

The bottom-quark mass from non-relativistic sum rules at NNNLO

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Radcor-Loopfest 2015

Why precision?

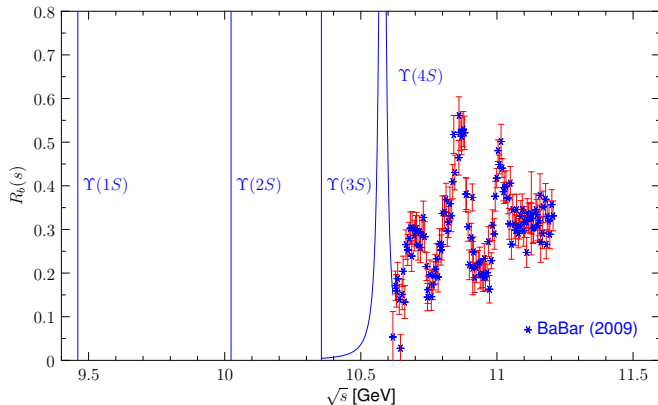
Higgs branching ratios at ILC:

mode	BR	$\sigma \cdot BR$ (fb)	$N_{evt}/250 \text{ fb}^{-1}$	$\Delta(\sigma BR)/(\sigma BR)$	$\Delta BR/BR$
$h \rightarrow b\bar{b}$	65.7%	232.8	58199	1.0%	2.7%
$h \rightarrow c\bar{c}$	3.6%	12.7	3187	6.9%	7.3%
$h \rightarrow gg$	5.5%	19.5	4864	8.5%	8.9%
$h \rightarrow WW^*$	15.0%	53.1	13281	8.1%	8.5%
$h \rightarrow \tau^+\tau^-$	8.0%	28.2	7050	3.6%	4.4%
$h \rightarrow ZZ^*$	1.7%	6.1	1523	26%	26%
$h \rightarrow \gamma\gamma$	0.29%	1.02	255	23-30%	23-30%

[ILC TDR 2013]

$$\Gamma(h \rightarrow b\bar{b}) \propto m_b^2$$

Bottom production at e^+e^- colliders



$$R_b(s) = \frac{\sigma(e^+e^- \rightarrow b\bar{b}X)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

Non-relativistic sum rules

- Determine bottom mass from moments:

$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \frac{R_b(s)}{s^{n+1}}$$

$$\mathcal{M}_n^{\text{th}} = \mathcal{M}_n^{\text{exp}}$$

Non-relativistic sum rules

- ▶ Determine bottom mass from moments:

$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \frac{R_b(s)}{s^{n+1}}$$
$$\mathcal{M}_n^{\text{th}} = \mathcal{M}_n^{\text{exp}}$$

- ▶ large $n \sim 10$:
 - + good data
 - + sensitivity to m_b
 - sensitivity to non-perturbative structure

Need accurate description of **non-relativistic bound states**

Effective theory framework

Scales: $m_b \gg m_b v \gg m_b v^2 > \Lambda_{\text{QCD}}$

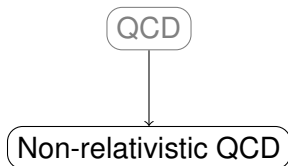
- ▶ **hard modes:** $k \sim m_b$
- ▶ **soft modes:** $k \sim m_b v$
- ▶ **potential modes:** $k_0 \sim m_b v^2, \vec{k} \sim m_b v$
- ▶ **ultrasoft modes:** $k \sim m_b v^2$

QCD

Effective theory framework

Scales: $m_b \gg m_b v \gg m_b v^2 > \Lambda_{\text{QCD}}$

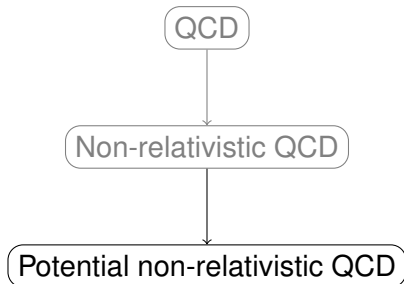
- ▶ hard modes: $k \sim m_b \rightarrow$ (local) effective vertices
- ▶ soft modes: $k \sim m_b v$
- ▶ potential modes: $k_0 \sim m_b v^2, \vec{k} \sim m_b v$
- ▶ ultrasoft modes: $k \sim m_b v^2$



Effective theory framework

Scales: $m_b \gg m_b v \gg m_b v^2 > \Lambda_{\text{QCD}}$

- ▶ hard modes: $k \sim m_b \rightarrow$ (local) effective vertices
- ▶ soft modes: $k \sim m_b v \rightarrow$ (non-local) potentials
- ▶ potential light particle modes \rightarrow (non-local) potentials
- ▶ potential bottom quark modes: $k_0 \sim m_b v^2, \vec{k} \sim m_b v$
- ▶ ultrasoft modes: $k \sim m_b v^2$



Potential non-relativistic QCD

[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger(x) \left(i\partial_0 + \frac{\partial^2}{2m} \right) \psi(x) \\ & + \int d^3\mathbf{r} [\psi^\dagger\psi](x + \mathbf{r}) \left(-\frac{C_F\alpha_s}{r} \right) [\chi^\dagger\chi](x) \\ & + \mathcal{L}_{\text{anti-quark}} + \{\text{NLO}\}\end{aligned}$$

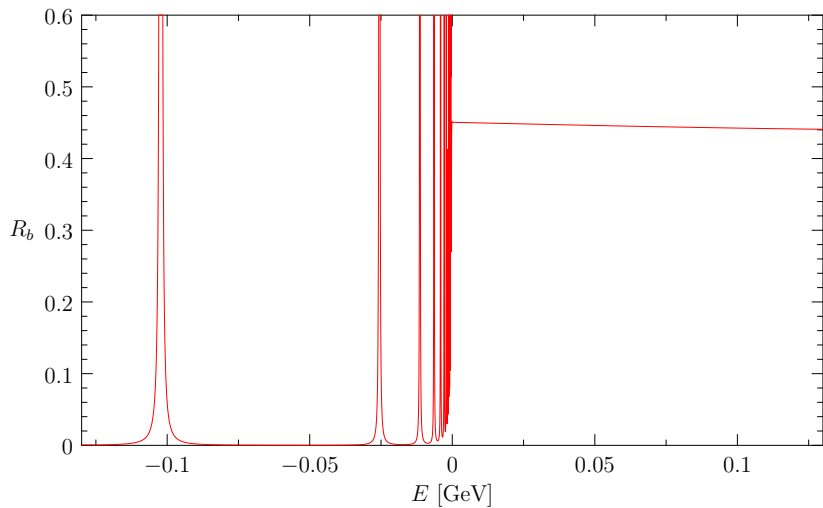
- ▶ Power counting: $\alpha_s \sim v$ ($\sim 1/\sqrt{n}$)
- ▶ Equation of motion $\hat{=}$ Coulomb Schrödinger equation



- ▶ Poles of propagator $\hat{=}$ bound states

Potential non-relativistic QCD

R_b at leading order



Potential non-relativistic QCD

NNNLO

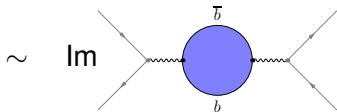
$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger \left(i\partial_0 + g_s A_0(t, \mathbf{0}) + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3} \right) \psi \\ & + \int d^3\mathbf{r} [\psi^\dagger \psi](x + \mathbf{r}) \left(-\frac{C_F \alpha_s}{r} + \delta V(r) \right) [\chi^\dagger \chi](x) \\ & - g_s \psi^\dagger \mathbf{x} \cdot \mathbf{E}(t, \mathbf{0}) \psi \\ & + \mathcal{L}_{\text{anti-quark}} + \mathcal{L}_{\text{gluon}} + \{\text{N}^4\text{LO}\}\end{aligned}$$

Beyond LO:

- ▶ Corrections to kinetic energy
- ▶ Corrections to potential [Anzai, Kiyo, Sumino 09; Smirnov, Smirnov, Steinhauser 09]
- ▶ Ultrasoft gluons

Calculating $b\bar{b}$ production

$$R_b \propto \int \left| \begin{array}{c} e^- \\ \nearrow \\ \bullet \\ \nwarrow \\ e^+ \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \bar{b} \\ \nwarrow \\ \bullet \\ \nearrow \\ b \end{array} \right|^2$$



Calculating $b\bar{b}$ production

$$R_b \propto \int \left| \begin{array}{c} e^- \\ \swarrow \\ \bullet \\ \nearrow \\ e^+ \end{array} \text{---} \text{wavy} \text{---} \begin{array}{c} \bar{b} \\ \swarrow \\ \triangle \\ \nearrow \\ b \end{array} \right|^2 \sim \text{Im} \left[\begin{array}{c} \bar{b} \\ \swarrow \\ \bullet \\ \nearrow \\ b \end{array} \right]$$

Effective coupling to photon:

$$\vec{j}_i^v = c_v \psi^\dagger \sigma_i \chi + \frac{d_v}{6m_b^2} \psi^\dagger \sigma_i \mathbf{D}^2 \chi$$

[Marquard, Piclum, Seidel, Steinhauser 14]

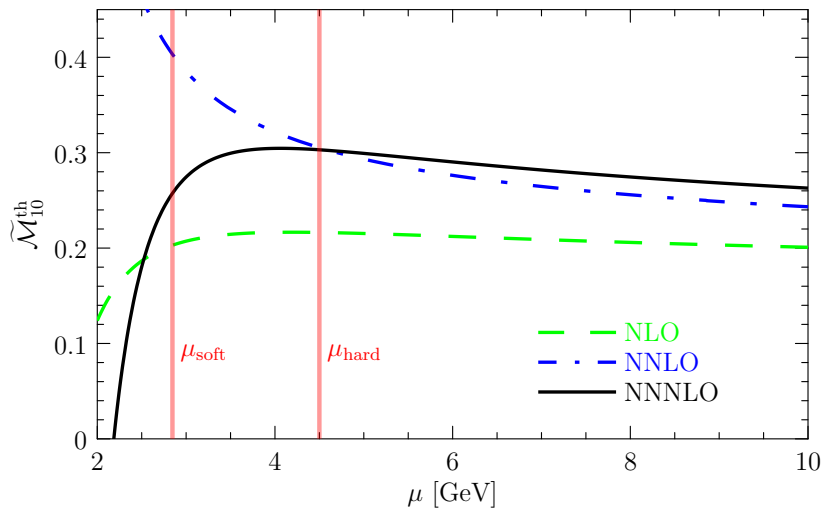
Cross section in PNRQCD at NNNLO:

$$\begin{array}{c} \begin{array}{c} \bar{b} \\ \swarrow \\ \bullet \\ \nearrow \\ b \end{array} \\ \text{---} \text{wavy} \end{array} = \begin{array}{c} \text{---} \text{wavy} \text{---} \begin{array}{c} \bullet \\ \text{---} \text{green} \text{---} \bullet \end{array} \\ c_v, d_v \end{array} + \begin{array}{c} \text{---} \text{wavy} \text{---} \begin{array}{c} \bullet \\ \text{---} \text{green} \text{---} \text{yellow} \text{---} \bullet \end{array} \\ c_v, d_v, \delta V \end{array} + \begin{array}{c} \text{---} \text{wavy} \text{---} \begin{array}{c} \bullet \\ \text{---} \text{green} \text{---} \text{yellow} \text{---} \text{yellow} \text{---} \bullet \end{array} \end{array} \\ + \begin{array}{c} \text{---} \text{wavy} \text{---} \begin{array}{c} \bullet \\ \text{---} \text{green} \text{---} \text{yellow} \text{---} \text{yellow} \text{---} \text{yellow} \text{---} \bullet \end{array} \end{array} + \begin{array}{c} \text{---} \text{wavy} \text{---} \begin{array}{c} \bullet \\ \text{---} \text{green} \text{---} \text{pink} \text{---} \bullet \end{array} \end{array} + \dots$$

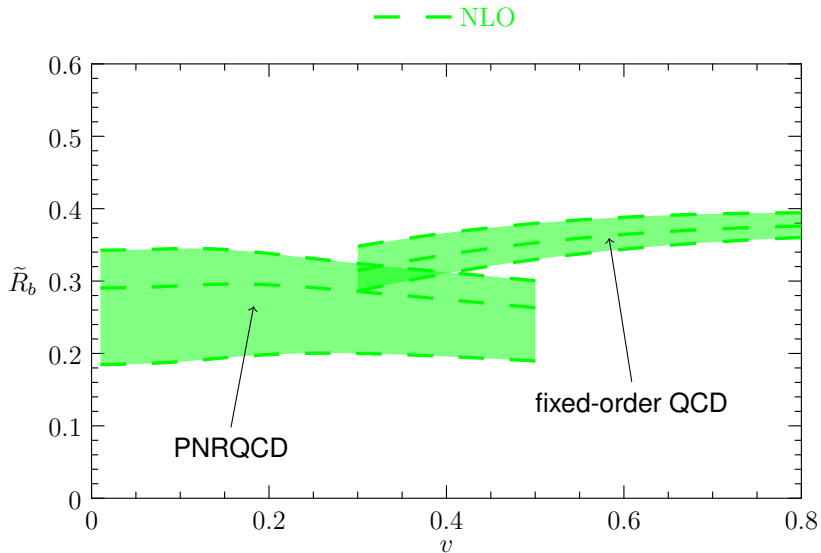
[Beneke, Kiyo, Kniehl, Schuller, Penin, ... 02-14]

Results

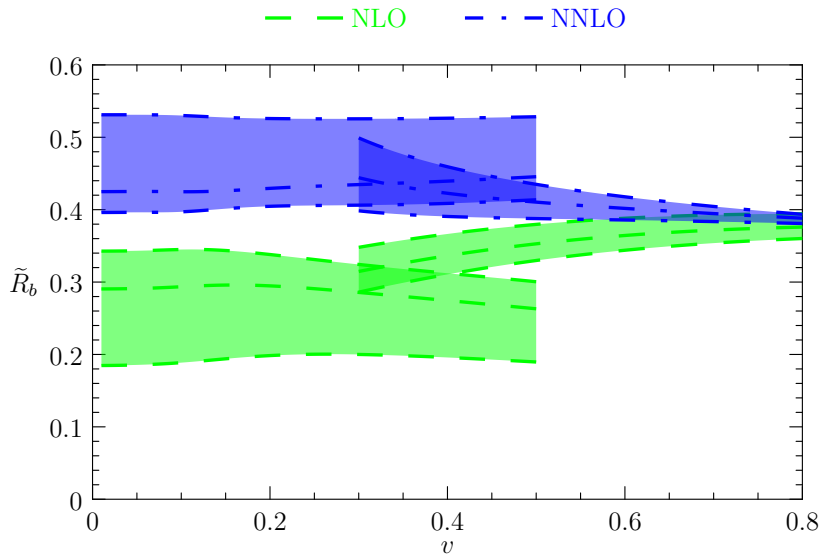
Choosing the scale



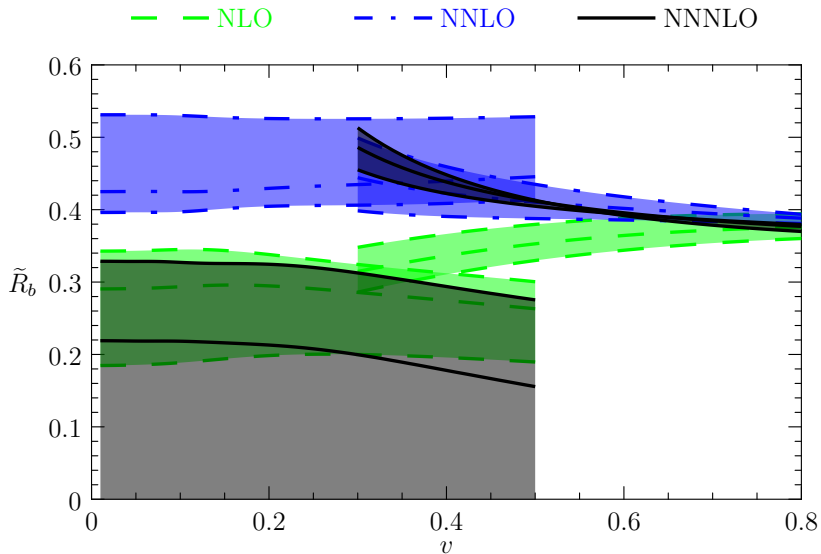
Continuum cross section



Continuum cross section

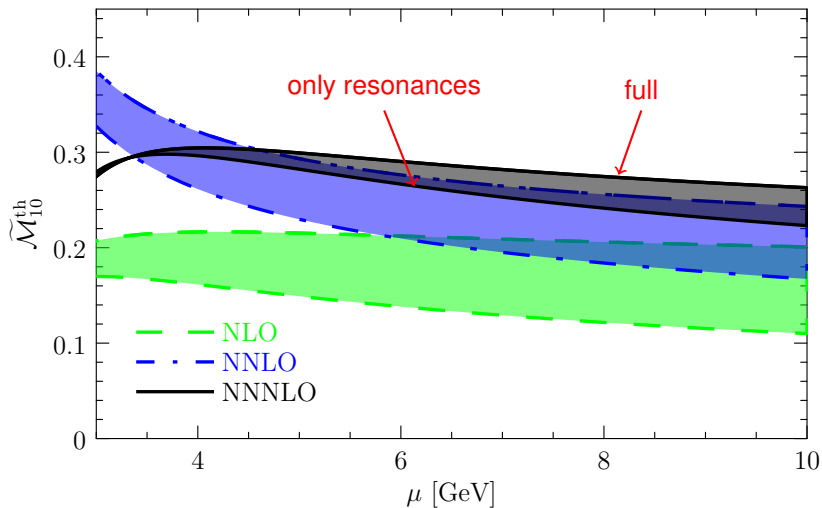


Continuum cross section



Continuum cross section

Impact on $\mathcal{M}_{10}^{\text{th}}$



Charm effects

- ▶ $m_c \sim m_b v$: **soft** \Rightarrow Correction to potential

$$\delta V_{m_c} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

The diagram shows three Feynman diagrams representing corrections to the potential δV_{m_c} . Each diagram consists of two horizontal green lines representing quark lines, with four vertices marked by green dots. The diagrams are connected by plus signs and followed by an ellipsis. Diagram 1: A vertical gluon line (represented by a vertical line with small circles) connects the two vertices on each horizontal line. An orange loop with arrows is attached to the top vertex of the gluon line. Diagram 2: A vertical gluon line connects the two vertices on each horizontal line. A green loop with arrows is attached to the bottom vertex of the gluon line. Diagram 3: A vertical gluon line connects the two vertices on each horizontal line. An orange loop with arrows is attached to the top vertex of the gluon line, and a horizontal gluon line with arrows connects the two vertices of this loop.

[Melles 00]

- ▶ Expect significant mass shift $\sim -30 \text{ MeV}$ [Hoang 00]

Charm effects

- ▶ $m_c \sim m_b v$: **soft** \Rightarrow Correction to potential

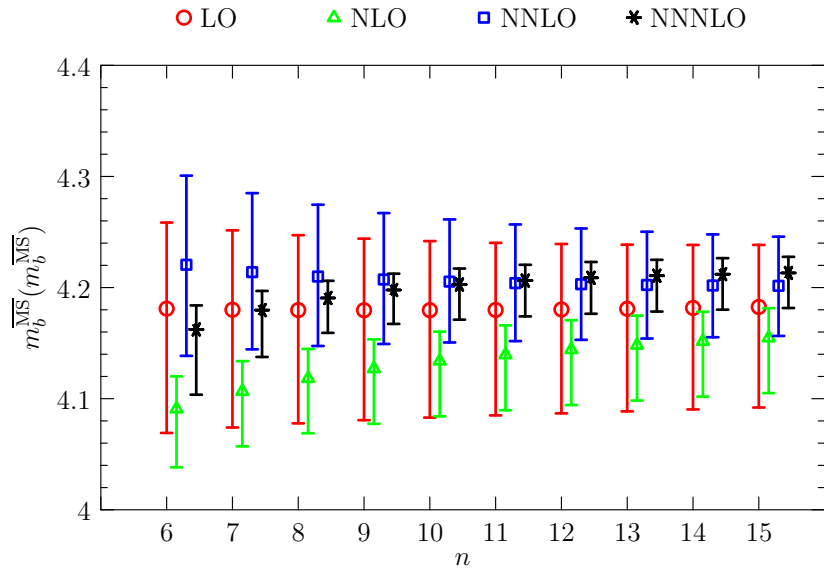
$$\delta V_{m_c} = \text{[Diagram 1]} + \text{[Diagram 2]} + \text{[Diagram 3]} + \dots$$

[Melles 00]

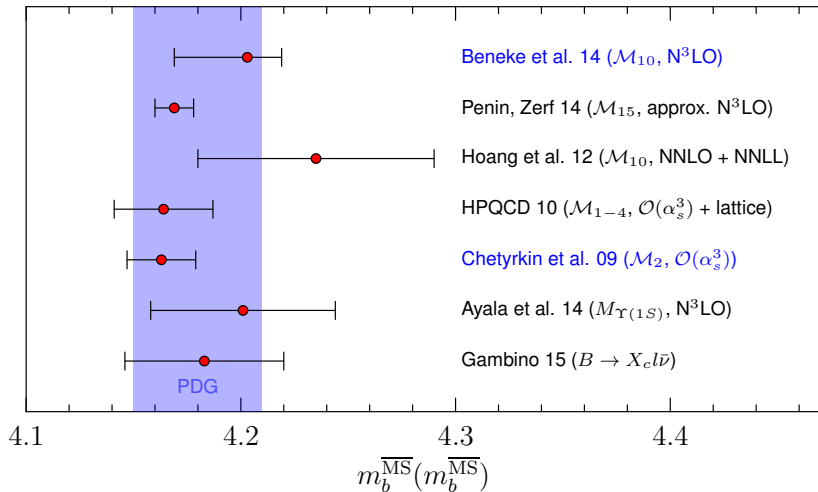
- ▶ Expect significant mass shift $\sim -30 \text{ MeV}$ [Hoang 00]

We find $\sim -3 \text{ MeV}$ (at NNLO)

The bottom quark mass



The bottom quark mass



Conclusions

- ▶ Precise bottom quark mass from non-relativistic sum rules at NNNLO:

$$m_b^{\text{PS}}(2 \text{ GeV}) = 4.532_{-0.039}^{+0.013} \text{ GeV}$$

$$m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = 4.203_{-0.034}^{+0.016} \text{ GeV}$$

- ▶ Uncertainty dominated by theory
- ▶ More work needed for $m_b^{\overline{\text{MS}}}(m_H)$: α_s, β_4

Backup

Expansion in kinetic energy

$$\mathcal{M}_n^{\text{th}} = \frac{12\pi^2 n_c e_b^2}{m_b^2} \sum_{N=1}^{\infty} \frac{Z_N}{(2m_b + E_N)^{2n+1}} + \int_{4m_b^2}^{\infty} ds \frac{R_b(s)}{s^{n+1}}$$

Expansion in kinetic energy

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From $\frac{4}{s} = \frac{1}{m_b^2} \left(1 - \frac{E}{m} + \dots\right)$

$$\tilde{\mathcal{M}}_n^{\text{th}} = 48\pi^2 n_c e_b^2 \sum_{N=1}^{\infty} \frac{\tilde{Z}_N}{(2m_b + E_N)^{2n+3}} + \int_{4m_b^2}^{\infty} ds \frac{\tilde{R}_b(s)}{s^{n+1}}$$

Expansion in kinetic energy

$$\mathcal{M}_n^{\text{th}} = \frac{12\pi^2 n_c e_b^2}{m_b^2} \sum_{N=1}^{\infty} \frac{Z_N}{(2m_b + E_N)^{2n+1}} + \int_{4m_b^2}^{\infty} ds \frac{R_b(s)}{s^{n+1}}$$

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$$\widetilde{\mathcal{M}}_n^{\text{th}} = 48\pi^2 n_c e_b^2 \sum_{N=1}^{\infty} \frac{\widetilde{Z}_N}{(2m_b + E_N)^{2n+3}} + \int_{4m_b^2}^{\infty} ds \frac{\widetilde{R}_b(s)}{s^{n+1}}$$

- ▶ Difference formally N⁴LO, but shifts mass by **26 MeV**
- ▶ Possibly subleading renormalons?

$$E_N = E_N^{\text{LO}} + E_N^{(1)} + E_N^{(2)} + \dots, \quad E_N^{(i)} \approx \Lambda_{\text{QCD}}$$

$$\frac{E_N}{m_b} = \frac{E_N^{\text{LO}}}{m_b} + \frac{E_N^{(1)}}{m_b} + \underbrace{\frac{E_N^{(2)}}{m_b} + \dots}_{\text{beyond NNNLO}}$$

but $\sim \Lambda_{\text{QCD}}/m_b$

The bottom quark mass

Low vs. high moments

