The bottom-quark mass from non-relativistic sum rules at NNNLO

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Why precision?

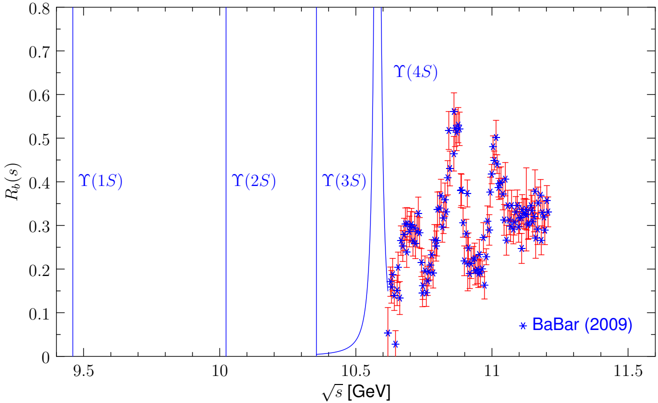
Higgs branching ratios at ILC:

mode	BR	$\sigma \cdot BR\,(\text{fb})$	$N_{evt}/250\,\text{fb}^{-1}$	$\Delta(\sigma\,BR)/(\sigma\,BR)$	$\Delta BR/BR$
$h \to b\bar{b}$	65.7%	232.8	58199	1.0%	2.7%
$h \to c\bar{c}$	3.6%	12.7	3187	6.9%	7.3%
$h \to gg$	5.5%	19.5	4864	8.5%	8.9%
$h \to WW^*$	15.0%	53.1	13281	8.1%	8.5%
$h \to \tau^+\tau^-$	8.0%	28.2	7050	3.6%	4.4%
$h \to ZZ^*$	1.7%	6.1	1523	26%	26%
$h \to \gamma\gamma$	0.29%	1.02	255	23-30%	23-30%

[ILC TDR 2013]

$$\Gamma(h \to b\bar{b}) \propto m_b^2$$

Bottom production at e^+e^- colliders



$$R_b(s) = \frac{\sigma(e^+e^- \to b\bar{b}\,X)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Non-relativistic sum rules

- Determine bottom mass from moments:

$$\mathcal{M}_n = \int_{s_0}^{\infty} ds \, \frac{R_b(s)}{s^{n+1}}$$

$$\mathcal{M}_n^{\text{th}} = \mathcal{M}_n^{\text{exp}}$$

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- large $n \sim 10$:
  - $+$ good data
  - $+$ sensitivity to m_b
  - $-$ sensitivity to non-perturbative structure

Need accurate description of non-relativistic bound states

Effective theory framework

Scales: $m_b \gg m_b v \gg m_b v^2 > \Lambda_{\mathrm{QCD}}$

- hard modes: $k \sim m_b$
- soft modes: $k \sim m_b v$
- potential modes: $k_0 \sim m_b v^2, \vec{k} \sim m_b v$
- ultrasoft modes: $k \sim m_b v^2$

QCD

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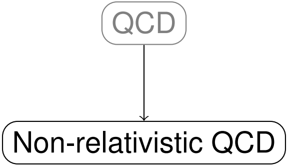

Effective theory framework

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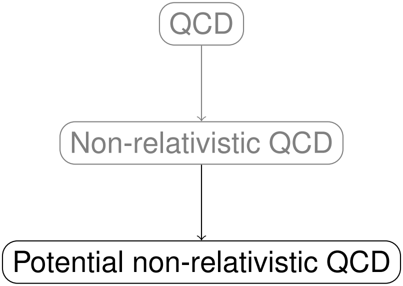
- hard modes: $k \sim m_b \rightarrow$ (local) effective vertices
- soft modes: $k \sim m_b v \rightarrow$ (non-local) potentials
- potential light particle modes $\rightarrow$ (non-local) potentials
- potential bottom quark modes: $k_0 \sim m_b v^2, \vec{k} \sim m_b v$
- ultrasoft modes: $k \sim m_b v^2$

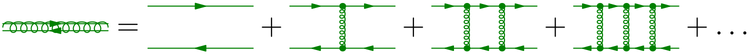


Potential non-relativistic QCD

[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger(x)\left(i\partial_0 + \frac{\partial^2}{2m}\right)\psi(x) + \int d^3\mathbf{r}\,[\psi^\dagger\psi](x+\mathbf{r})\left(-\frac{C_F\alpha_s}{r}\right)[\chi^\dagger\chi](x) + \mathcal{L}_{\text{anti-quark}} + \{\text{NLO}\}$$

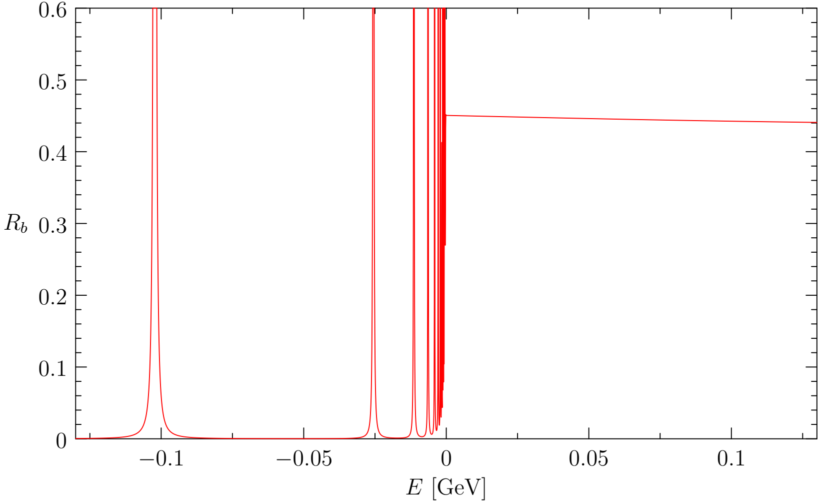
- Power counting: $\alpha_s \sim v\ (\sim 1/\sqrt{n})$
- Equation of motion $\hat{=}$ Coulomb Schrödinger equation



- Poles of propagator $\hat{=}$ bound states

Potential non-relativistic QCD

R_b at leading order



Potential non-relativistic QCD

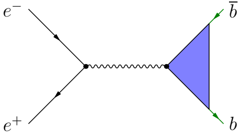
NNNLO

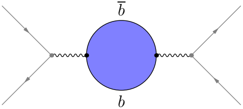
$$
\begin{aligned}
\mathcal{L}_{\text{PNRQCD}} = {} & \psi^\dagger\left(i\partial_0 + g_s A_0(t,\mathbf{0}) + \frac{\partial^2}{2m} + \frac{\partial^4}{8m^3}\right)\psi \\
& + \int d^3\mathbf{r}\,[\psi^\dagger\psi](x+\mathbf{r})\left(-\frac{C_F\alpha_s}{r} + \delta V(r)\right)[\chi^\dagger\chi](x) \\
& - g_s\psi^\dagger\mathbf{x}\cdot\mathbf{E}(t,\mathbf{0})\psi \\
& + \mathcal{L}_{\text{anti-quark}} + \mathcal{L}_{\text{gluon}} + \{\text{N}^4\text{LO}\}
\end{aligned}
$$

Beyond LO:

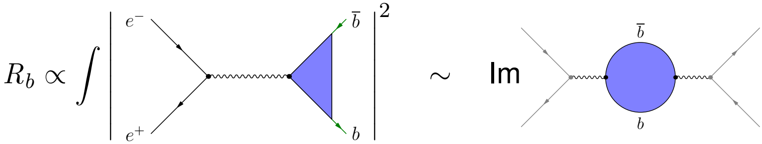
- Corrections to kinetic energy
- Corrections to potential [Anzai, Kiyo, Sumino 09; Smirnov, Smirnov, Steinhauser 09]
- Ultrasoft gluons

Calculating $b\overline{b}$ production

$R_b \propto \int \Bigg|$ 

$\Bigg|^2 \sim \text{Im}$ 

Calculating $b\bar{b}$ production

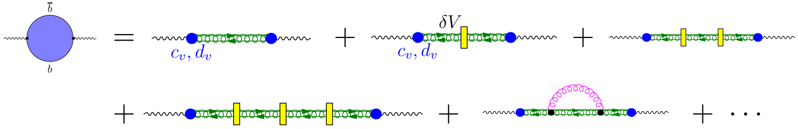


Effective coupling to photon:

$$j_i^v = c_v \psi^\dagger \sigma_i \chi + \frac{d_v}{6m_b^2} \psi^\dagger \sigma_i \mathbf{D}^2 \chi$$

[Marquard, Piclum, Seidel, Steinhauser 14]

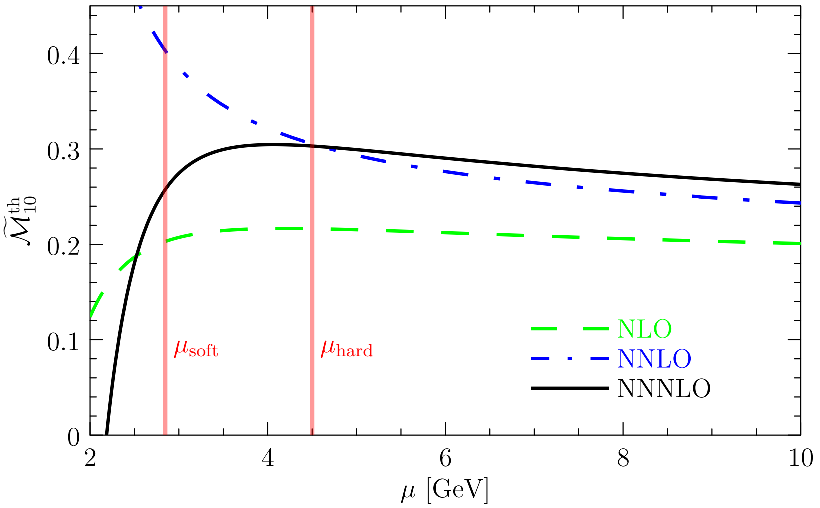
Cross section in PNRQCD at NNNLO:



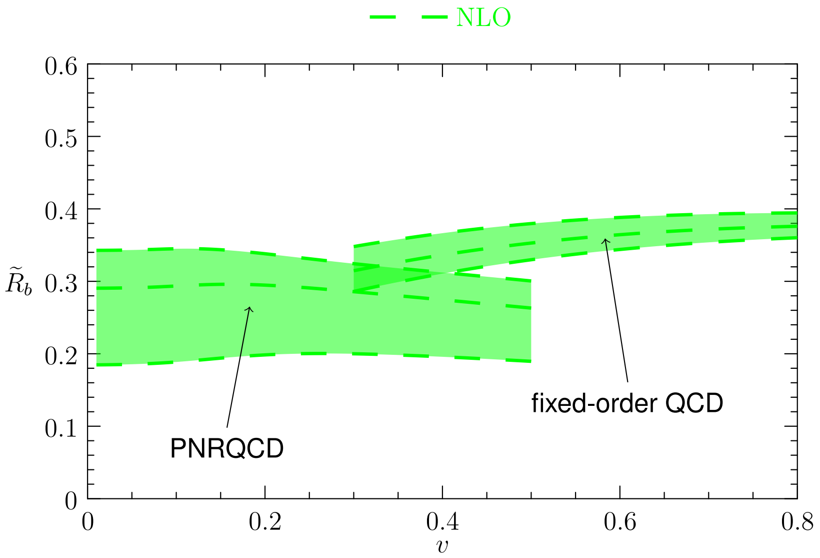
[Beneke, Kiyo, Kniehl, Schuller, Penin, . . . 02-14]

Results

Choosing the scale



Continuum cross section



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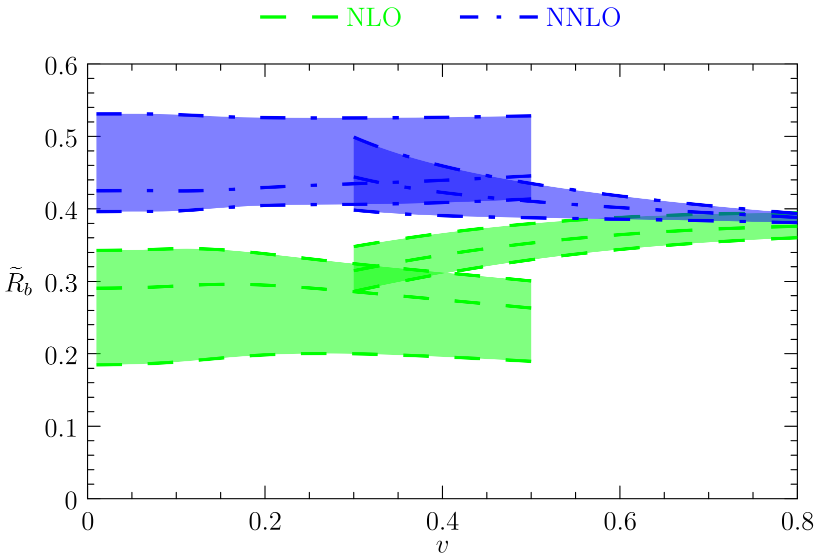

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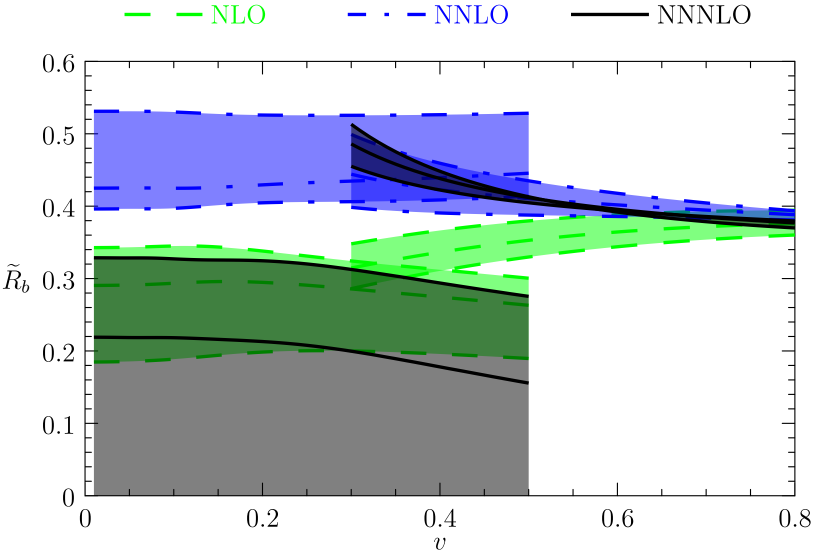
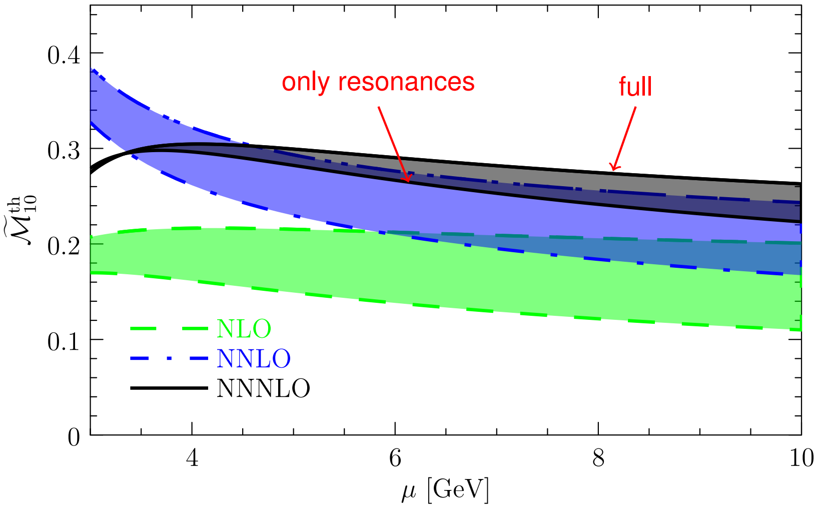


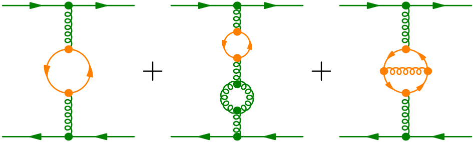
Continuum cross section

Impact on $\mathcal{M}_{10}^{\text{th}}$



Charm effects

- $m_c \sim m_b v$: soft $\Rightarrow$ Correction to potential

$$\delta V_{m_c} =$$  $$+ \cdots$$

[Melles 00]

- Expect significant mass shift $\sim -30\,\text{MeV}$ [Hoang 00]

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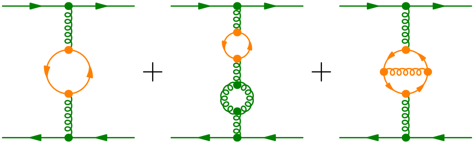


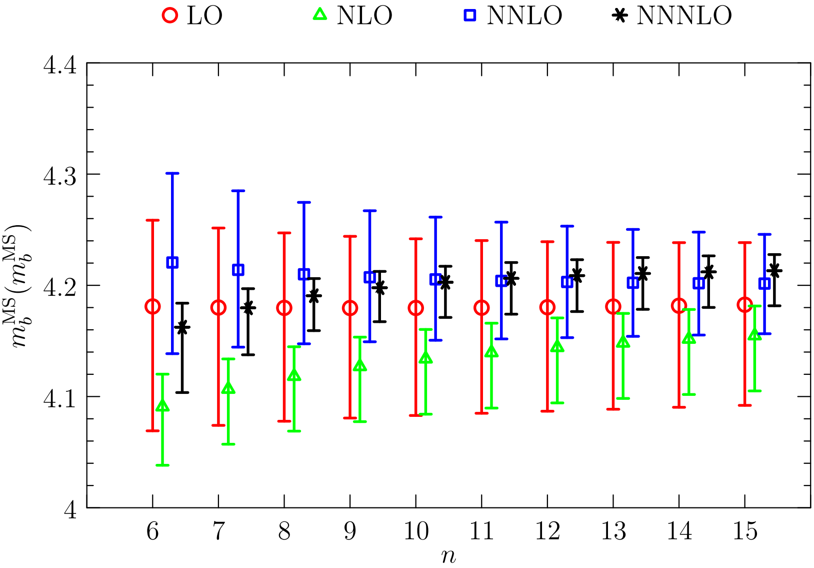
$+ \cdots$

[Melles 00]

- Expect significant mass shift $\sim -30\,$MeV [Hoang 00]

We find $\sim -3\,$MeV (at NNLO)

The bottom quark mass



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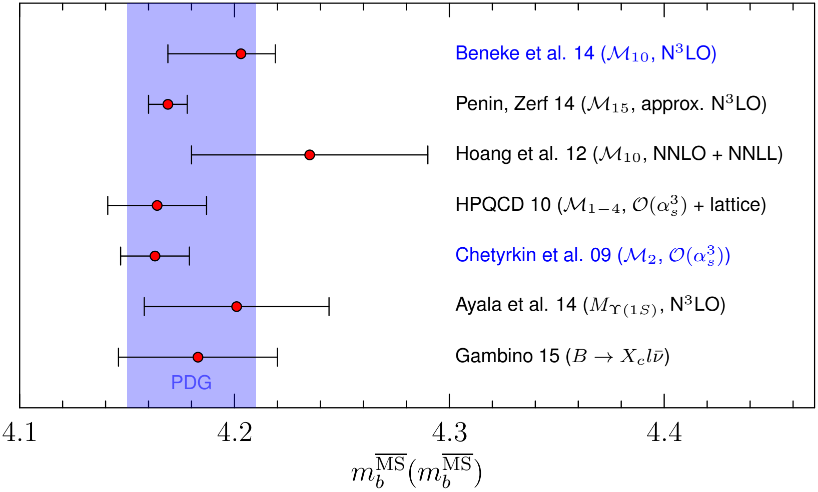

Conclusions

- Precise bottom quark mass from non-relativistic sum rules at NNNLO:

$$m_b^{\text{PS}}(2\,\text{GeV}) = 4.532^{+0.013}_{-0.039}\,\text{GeV}$$
$$m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) = 4.203^{+0.016}_{-0.034}\,\text{GeV}$$

- Uncertainty dominated by theory
- More work needed for $m_b^{\overline{\text{MS}}}(m_H)$: α_s, β_4

Backup

Expansion in kinetic energy

$$\mathcal{M}_n^{\text{th}} = \frac{12\pi^2 n_c e_b^2}{m_b^2} \sum_{N=1}^{\infty} \frac{Z_N}{(2m_b + E_N)^{2n+1}} + \int_{4m_b^2}^{\infty} ds\, \frac{R_b(s)}{s^{n+1}}$$

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From $\frac{4}{s} = \frac{1}{m_b^2}\left(1 - \frac{E}{m} + \ldots\right)$

$$\widetilde{\mathcal{M}}_n^{\text{th}} = 48\pi^2 n_c e_b^2 \sum_{N=1}^{\infty} \frac{\widetilde{Z}_N}{(2m_b + E_N)^{2n+3}} + \int_{4m_b^2}^{\infty} ds \, \frac{\widetilde{R}_b(s)}{s^{n+1}}$$

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- Difference formally N^4LO, but shifts mass by $26\,\text{MeV}$
- Possibly subleading renormalons?

$$E_N = E_N^{\text{LO}} + E_N^{(1)} + E_N^{(2)} + \dots, \qquad E_N^{(i)} \approx \Lambda_{\text{QCD}}$$

$$\frac{E_N}{m_b} = \frac{E_N^{\text{LO}}}{m_b} + \frac{E_N^{(1)}}{m_b} + \underbrace{\frac{E_N^{(2)}}{m_b} + \dots}_{\substack{\text{beyond NNNLO} \\ \text{but } \sim\Lambda_{\text{QCD}}/m_b}}$$

The bottom quark mass

Low vs. high moments