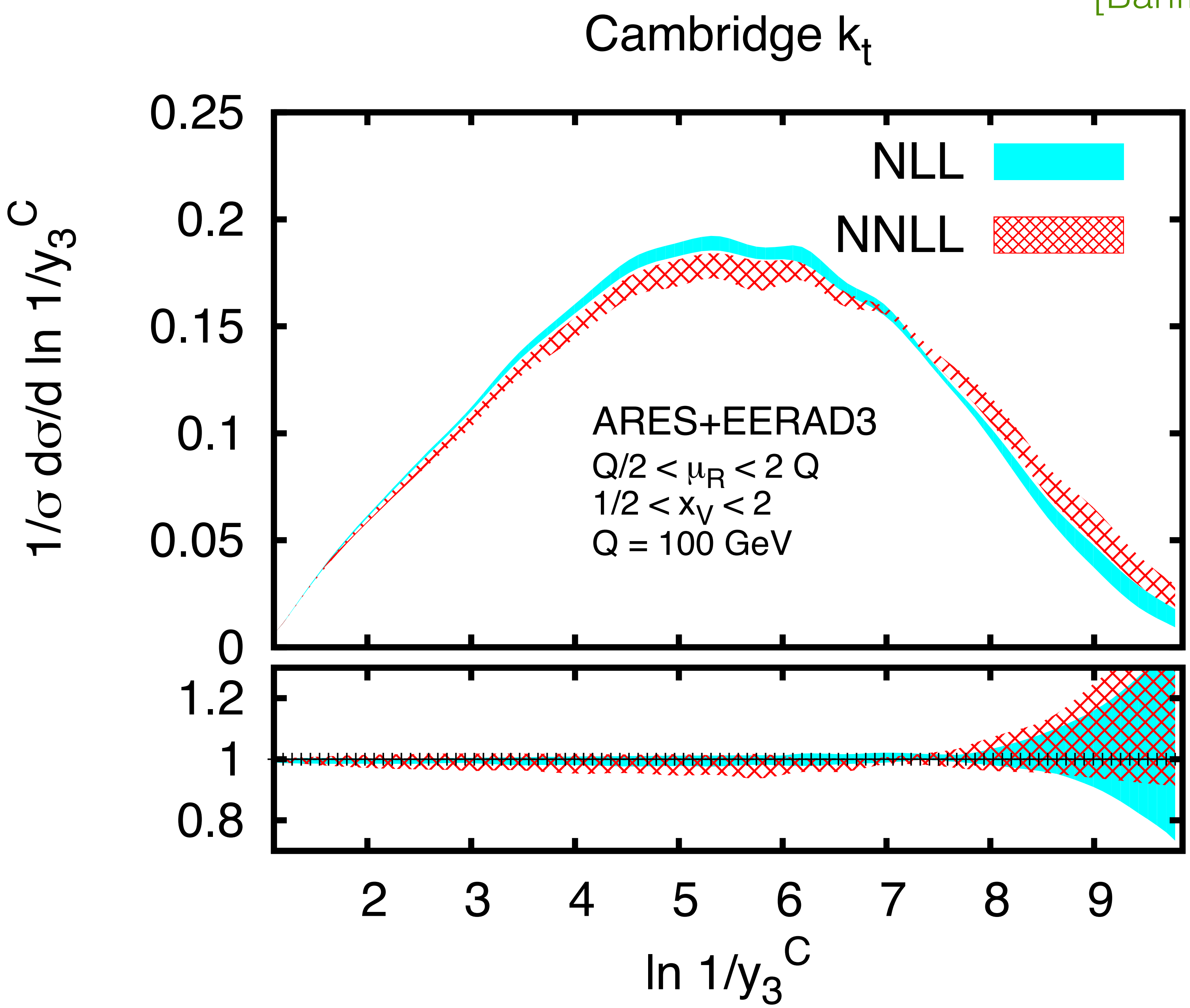
$$y_{ij}^{(C)} = y_{ij}^{(D)}\,, \qquad v_{ij}^{(C)} = 2\,(1-\cos\theta_{ij})\,, \quad + \text{ soft freezing}$$

- Generally complex logarithmic structure (all corrections can be non trivial)

- Clustering of emissions far in rapidity (e.g. soft and hard-collinear) is allowed - possible issues with factorisation

The two-jet rate in e+e-**

[Banfi, McAslan, Monni, Zanderighi]



- The Cambridge algorithm gets sensitive to multiple emissions at NNLL: sizeable corrections (uncertainties underestimated at NLL)

- Checks ongoing for Durham algorithm - more involved structure

** Preliminary plot

Conclusions

- Novel general method for the resummation of any rIRC safe, global two-scales observable at NNLL order

    - rIRC safety as only applicability condition

    - Contribution of resolved real radiation formulated in terms of four dimensional integrals, which is suitable for efficient numerical implementation, currently automated for final-state radiation (ARES)

    - NNLL corrections systematically derived, each correction has clear physical interpretation. Method extendable to higher orders

- Formulation in a Parton Shower framework - important hints on what's needed for a NNLL parton shower

- Modulo technical work, NNLL resummation for this (broad) class of observables in processes with 2 coloured Born legs is a theoretically solved problem

Conclusions

- Novel general method for the resummation of any rIRC safe, global two-scales observable at NNLL order
    - rIRC safety as only applicability condition
    - Contribution of resolved real radiation formulated in terms of four dimensional integrals, which is suitable for efficient numerical implementation, currently automated for final-state radiation (ARES)
    - NNLL corrections systematically derived, each correction has clear physical interpretation. Method extendable to higher orders
- Formulation in a Parton Shower framework - important hints on what's needed for a NNLL parton shower
- Modulo technical work, NNLL resummation for this (broad) class of observables in processes with 2 coloured Born legs is a theoretically solved problem

Outlook

- A number of problems need to be attacked in order to treat the most general resummable observable

- Extension to more than two hard legs requires a parametrisation of QCD interference between different hard emitters (trivial in the 2-legs case)
  - simple at NLL

    [Botts, Sterman (1989)]
    [Sterman et al. (1996 - 2000)]
    [Banfi, Marchesini, et al. (2000 - 2002)]
    [Bonciani, Catani, Mangano, Nason (2003)]
  - slightly more technical at NNLL, but not overly complex (singular structure for multi-jet amplitudes known)

    [Catani; Dixon, Magnea, Sterman; Gardi, Magnea; Becher, Neubert]

- Many jet observables (e.g. > 0 jet rates) at the LHC defined in a non-global way
  - solution for NNLL non-global logarithms

    recent progress in [Caron-Huot (2015)]. See also [Larkoski, Moult, Neill (2015)]

- Some observables contain more sources of large logarithms at once (e.g. H +1 jet rate; small-R effects in H+0 jet rate)

  see e.g. [Liu, Petriello (2012)][Dasgupta, Dreyer, Salam, Soyez (2014)]
  - not clear how to resum them simultaneously at NNLL in a general way

Thank you for your attention

Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\; \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- The standard requirement of IRC safety implies that

$$\lim_{\zeta_{m+1}\to 0} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m), \kappa_{m+1}(\bar{v}\zeta_{m+1})) = V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m))$$

$$\lim_{\mu\to 0} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m)) = V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m))$$

- We limit ourselves to *continuously* global observables*, i.e. the transverse momentum dependence is the same everywhere (it ensures the absence of non-global logarithms)

*Not a real limitation, although currently NNLL structure of non-global logarithms unknown

Requirements on the observable

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$$V(\{\tilde{p}\}, \kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia}, \kappa_{ib}\}(\zeta, \mu)\,,\ \mu^2 = (\kappa_{ia} + \kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety [Banfi, Salam, Zanderighi]

$$\lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (1)$$

  - The above limit must be well defined and non-zero (except possibly in a phase space region of zero measure)
  - Condition (1) simply requires the observable to scale in the same fashion for multiple emissions as for a single emission (IRC divergences have an exponential form)
  - It is enough to ensure the exponentiation of double logarithms to all orders

Requirements on the observable

- Parametrisation for single emission and collinear splitting

$$V(\{\tilde{p}\},\kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia},\kappa_{ib}\}(\zeta,\mu)\,,\ \mu^2 = (\kappa_{ia}+\kappa_{ib})^2/\kappa_{ti}^2$$

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$$\lim_{\zeta_{m+1}\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_m(\bar{v}\zeta_m),\kappa_{m+1}(\bar{v}\zeta_{m+1}))$$

$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{a})$$

$$\lim_{\mu\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\{\kappa_{ia},\kappa_{ib}\}(\bar{v}\zeta_i,\mu),\ldots\kappa_m(\bar{v}\zeta_m))$$

$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\ldots,\kappa_i(\bar{v}\zeta_i),\ldots\kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$

Requirements on the observable

- Parametrisation for single emission and collinear splitting

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$$\lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (1)$$

$$\lim_{\zeta_{m+1}\to 0} \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m), \kappa_{m+1}(\bar{v}\zeta_{m+1}))$$

$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{a})$$

$$\lim_{\mu\to 0} \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \{\kappa_{ia}, \kappa_{ib}\}(\bar{v}\zeta_i, \mu), \ldots \kappa_m(\bar{v}\zeta_m))$$

$$= \lim_{\bar{v}\to 0} \frac{1}{\bar{v}} V(\{\tilde{p}\}, \kappa_1(\bar{v}\zeta_1), \ldots, \kappa_i(\bar{v}\zeta_i), \ldots \kappa_m(\bar{v}\zeta_m)) \qquad (2.\mathrm{b})$$

Requirements on the observable

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$$V(\{\tilde{p}\},\kappa_i(\zeta_i)) = \zeta_i\,; \qquad \kappa_i(\zeta) \to \{\kappa_{ia},\kappa_{ib}\}(\zeta,\mu)\,,\ \mu^2 = (\kappa_{ia}+\kappa_{ib})^2/\kappa_{ti}^2$$

- Impose the following conditions, known as recursive IRC (rIRC) safety [Banfi, Salam, Zanderighi]

- Conditions (2.a) and (2.b), in addition to plain IRC safety, require that for sufficiently small $\bar{v}$ there exists some ϵ that can be chosen *independently* of $\bar{v}$ such that we can neglect any emissions at scales $\sim \epsilon\bar{v}$

- The order with which one takes the limit is different in fixed-order and resummed calculations, and the final result must not change

$$\lim_{\mu\to 0}\lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\dots,\{\kappa_{ia},\kappa_{ib}\}(\bar{v}\zeta_i,\mu),\dots\kappa_m(\bar{v}\zeta_m))$$
$$= \lim_{\bar{v}\to 0}\frac{1}{\bar{v}}V(\{\tilde{p}\},\kappa_1(\bar{v}\zeta_1),\dots,\kappa_i(\bar{v}\zeta_i),\dots\kappa_m(\bar{v}\zeta_m)) \qquad (2.b)$$