HIGGS PRODUCTION: CALCULATING AT N3LO

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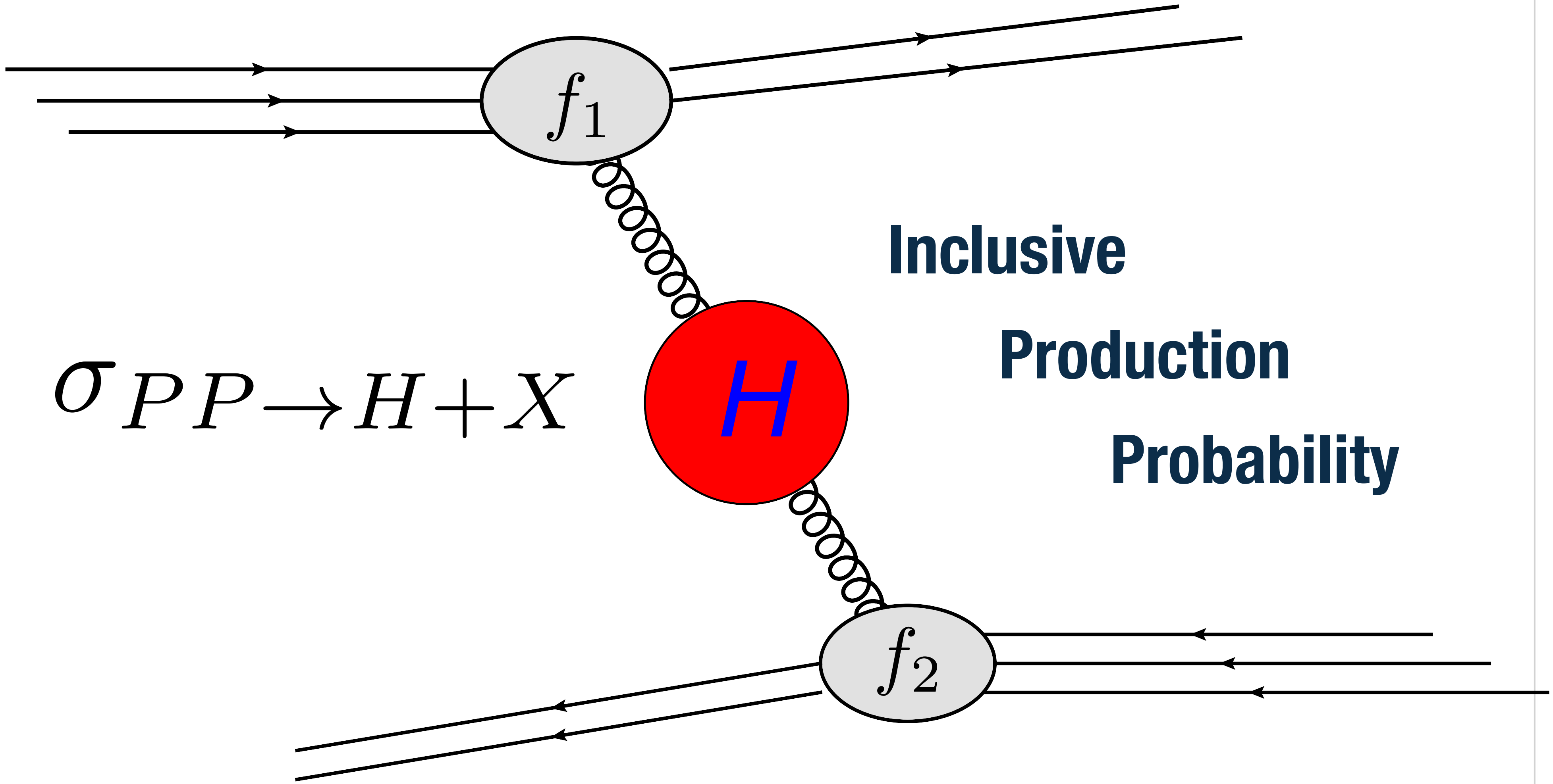
Achilleas Lazopoulos, BM

HIGGS PRODUCTION



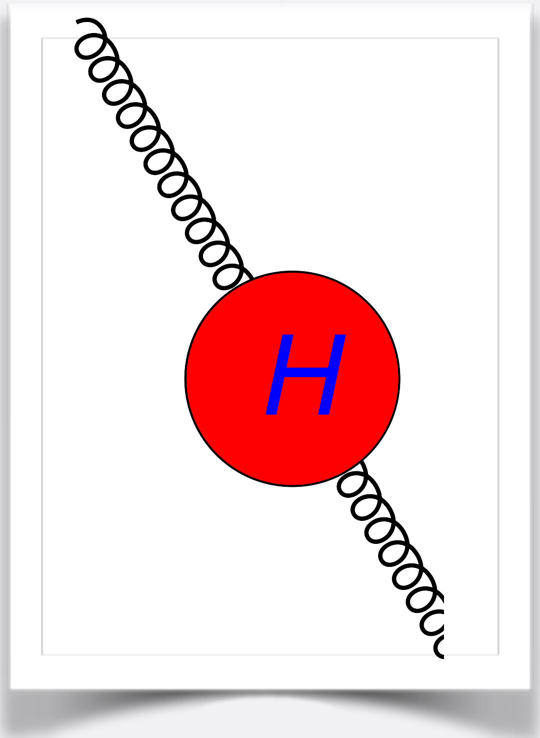
$$\sigma_{PP\to H+X}$$

Inclusive

Production

Probability

HIGGS PRODUCTION

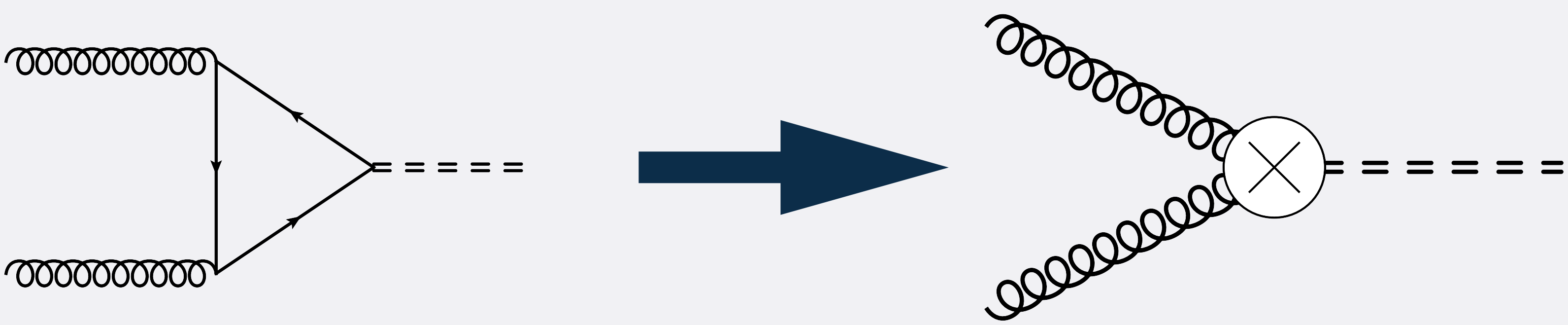


Calculate from perturbative QFT+PDF

$$\sigma_{PP\to H+X} = \int dx_1 dx_2 f_1(x_1) f_2(x_2) \hat{\sigma}(x_1 x_2)$$

QCD dominates at the LHC

Gluon-Fusion



Heavy Top Approximation

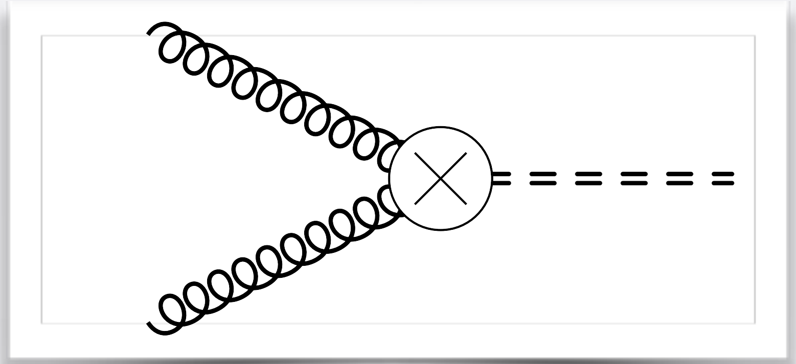
GLUON FUSION

1978 **LO** $\hat{\sigma}^{8TeV} = 9.6pb$ $\pm 25\%$

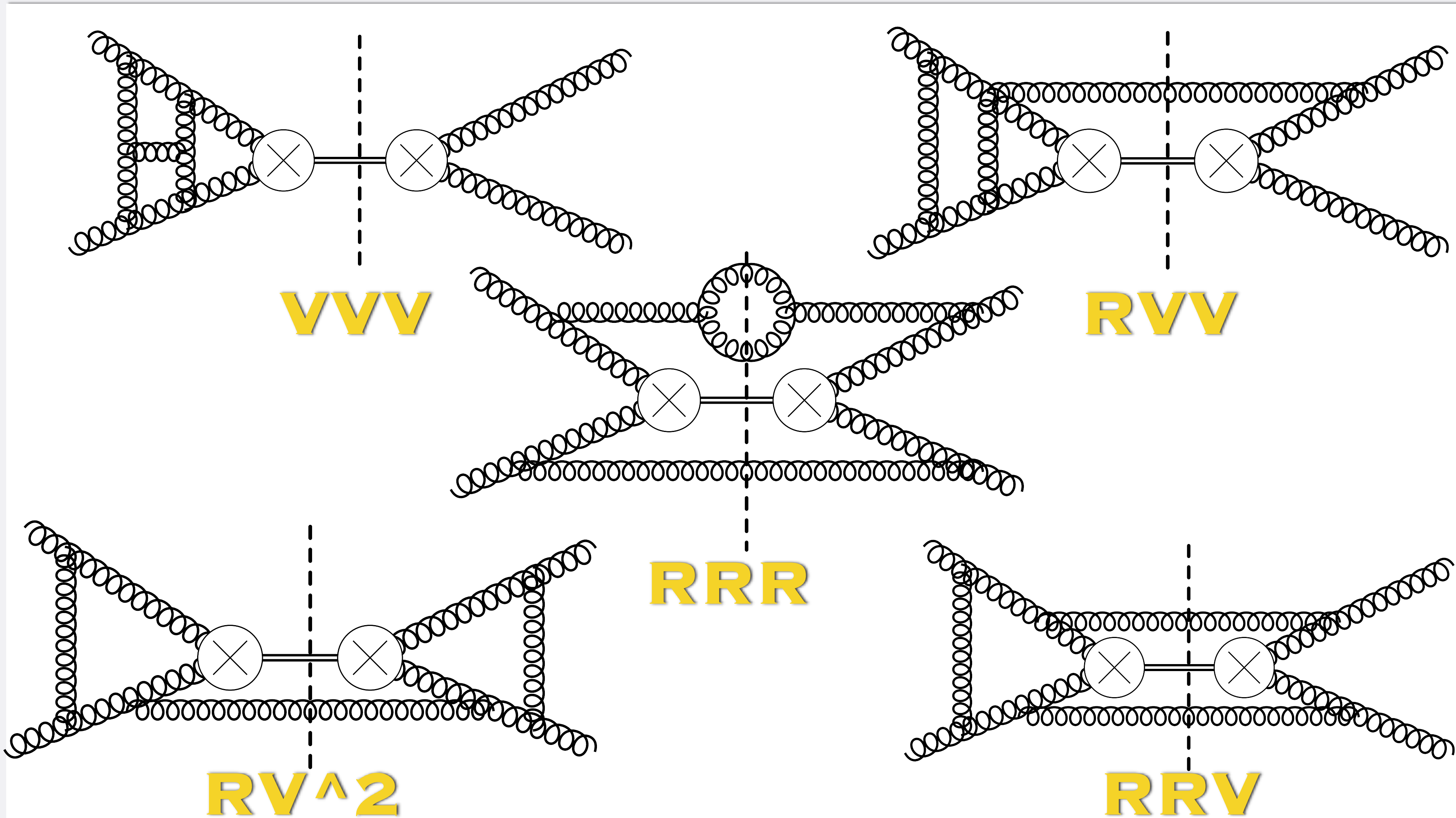
1991 **NLO** $\hat{\sigma}^{8TeV} = 16.7pb$ $\pm 20\%$

2002 **NNLO** $\hat{\sigma}^{8TeV} = 19.6pb$ $\pm 9\%$

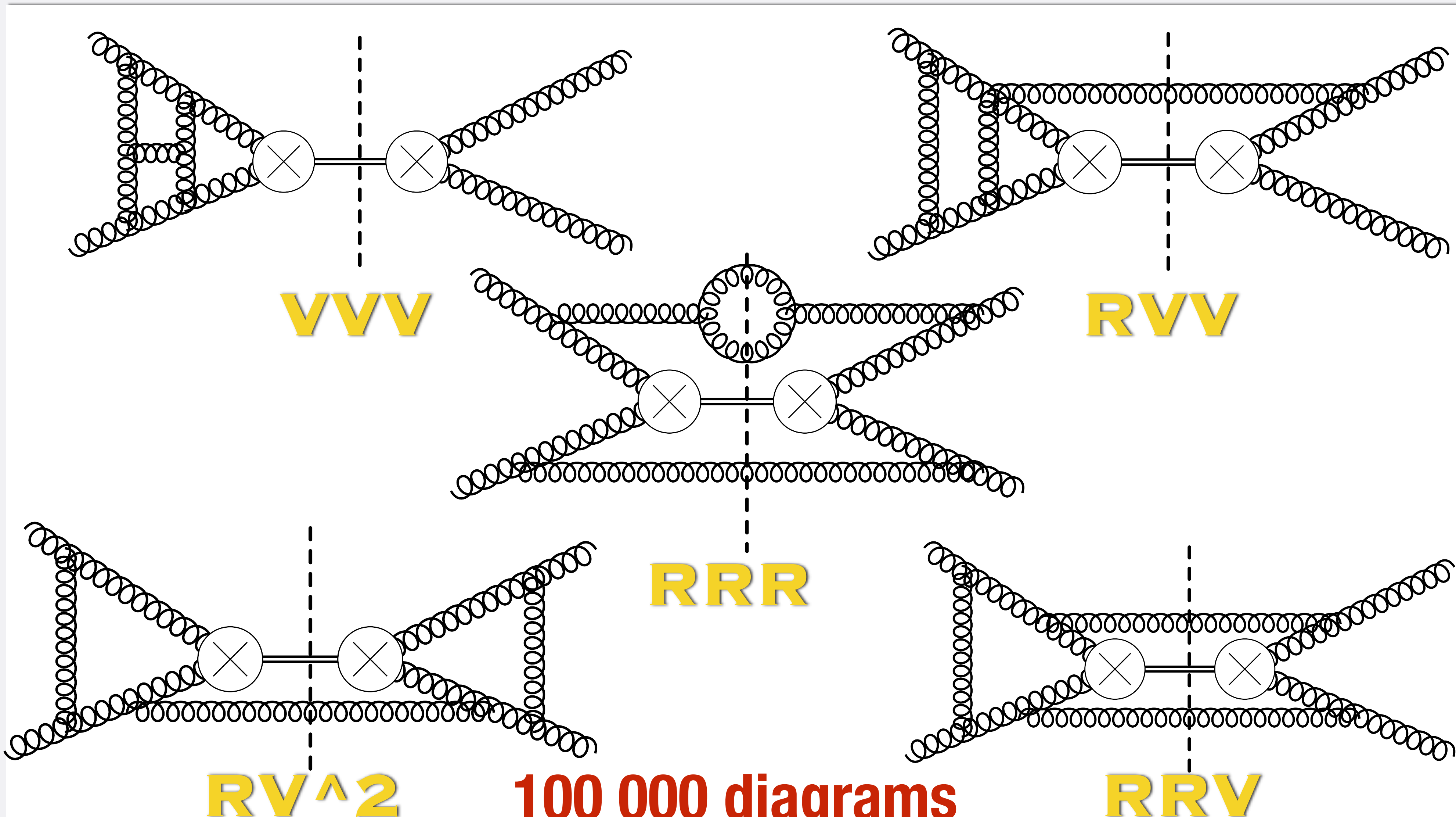
2015 **N3LO**



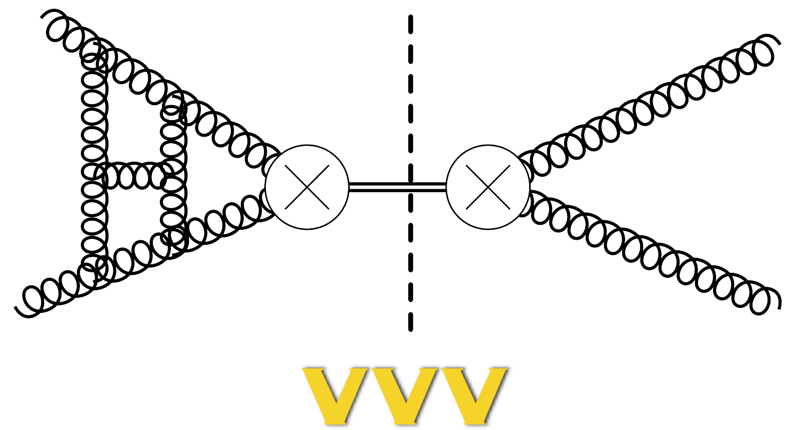
FEYNMAN DIAGRAMS



FEYNMAN DIAGRAMS



THE INTEGRAND



Loop Integrals

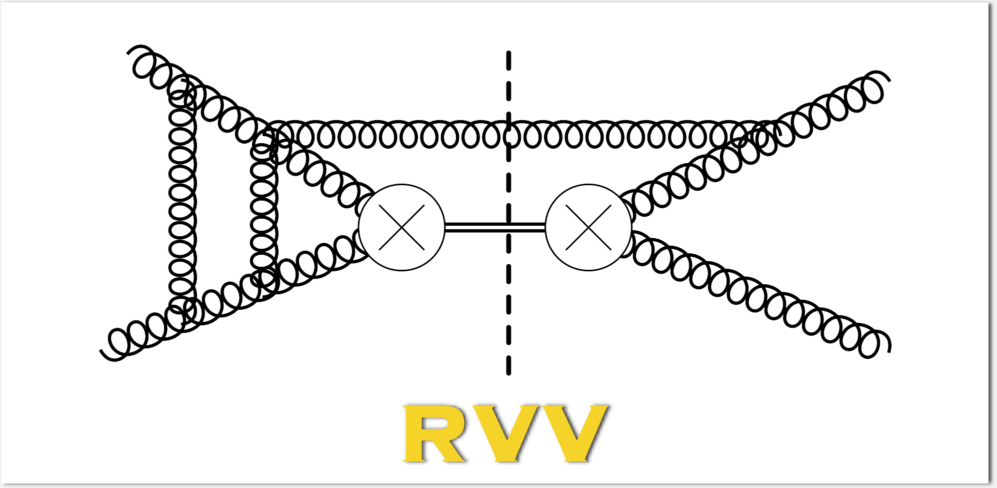
$$I = \int \frac{d^d p_h}{(2\pi)^d} \frac{d^d p_4}{(2\pi)^d} \frac{d^d p_5}{(2\pi)^d} \frac{d^d p_6}{(2\pi)^d} \delta_+(p_h^2 - m_h^2)\delta^d(p_1 + p_2 - p_h \qquad)$$

$$\times \left|\mathcal{M}(p_i)\right|^2$$

VVV Integrand

THE INTEGRAND

One parton in the final state



Loop Integrals

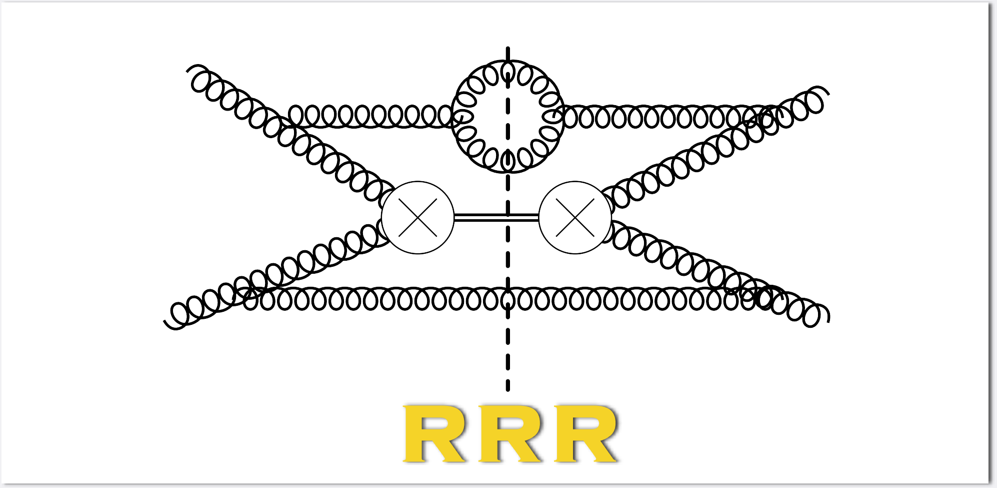
$$I = \int \frac{d^d p_h}{(2\pi)^d} \frac{d^d p_4}{(2\pi)^d} \frac{d^d p_5}{(2\pi)^d} \frac{d^d p_6}{(2\pi)^d} \delta_+(p_h^2 - m_h^2) \delta^d(p_1 + p_2 - p_h - p_4 \qquad)$$

$$\times |\mathcal{M}(p_i)|^2 \times \delta_+(p_4^2)$$

RVV Integrand

THE INTEGRAND

No loop left!



$$I = \int \frac{d^d p_h}{(2\pi)^d} \frac{d^d p_4}{(2\pi)^d} \frac{d^d p_5}{(2\pi)^d} \frac{d^d p_6}{(2\pi)^d} \delta_+(p_h^2 - m_h^2) \delta^d(p_1 + p_2 - p_h - p_4 - p_5 - p_6)$$

$$\times |\mathcal{M}(p_i)|^2 \times \delta_+(p_4^2) \times \delta_+(p_5^2) \times \delta_+(p_6^2)$$

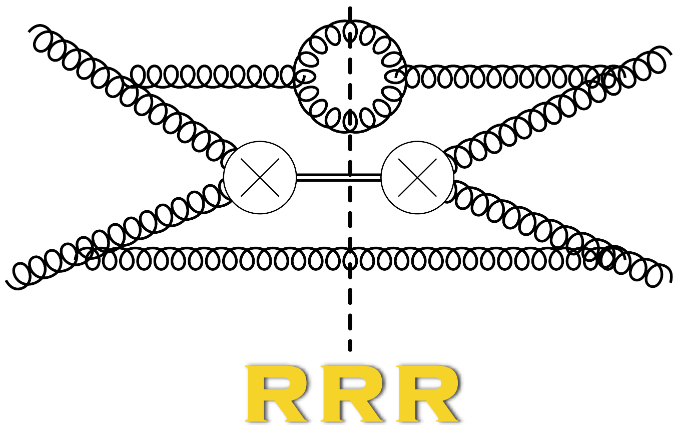
RRR Integrand

THE INTEGRAND

No loop left!

Integrals depend only on

s and $z = \frac{m_h^2}{s}$



$$I = \int \frac{d^d p_h}{(2\pi)^d} \frac{d^d p_4}{(2\pi)^d} \frac{d^d p_5}{(2\pi)^d} \frac{d^d p_6}{(2\pi)^d} \delta_+(p_h^2 - m_h^2) \delta^d(p_1 + p_2 - p_h - p_4 - p_5 - p_6)$$

$$\times |\mathcal{M}(p_i)|^2 \times \delta_+(p_4^2) \times \delta_+(p_5^2) \times \delta_+(p_6^2)$$

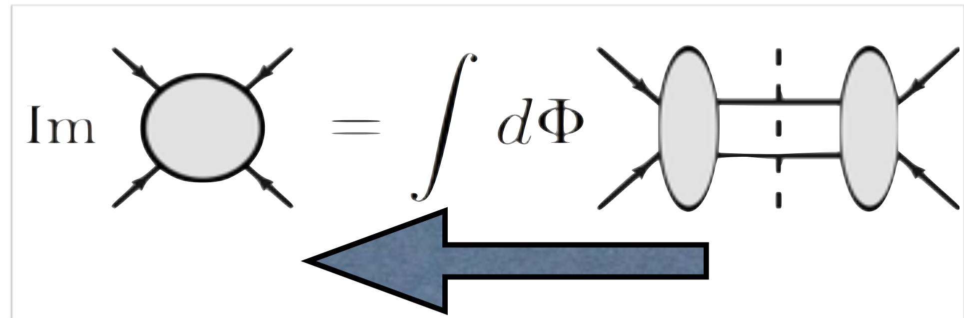
RRR Integrand

Exploit similarity!

REDUCING COMPLEXITY

Reverse Unitarity

Optical theorem



Cutkosky's Rule

$$\delta^+(p^2) \to \left[\frac{1}{p^2}\right]_c \sim \frac{1}{p^2+i\epsilon} - \frac{1}{p^2-i\epsilon}$$

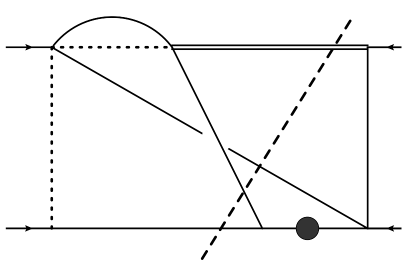
Treat cuts almost as ordinary propagators!

REDUCING COMPLEXITY

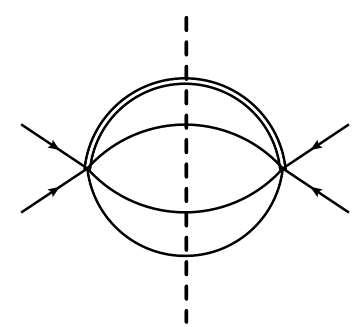
Use “Integration-By-Part” (IBP) Identities

$$\int \frac{d^d p_4}{(2\pi)^d} \frac{\partial}{\partial p_4^\mu} \left(p_4^\mu \left[\frac{1}{p_4^2} \right]_c |\mathcal{M}(p_4, \ldots)|^2 \ldots \right) = 0$$

→ **“Master Integrals”**



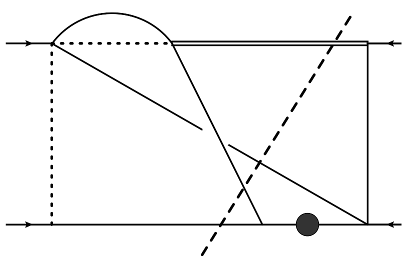
$$= - \frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$



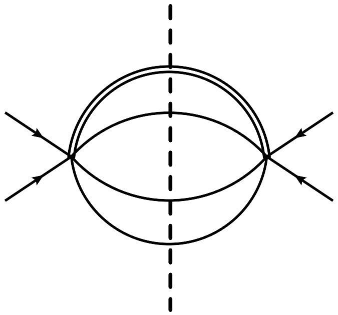
REDUCING COMPLEXITY

Use “Integration-By-Part” Identities

→ “Master Integrals”



$$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$

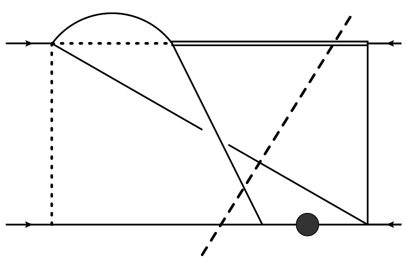


Does that simplify life?	before	after
NNLO	50 000	27

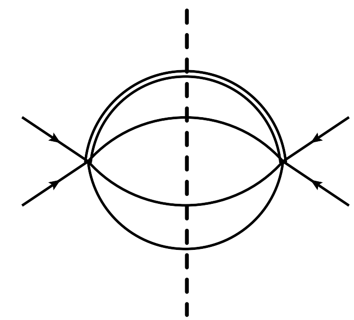
REDUCING COMPLEXITY

Use "Integration-By-Part" Identities

→ "Master Integrals"

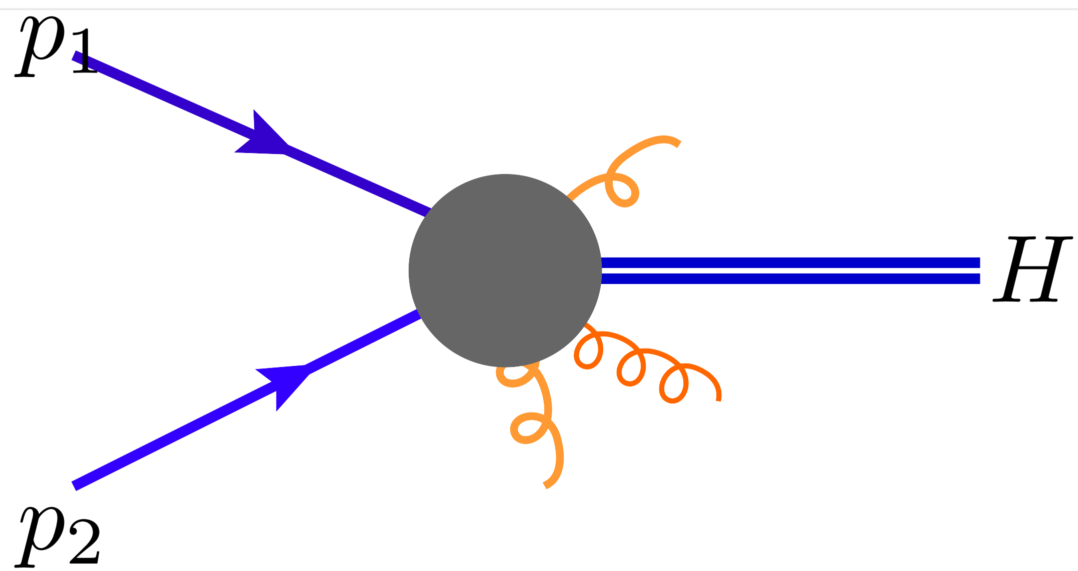


$$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$



Does that simplify life?	before	after
NNLO	50 000	27
N3LO	517 531 178	1028

THRESHOLD EXPANSION



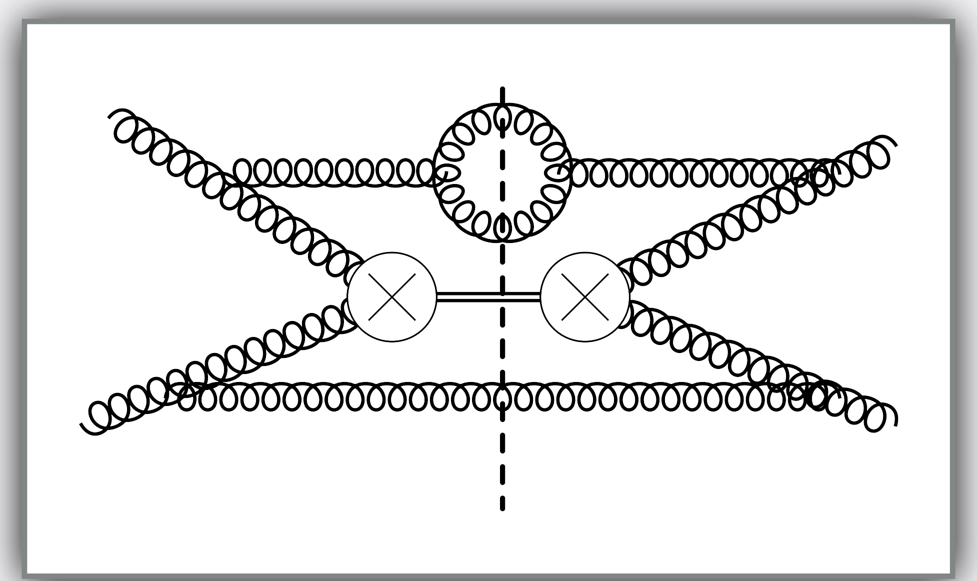
$$z = \frac{m_H^2}{\hat{s}} \sim 1$$

Expand around the production threshold of the Higgs

$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

MASTER INTEGRALS

Method #1: Momentum Space Expansion



Rescale final state momenta

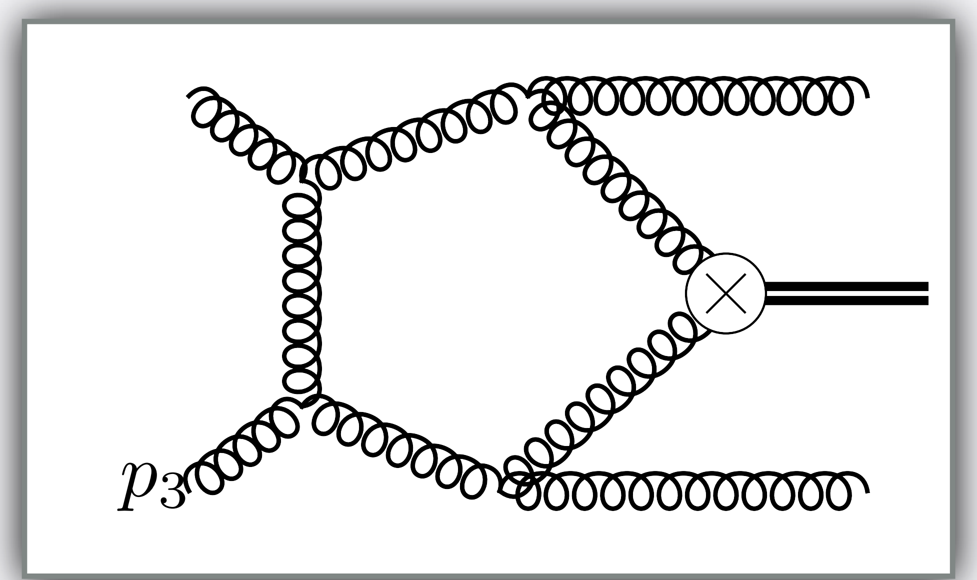
$$p_f \to (1-z)p_f$$

Expand integrand and measure

Ready to compute RRR!

MASTER INTEGRALS

Method #1: Momentum Space Expansion



Not so easy with loops: RRV

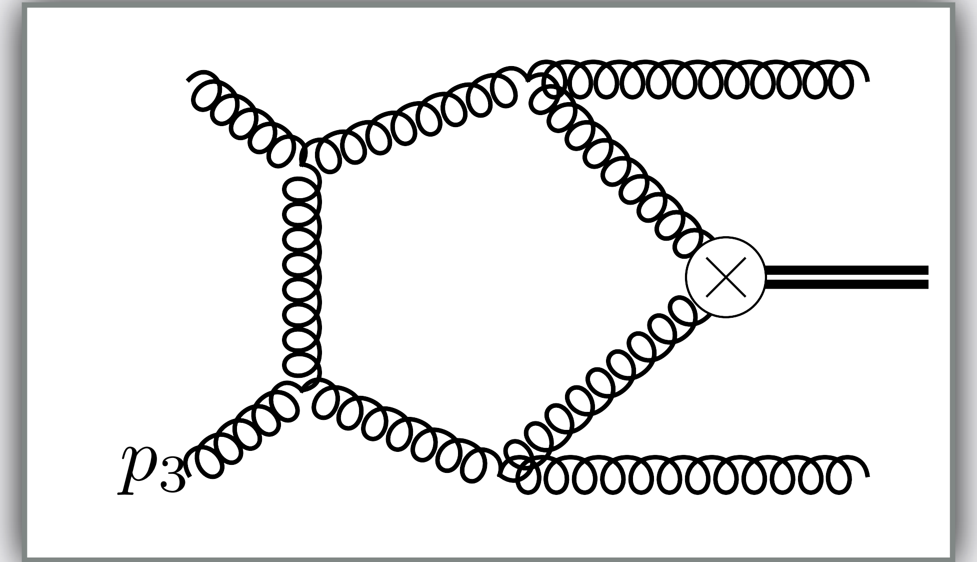
Loop momentum not constrained

Split in regions

Soft	Coll 1	Coll 2	Hard
$k \to \bar{z}k$	$k \to k\|\|p_1$	$k \to k\|\|p_2$	k

MASTER INTEGRALS

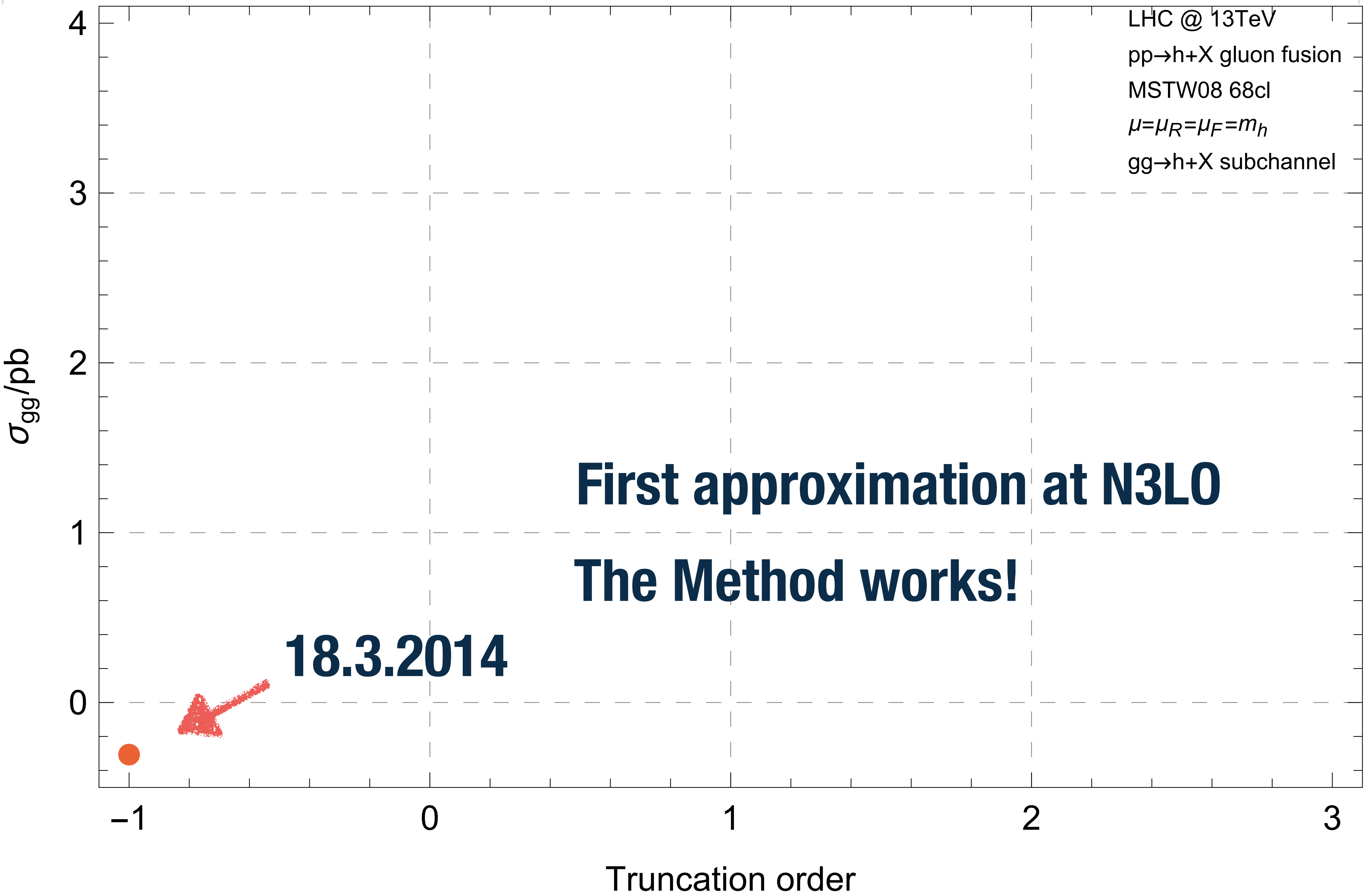
Method #1: Momentum Space Expansion



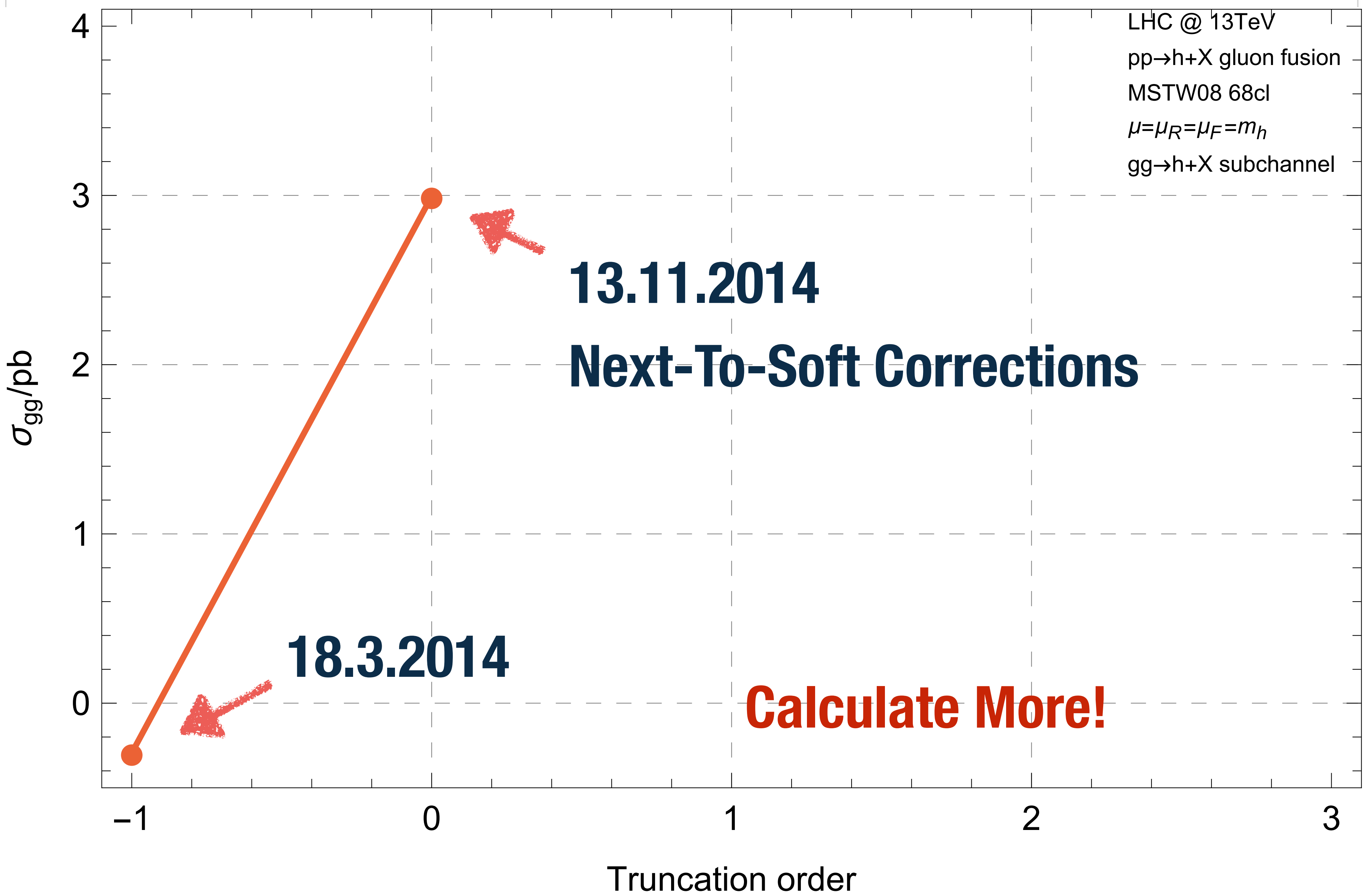
parametrize loop momentum

$$p_l \to p_1\alpha + p_2\beta + p_{l,\perp}$$

Hard	$\alpha \to \alpha, \quad \beta \to \beta, \quad p_{l,\perp}^2 \to p_{l,\perp}^2$
Coll 1	$\alpha \to \alpha, \quad \beta \to (1-z)\beta, \quad p_{l,\perp}^2 \to (1-z)p_{l,\perp}^2$
Coll 2	$\alpha \to (1-z)\alpha, \quad \beta \to \beta, \quad p_{l,\perp}^2 \to (1-z)p_{l,\perp}^2$
Soft	$\alpha \to (1-z)\alpha, \quad \beta \to (1-z)\beta, \quad p_{l,\perp}^2 \to (1-z)p_{l,\perp}^2$



LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
First approximation at N3LO
The Method works!
18.3.2014
σ_{gg}/pb
Truncation order
4
3
2
1
0
−1
0
1
2
3



LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
13.11.2014
Next-To-Soft Corrections
18.3.2014
Calculate More!
σ_{gg}/pb
Truncation order
4
3
2
1
0
−1
0
1
2
3

MASTER INTEGRALS

Method #2: Differential Equations

$$z = \frac{m_h^2}{s}$$

Only two variables: s and z

Dependence on s trivial: „Energy - dimension“

Integrand depends on z
only via Higgs on-shell constraint

$$\frac{\partial}{\partial z}\delta(p_h^2 - sz) \rightarrow s\left(\frac{1}{p_h^2 - sz}\right)_c^2$$

Relate the differential with IBP identities to Master-Integrals

$$\frac{\partial}{\partial z}\vec{M} = A(z)\vec{M}$$

MASTER INTEGRALS

Method #2: **Differential Equations**

$$\frac{\partial}{\partial z}\vec{M} = A(\epsilon, z)\vec{M}$$

- System of coupled differential equations
- Coefficient Matrix A is not constant

One solution:

Find a way to decouple order by order in ϵ

$$\frac{\partial}{\partial z}\vec{M} = \epsilon A(\epsilon, z)\vec{M}$$

MASTER INTEGRALS

Solve $$\frac{\partial}{\partial z}\vec{M} = A(\epsilon, z)\vec{M}$$

with a generalised power series Ansatz

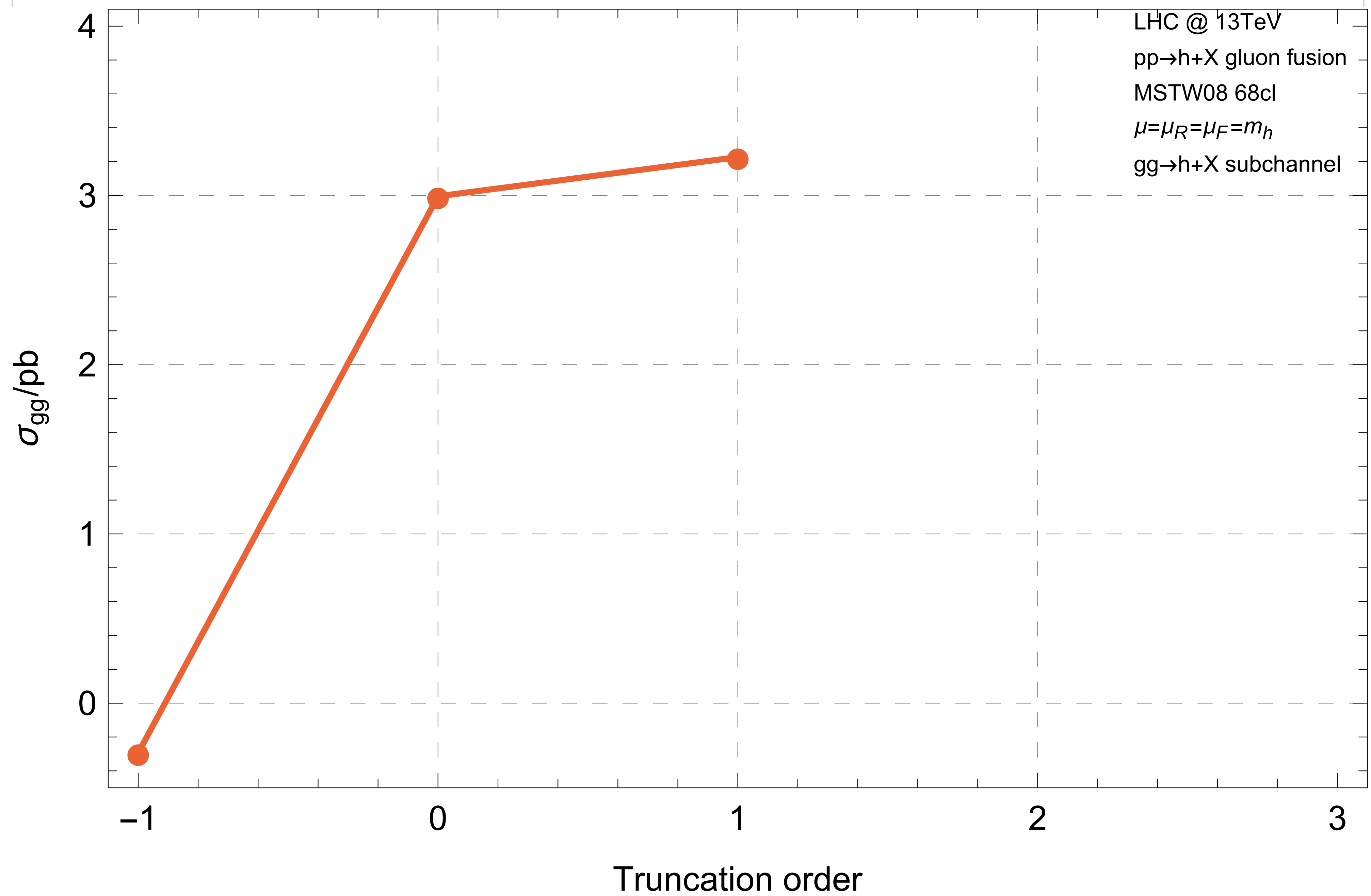
$$M_i = \sum_j \sum_{k=2}^{6} c_{ijk}(1-z)^{(j-k\epsilon)}$$

1028 Differential Equations

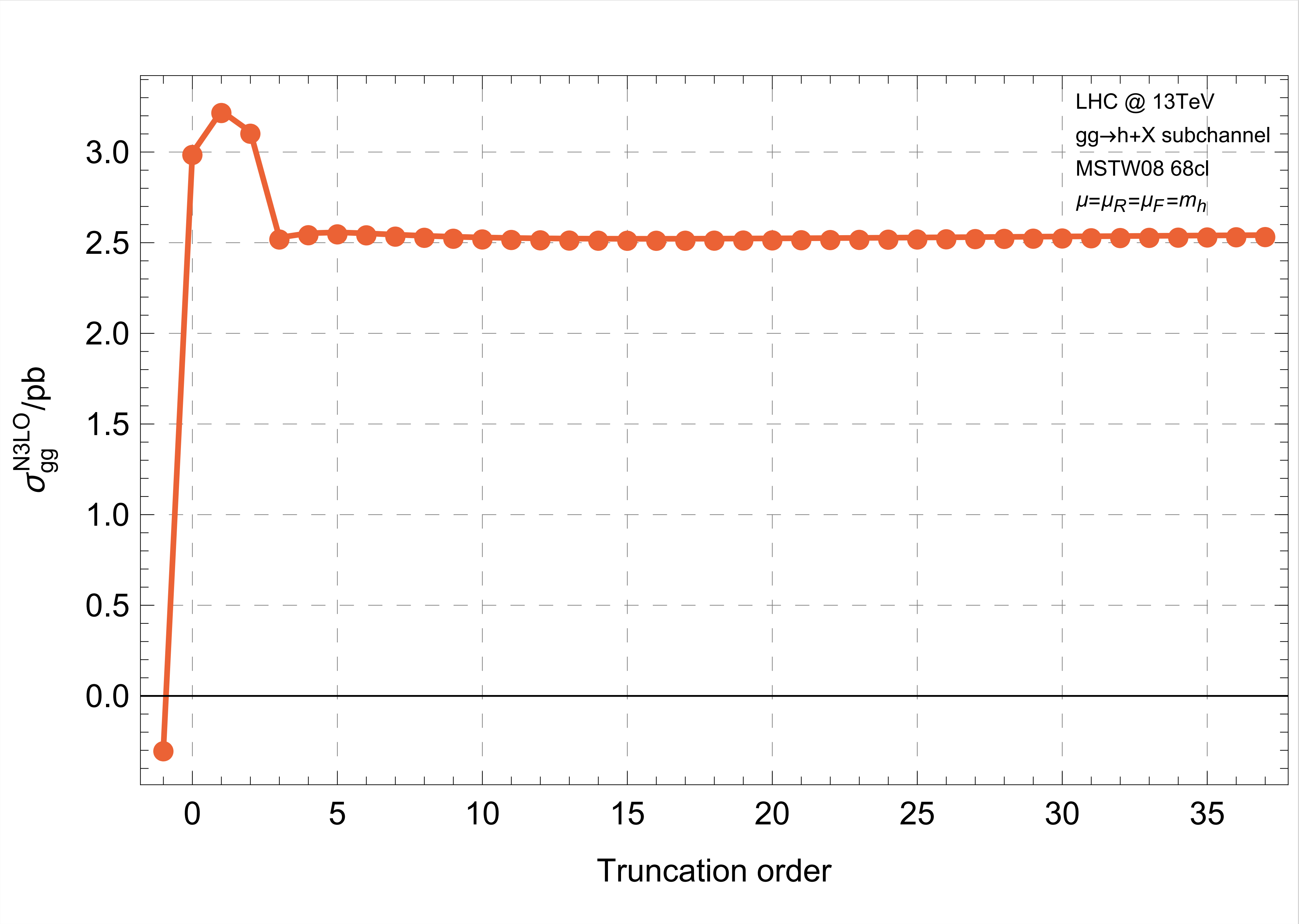
need 1028 Boundary Conditions!

Combine with Expansion-by-Regions

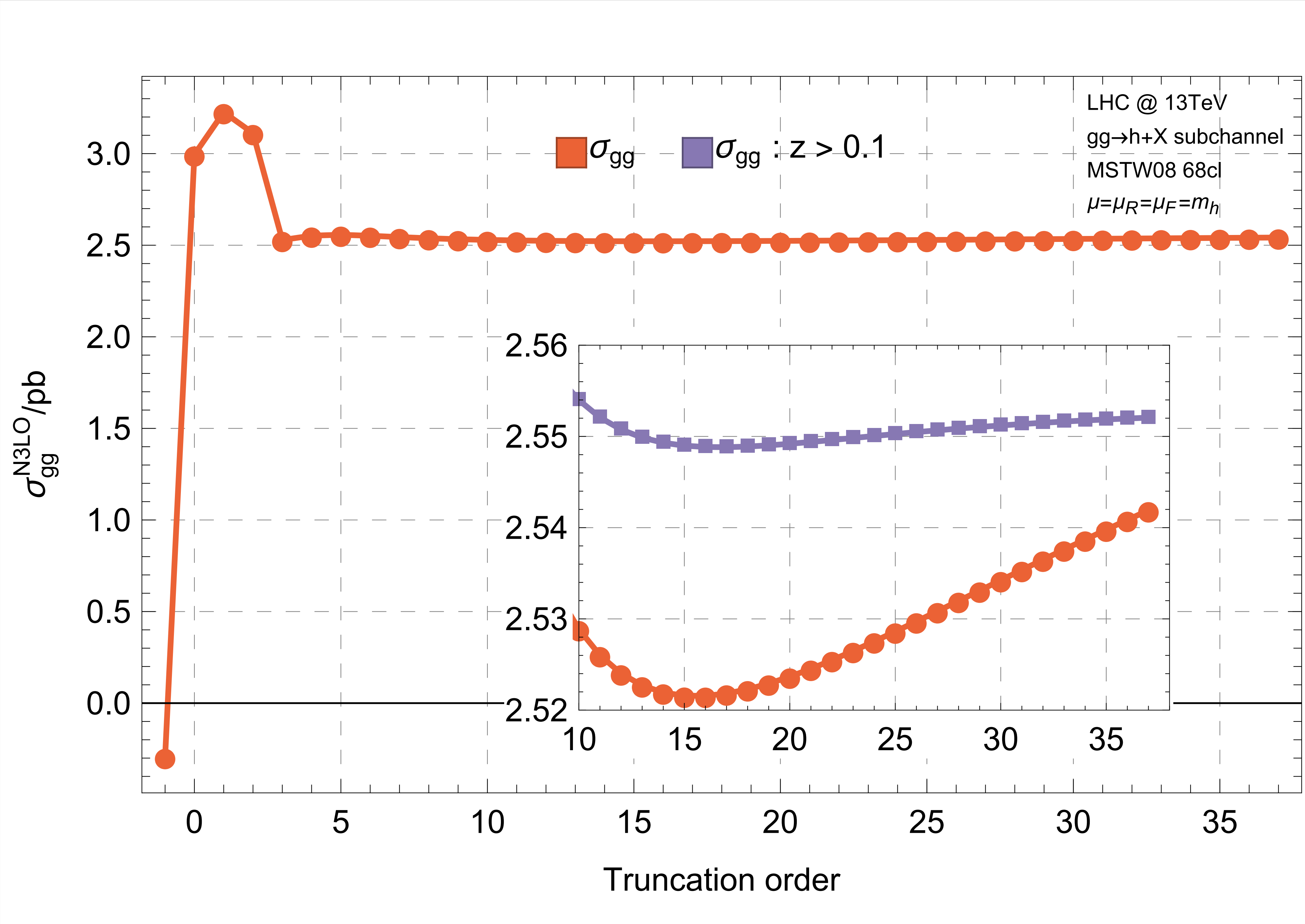
1028 → 78



LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
μ=μR=μF=mh
gg→h+X subchannel
σgg/pb
Truncation order
4
3
2
1
0
−1
0
1
2
3



LHC @ 13TeV
gg→h+X subchannel
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
σ_{gg}^{N3LO}/pb
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
35
Truncation order



LHC @ 13TeV
gg→h+X subchannel
MSTW08 68cl
μ=μR=μF=mh
σgg
σgg : z > 0.1
σgg N3LO/pb
Truncation order
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
35
2.56
2.55
2.54
2.53
2.52

THE METHODS

- Technology to handle hundreds of thousands of Feynman diagrams
- Improved algorithms to reduce the complexity of integrands to a manageable size
- Techniques to perform integrand and integral expansions
- Novel methods of computing Feynman Integrals

Ready to predict at N3LO!

CONCLUSIONS

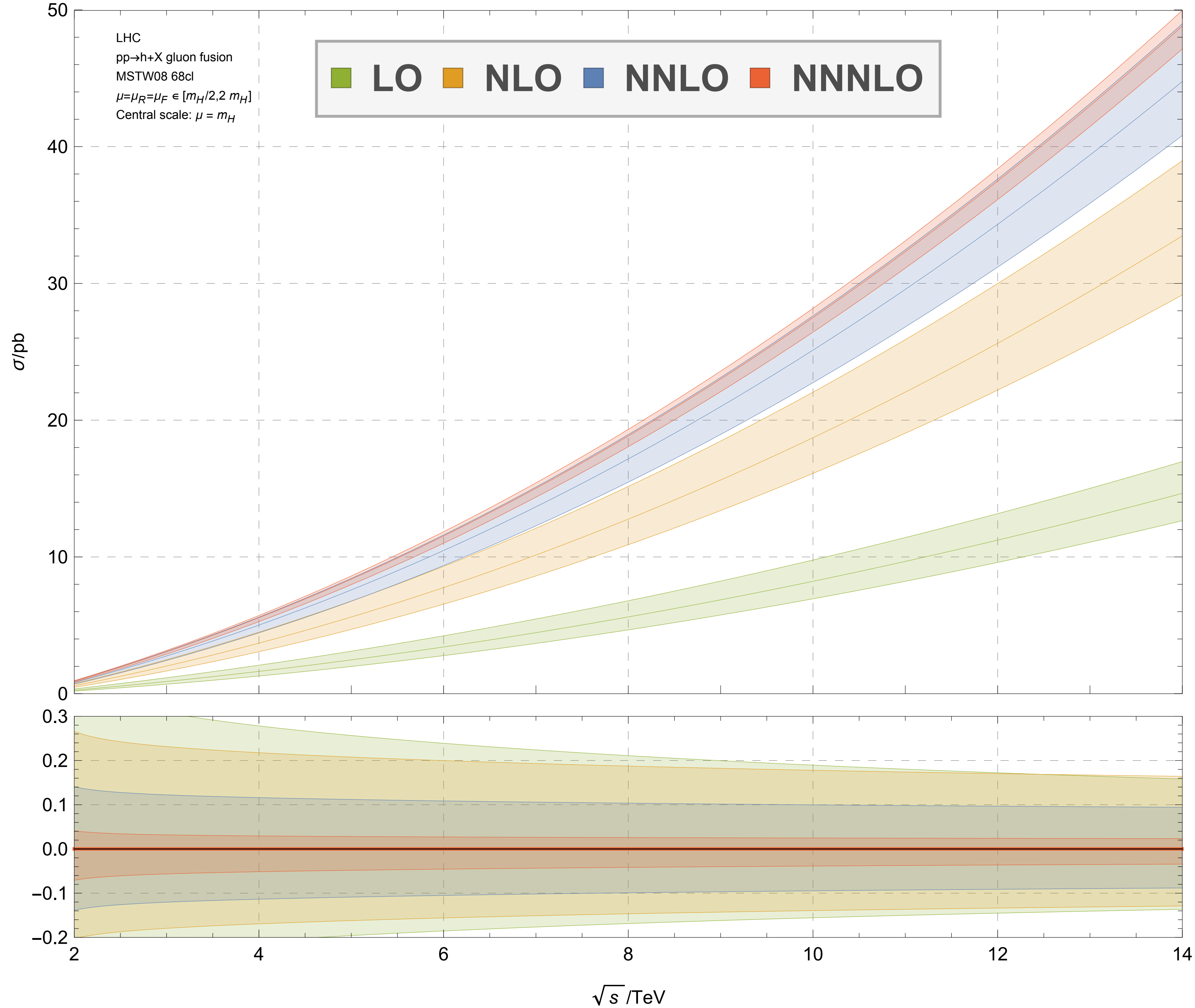
First complete calculation of an observable at N3LO

30 orders in the threshold expansion for Gluon Fusion

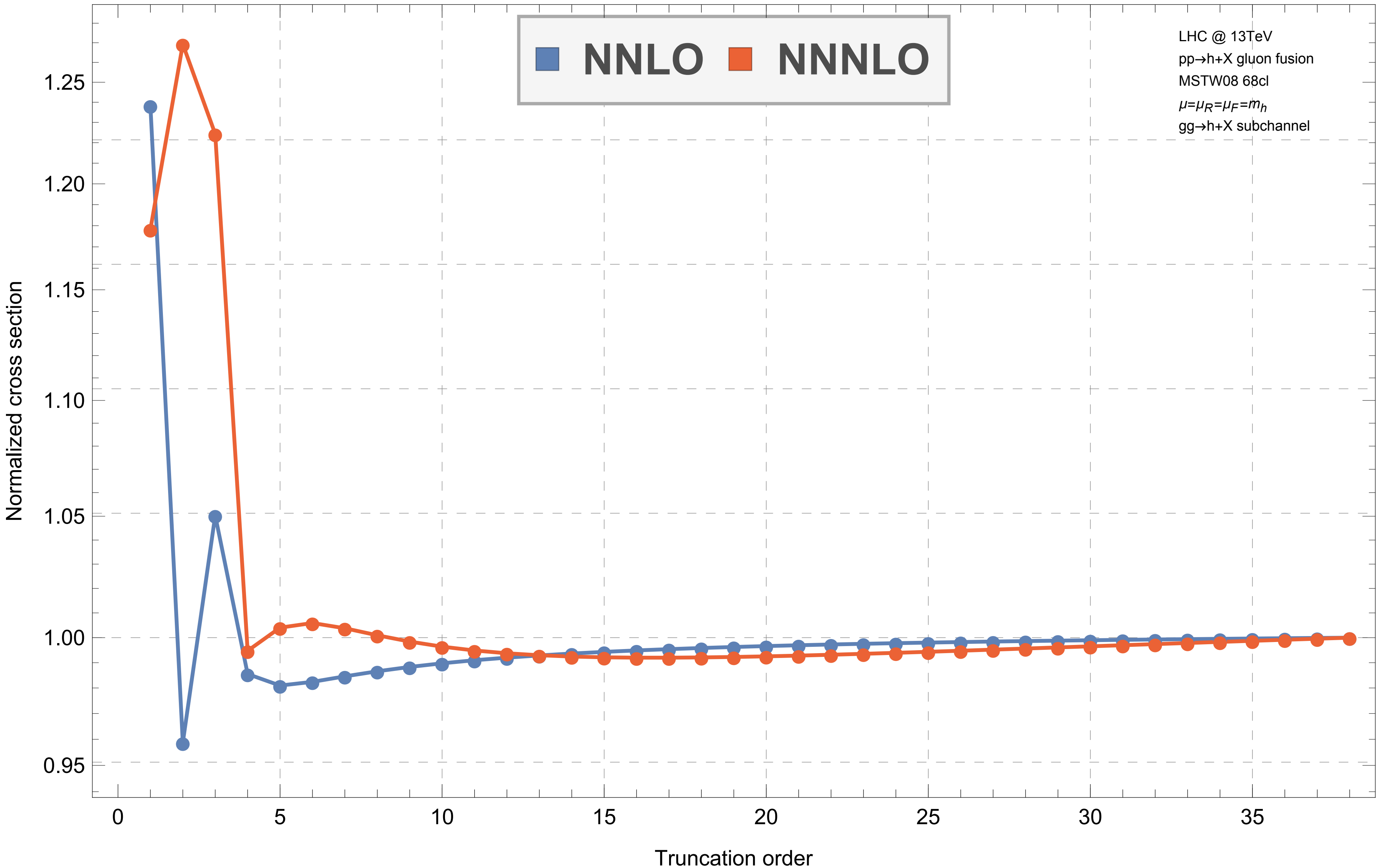
Drastic reduction of scale dependence

Direct impact on LHC phenomenology

This is the dawn of the age of N3LO precision



LO
NLO
NNLO
NNNLO
LHC
pp→h+X gluon fusion
MSTW08 68cl
μ=μR=μF ∈ [mH/2,2 mH]
Central scale: μ = mH
σ/pb
√s /TeV
0
10
20
30
40
50
−0.2
−0.1
0.0
0.1
0.2
0.3
2
4
6
8
10
12
14



NNLO
NNNLO
LHC @ 13TeV
pp→h+X gluon fusion
MSTW08 68cl
$\mu=\mu_R=\mu_F=m_h$
gg→h+X subchannel
Normalized cross section
Truncation order
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0
5
10
15
20
25
30
35

PARTON LUMINOSITY

$$\mathcal{L}_{12}(z) = \int_{\tau/z}^{1} dx_1 f_1(x_1) f_2(\frac{\tau}{z x_1})$$