

N³LO calculations for top-quark differential cross sections near partonic threshold

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- Higher-order corrections
- Total $t\bar{t}$ cross sections
- Top p_T distributions
- Top rapidity distributions
- Top forward-backward asymmetry
- Single-top production cross sections
- Single-top p_T distributions
- FCNC top production

Higher-order corrections

QCD corrections significant for $t\bar{t}$ and single-top production

Soft-gluon corrections are important

Soft terms: $\left[\frac{\ln^k(s_4/m_t^2)}{s_4} \right]_+$ with $k \leq 2n - 1$, s_4 distance from threshold

Resum these soft corrections - factorization and RGE

NNLL accuracy—two-loop soft anomalous dimensions

Approximate $N^3\text{LO}$ (aN³LO) differential cross sections from expansion of resummed expressions

Calculation is for **partonic threshold for the double-differential cross section** using the standard moment-space resummation in perturbative QCD

Latest results for $t\bar{t}$:

total cross section: Phys. Rev. D 90, 014006 (2014) [arXiv:1405.7046 [hep-ph]]

p_T and y distributions: Phys. Rev. D 91, 031501 (2015) [arXiv:1411.2633 [hep-ph]]

A_{FB} : Phys. Rev. D 91, 071502 (2015) [arXiv:1501.01581 [hep-ph]]

Partonic threshold approximation

Approximation works very well for LHC and Tevatron energies
less than 1% difference between approximate and exact cross sections
at both NLO and NNLO

also true for p_T and rapidity distributions and A_{FB}

NK, PRD 82, 114030 (2010) and PRD84, 011504 (2011)

fixed-order expansion - no prescription is used

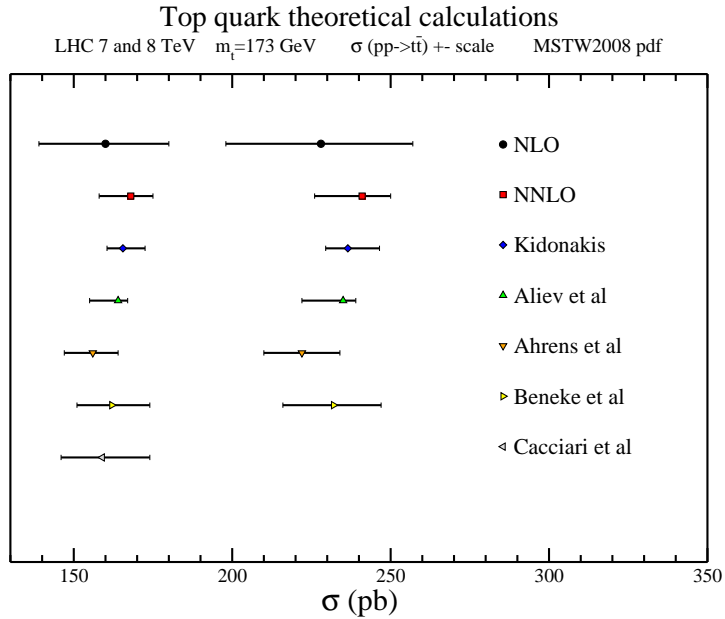
stability of the theoretical NNLO approximate result in this
double-differential pQCD resummation approach over the past decade

the reliability of the NNLO approximate result and near-identical
value to exact NNLO is very important for several reasons

- provides confidence of application to other processes
- used as background for many analyses (Higgs, etc)

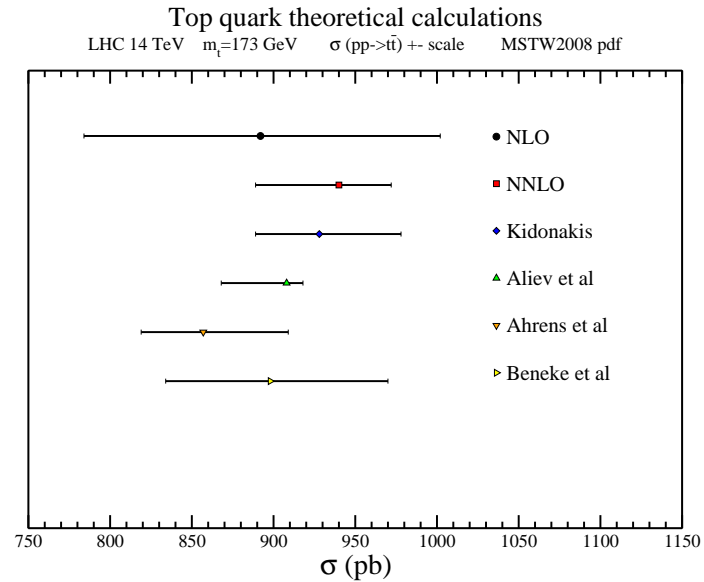
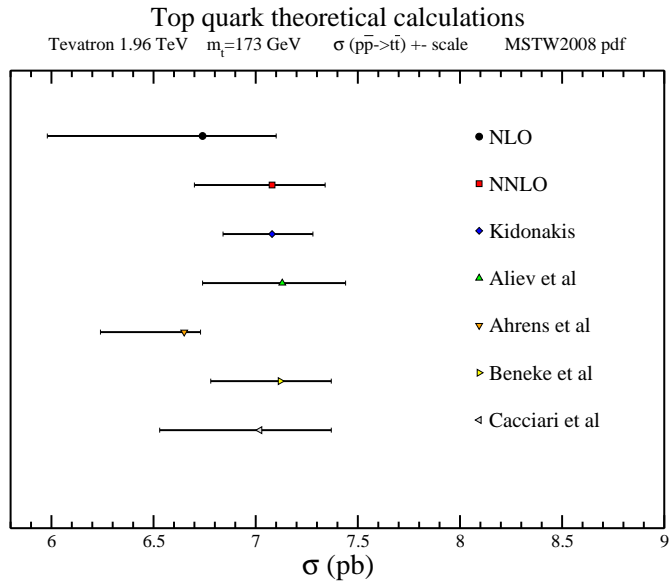
add aN^3LO corrections for best result

NK, 2014 and 2015

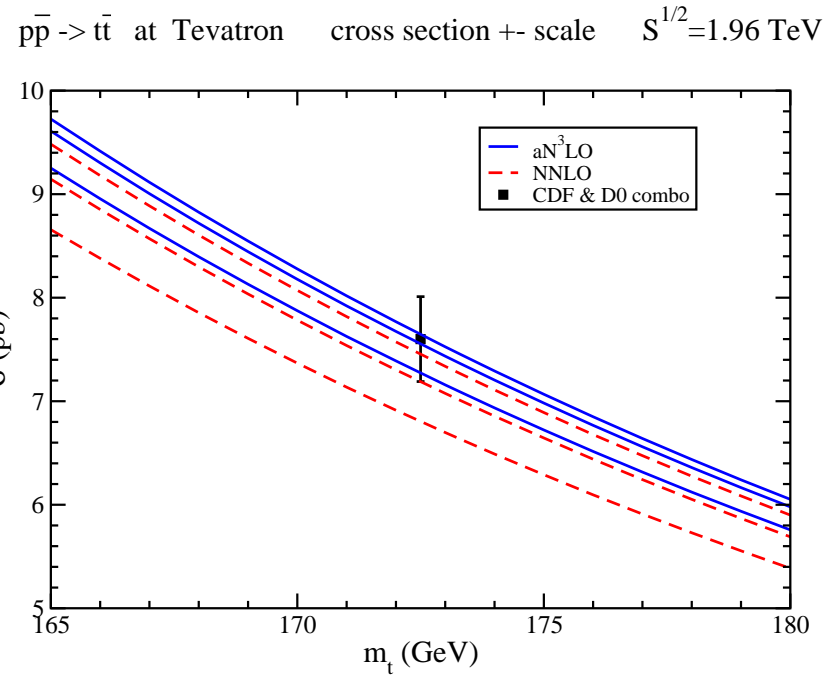
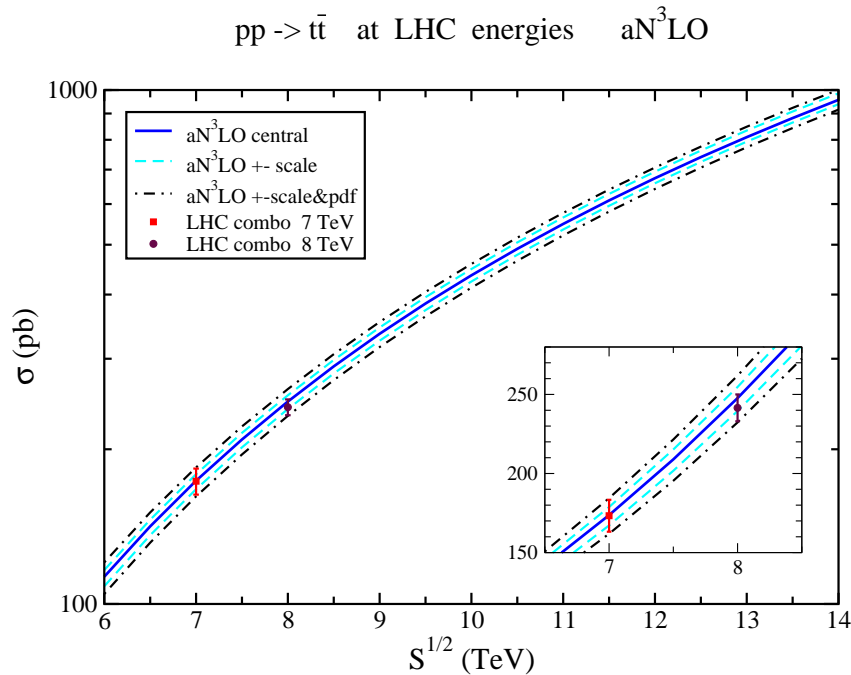


**Comparison of various NNLO approx approaches
all with the same choice of parameters**

Kidonakis, PRD 82, 114030 (2010) differential-pQCD
Aliev et al, CPC 182, 1034 (2011) total-pQCD
Ahrens et al, PLB 703, 135 (2011) differential -SCET
Beneke et al, NPB 855, 695 (2012) total-SCET
Cacciari et al, PLB 710, 612 (2012) total-pQCD



Top-pair cross sections at the LHC and the Tevatron



aN³LO total $t\bar{t}$ cross sections with $m_t = 173.3$ GeV

Tevatron 1.96 TeV: $7.37^{+0.09+0.38}_{-0.27-0.28}$ pb

LHC 7 TeV: 174^{+5+9}_{-7-10} pb

LHC 8 TeV: 248^{+7+12}_{-8-13} pb

LHC 13 TeV: 810^{+24+30}_{-16-32} pb

LHC 14 TeV: 957^{+28+34}_{-19-36} pb

Relative size of perturbative corrections

$$\sigma^{\text{aN}^3\text{LO}} = \sigma^{(0)} \left[1 + \frac{\sigma^{(1)}}{\sigma^{(0)}} + \frac{\sigma^{(2)}}{\sigma^{(0)}} + \frac{\sigma^{(3)}}{\sigma^{(0)}} \right]$$

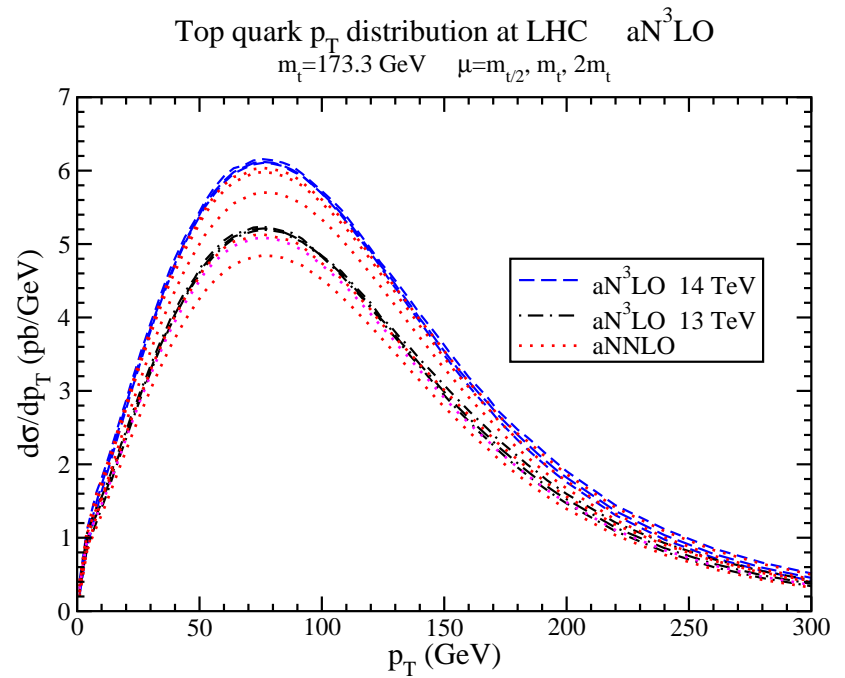
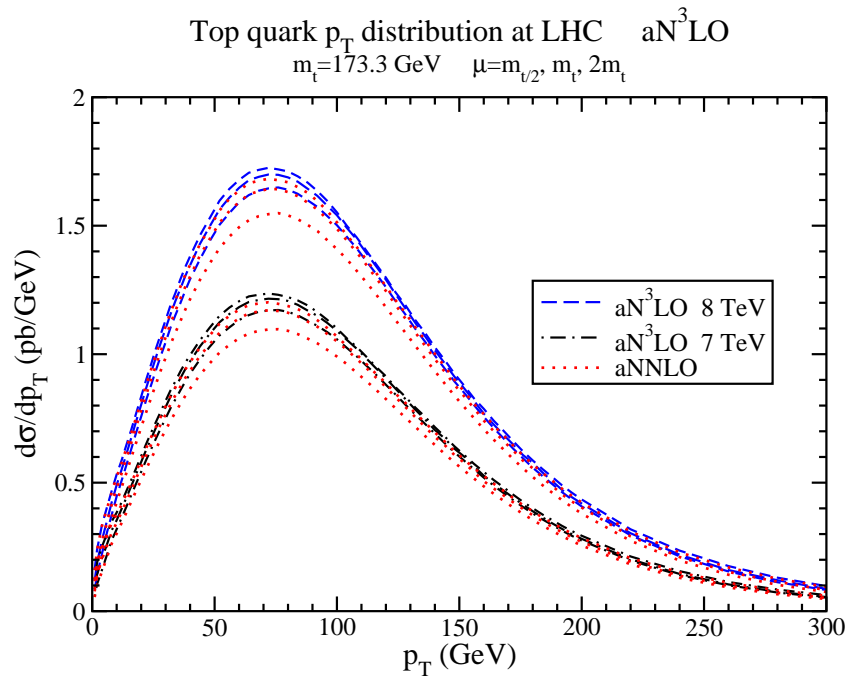
Fractional contributions to the perturbative series for the $t\bar{t}$ cross section					
corrections	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 13 TeV	LHC 14 TeV
$\sigma^{(1)}/\sigma^{(0)}$	0.236	0.470	0.476	0.493	0.496
$\sigma^{(2)}/\sigma^{(0)}$	0.106	0.178	0.177	0.172	0.170
$\sigma^{(3)}/\sigma^{(0)}$	0.068	0.066	0.059	0.045	0.043

At 14 TeV $\sigma^{\text{aN}^3\text{LO}} = 1.709 \sigma^{(0)}$

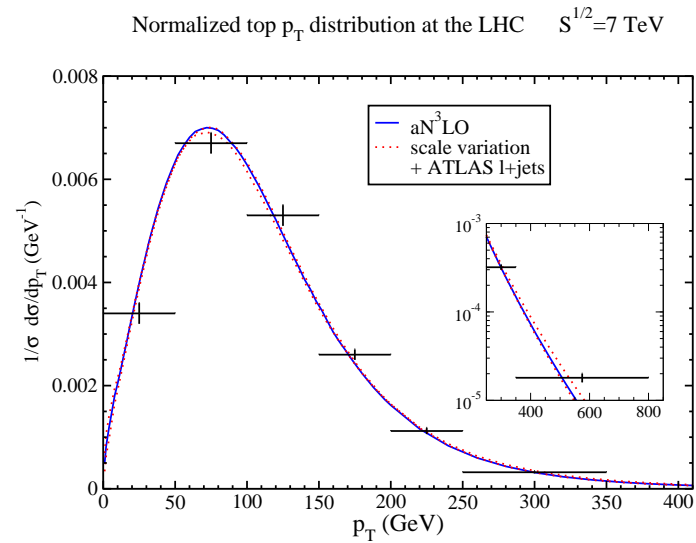
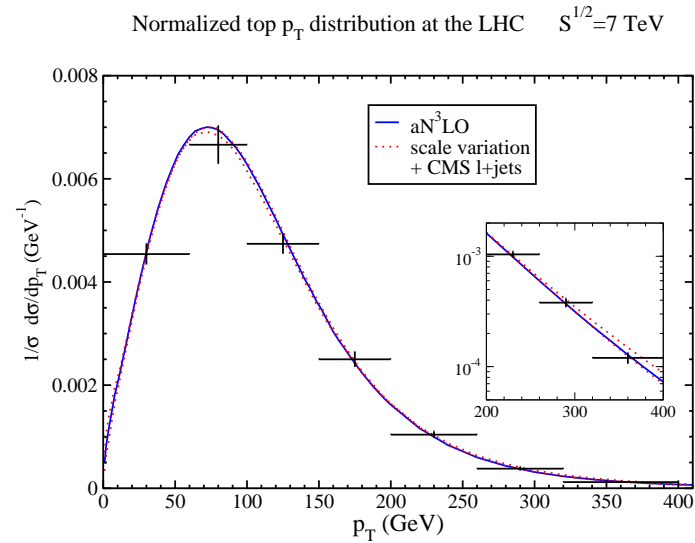
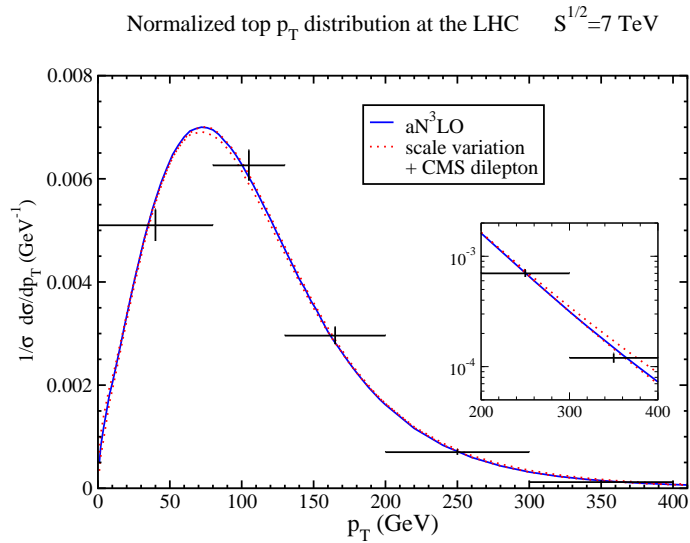
Series approximated well by $\sum_{n=1}^4 1/n! = 1.708 \dots$

It is amusing to note that $\sum_{n=1}^{\infty} 1/n! = e - 1 = 1.718 \dots$

Top quark p_T distribution at the LHC

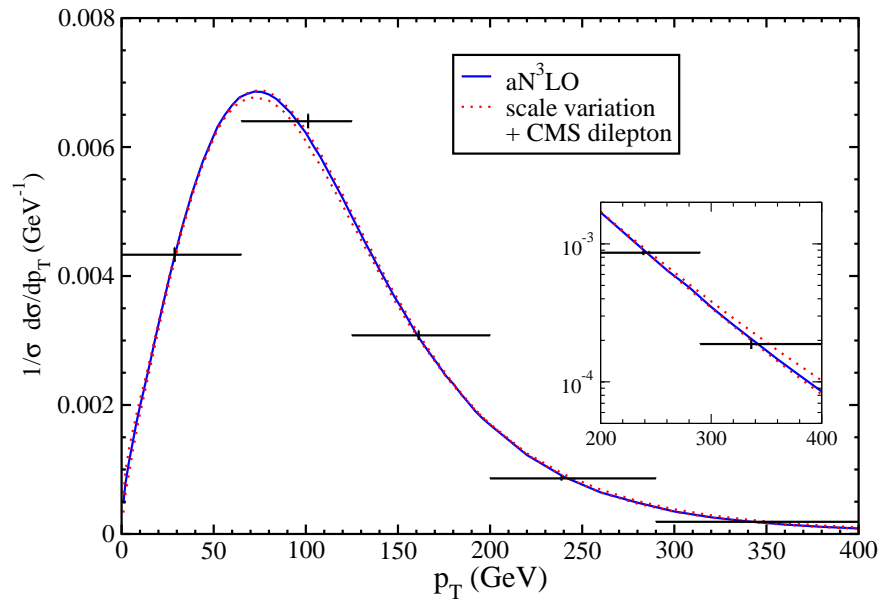


Normalized top quark p_T distribution at 7 TeV LHC

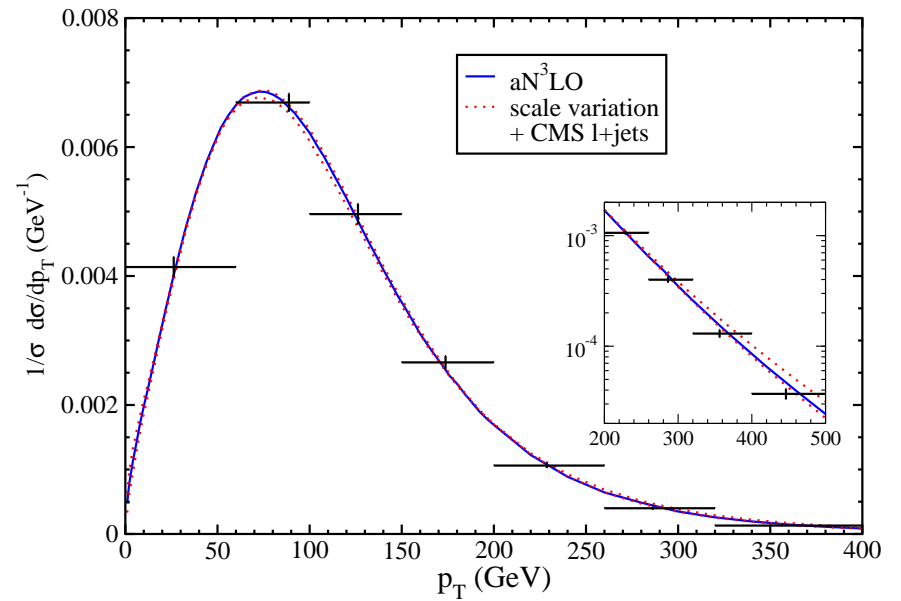


Normalized top quark p_T distribution at 8 TeV LHC

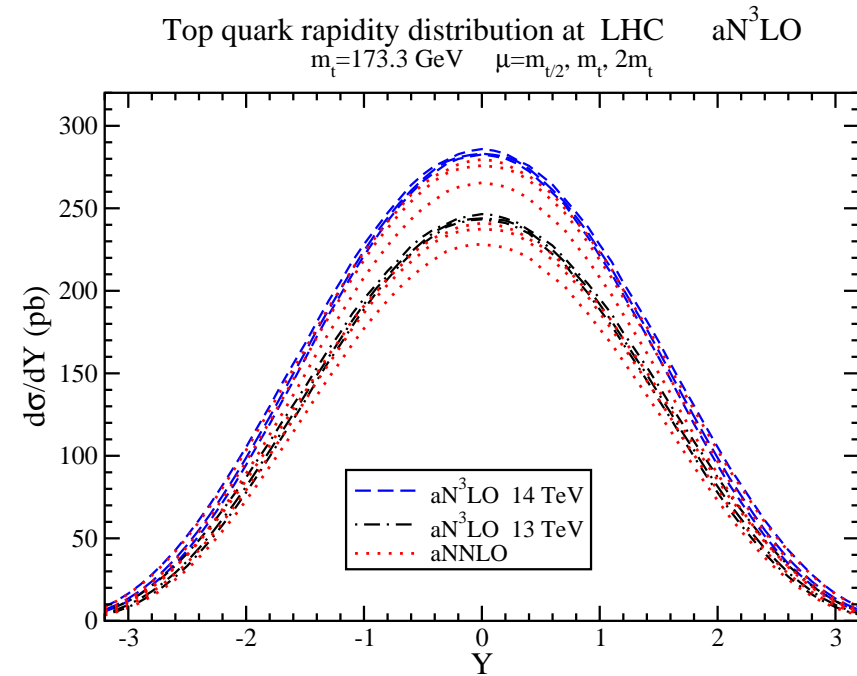
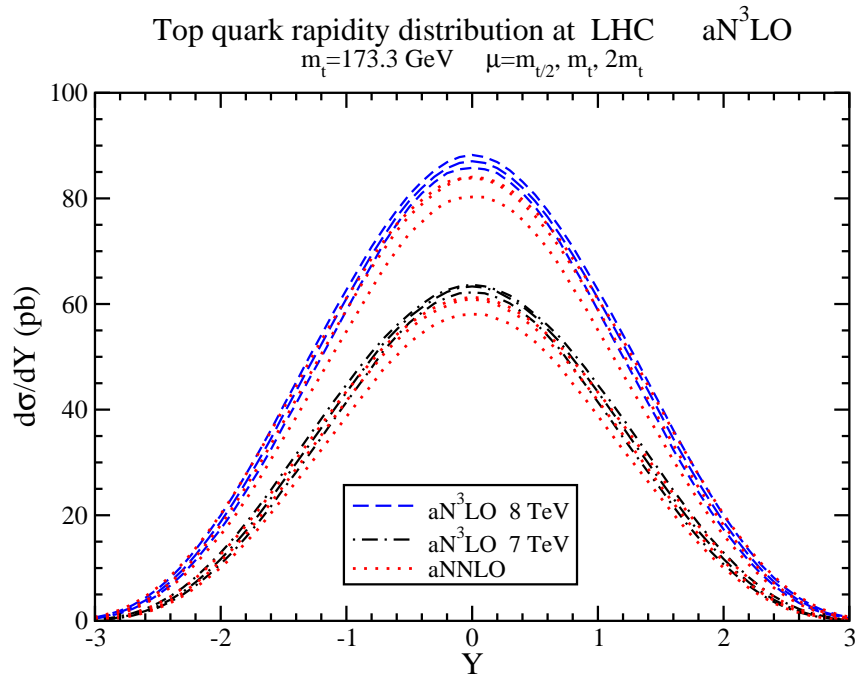
Normalized top p_T distribution at the LHC $S^{1/2}=8$ TeV



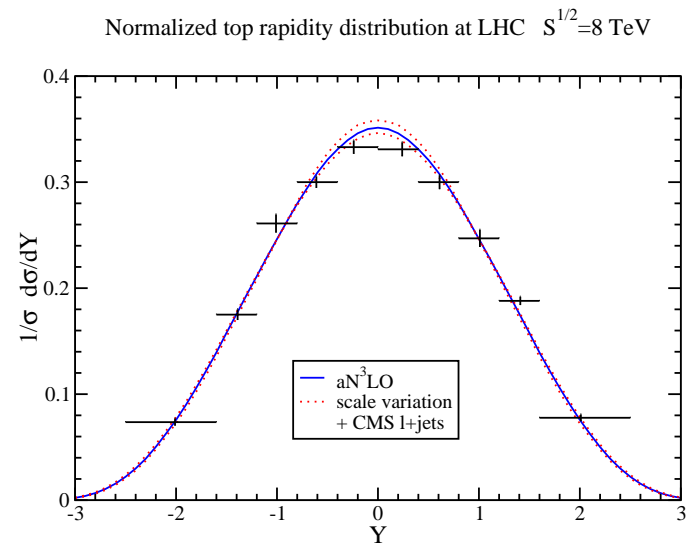
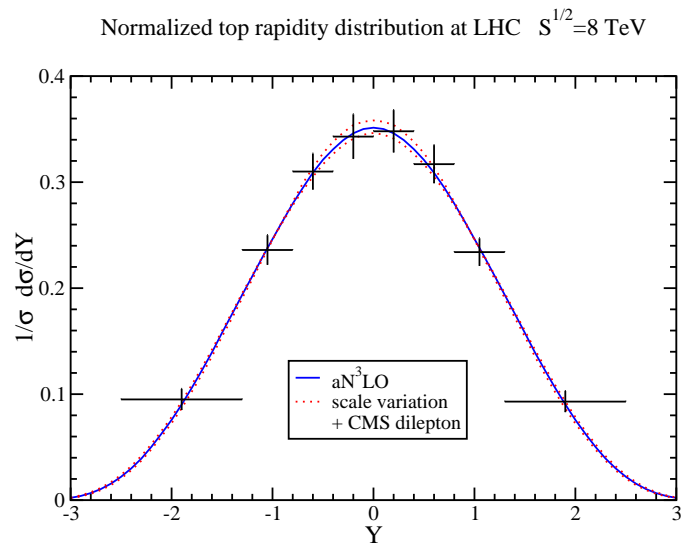
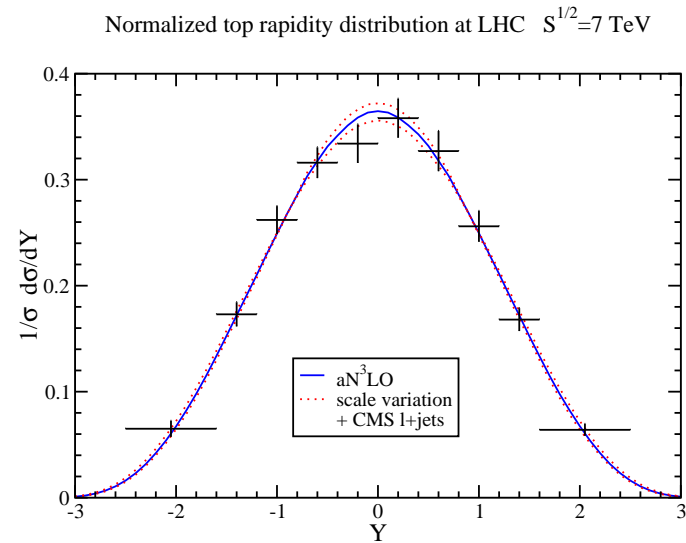
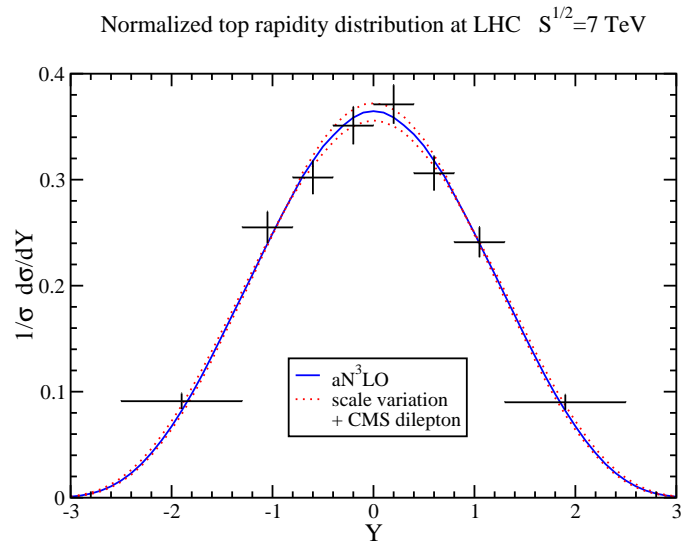
Normalized top p_T distribution at the LHC $S^{1/2}=8$ TeV



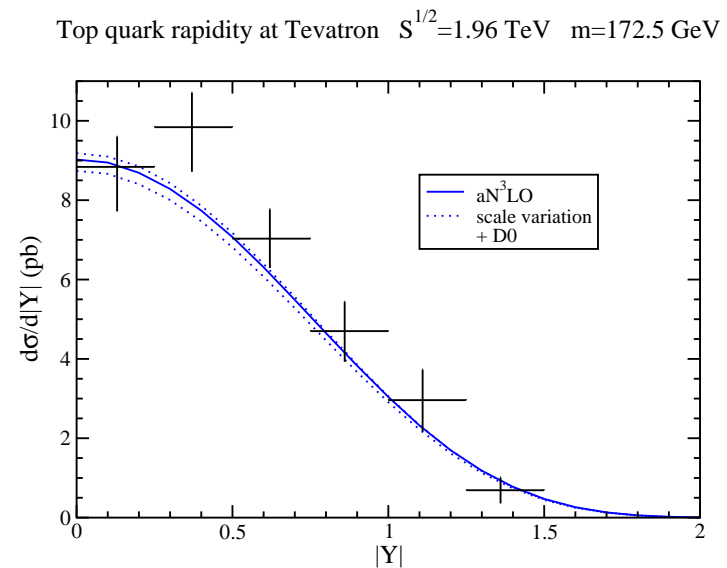
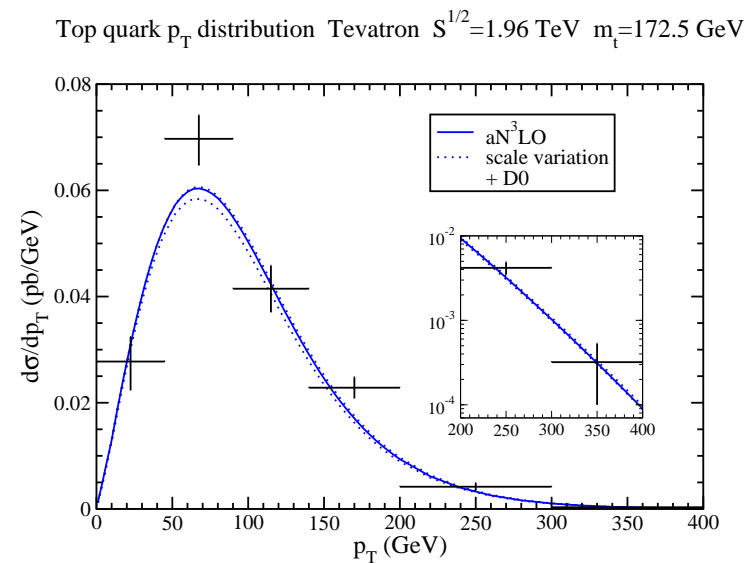
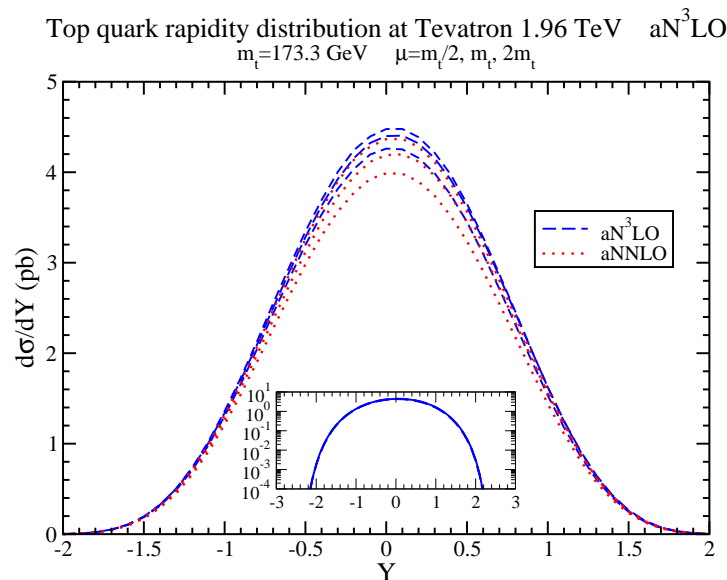
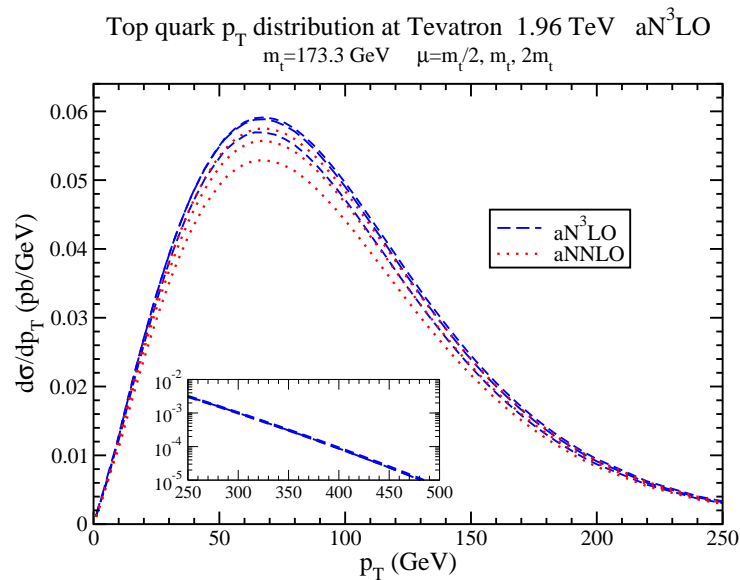
Top quark rapidity distribution at the LHC



Normalized top quark rapidity distribution at the LHC



Top quark p_T and rapidity distributions at the Tevatron



Top forward-backward asymmetry at the Tevatron

$$A_{\text{FB}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)} \equiv \frac{\Delta\sigma}{\sigma} \quad (1)$$

$$A_{\text{FB}} = \frac{\Delta\sigma^{\text{EW}} + \alpha_s^3 \Delta\sigma^{(1)} + \alpha_s^4 \Delta\sigma^{(2)} + \alpha_s^5 \Delta\sigma^{(3)} + \dots}{\alpha_s^2 \sigma^{(0)} + \alpha_s^3 \sigma^{(1)} + \alpha_s^4 \sigma^{(2)} + \alpha_s^5 \sigma^{(3)} + \dots} \quad (2)$$

$$A_{\text{FB}} = \frac{\Delta\sigma^{\text{EW}}}{\alpha_s^2 \sigma^{(0)}} + \alpha_s \frac{\Delta\sigma^{(1)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{\text{EW}} \sigma^{(1)}}{\alpha_s (\sigma^{(0)})^2} + \alpha_s^2 \left[\frac{\Delta\sigma^{(2)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{(1)} \sigma^{(1)}}{(\sigma^{(0)})^2} \right] \\ + \frac{\Delta\sigma^{\text{EW}}}{(\sigma^{(0)})^3} \left[(\sigma^{(1)})^2 - \sigma^{(0)} \sigma^{(2)} \right] + \alpha_s^3 \left[\frac{\Delta\sigma^{(3)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{(2)} \sigma^{(1)}}{(\sigma^{(0)})^2} + \frac{\Delta\sigma^{(1)} (\sigma^{(1)})^2}{(\sigma^{(0)})^3} - \frac{\Delta\sigma^{(1)} \sigma^{(2)}}{(\sigma^{(0)})^2} \right] + \dots \quad (3)$$

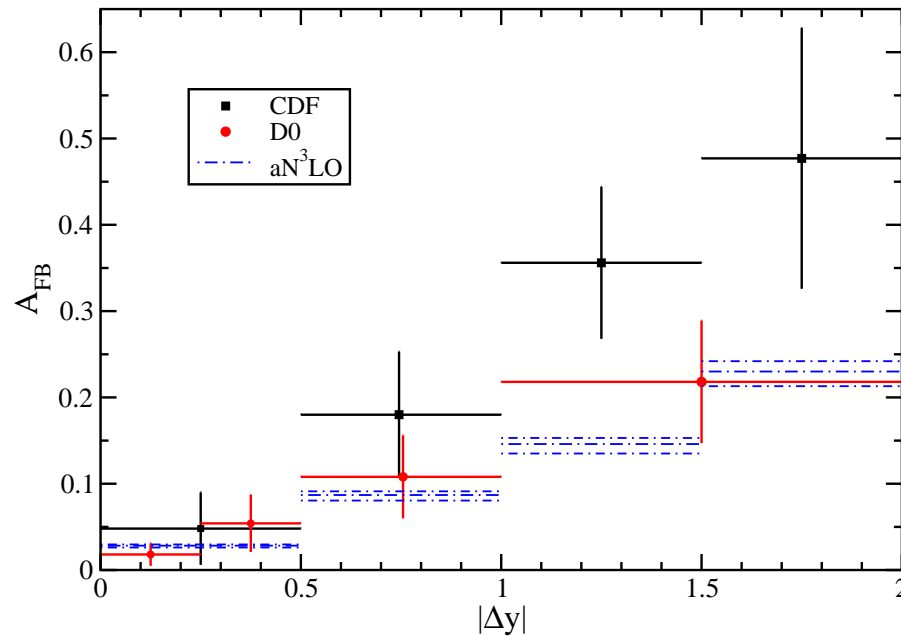
Top-quark asymmetry at the Tevatron		
aN ³ LO A_{FB} %	$p\bar{p}$ frame	$t\bar{t}$ frame
QCD only Eq. (2)	$5.6^{+0.3}_{-0.4}$	$8.1^{+0.4}_{-0.6}$
QCD only Eq. (3)	6.0 ± 0.1	8.7 ± 0.2
QCD+EW Eq. (2)	$6.4^{+0.5}_{-0.6}$	$9.4^{+0.7}_{-0.9}$
QCD+EW Eq. (3)	6.8 ± 0.3	10.0 ± 0.6

large corrections: aN³LO/NNLO ratio is 1.08 [Eq. (2)] or 1.05 [Eq. (3)]

Top differential A_{FB} at the Tevatron

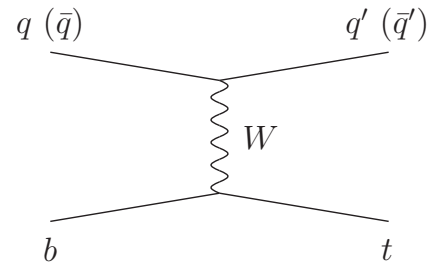
$$A_{\text{FB}}^{\text{bin}} = \frac{\sigma_{\text{bin}}^+(\Delta y) - \sigma_{\text{bin}}^-(\Delta y)}{\sigma_{\text{bin}}^+(\Delta y) + \sigma_{\text{bin}}^-(\Delta y)} \quad \text{with} \quad \Delta y = y_t - y_{\bar{t}}$$

A_{FB} at Tevatron $S^{1/2}=1.96 \text{ TeV}$ $m_t=173.3 \text{ GeV}$

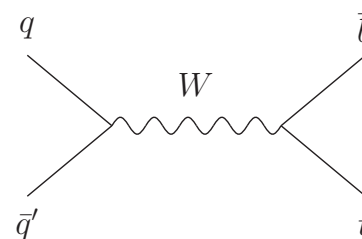


Single-top partonic processes at LO

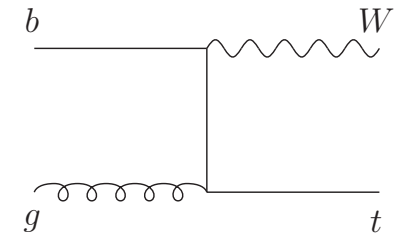
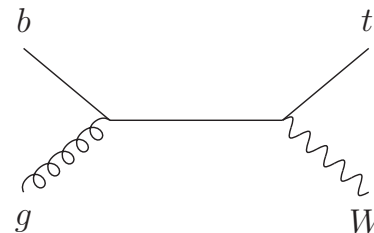
- **t channel:** $qb \rightarrow q't$ and $\bar{q}b \rightarrow \bar{q}'t$
dominant at Tevatron and LHC



- **s channel:** $q\bar{q}' \rightarrow \bar{b}t$
small at Tevatron and LHC



- **associated tW production:** $bg \rightarrow tW^-$
very small at Tevatron, significant at LHC



Single top t -channel production at aNNLO at the LHC

$m_t = 173.3 \text{ GeV}$

LHC	t	\bar{t}	Total (pb)
8 TeV	$55.9^{+2.1}_{-0.3} \pm 1.1$	$30.6 \pm 0.7^{+0.9}_{-1.1}$	$86.5^{+2.8+2.0}_{-1.0-2.2}$
13 TeV	$136^{+3}_{-1} \pm 3$	$82^{+2}_{-1} \pm 2$	$218^{+5}_{-2} \pm 5$
14 TeV	$154^{+4}_{-1} \pm 3$	94^{+2+2}_{-1-3}	248^{+6+5}_{-2-6}

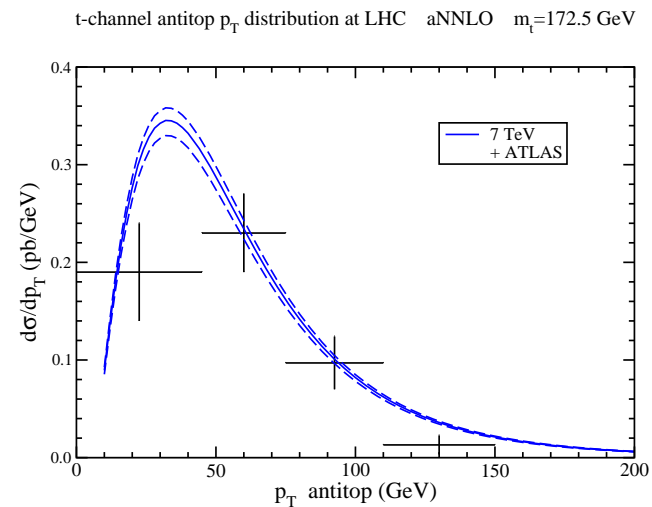
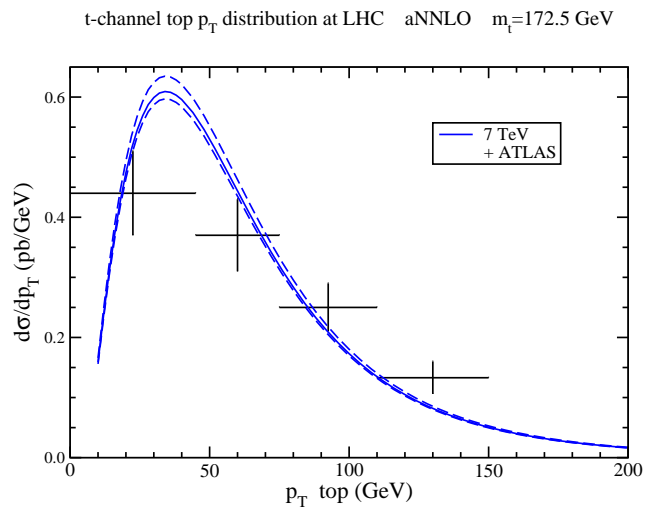
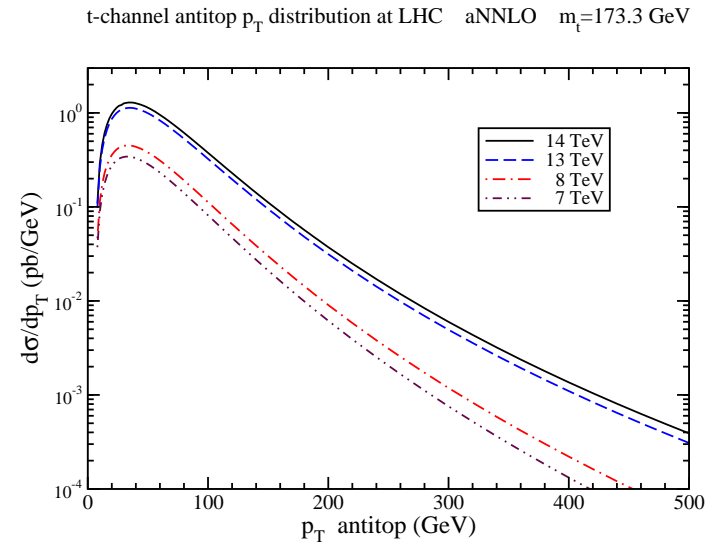
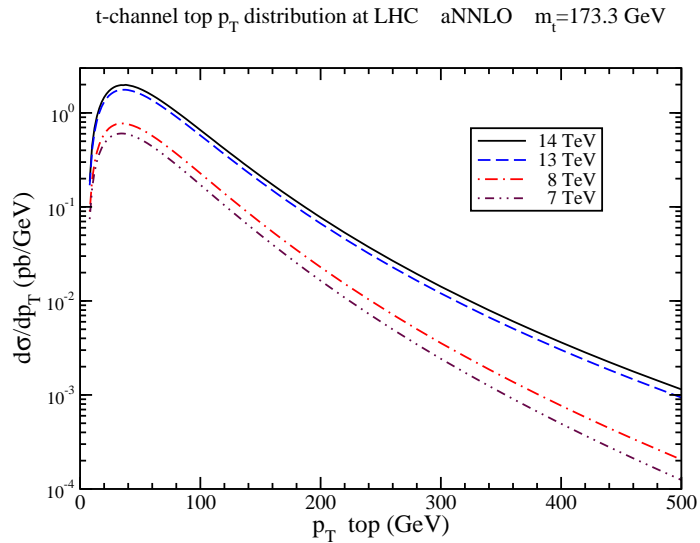
\pm scale \pm pdf errors with MSTW2008 NNLO pdf 90% CL

ratio $\sigma(t)/\sigma(\bar{t}) = 1.82^{+0.10}_{-0.09}$ at 8 TeV

- compares well with CMS result $1.95 \pm 0.10 \pm 0.19$

agrees with NNLO result in 1404.7116 [hep-ph]

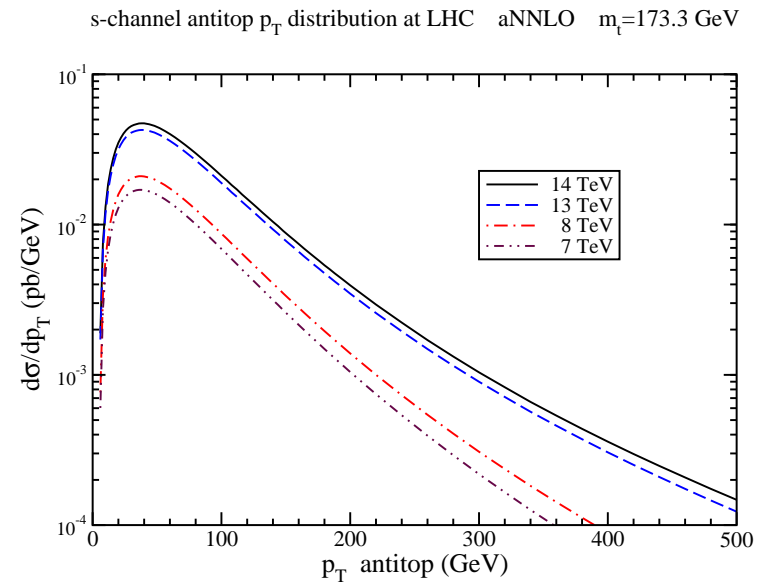
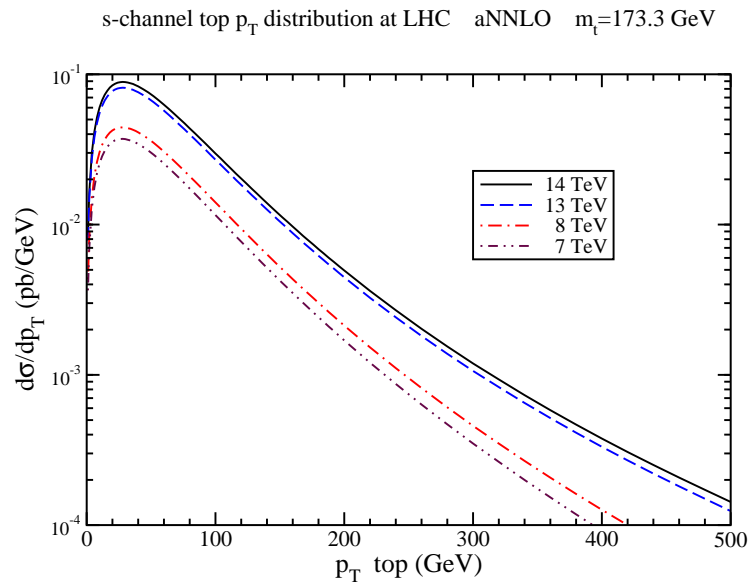
Single top t -channel aNNLO p_T distributions at the LHC



Single top s -channel production at aNNLO at the LHC

LHC	t	\bar{t}	Total (pb)
8 TeV	$3.75 \pm 0.07 \pm 0.13$	$1.90 \pm 0.01 \pm 0.08$	$5.65 \pm 0.08 \pm 0.21$
13 TeV	$7.07 \pm 0.13^{+0.24}_{-0.22}$	$4.10 \pm 0.05^{+0.14}_{-0.16}$	$11.17 \pm 0.18 \pm 0.38$
14 TeV	$7.79 \pm 0.14^{+0.31}_{-0.24}$	$4.57 \pm 0.05^{+0.18}_{-0.17}$	$12.35 \pm 0.19^{+0.49}_{-0.41}$

($m_t = 173.3$ GeV) \pm scale \pm pdf errors with MSTW2008 NNLO pdf 90% CL



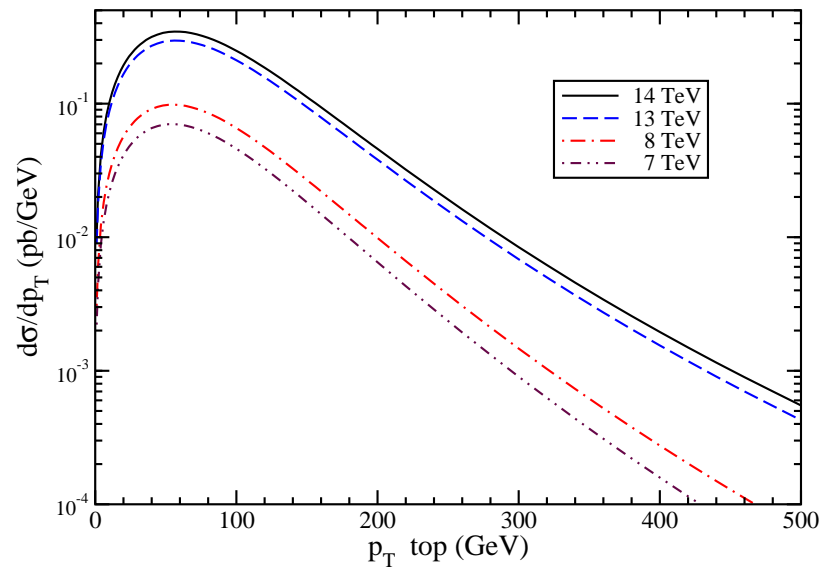
Associated tW^- production at aNNLO at the LHC

$m_t = 173.3 \text{ GeV}$

LHC	tW^-	$tW^- + \bar{t}W^+$ (pb)
8 TeV	$11.0 \pm 0.3 \pm 0.7$	$22.0 \pm 0.6 \pm 1.4$
13 TeV	$35.20 \pm 0.9^{+1.6}_{-1.7}$	$70.40 \pm 1.8^{+3.2}_{-3.4}$
14 TeV	$41.6 \pm 1.0^{+1.5}_{-2.3}$	$83.1 \pm 2.0^{+3.1}_{-4.6}$

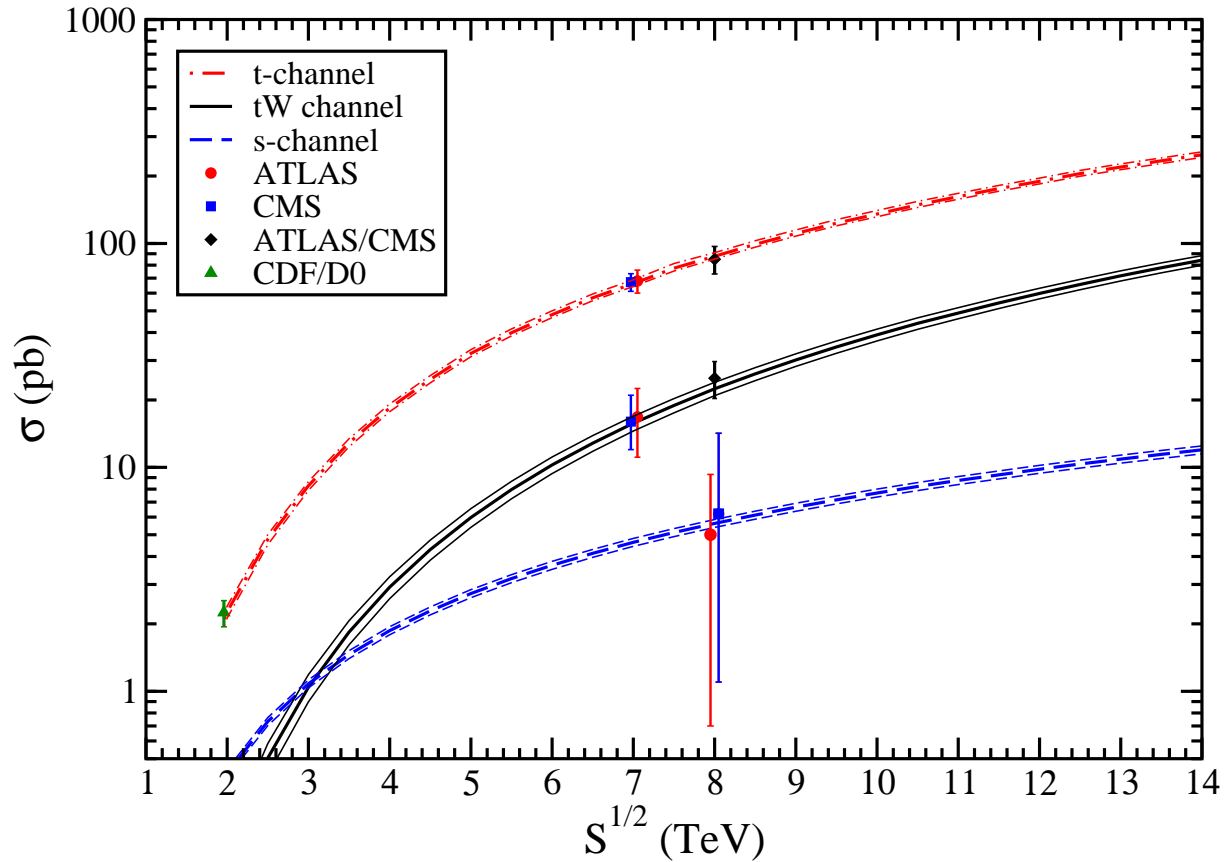
\pm scale \pm pdf errors with MSTW2008 NNLO pdf 90% CL

Top p_T distribution in tW^- production at LHC aNNLO $m_t=173.3 \text{ GeV}$



Single-top cross sections

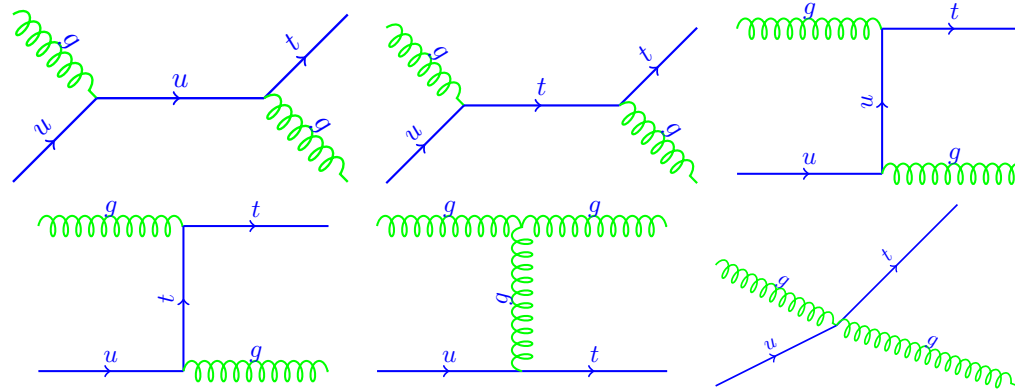
aNNLO single-top cross sections +-scale&pdf $m_t=172.5$ GeV



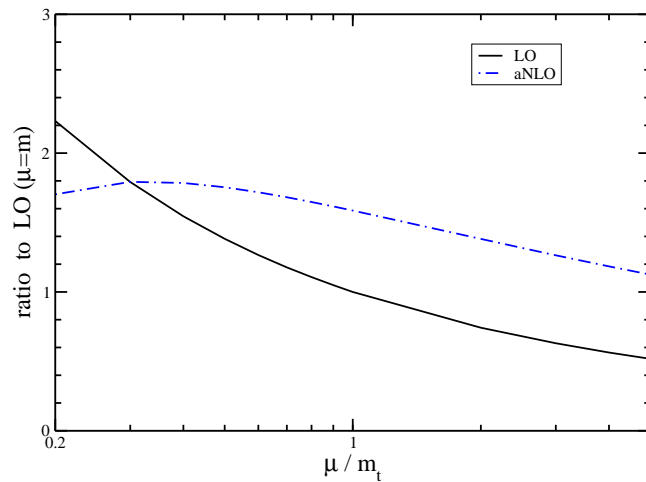
Excellent agreement of theory with data for all three channels

FCNC top production

anomalous gluon couplings



gu -> tg at the LHC $S^{1/2} = 7 \text{ TeV}$



N. Kidonakis and E. Martin, Phys. Rev. D 90, 054021 (2014)

Summary

- soft-gluon corrections for $t\bar{t}$ and single-top production through N³LO
- total cross sections
- top quark p_T and rapidity distributions
- top quark forward-backward asymmetry
- corrections are significant at the LHC and Tevatron
- excellent agreement with LHC and Tevatron data