Two-loop QCD amplitudes for Higgs $\to b + \bar{b} + g$

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- Higgs+1 jet
- Two loop amplitudes $b + \bar{b} \to H + g$
- IBP, LI, MI
- Two loop IR structure
- Summary

with *Taushif Ahmed, Maguni Mahakhud, Narayan Rana and V. Ravindran*

RADCOR 2015

Run-I@LHC

- At the experimentally accessible energy scales, the Run-I@LHC has established the SM framework as the true theory of electroweak interactions
- The discovered new boson at 125 GeV behaves like the SM scalar and its mass fixes the last free parameter of the Lagrangian
- Negative results from the searches for signals of new physics tightly constrain many new physics scenarios, surviving parameter space is no longer appropriate to address the physics problems they were intented to solve
- Experimental program at the LHC relies heavily on the precise theoretical predictions for the relevant signals and the many QCD backgrounds
- Remarkable agreement between the predicted SM values and the measured cross sections spanning a broad range is a significant validation of the theoretical framework

The Higgs

- Post discovery of a new particle

$$m_H = 125.09 \pm 0.24 \, \text{GeV}$$

ATLAS & CMS combined measurement in the $H \to \gamma\gamma$ and $H \to ZZ \to 4\ell$ channels for $\sqrt{s} = 7, 8$ TeV

PRL 114 (2015) 191803

- With increasing dataset, emphasis shifted to determining its properties and testing the consistency of the SM against the data
- Spin, Charge conjugation and Parity probed by examining the angular distributions of the decay channels $H \to \gamma\gamma$, $H \to ZZ$, $H \to WW$. Data favours a CP-even, spin-zero particle
- Strengths of the couplings with gauge bosons and fermions explored for a number of benchmark models
- Results consistent with expectation of a SM Higgs boson

Precision studies

- Precise theoretical predictions of the Higgs boson observable will be key to precision studies of its properties at the LHC
- New physics models look for deviation from the SM predictions and constraining these models needs precise theoretical inputs
- Theoretical uncertainties as a result of the missing higher order terms in pQCD at the LHC energies are large and is comparable to the experimental errors
- To study the properties of the Higgs boson, differential distribution of the Higgs boson play an important role
- Observables with jet vetos enhance the significance of the signal considerably enabling the study the properties of Higgs couplings

Exclusive observables

- NNLO predictions of the Higgs bosons with one jet through effective gluon fusion process is available

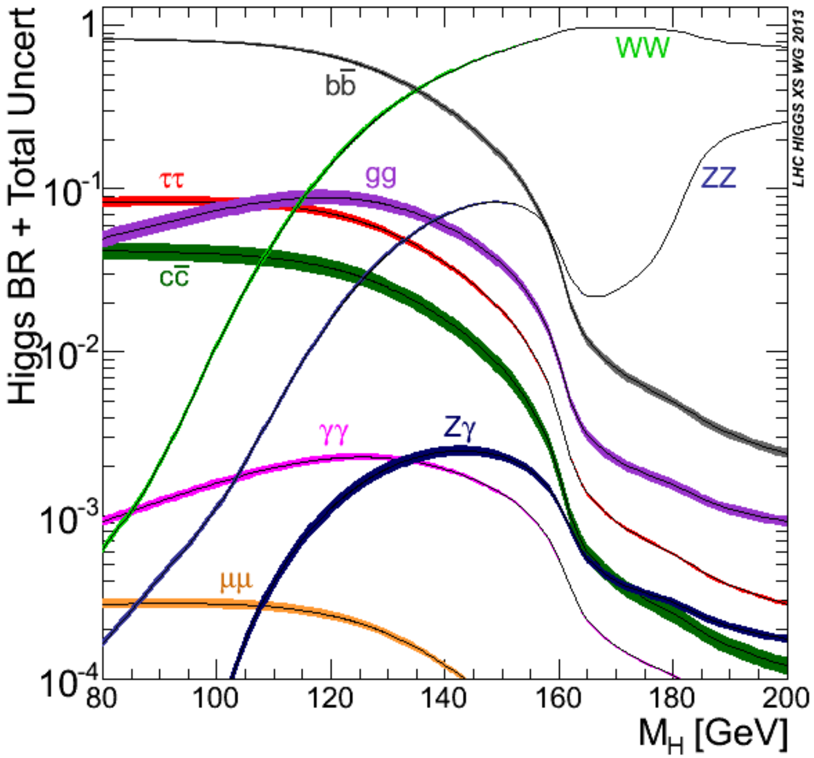
Gehrmann, Jaquier, Glover, Koukoutsakis

Boughezal, Caola, Melnikov, Petriello, Schulze

- As the experimental accuracy improves it is important to include the sub-dominant contributions to the production
- Higgs through $b\bar{b}$ annihilation with 1-jet is one such process and is known only upto NLO level
- Here we present one of the ingradients for the production Higgs+1 jet in $b\bar{b}$ anhillation *viz*. two loop QCD amplitudes

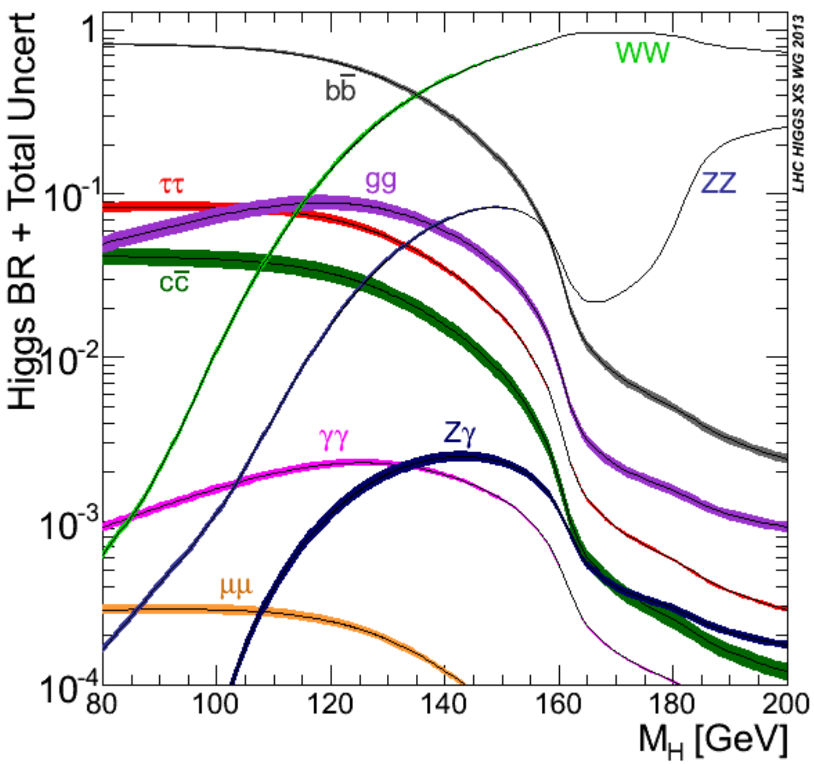
Branching fraction of SM Higgs

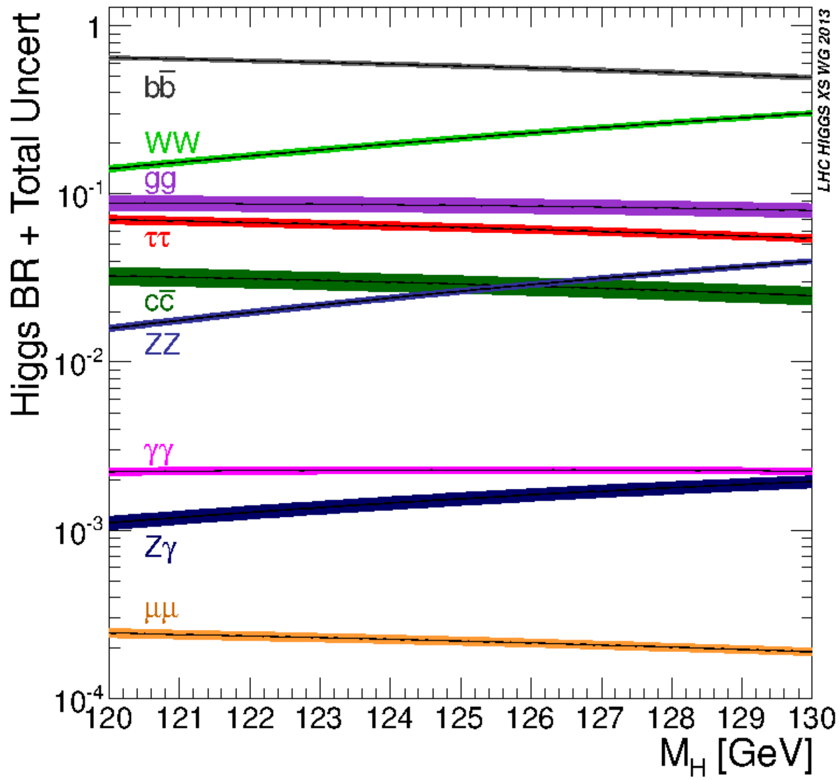
- **Branching fraction of SM Higgs as a function of Higgs mass**



Branching fraction of SM Higgs

- Branching fraction of SM Higgs as a function of Higgs mass





- Mass range of interest $M_H = 120 - 130$ GeV, significant contribution comes from H$\rightarrow$ $b\bar{b}$, $\tau^+\tau^-$, WW^* and ZZ^*

Interaction part of the action

b-quarks with ○ scalar Higgs (SM) and ○ pseudoscalar Higgs (MSSM)

$$S_I^b = -\lambda \int d^4x\, \phi(x)\, \overline{\psi}_b(x)\, \psi_b(x) \qquad \text{SM}$$

$$= -\tilde{\lambda} \int d^4x\, \tilde{\phi}(x)\, \overline{\psi}_b(x)\, \gamma_5\, \psi_b(x) \qquad \text{MSSM}$$

Yukawa coupling: $\lambda = \frac{\sqrt{2}}{v} m_b \qquad \tilde{\lambda} = \begin{cases} -\lambda \frac{\sin\alpha}{\cos\beta} & \tilde{\phi} = h \\ \lambda \frac{\cos\alpha}{\cos\beta} & \tilde{\phi} = H \\ \lambda \tan\beta & \tilde{\phi} = A \end{cases}$

- **α: mixing angle between weak and mass eigenstates of neutral scalars**
- **Scale of the problem is the Higgs boson mass and b-quark mass is much smalller, hence both in the phase space integral and matrix elements, the b-quark mass is treated as massless like the light quark flavours, while retaining the b-quark mass in the Yukawa coupling**

Variable Flavour Scheme (VFS)

- In the VFS, one assumes the initial state b-quarks inside the proton, as a result of emission of collinear $b\,\bar{b}$ states from the gluons intrinsically present inside the proton
- Being collinear, give large logs, which need to be resummed. The resummed contribution is the source for non-vanishing b and $\bar{b}$ parton distribution functions inside the proton in the VFS scheme. Active flavours $n_f = 5$
- Fully inclusive cross section for Higgs production in association with bottom quark to NNLO level accuracy is known in the VFS

Harlander, Kilgore

Higgs decay $H(q) \to b(p_1) + \bar{b}(p_2) + g(p_3)$

- **Mandelstam variables**

$$s \equiv (p_1 + p_2)^2 > 0, \quad t \equiv (p_2 + p_3)^2 > 0, \quad u \equiv (p_1 + p_3)^2 > 0$$

$$s + t + u = M_H^2 \equiv Q^2 > 0$$

- **Dimensionless invariants**

$$0 < x \equiv s/Q^2 < 1, \quad 0 < y \equiv u/Q^2 < 1, \quad 0 < z \equiv t/Q^2 < 1$$

$$x + y + z = 1$$

- **Two-loop four-point functions with one off-shell external leg and massless internal propagators can be expressed in terms of HPLs and 2dHPLs as functions of dimensionless invariants**

Analytical continuation: Higgs+1 jet production $Q^2 = M_H^2 > 0$

- Continuation from the Euclidean region to any physical Minkowskian region requires in general the analytic continuation

$$1 \to 3 \qquad \Longleftrightarrow \qquad 2 \to 2$$

- $\bar{b}(-p_1) + b(-p_2) \to g(p_3) + H(p_4) \qquad s > 0, \quad t < 0, \quad u < 0$

$$0 < u_1 \equiv -\frac{u}{s} < 1, \qquad 0 < v_1 \equiv \frac{Q^2}{s} < 1$$

- $b(-p_2) + g(-p_3) \to b(p_1) + H(p_4) \qquad s < 0, \quad t > 0, \quad u < 0$

$$0 < u_2 \equiv -\frac{u}{t} < 1, \qquad 0 < v_2 \equiv \frac{Q^2}{t} < 1$$

- $\bar{b}(-p_1) + g(-p_3) \to \bar{b}(p_2) + H(p_4) \qquad s < 0, \quad t < 0, \quad u > 0$

$$0 < u_3 \equiv -\frac{t}{u} < 1, \qquad 0 < v_3 \equiv \frac{Q^2}{u} < 1$$

- Relations for HPLs and 2dHPLs needed for analytic continuation of the 2-loop, 4-point master integrals to kinematics of all $2 \to 2$ scatterings with one off-shell external leg

Gehrmann and Remiddi NPB640 (2002) 379

Amplitude $|\mathcal{M}\rangle = \mathcal{S}_\mu(b,\bar{b};g)\varepsilon^\mu$ of $H(q) \to b(p_1)+\bar{b}(p_2)+g(p_3)$

- Using equations of motion and $p_3 \cdot \varepsilon = 0$ the general form

$$\mathcal{S}_\mu(b,\bar{b};g) = \bar{u}(p_1)\Big\{A'\, p_{1\mu} + A''\, p_{2\mu} + A_2\, \not{p}_3\gamma_\mu\Big\}v(p_2)$$

- QCD Ward identities $\Rightarrow A'/p_2.p_3 = -A''\,/p_1.p_3 \equiv A_1$. Amplitude reduces

$$\mathcal{S}_\mu(b,\bar{b};g)\,\varepsilon^\mu = \bar{u}(p_1)\Big\{A_1\,(p_2.p_3\,p_{1\mu} - p_1.p_3\,p_{2\mu}) + A_2\,\not{p}_3\gamma_\mu\Big\}v(p_2)\,\varepsilon^\mu$$
$$\equiv A_1\,\mathrm{T}_1 + A_2\,\mathrm{T}_2$$

- Coefficients A_m $(m=1,2)$ can be expanded in powers of $a_s = g_s^2/16\pi^2$

$$A_m = \frac{\lambda}{\mu_R^\epsilon}\,4\pi\sqrt{a_s}T^a_{ij}\Big\{A_m^{(0)} + a_s A_m^{(1)} + a_s^2 A_m^{(2)} + \mathcal{O}(a_s^3)\Big\}$$

Coefficients $A^{(l)}_{1,2}$ completely specify the amplitude order by order in perturbation theory

Projection Operators

- Using appropriate d-dimensional projection operators and summing over the spin, coefficients $A_{1,2}$ can be extracted to a particular order in pQCD

$$A_m = \sum_{\text{spins}} \mathcal{P}(A_m)\, \mathcal{S}_\mu(b,\bar{b};g)\, \varepsilon^\mu$$

- Projection operators

$$\mathcal{P}(A_1) = \frac{2(d-2)}{s^2\, t\, u\,(d-3)} \mathrm{T_1}^\dagger + \frac{1}{s\, t\, u\,(d-3)} \mathrm{T_2}^\dagger$$
$$\mathcal{P}(A_2) = \frac{1}{s\, t\, u\,(d-3)} \mathrm{T_1}^\dagger + \frac{1}{2\, t\, u\,(d-3)} \mathrm{T_2}^\dagger$$

- By suitably crossing the Higgs decay $(1 \to 3)$ amplitudes can be related to Higgs + 1 jet $(2 \to 2)$ production amplitude, with the $A_{1,2}$ now expressed in terms of u_i and v_i

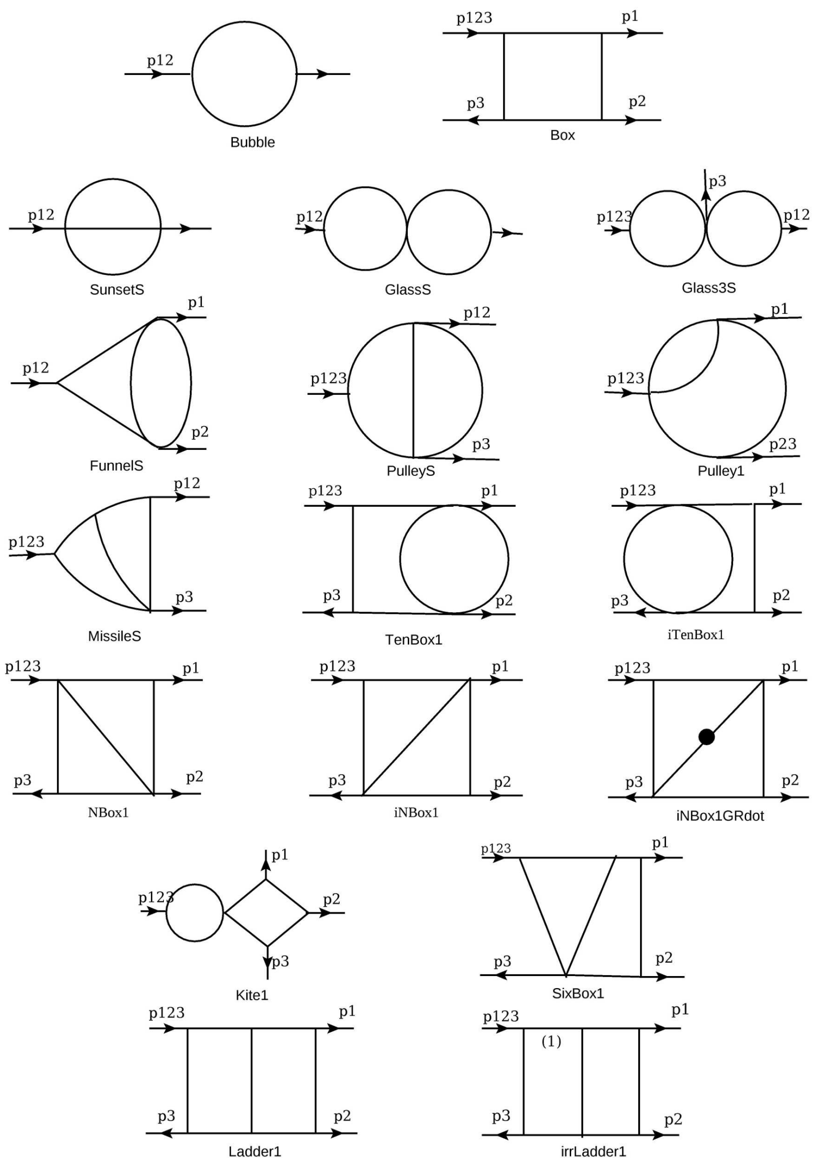
Calculation of unrenormalised ampliudes $|\hat{M}^{(l)}\rangle$

Process $H \to b + \bar{b} + g$ to 2-loop

$$|\mathcal{M}\rangle = \frac{\hat{\lambda}}{\mu_0^{\epsilon}} S_{\epsilon} \left(\frac{\hat{a}_s}{\mu_0^{\epsilon}} S_{\epsilon}\right)^{\frac{1}{2}} \left\{ |\hat{\mathcal{M}}^{(0)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^{\epsilon}} S_{\epsilon}\right) |\hat{\mathcal{M}}^{(1)}\rangle + \left(\frac{\hat{a}_s}{\mu_0^{\epsilon}} S_{\epsilon}\right)^2 |\hat{\mathcal{M}}^{(2)}\rangle + \mathcal{O}(\hat{a}_s^3) \right\}$$

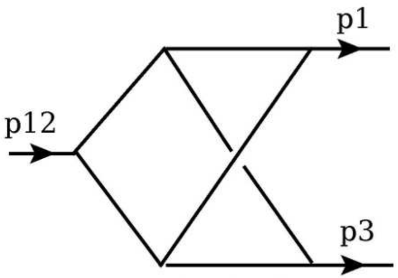
- **Dimensional regularisation $d = 4 + \epsilon$**
- **Gauge choice: ∘ External gluon (axial) ∘ Internal gluon (Feynman)**
- **Feynmann diagrams are generate using QGRAF**
  - **Tree level 2**
  - **1-loop level 13**
  - **2-loop level 251**
- **Convert raw QGRAF symbolic output to FORM readable format incorporating the Feynman rules (in-house FORM codes)**
- **The Integrals are reduced to master integrals (MI) using IBP and LI identities (mathematica packages FIRE and LiteRed)**

Planar topologies of master integrals

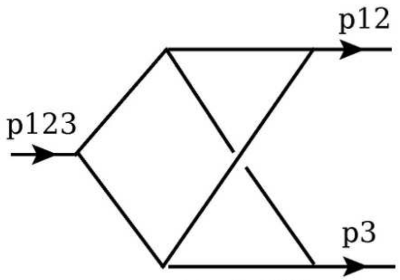


Gehrmann and Remiddi

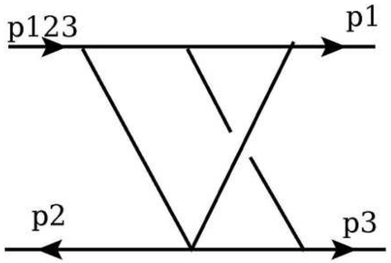
Non-planar topologies of master integrals



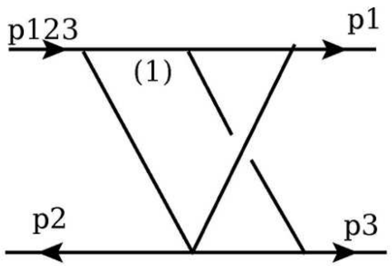
RocketS



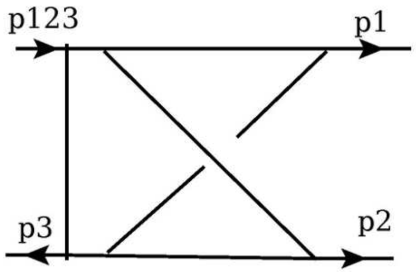
Rocket3S



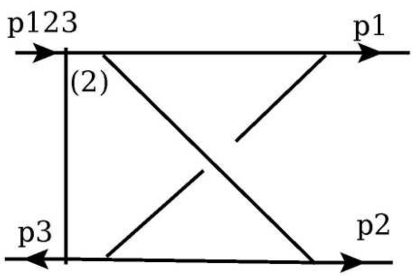
VXBox1p



irrVXBox1p



iXBox1



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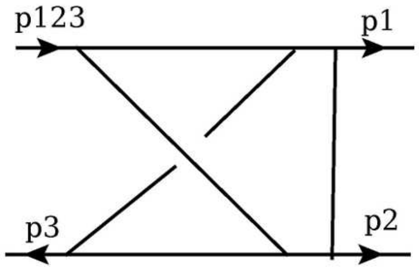
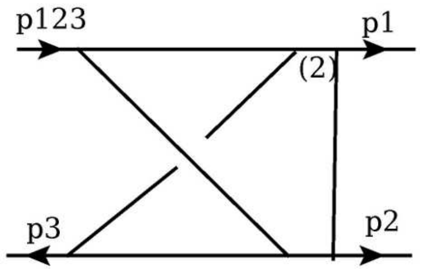


XBox1



irrXBox1

Gehrmann and Remiddi

1-loop

- **Integral belongs to one of the following sets:**

$$\{\mathcal{D},\ \mathcal{D}_1,\ \mathcal{D}_{12},\ \mathcal{D}_{123}\}\,,\{\mathcal{D},\ \mathcal{D}_2,\ \mathcal{D}_{23},\ \mathcal{D}_{123}\}\,,\{\mathcal{D},\ \mathcal{D}_3,\ \mathcal{D}_{31},\ \mathcal{D}_{123}\}$$

$$\mathcal{D} = k_1^2,\ \mathcal{D}_i = (k_1 - p_i)^2,\ \mathcal{D}_{ij} = (k_1 - p_i - p_j)^2,\ \mathcal{D}_{ijk} = (k_1 - p_i - p_j - p_k)^2$$

- **Scalar products $\{S_{ij}\}$ with loop momenta k_1 and external momenta p_i can be expressed, in terms of $\mathcal{D}$'s in a set. Each set $\{\mathcal{D}\}$ form a complete basis and are linearly independent**

- **At 1-loop, number of $\{S_{ij}\}$ = max number of propagators. Does not hold for 2-loop and additional auxilliary denominators need to be introcuced**

$$\mathcal{I} = \int \prod_{\ell=1}^{l} d^d k_\ell \frac{\{\mathcal{S}_{ij}\}}{\mathcal{D}_1^{n_1}\cdots\mathcal{D}_m^{n_m}} \qquad \begin{cases} NLO & m \le 4 & \{\mathcal{S}_{ij}\} = 4 \\ NNLO & m \le 7 & \{\mathcal{S}_{ij}\} = 9 \end{cases}$$

Two Loop

- Nine independent scalar products involving loop momenta $k_{1,2}$ and external momenta $p_{1,2,3}$. Feynman integral contain terms belonging to one of the following six sets:

$$\{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;1},\ \mathcal{D}_{2;1},\ \mathcal{D}_{1;12},\ \mathcal{D}_{2;12},\ \mathcal{D}_{1;123},\ \mathcal{D}_{2;123}\}\,,$$

$$\{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;1},\ \mathcal{D}_{2;1},\ \mathcal{D}_{1;12},\ \mathcal{D}_{2;12},\ \mathcal{D}_{1;123},\ \mathcal{D}_{0;3}\}\,,$$

$$\{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;2},\ \mathcal{D}_{2;2},\ \mathcal{D}_{1;23},\ \mathcal{D}_{2;23},\ \mathcal{D}_{1;123},\ \mathcal{D}_{2;123}\}\,,$$

$$\{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;2},\ \mathcal{D}_{2;2},\ \mathcal{D}_{1;23},\ \mathcal{D}_{2;23},\ \mathcal{D}_{1;123},\ \mathcal{D}_{0;1}\}\,,$$

$$\{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;3},\ \mathcal{D}_{2;3},\ \mathcal{D}_{1;31},\ \mathcal{D}_{2;31},\ \mathcal{D}_{1;123},\ \mathcal{D}_{2;123}\}\,,$$

$$\{\mathcal{D}_0,\ \mathcal{D}_1,\ \mathcal{D}_2,\ \mathcal{D}_{1;3},\ \mathcal{D}_{2;3},\ \mathcal{D}_{1;31},\ \mathcal{D}_{2;31},\ \mathcal{D}_{1;123},\ \mathcal{D}_{0;2}\}$$

$$\mathcal{D}_0 = (k_1 - k_2)^2,\ \mathcal{D}_\alpha = k_\alpha^2,\ \mathcal{D}_{\alpha;i} = (k_\alpha - p_i)^2,\ \mathcal{D}_{\alpha;ij} = (k_\alpha - p_i - p_j)^2,$$

$$\mathcal{D}_{0;i} = (k_1 - k_2 - p_i)^2,\ \mathcal{D}_{\alpha;ijk} = (k_\alpha - p_i - p_j - p_k)^2 \qquad \alpha = 1, 2;\, i = 1, 2, 3$$

UV divergences; $d = 4 + \epsilon$; $\overline{\mathrm{MS}}$ scheme

- **Bare couplings related to renormalised** $\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon = \frac{a_s}{\mu_R^\epsilon} Z(\mu_R^2)$

$$\frac{\hat{a}_s}{\mu_0^\epsilon} S_\epsilon = \frac{a_s}{\mu_R^\epsilon} \left[1 + a_s \left(\frac{1}{\epsilon} r_{a_{1;1}} \right) + a_s^2 \left(\frac{1}{\epsilon^2} r_{a_{2;2}} + \frac{1}{\epsilon} r_{a_{2;1}} \right) + \mathcal{O}(a_s^3) \right]$$

$$S_\epsilon = \exp\left[\frac{\epsilon}{2} (\gamma_E - \ln 4\pi) \right] \ , \quad r_{a_{1;1}} = 2\beta_0 \ , \quad r_{a_{2;2}} = 4\beta_0^2 \ , \quad r_{a_{2;1}} = \beta_1$$

$$\beta_0 = \left(\frac{11}{3} C_A - \frac{4}{3} T_F n_f \right), \qquad \beta_1 = \left(\frac{34}{3} C_A^2 - \frac{20}{3} C_A T_F n_f - 4 C_F T_F n_f \right)$$

$$\frac{\hat{\lambda}}{\mu_0^\epsilon} S_\epsilon = \frac{\lambda}{\mu_R^\epsilon} \left[1 + a_s \left(\frac{1}{\epsilon} r_{\lambda_{1;1}} \right) + a_s^2 \left(\frac{1}{\epsilon^2} r_{\lambda_{2;2}} + \frac{1}{\epsilon} r_{\lambda_{2;1}} \right) + \mathcal{O}(a_s^3) \right]$$

$$r_{\lambda_{1;1}} = 6 C_F \ , \ r_{\lambda_{2;2}} = \left(18 C_F^2 + 6 \beta_0 C_F \right) , \ r_{\lambda_{2,1}} = \left(\frac{3}{2} C_F^2 + \frac{97}{6} C_F C_A - \frac{10}{3} C_F T_F n_f \right)$$

UV

- **Using the bare couplings**

$$
\begin{aligned}
|\mathcal{M}\rangle &= \frac{\lambda}{\mu_R^{\epsilon}}(a_s)^{\frac{1}{2}}\left(|\mathcal{M}^{(0)}\rangle + a_s|\mathcal{M}^{(1)}\rangle + a_s^2|\mathcal{M}^{(2)}\rangle + \mathcal{O}(a_s^3)\right)\\
|\mathcal{M}^{(0)}\rangle &= \left(\frac{1}{\mu_R^{\epsilon}}\right)^{\frac{1}{2}}|\hat{\mathcal{M}}^{(0)}\rangle\\
|\mathcal{M}^{(1)}\rangle &= \left(\frac{1}{\mu_R^{\epsilon}}\right)^{\frac{3}{2}}\left[\,|\hat{\mathcal{M}}^{(1)}\rangle + \mu_R^{\epsilon}\left(\frac{r_{a_1}}{2} + r_{\lambda_1}\right)|\hat{\mathcal{M}}^{(0)}\rangle\,\right]\\
|\mathcal{M}^{(2)}\rangle &= \left(\frac{1}{\mu_R^{\epsilon}}\right)^{\frac{5}{2}}\left[\,|\hat{\mathcal{M}}^{(2)}\rangle + \mu_R^{\epsilon}\left(\frac{3r_{a_1}}{2} + r_{\lambda_1}\right)|\hat{\mathcal{M}}^{(1)}\rangle\right.\\
&\qquad \left. + \mu_R^{2\epsilon}\left(\frac{r_{a_2}}{2} - \frac{r_{a_1}^2}{8} + \frac{r_{a_1}}{2}r_{\lambda_1} + r_{\lambda_2}\right)|\hat{\mathcal{M}}^{(0)}\rangle\,\right]
\end{aligned}
$$

$r_{a_1} = \left(\frac{1}{\epsilon}r_{a_{1;1}}\right)$, $r_{a_2} = \left(\frac{1}{\epsilon^2}r_{a_{2;2}} + \frac{1}{\epsilon}r_{a_{2;1}}\right)$, $r_{\lambda_1} = \left(\frac{1}{\epsilon}r_{\lambda_{1;1}}\right)$, $r_{\lambda_2} = \left(\frac{1}{\epsilon^2}r_{\lambda_{2;2}} + \frac{1}{\epsilon}r_{\lambda_{2;1}}\right)$

$|\hat{\mathcal{M}}^{(l)}\rangle$ **unrenormalised color-space vector represents the $l^{\rm th}$ loop amplitude**

Infrared factorisation: Catani's formula

- IR divergent structure of amplitudes well understood. Renormalised amplitudes

$$|\mathcal{M}^{(1)}\rangle = 2\,\mathbf{I}_b^{(1)}(\epsilon)\,|\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(1)fin}\rangle$$

$$|\mathcal{M}^{(2)}\rangle = 2\,\mathbf{I}_b^{(1)}(\epsilon)\,|\mathcal{M}^{(1)}\rangle + 4\,\mathbf{I}_b^{(2)}(\epsilon)\,|\mathcal{M}^{(0)}\rangle + |\mathcal{M}^{(2)fin}\rangle$$

- Universal substraction operators $\mathbf{I}^{(i)}$

$$\mathbf{I}_b^{(1)}(\epsilon) = \frac{1}{2}\frac{e^{-\frac{\epsilon}{2}\gamma_E}}{\Gamma(1+\frac{\epsilon}{2})}\Bigg\{\Big(\frac{4}{\epsilon^2}-\frac{3}{\epsilon}\Big)(C_A-2C_F)\Big[\Big(-\frac{s}{\mu_R^2}\Big)^{\frac{\epsilon}{2}}\Big]$$

$$+\Big(-\frac{4C_A}{\epsilon^2}+\frac{3C_A}{2\epsilon}+\frac{\beta_0}{2\epsilon}\Big)\Big[\Big(-\frac{t}{\mu_R^2}\Big)^{\frac{\epsilon}{2}}+\Big(-\frac{u}{\mu_R^2}\Big)^{\frac{\epsilon}{2}}\Big]\Bigg\},$$

$$\mathbf{I}_b^{(2)}(\epsilon) = -\frac{1}{2}\mathbf{I}_b^{(1)}(\epsilon)\Big[\mathbf{I}_b^{(1)}(\epsilon)-\frac{2\beta_0}{\epsilon}\Big] + \frac{e^{\frac{\epsilon}{2}\gamma_E}\,\Gamma(1+\epsilon)}{\Gamma(1+\frac{\epsilon}{2})}\Big[-\frac{\beta_0}{\epsilon}+K\Big]\,\mathbf{I}_b^{(1)}(2\epsilon)$$

$$+\Big(2\mathbf{H}_q^{(2)}(\epsilon)+\mathbf{H}_g^{(2)}(\epsilon)\Big)$$

Catani PLB427 (1998) 161; Sterman et. al. PLB552 (2003) 48

IR

$$\mathbf{H}_q^{(2)}(\epsilon) = \frac{1}{\epsilon}\Big\{C_A C_F\Big(-\frac{245}{432}+\frac{23}{16}\zeta_2-\frac{13}{4}\zeta_3\Big)+C_F^2\Big(\frac{3}{16}-\frac{3}{2}\zeta_2+3\zeta_3\Big)$$

$$+C_F n_f\Big(\frac{25}{216}-\frac{1}{8}\zeta_2\Big)\Big\}$$

$$\mathbf{H}_g^{(2)}(\epsilon) = \frac{1}{\epsilon}\Big\{C_A^2\Big(-\frac{5}{24}-\frac{11}{48}\zeta_2-\frac{1}{4}\zeta_3\Big)+C_A n_f\Big(\frac{29}{54}+\frac{1}{24}\zeta_2\Big)-\frac{1}{4}C_F n_f-\frac{5}{54}n_f^2\Big\}$$

$$K = \Big(\frac{67}{18}-\frac{\pi^2}{6}\Big)C_A-\frac{10}{9}T_F n_f$$

Born amplitude $|\mathcal{M}^{(0)}\rangle$ and the finite parts $|\mathcal{M}^{(l)fin}\rangle, l=1,2$ are process dependent, needs to be explicitly computed

l^{th} loop amplitude $|\mathcal{M}^{(l)}\rangle = 4\pi\, T^a_{ij}\{A_1^{(l)}\mathrm{T}_1 + A_2^{(l)}\mathrm{T}_2\}$

- Renormalised coefficients $A_m^{(l)}$ in terms of their bare counterparts $\hat{A}_m^{(l)}$

$$A_m^{(0)} = \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{1}{2}} \hat{A}_m^{(0)}$$

$$A_m^{(1)} = \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{3}{2}} \Big[\, \hat{A}_m^{(1)} + \mu_R^\epsilon \Big(\frac{r_{a_1}}{2} + r_{\lambda_1}\Big) \hat{A}_m^{(0)} \,\Big]$$

$$A_m^{(2)} = \left(\frac{1}{\mu_R^\epsilon}\right)^{\frac{5}{2}} \Big[\, \hat{A}_m^{(2)} + \mu_R^\epsilon \Big(\frac{3 r_{a_1}}{2} + r_{\lambda_1}\Big) \hat{A}_m^{(1)} + \mu_R^{2\epsilon} \left(\frac{r_{a_2}}{2} - \frac{r_{a_1}^2}{8} + \frac{r_{a_1}}{2} r_{\lambda_1} + r_{\lambda_2}\right) \hat{A}_m^{(0)} \,\Big]$$

- Subtracting terms proportional to universal part $I_b^{(l)}$, Finite parts of $A_m^{(l)}$

$$A_m^{(1)} = 2\, \mathrm{I}_b^{(1)}(\epsilon)\ A_m^{(0)} + A_m^{(1)fin}$$

$$A_m^{(2)} = 2\, \mathrm{I}_b^{(1)}(\epsilon)\ A_m^{(1)} + 4\, \mathrm{I}_b^{(2)}(\epsilon)\ A_m^{(0)} + A_m^{(2)fin}$$

- IR poles structure agree exactly, providing a crucial test of our computation

Finite parts of $A_m^{(l)fin}$

$$A_m^{(l)fin} = \sum_{n=0}^{l} A_m^{(0)} \, \mathcal{B}_{m;n}^{(l)} \ln^n\left(-\frac{Q^2}{\mu^2}\right)$$

$$A_1^{(0)} = -\frac{4i}{t\,u} \qquad \text{and} \qquad A_2^{(0)} = i\left(\frac{1}{t} + \frac{1}{u}\right)$$

- Coefficients $\mathcal{B}_{m;n}^{(l)}$ are given in the paper.
- Without using projectors, we independently computed $\langle \mathcal{M}^{(0)} | \mathcal{M}^{(l)} \rangle$ for $l = 1, 2$, as an additional check
- Corresponding coefficients for Higgs+ 1 jet production at hadrons colliders are also provided in the arXiv submission using analytically continued HPLs and 2d HPLs

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Summary

- **The potential sub dominant contribution to Higgs + 1 jet production at LHC comes from $b + \bar{b} \to H + g$**
- **Amplitudes for the partonic process $H \to b + \bar{b} + g$ and processes related by crossing are presented to 2-loop level in QCD**
- **The crossing processes, contribute to exclusive observables involving Higgs boson and a jet**
- **The IR structure of the amplitudes are in accordance with the prediction of Catani upto two loop level**