Threshold Resummation for Hadronic Collisions With 3 Particles in the Final State

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Introduction

- Precision predictions for processes with more than 2 legs in the final state are of unquestionable importance for the LHC physics
- With higher-order fixed-order calculations not always feasible, resummation methods offer systematic improvements
- For the scattering processes relevant at the LHC resummed predictions obtained with standard (direct QCD) techniques predominantly applied to hard-scattering processes of at most 2→2 type
- Recently, progress in automatization of resummation for processes which involve up to 5 hard partons in the final state *[Gerwick, Schumann, Höche, Marzani'15]*, based on the CAESAR approach *[Banfi, Salam, Zanderighi'02-10]*→ event shapes
- N-jet cross sections resummed in SCET *[Stewart, Tackmann, Waalewijn, Kelley, Schwartz, Berger, Marcantonini, Liu, Mantry, Petriello, Alioli, Bauer, Berggren, Hornig, Vermilion, Walsh, Zuberi, …]*

General Formalism

- Guideline: general formalism developed for 2→2 *[Kidonakis, Oderda, Sterman'98], [Laenen, Oderda, Sterman'98]*

Distance to production threshold measured with dimensionless infra-red safe weight *w* *[Contopanagos, Laenen, Sterman'96]*

For 2→2 we have e.g.

PIM: pair-invariant mass kinematics

$$w_{PIM} = 1 - z = 1 - \frac{M^2}{\hat{s}}$$

1PI: 1-particle inclusive kinematics

$$w_{1PI} = \frac{s_4}{\hat{s}} = \frac{s + t + u}{\hat{s}}$$

(for massless particles/jets in the final state)

General Formalism

Factorization principle

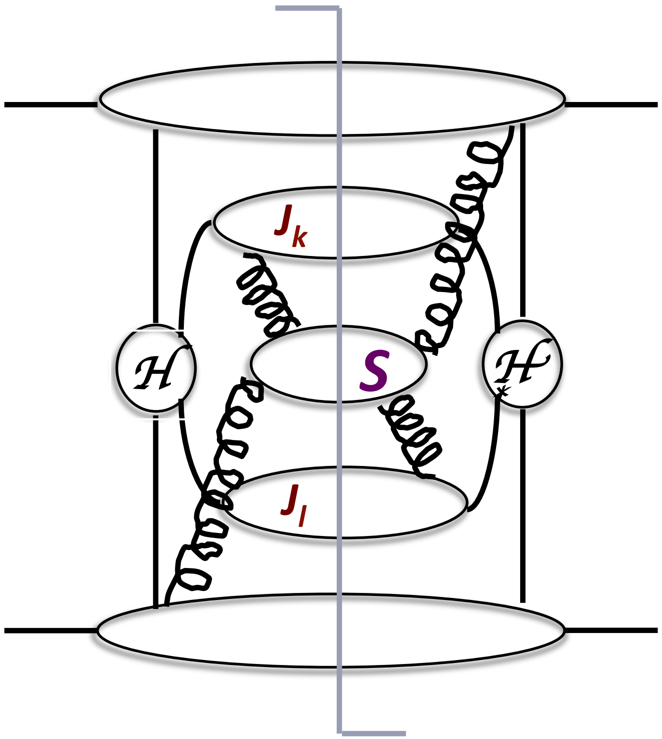
$$\hat{\sigma}_{ab\to kl} = H_{IJ} \otimes E_a \otimes E_b \otimes S_{JI} \otimes J_k \otimes J_l$$

realized via (valid near threshold)

$$W = w_a c_a + w_b c_b + w_s + w_k + w_l$$

total weight

individual weights for each of the factorized functions, vanish at threshold



PIM: $c_a = c_b = 1$

1PI: $c_a = \frac{u}{t+u}$ $\quad c_b = \frac{t}{t+u}$

General Formalism

$$\frac{d\sigma_{AB\to kl}}{d\Pi} = \sum_{a,b} \int dx_a dx_b\, f_{a/A}(x_a^2, \mu^2) f_{b/B}(x_b^2, \mu^2)\, \Omega_{ab\to kl}(w, \hat{\Pi}, \mu^2, \{m^2\})$$

$$\frac{d\sigma_{ab\to kl}}{d\hat{\Pi}} = H_{ab\to kl,IJ} \int dw_a\, dw_b\, dw_s\, dw_k\, dw_l\, \delta(W - c_a w_a - c_b w_b - w_s - w_k - w_l)$$
$$\psi_{a/a}(w_a, Q/\mu, \dots)\psi_{b/b}(w_b, Q/\mu, \dots) S_{ab\to kl,JI}(w_s, Q/\mu, \dots) J_k(w_k, Q/\mu, \dots) J_l(w_l, Q/\mu, \dots)$$

Laplace transform: $\tilde{F}(N) = \int_0^\infty d\omega\ \mathrm{e}^{-N\omega} F(\omega)$

incoming collinear radiation

$$\tilde{\Omega}_{ab\to kl}(N, Q/\mu, \dots) = H_{ab\to kl,IJ} \frac{\tilde{\psi}_{a/a}(c_a N, Q/\mu, \dots)\tilde{\psi}_{b/b}(c_b N, Q/\mu, \dots)}{\tilde{f}_{a/a}(c_a N, Q/\mu, \dots)\tilde{f}_{b/b}(c_b N, Q/\mu, \dots)}$$
$$\tilde{S}_{ab\to kl,JI}(N, Q/\mu, \dots) J_k(N, Q/\mu, \dots) J_l(N, Q/\mu, \dots)$$

soft wide-angle emission

outgoing collinear radiation

General Formalism For 2→3

- Factorization principle holds for any number of jets/particles in the final state *[Kidonakis, Oderda, Sterman'98][Bonciani, Catani, Mangano, Nason'03]* but adding one more particle/jet requires adjusting for
  - colour structure of the underlying hard scattering, if more than three coloured partons participating: affects hard *H* and soft *S* functions

$$\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)S^{(N)}_{JI}=-\Gamma^{\dagger}_{JK}S(N)_{KI}-S(N)_{JL}\Gamma_{LI}$$

→ soft anomalous dimension

  - more complicated kinematics: affects *H*, *S* (anomalous dimension) and the arguments of incoming jet functions (coefficients c_a, c_b)

Threshold and Kinematics

- 3-particle invariant mass kinematics

$$w = 1 - \frac{(p_3 + p_4 + p_5)^2}{\hat{s}} \qquad c_a = c_b = 1$$

→ Invariant mass of the final state

- 1-particle inclusive kinematics

$$w = \frac{(p_1 + p_2 - p_5)^2 - s_{34}}{\hat{s}} \qquad c_{a,b} = c_{a,b}(s_{34}, t_{15}, t_{25})$$

→ $p_{T,5}$, y_5

- 2-particle inclusive kinematics

$$w = \frac{(p_1 + p_2 - p_3 - p_4)^2 - m_5^2}{\hat{s}} \qquad c_{a,b} = c_{a,b}(s_{34}, t_{13}, t_{23}, t_{14}, t_{24})$$

→ $p_{T,34}$, y_{34}, M_{34}

A First Step

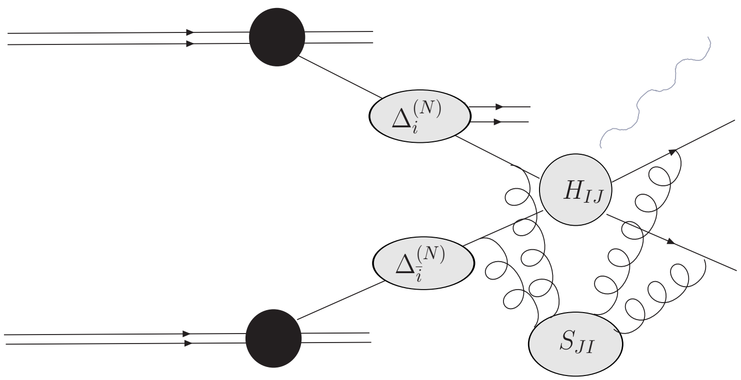
- In principle, if the decomposition of the hard function into color channels known, all ingredients to perform threshold resummation for a selected 2→3 observable available

- Plenty of technicalities → as a first step, focus on
  - NLL resummation at absolute production threshold $\hat{s} \to m_3 + m_4 + m_5$

$$w = \beta^2 = 1 - \frac{(m_3 + m_4 + m_5)^2}{\hat{s}}$$

  - final state with only 2 massive colored particles $pp \to Q\bar{Q}B$
    - no final state jets
    - color structure same as for the $Q\bar{Q}$ production

Threshold Resummation for Q Qbar B



Colour space basis in which Γ_{IJ} is diagonal in the threshold limit

$$\hat{\sigma}^{(\mathrm{res},N)}_{ab\to klB} = \sum_I \underbrace{\hat{\sigma}^{(0,N)}_{ab\to klB,I}\, C_{ij\to klB,I}}_{\text{hard function } H_{ab\to kl B,I}} \, \Delta_a^{(N+1)} \Delta_b^{(N+1)} \, \Delta^{(\mathrm{soft},N)}_{ab\to klB,I}$$

hard function $H_{ab \to kl\,B,I}$

incoming jet factors, known

soft-wide angle emission

$$\log \Delta^{(\mathrm{soft},N)}_{ab\to klB,I} = \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ij\to klB,I}(\alpha_s(Q^2(1-z)^2)) \qquad D_{ij\to klB,I} = \lim_{\beta\to 0} \frac{\pi}{\alpha_s} 2\mathrm{Re}(\bar{\Gamma}_{II})$$

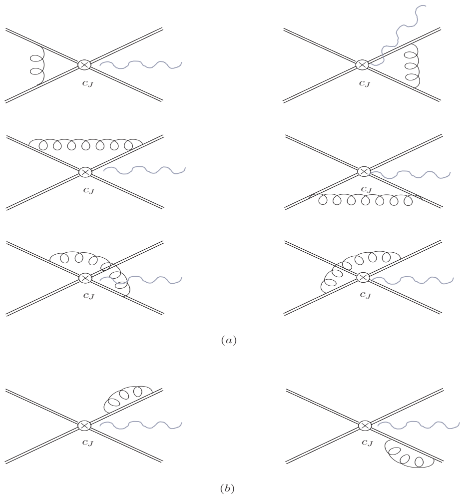
At NLL accuracy $C_{ab \to kl\,B,I} = 1$

Soft Anomalous Dimension

- Soft anomalous dimensions known at two loops for any number of masless/massive legs *[Mert-Aybat, Dixon, Sterman'06] [Becher, Neubert'09] [Mitov, Sterman, Sung'09-'10] [Ferroglia, Neubert, Pecjak, Yang'09] [Beneke, Falgari, Schwinn'09], [Czakon, Mitov, Sterman'09] [Kidonakis'10]*

  - For NLL need only 1-loop

$$\Gamma(g) = -\frac{g}{2}\frac{\partial}{\partial g}\mathrm{Res}_{\epsilon\to 0} Z(g,\epsilon)$$



- N.B. color structure also known explicitly for 2→3 *[Sjödahl'08]*

Anomalous Dimension

$$\Gamma_{q\bar{q}\to klB} = \frac{\alpha_s}{\pi}\begin{bmatrix} -C_F(L_{\beta,kl}+1) & \frac{C_F}{C_A}\Omega_3 \\ 2\Omega_3 & \frac{1}{2}(C_A-2C_F)(L_{\beta,kl}+1)+C_A\Lambda_3+(8C_F-3C_A)\Omega_3 \end{bmatrix}$$

$$\Gamma_{gg\to klB} = \frac{\alpha_s}{\pi}\begin{bmatrix} -C_F(L_{\beta,kl}+1) & 0 & \Omega_3 \\ 0 & \frac{1}{2}((C_A-2C_F)(L_{\beta,kl}+1)+C_A\Lambda_3) & \frac{N_c}{2}\Omega_3 \\ 2\Omega_3 & \frac{N_c^2-4}{2N_c}\Omega_3 & \frac{1}{2}((C_A-2C_F)(L_{\beta,kl}+1)+C_A\Lambda_3) \end{bmatrix}$$

$$L_{\beta,kl} = \frac{\kappa^2+\beta_{kl}^2}{2\kappa\beta_{kl}}\left(\log\left(\frac{\kappa-\beta_{kl}}{\kappa+\beta_{kl}}\right)+i\pi\right)$$

$$\beta_{kl} = \sqrt{1-\frac{(m_k+m_l)^2}{s_{kl}}}$$

$$T_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{t}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$U_i(m) = \frac{1}{2}\left(\ln((m^2-\hat{u}_i)^2/(m^2\hat{s}))-1+i\pi\right)$$

$$\Lambda_3 = (T_1(m_k)+T_2(m_l)+U_1(m_l)+U_2(m_k))/2$$

$$\Omega_3 = (T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2$$

$$t_1 = (p_i-p_k)^2 \qquad t_2 = (p_j-p_l)^2$$

$$u_1 = (p_i-p_l)^2 \qquad u_2 = (p_j-p_k)^2$$

Anomalous Dimension

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$$\Omega_3 = (T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2$$

$$t_1 = (p_i-p_k)^2 \qquad t_2 = (p_j-p_l)^2$$

$$u_1 = (p_i-p_l)^2 \qquad u_2 = (p_j-p_k)^2$$

Reduces to the 2->2 case in the limit $p_B \to 0$, $m_B \to 0$

Coefficients $D^{(1)}_{ab\to kl\,B,I}$ governing soft emission same as for the QQbar process: soft emission at the absolute threshold driven only by the color structure

Application: Associated Higgs Production with TTBar Pair

- $pp \to t\bar{t}H$ is a direct probe of the strength of the top-Yukawa coupling without making any assumptions regarding its nature

- NLO available since some time *[Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas '01-'02][Reina, Dawson'01][Reina, Dawson, Wackeroth'02] [Dawson,Orr,Reina,Wackeroth'03] [Dawson, Jackson, Orr, Reina, Wackeroth'03]*

- NLO interfaced to parton showers in aMC@NLO *[Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli'11]*and POWHEG-Box *[Garzelli, Kardos,Papadopoulos,Trocsanyi'11] [Hartanto, Jäger,Reina, Wackerothi'15]* as well as SHERPA *[Gleisberg, Höche, Krauss, Schönherr, Schumann, Siegert, Winter]*

- NLO EW corrections available *[Frixione, Hirschi, Pagani,Shao,Zaro'14][Zhang, Ma,Zhang,Chen,Guo'14]*

- Top and Higgs decays: NLO QCD for $W^+ W^-$ *b bbar H,* studies of interference and background processes *[-> A. Denner's talk]*

NLO IN THE THRESHOLD LIMIT

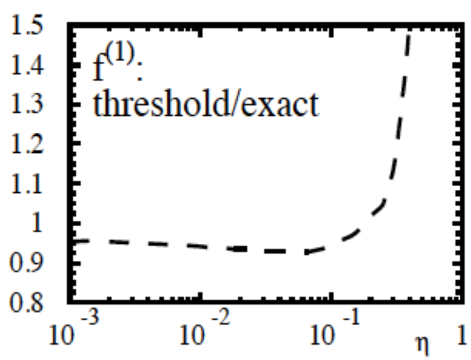
$$\hat{\sigma}^{\text{NLO,log}}_{ab\to t\bar{t}H,I} \;=\; \hat{\sigma}^{\text{LO}}_{ab\to t\bar{t}H,I}\frac{2\alpha_s}{\pi}\left[2C^a\left(2\log^2\beta - 3\log\beta - 2\log\beta\log\left(\frac{\mu}{2m_t+m_H}\right)\right) + D^{(1)}_{ab\to t\bar{t}H,I}\log\beta\right]$$

Scaling functions:

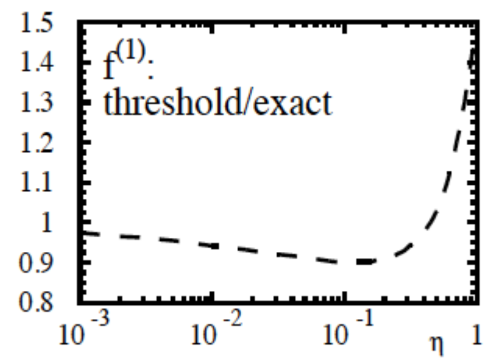
$$\hat{\sigma}^{\text{NLO}}_{ab\to t\bar{t}H} = \frac{\alpha_s(\mu^2)}{\mu_0^2}\left[f^{(0}_{ab}(\eta) + 4\pi\alpha_s(\mu^2)\left[f^{(1)}_{ab}(\eta) + \bar{f}^{(1)}_{ab}(\eta)\log\left(\frac{\mu^2}{\mu_0^2}\right)\right]\right]$$

$$\eta = \frac{\hat{s}}{4\mu_0^2} - 1 \qquad\qquad \mu_0 = m_t + m_H/2$$

$f_{q\bar{q}}$



f_{gg}



[Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas '02]

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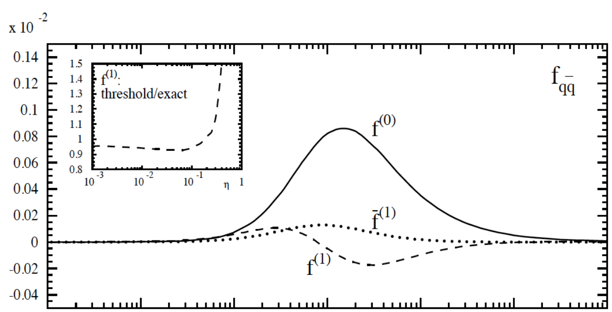
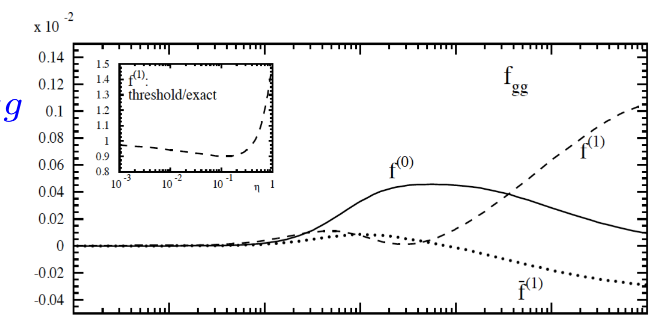
$$\eta = \frac{\hat{s}}{4\mu_0^2} - 1 \qquad \mu_0 = m_t + m_H/2$$

Drawback: σ^{LO} proportional to β^4 in the threshold limit (phase-space effect)

$f_{q\bar{q}}$



f_{gg}



[Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas ’02]

Results

[AK, Motyka, Stebel, Theeuwes, in progress]

MMHT2014NLO

preliminary

$\sqrt{S}$ [TeV]	NLO [fb]	NLL+NLO [fb]	pdf uncertainty	K-factor
8	$131.4^{+3.6\%}_{-9.0\%}$	$135.1^{+1.3\%}_{-5.0\%}$	$^{+3.0\%}_{-2.7\%}$	1.028
13	$506.3^{+5.7\%}_{-9.3\%}$	$517.0^{+4.0\%}_{-5.9\%}$	$^{+2.3\%}_{-2.3\%}$	1.021
14	$610.9^{+6.5\%}_{-9.1\%}$	$623.4^{+4.9\%}_{-5.6\%}$	$^{+2.3\%}_{-2.2\%}$	1.020

Scale uncertainty defined by variation between $\mu=\mu_0/2$ and $\mu=2\mu_0$, $\mu_0=m_t + m_H/2$
K-factor measures ratio NLL+NLO to NLO

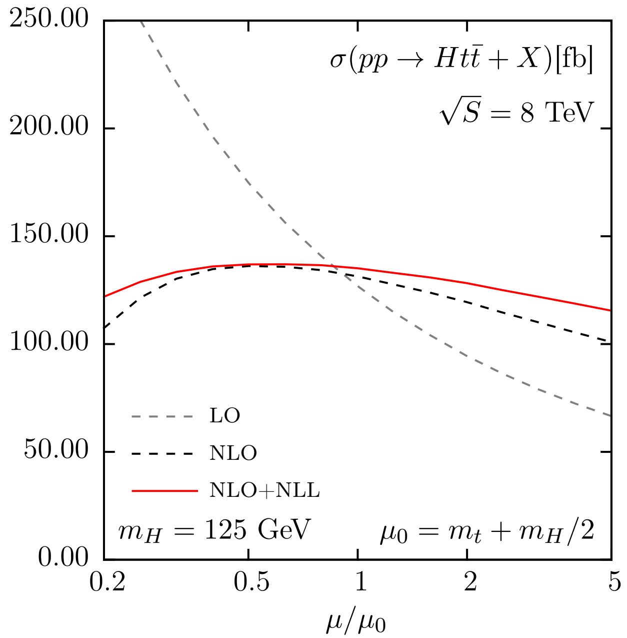
NLO obtained with aMC@NLO *[Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro]*

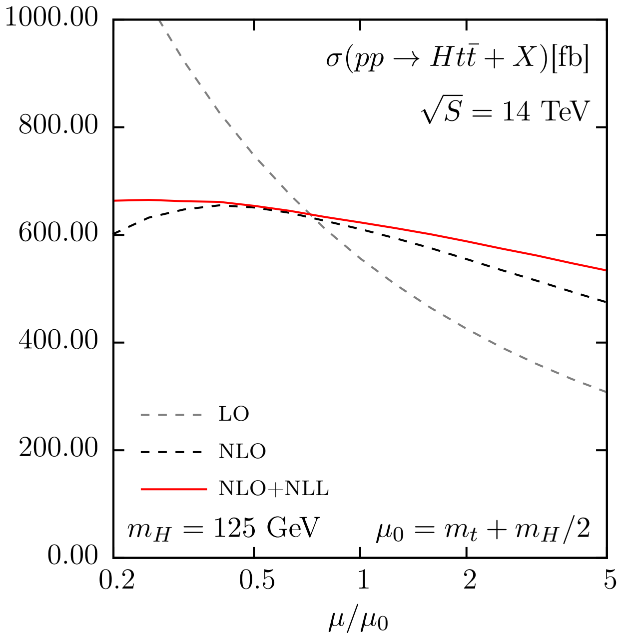
Scale Dependence

[AK, Motyka, Stebel, Theeuwes, in progress]

MMHT2014NLO

preliminary





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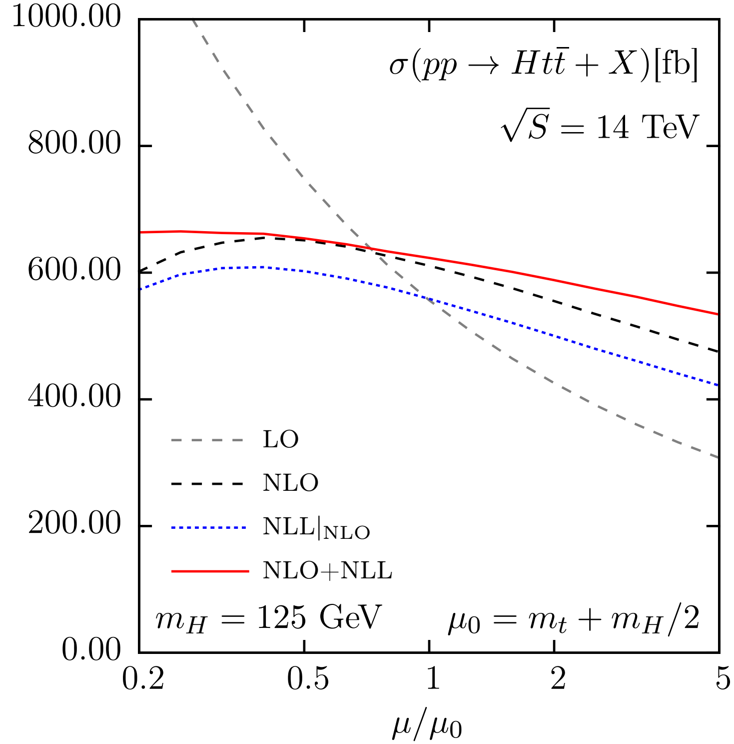

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Summary

- Applications of threshold resummation techniques to the class of 2->3 processes lead to interesting phenomenology and improved precision of the predictions
- Current knowledge makes it feasible to develop such applications of the standard resummation approach at least at NLL accuracy
- First application: threshold resummed predictions for $pp \to t\bar{t}H$ total cross section show reduced scale dependence

BACKUP

Higher Orders At Threshold

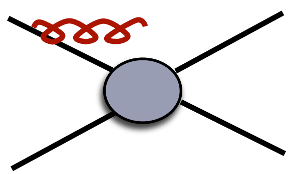
In analogy to top-pair production

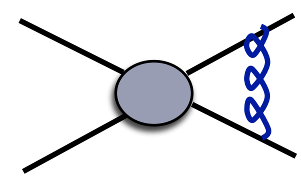
General structure of the NLO correction in the threshold limit $\beta \to 0, \quad \beta^2 = 1 - 4m^2/\hat{s}$

$$\Delta\hat{\sigma}_i^{\mathrm{NLO}} \sim \alpha_s \; \hat{\sigma}_i^{\mathrm{LO}} \left\{ A^{(i)} \log^2(\beta^2) + B^{(i)} \log(\beta^2) + C^{(i)} \frac{1}{\beta} + D^{(i)} \right\}$$

Soft/collinear gluon emission

Coulomb gluons





At higher orders:

$\sim \alpha_s^n \log^{2n}(\beta)$

$\sim \alpha_s^n / \beta^n$

Both types of corrections can be resummed to all orders

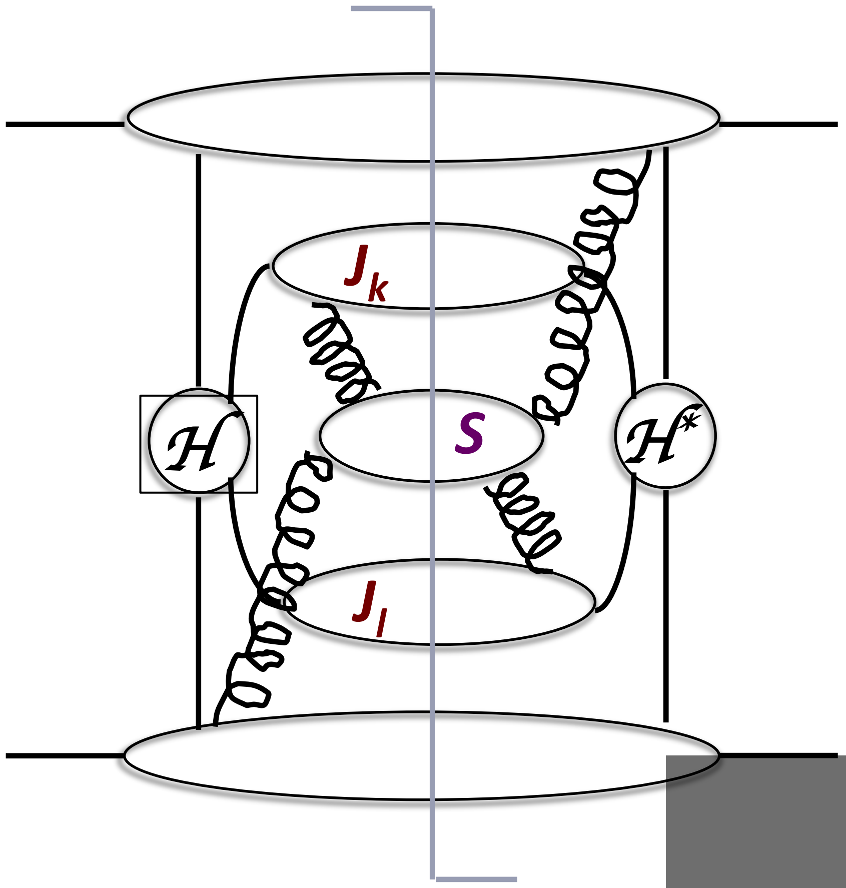
Resummation-improved NLL+NLO total cross section

- NLL resummed expression has to be matched with the full NLO result

$$\sigma^{(\text{match})}_{h_1 h_2 \to kl}(\rho, \{m^2\}, \mu^2) = \sum_{i,j=q,\bar{q},g} \int_{C_{MP}-i\infty}^{C_{MP}+i\infty} \frac{dN}{2\pi i} \rho^{-N} f^{(N+1)}_{i/h_1}(\mu^2)\, f^{(N+1)}_{j/h_2}(\mu^2) \times \left[\hat{\sigma}^{(\text{res},N)}_{ij\to kl}(\{m^2\},\mu^2) - \hat{\sigma}^{(\text{res},N)}_{ij\to kl}(\{m^2\},\mu^2)\Big|_{NLO} \right] + \sigma^{\text{NLO}}_{h_1 h_2 \to kl}(\rho, \{m^2\}, \mu^2)\,,$$

- Inverse Mellin transform evaluated using a contour in the complex *N* space according to 'Minimal Prescription' *[Catani, Mangano, Nason Trentadue'96]*

- NLO cross sections evaluated with PROSPINO



J_k
$\mathcal{H}$
S
$\mathcal{H}^*$
J_l