Jet production in the colorful NNLO framework

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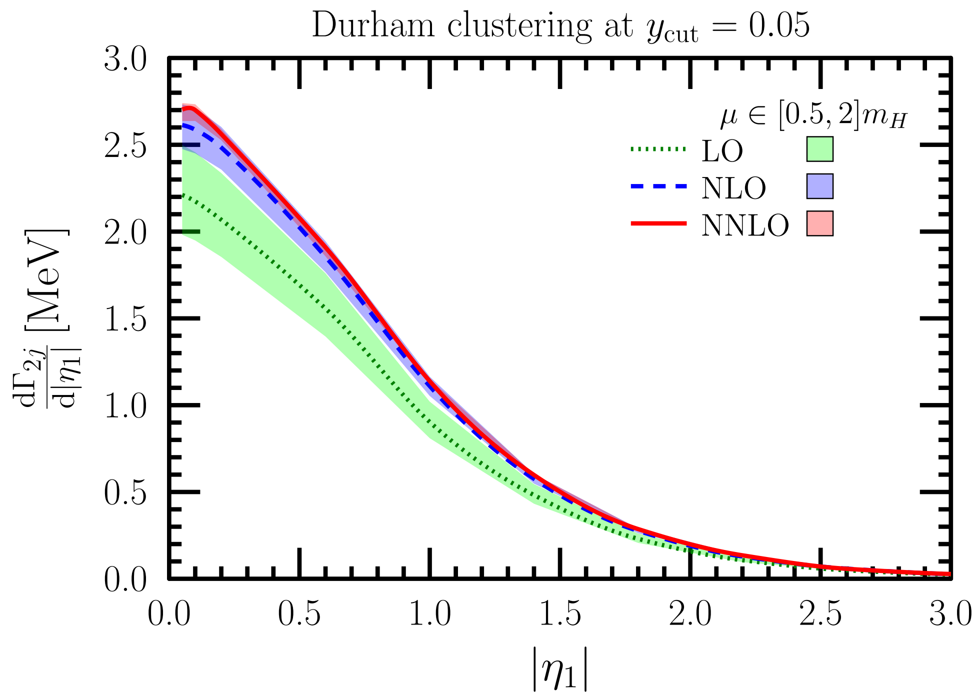
QCD matters

- High precision experiments demand high precision predictions.
- Relatively large coupling ⇒ final state is QCD dominated.
- Key processes have irreducible QCD background and/or QCD corrections.



- Large scale uncertaint(y)(ies).

$H \to b\bar{b}$ decay @ NNLO
Del Duca et al. arXiv:1501.07226

Road to high precision predictions

How to increase precision?

Go one step further in the perturbative expansion:

$$\sigma = \sigma^{\mathrm{LO}} + \sigma^{\mathrm{NLO}} + \sigma^{\mathrm{NNLO}} + \ldots$$

The aim: numerical calculation → kinematical singularities due to soft/collinear parton emissions are treated with local subtractions.

@NLO:

$$\sigma^{\mathrm{NLO}} = \int_{m+1} \Big[\underbrace{\mathrm{d}\sigma^{\mathrm{R}}_{m} J_{m+1} - \mathrm{d}\sigma^{\mathrm{R,A}}_{m+1} J_{m}}_{\text{Singly unresolved}} \Big] + \int_{m} \left[\mathrm{d}\sigma^{\mathrm{V}}_{m} + \int_{1} \mathrm{d}\sigma^{\mathrm{R,A}}_{m+1} \right] J_{m}$$

Singly unresolved

The Colorful NNLO framework

The Colorful NNLO framework

The Colorful NNLO framework was worked out for colorless initial states by Del Duca, Somogyi, Trocsanyi et al., for detailed information see Gabor's talk...

@NNLO:

$$\sigma^{\rm NNLO} =$$

$$= \int_{m+2} \left\{ \mathrm{d}\sigma^{\rm RR}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\rm RR,A_2}_{m+2} J_m - \left[\mathrm{d}\sigma^{\rm RR,A_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\rm RR,A_{12}}_{m+2} J_m \right] \right\}_{\varepsilon=0}$$

$$+ \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\rm RV}_{m+1} + \int_1 \mathrm{d}\sigma^{\rm RR,A_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] J_m \right\}_{\varepsilon=0}$$

$$+ \int_m \left\{ \mathrm{d}\sigma^{\rm VV}_m + \int_2 \left[\mathrm{d}\sigma^{\rm RR,A_2}_{m+2} - \mathrm{d}\sigma^{\rm RR,A_{12}}_{m+2} \right] + \int_1 \left[\mathrm{d}\sigma^{\rm RV,A_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\rm RR,A_1}_{m+2} \right)^{\rm A_1} \right] \right\}_{\varepsilon=0} J_m$$

The Colorful NNLO framework

@NNLO:

m-parton line:

$$\int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\}_{\varepsilon=0} J_m$$

- Free of kinematical singularities (jet-function!).
- Contains the integrals of the subtraction terms (see the talk of Gabor on thursday afternoon, too).
- State of the art: Two-loop amplitude (& integration of subtraction terms).

The Colorful NNLO framework

@NNLO:

m+1-parton line:

$$\int_{m+1}\left\{\left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1}+\int_1\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2}\right)J_{m+1}-\left[\mathrm{d}\sigma^{\mathrm{RV,A_1}}_{m+1}+\left(\int_1\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2}\right)^{\mathrm{A_1}}\right]J_m\right\}_{\varepsilon=0}$$

- Only singly unresolved regions present.
- But RV has different factorization properties (one-loop amplitude is involved).
- State of the art: fast and precise evaluation of the involved one-loop amplitude (even in the singly unresolved region!).

The Colorful NNLO framework

@NNLO:

m+2-parton line:

$$\int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A_2}}_{m+2} J_m - \left[\mathrm{d}\sigma^{\mathrm{RR,A_1}}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A_{12}}}_{m+2} J_m \right] \right\}_{\varepsilon=0}$$

- Both singly and doubly unresolved regions present.
- Also spurious ones! e.g.: kinematical singularities of A_2 in singly and kinematical singularities of A_1 in doubly unresolved regions.

The Colorful NNLO framework

m+2-parton line:

Bottleneck(s):

- RR SME still tree-level, but contains two partons more compared to the Born one.
- Several subtraction terms. For $e^+ e^- \to u\,\bar{u}\,g\,g\,g$ O(100).
- Generation of UB PS points (momentum mappings) and the corresponding UB SMEs.
- Application of physical cuts.
- Filling histograms.

The Colorful NNLO framework

Bottom line:

- Several subprocesses contribute.
- Various subtraction terms have to be calculated.
- Subtraction terms present a twofold problem: heavy bookkeeping due to the immense number of terms, a clever grouping is needed to ease up (a bit) the computational burden.

$\Rightarrow$More profitable to automatize subtractions!

A numerical implementation of the Colorful NNLO framework

A numerical implementation

For demonstration we take e+ e- to 3-jet production.

The code is organized directory-wise, for a new process a new folder has to be created:

```
Kaleu101212        genprocesses        rambo.f90
alpha.f90          genprocesses.f90    random.f90
ap.f90             hbb                 ranlux.f
boost.f90          iops.f90            regioncheck.f90
checklimits.f90    main.f90            regions.f90
checkpoles.f90     modules.f90         reshuffle.f90
debug.f90          multi_rambo.f90     sigma.f90
ee3jet             nnloinput.f         statistics.f90
fjcore.cc          observables.f90     subtractions.f90
fjcore.hh          phasespace.f90
```

A numerical implementation

The generation of all subprocesses is automatic:

```
! We define the process at the Born level:
   proc_LO(1) = 'e+'
   proc_LO(2) = 'e-'
   proc_LO(3) = 'j '
   proc_LO(4) = 'j '
   proc_LO(5) = 'j '
```

```
Number of subprocesses at the NNLO-RR level:   20
-11   11    5  -5    5  -5    0
-11   11    5  -5    4  -4    0
-11   11    3  -3    5  -5    0
-11   11    5  -5    2  -2    0
-11   11    1  -1    5  -5    0
-11   11    5  -5    0   0    0
-11   11    4  -4    4  -4    0
-11   11    3  -3    4  -4    0
-11   11    2  -2    4  -4    0
-11   11    1  -1    4  -4    0
-11   11    4  -4    0   0    0
-11   11    3  -3    3  -3    0
-11   11    3  -3    2  -2    0
-11   11    1  -1    3  -3    0
-11   11    3  -3    0   0    0
-11   11    2  -2    2  -2    0
-11   11    1  -1    2  -2    0
-11   11    2  -2    0   0    0
-11   11    1  -1    1  -1    0
-11   11    1  -1    0   0    0
```

A numerical implementation

Investigating for possible numerical relations between SMES:

```
 We found the following relations for the double real:
e+ e- -> b  b~ b  b~ g   : Irreducible
e+ e- -> b  b~ c  c~ g   : Irreducible
e+ e- -> s  s~ b  b~ g   : Irreducible
e+ e- -> b  b~ u  u~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> d  d~ b  b~ g   ~ 1.0000 e+ e- -> s  s~ b  b~ g
e+ e- -> b  b~ g  g  g   : Irreducible
e+ e- -> c  c~ c  c~ g   ~ 4.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> s  s~ c  c~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> u  u~ c  c~ g   ~ 4.0000 e+ e- -> s  s~ b  b~ g
e+ e- -> d  d~ c  c~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> c  c~ g  g  g   ~ 4.0000 e+ e- -> b  b~ g  g  g
e+ e- -> s  s~ s  s~ g   ~ 1.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> s  s~ u  u~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> d  d~ s  s~ g   ~ 1.0000 e+ e- -> s  s~ b  b~ g
e+ e- -> s  s~ g  g  g   ~ 1.0000 e+ e- -> b  b~ g  g  g
e+ e- -> u  u~ u  u~ g   ~ 4.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> d  d~ u  u~ g   ~ 1.0000 e+ e- -> b  b~ c  c~ g
e+ e- -> u  u~ g  g  g   ~ 4.0000 e+ e- -> b  b~ g  g  g
e+ e- -> d  d~ d  d~ g   ~ 1.0000 e+ e- -> b  b~ b  b~ g
e+ e- -> d  d~ g  g  g   ~ 1.0000 e+ e- -> b  b~ g  g  g
```

A numerical implementation

Automatic detection of all singular regions:

```
~~~~ Cirs ~~~~
iterm:    1 , b (3) -> b (3) || g (5) || g (6)
UBorn: e+ e- -> b  b~ g
                \->  b  g  g
iterm:    2 , b (3) -> b (3) || g (5) || g (7)
UBorn: e+ e- -> b  b~ g
                \->  b  g  g
iterm:    3 , b (3) -> b (3) || g (6) || g (7)
UBorn: e+ e- -> b  b~ g
                \->  b  g  g
iterm:    4 , b~(4) -> b~(4) || g (5) || g (6)
UBorn: e+ e- -> b  b~ g
                   \->  b~ g  g
iterm:    5 , b~(4) -> b~(4) || g (5) || g (7)
UBorn: e+ e- -> b  b~ g
                   \->  b~ g  g
iterm:    6 , b~(4) -> b~(4) || g (6) || g (7)
UBorn: e+ e- -> b  b~ g
                   \->  b~ g  g
iterm:    7 , g (5) -> g (5) || g (6) || g (7)
UBorn: e+ e- -> b  b~ g
                      \->  g  g  g
```

A numerical implementation

An NNLO calculation is extremely complex. Due to this complexity it is good practice to make as much checks as possible.

In our code the following ones are built in:

- Check upon individual subtraction terms, e.g.:

$$\lim_{p_i||p_r||p_s} \frac{\mathcal{C}_{irs}}{|\mathcal{M}_{\mathrm{RR}}|^2} = 1$$

- Checking bookkeeping and overall consistency by checking complete lines, e.g.:

$$\lim_{p_i||p_r,\, p_s\to 0} \frac{\mathcal{A}_1 + \mathcal{A}_2 - \mathcal{A}_{12}}{|\mathcal{M}_{\mathrm{RR}}|^2} = 1$$

A numerical implementation

Testing the subtraction terms in all limits (even in quad precision):

```
iterm:    5 , g (3) -> g (7) || b (3) || b~(6)
UBorn: e+ e- -> g  b~ b
                \->  g  b  b~
iexp=   1  , Cirs/RR=    1.00803271854102469359760256565251
iexp=   0  , Cirs/RR=    1.00499213240252449541142746655114
iexp=   0  , Cirs/RR=    1.00188210122417253945669893587992
iexp=  -1  , Cirs/RR=    1.00062799209472484026964163812994
iexp=  -2  , Cirs/RR=    1.00020195770962404594799550034461
iexp=  -3  , Cirs/RR=    1.00006420440823578844890246156009
iexp=  -4  , Cirs/RR=    1.00002033728517247971408852818993
iexp=  -5  , Cirs/RR=    1.00000643462401283847535738494549
iexp=  -6  , Cirs/RR=    1.00000203514784404384857605445244
iexp=  -7  , Cirs/RR=    1.00000064360436586785172322902143
iexp=  -8  , Cirs/RR=    1.00000020352898187202854767170299
iexp=  -9  , Cirs/RR=    1.00000006436185636550093621075798
iexp= -10  , Cirs/RR=    1.00000002035304016617958048905596
iexp= -11  , Cirs/RR=    1.00000000643619983435100933779238
iexp= -12  , Cirs/RR=    1.00000000203530543521165651669029
iexp= -13  , Cirs/RR=    1.00000000064362017775031206617771
iexp= -14  , Cirs/RR=    1.00000000020353075587311446332694
iexp= -15  , Cirs/RR=    1.00000000006435764917616243177692
iexp= -16  , Cirs/RR=    1.00000000002031691222958531635813
iexp= -17  , Cirs/RR=    1.00000000000562358122843217515565
```

A numerical implementation

Testing the whole m+2 parton line:

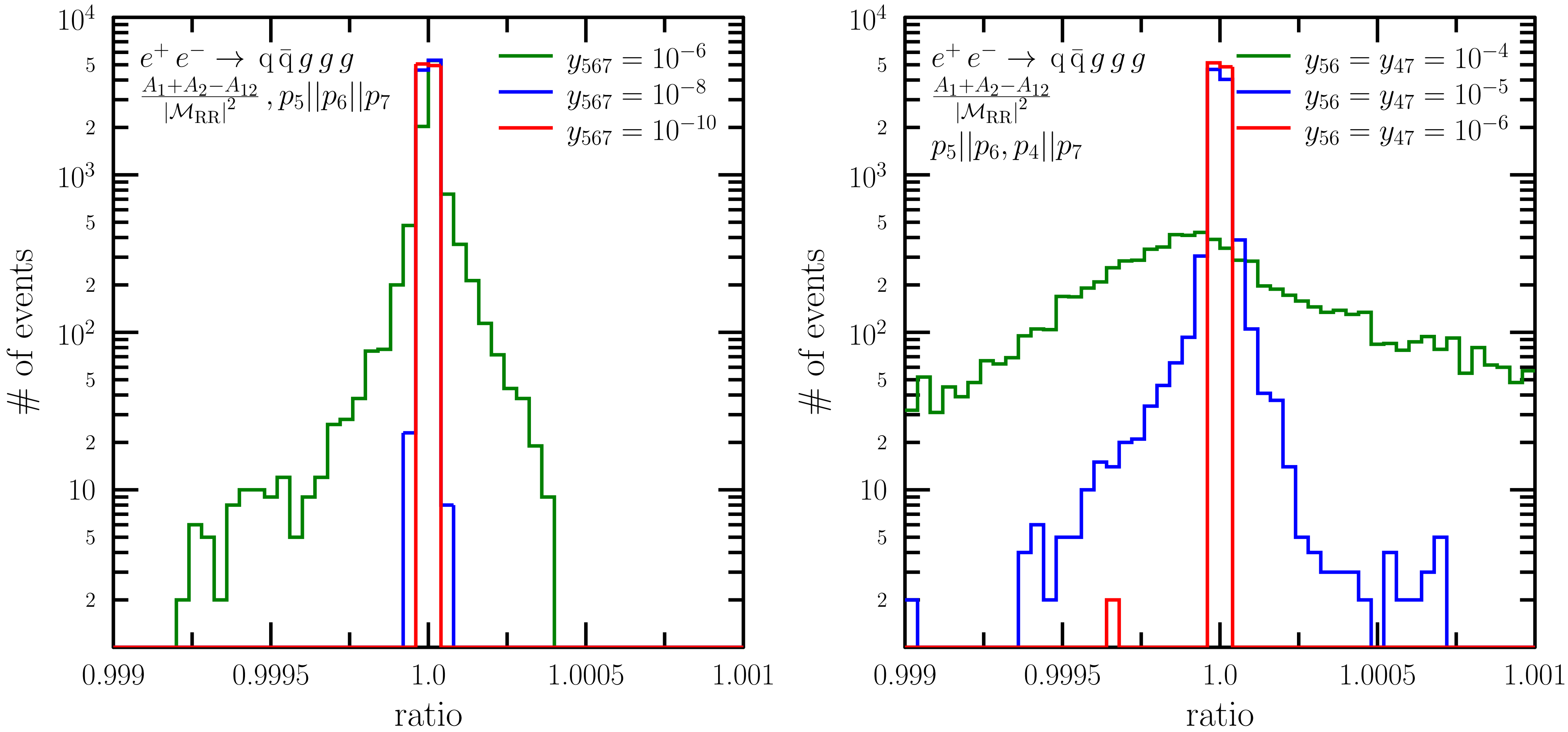
Doubly unresolved

```
CSirs: g (6) -> g (6) || g (7) , g (5) -> 0  VALID
iter no.  1 scale no.  1 1.06266634948744061310369102475825 *-WARN-*
iter no.  2 scale no.  1 .99933339118756664131317235085510 9
iter no.  3 scale no.  1 .9999360567162066793019613286621 79
iter no.  4 scale no.  1 .99999321715885735308166967682532 0
iter no.  5 scale no.  1 .99999928952733456236747237157707 3
iter no.  6 scale no.  1 .99999992795555748046415914784189 5
iter no.  7 scale no.  1 .99999999276423133274830626094779 4
iter no.  8 scale no.  1 .99999999927543467248458956328478 1
iter no.  9 scale no.  1 .99999999992751222931850440666947 9
iter no. 10 scale no.  1 .99999999999275023532799673566332 0
iter no. 11 scale no.  1 .99999999999927499230431132728220 4
iter no. 12 scale no.  1 .99999999999992749824289475291072 9
iter no. 13 scale no.  1 .99999999999999274979447470927552 7
iter no. 14 scale no.  1 .99999999999999927500384398391191 8
iter no. 15 scale no.  1 .99999999999999992767541466253552 1
```

Singly unresolved:

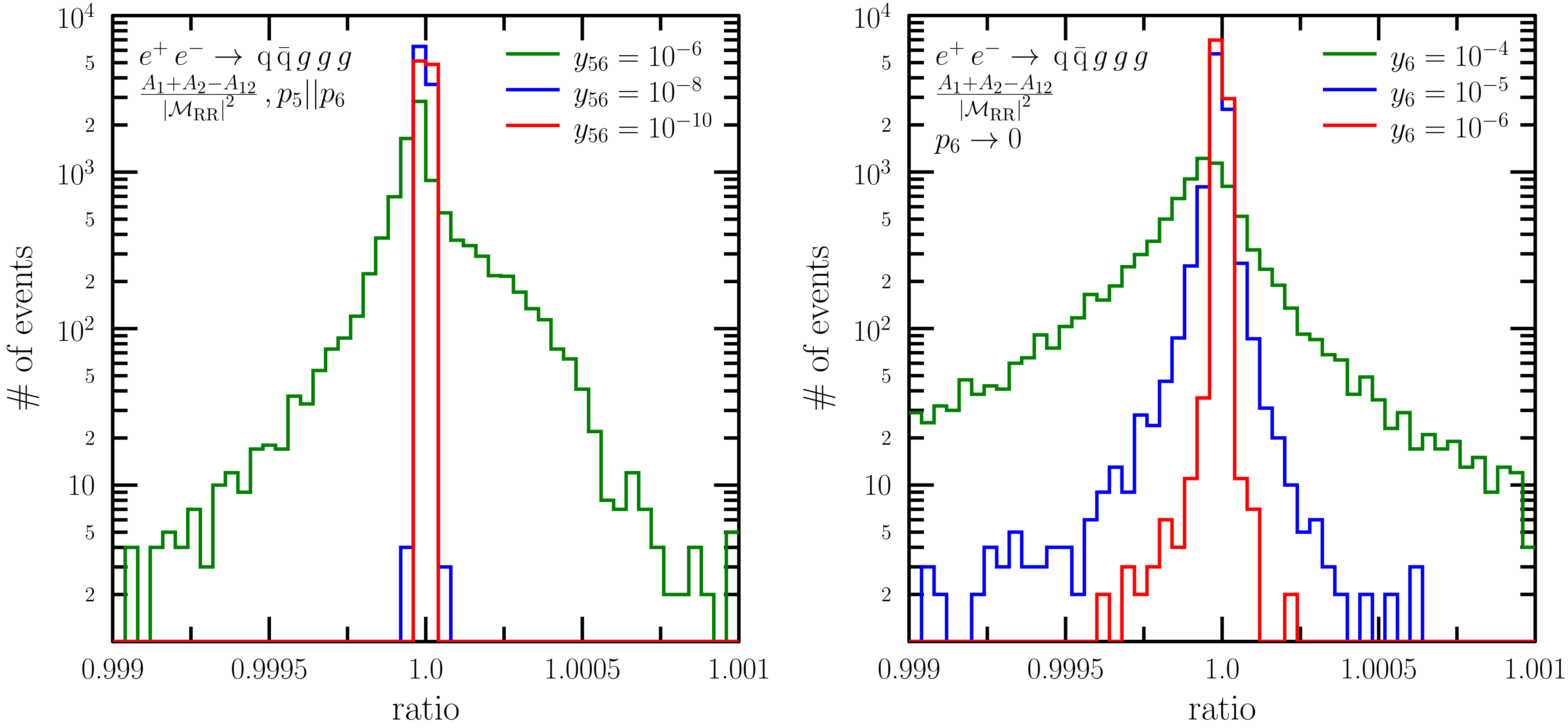
```
Cir: b (3) -> b (3) || g (7)  VALID
iter no.  1 scale no.  1 .96148670801871865442260647152993 8 *-WARN-*
iter no.  2 scale no.  1 1.00602959209786220837235112804777
iter no.  3 scale no.  1 1.00066580047174234782868128197356
iter no.  4 scale no.  1 1.00006749924864464471460885374332
iter no.  5 scale no.  1 1.00000675951123416892158622562722
iter no.  6 scale no.  1 1.00000067604739572862393476710447
iter no.  7 scale no.  1 1.00000006760570270606858225599869
iter no.  8 scale no.  1 1.00000000676057990234915689940388
iter no.  9 scale no.  1 1.00000000067605808655274887283141
iter no. 10 scale no.  1 1.00000000006760580961845340615602
iter no. 11 scale no.  1 1.00000000000676058097147183507127
iter no. 12 scale no.  1 1.00000000000067605809725802473631
iter no. 13 scale no.  1 1.00000000000006760580921822736597
iter no. 14 scale no.  1 1.00000000000000676057794954317165
iter no. 15 scale no.  1 1.00000000000000067615396661119602
```

A numerical implementation



Behavior of the m+2 parton line in a triple and double collinear limit.

A numerical implementation



Behavior of the m+2 parton line in a collinear and a soft limit.

Towards phenomenology

$e^+ e^- \rightarrow 3$ jets

Considering only $e^+ + e^- \to \gamma^* \to 3\,\mathrm{jets}$

NNLO* : no VV and I operators, hence non-physical, though still informative.

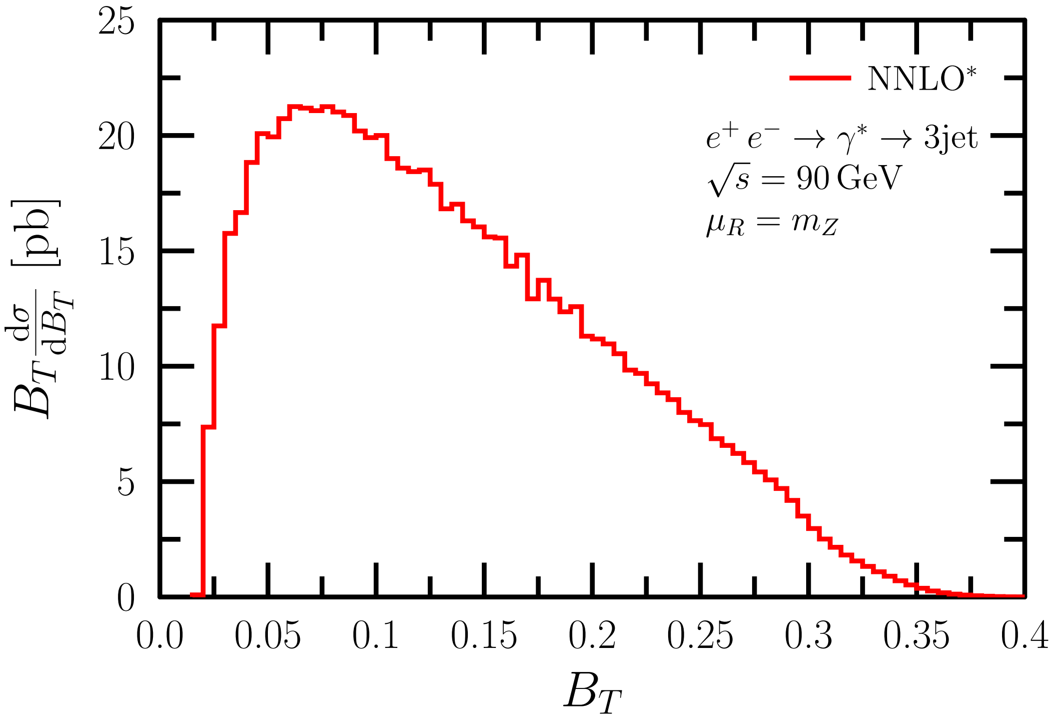
$$\sqrt{s} = 90\,\mathrm{GeV}$$
$$\mu_R = m_Z$$
$$m_Z = 91.1876\,\mathrm{GeV}\,,\ m_W = 80.385\,\mathrm{GeV}$$
$$\alpha_s(m_Z) = 0.118,\ 1/\alpha_{\mathrm{EM}} = 132.23$$

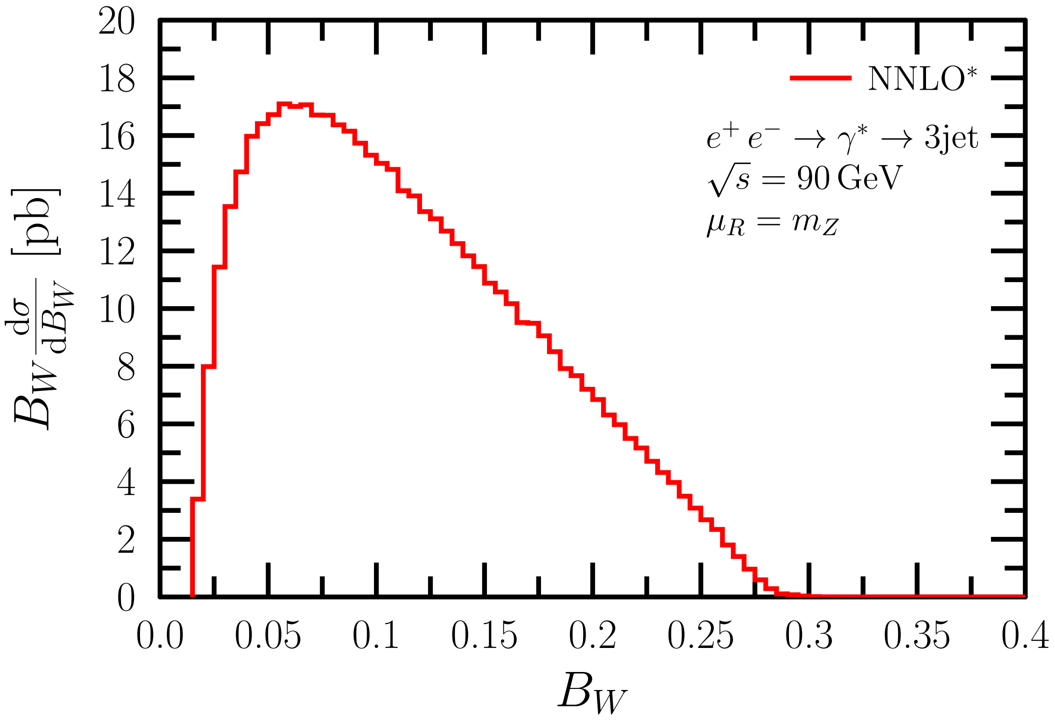
Calculating key event shape variables. For a precise definition, see: Gehrmann-De Ridder et al., arXiv:0711.4711 and also Kunszt et al. QCD at LEP, ETH-PT-89-39.

The whole calculation took 4+4 hours on 300 cores.

$e^+ e^- \to 3$ jets

Total and wide jet broadening:

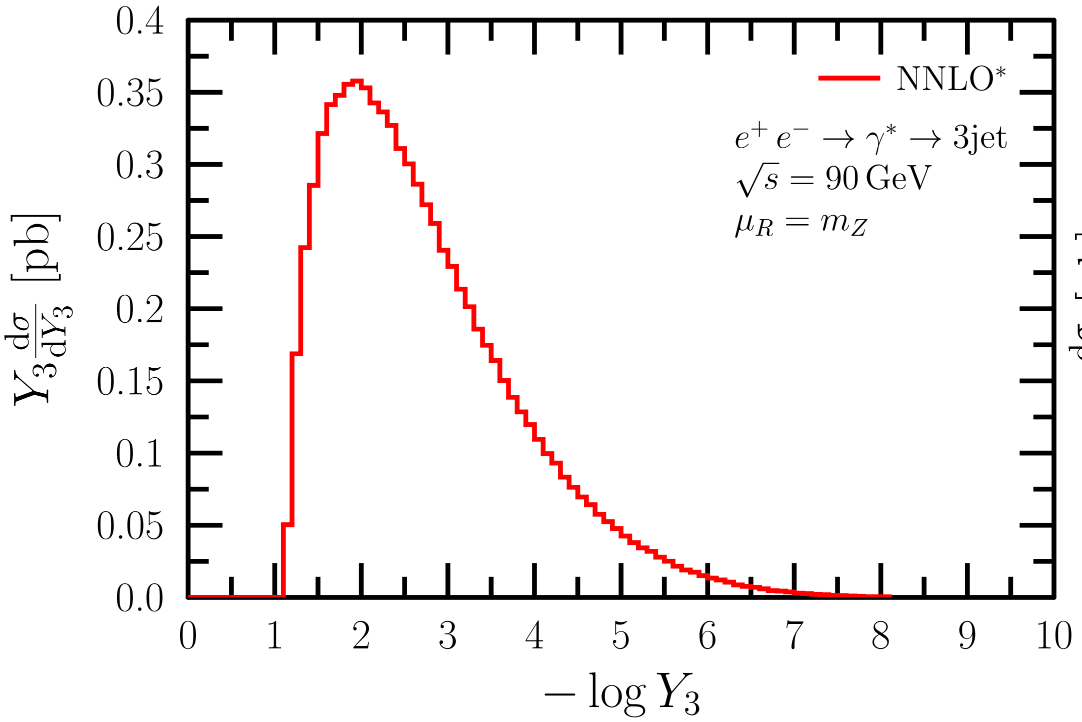


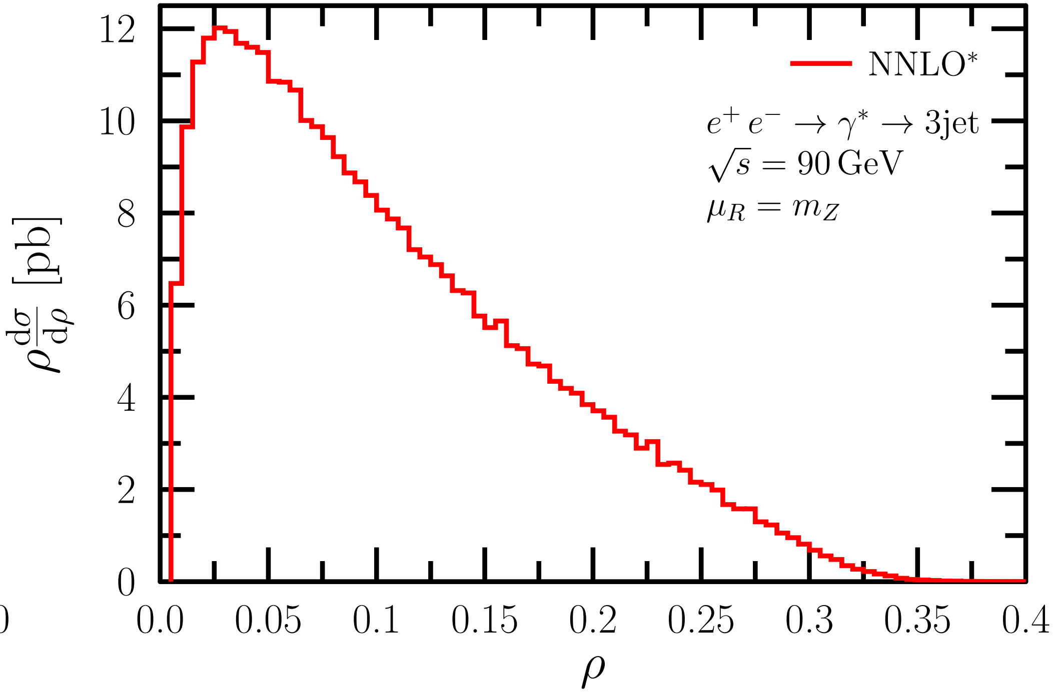


Preliminary

$e^+ e^- \rightarrow 3$ jets

The jet transition variable and large hemisphere invariant mass:

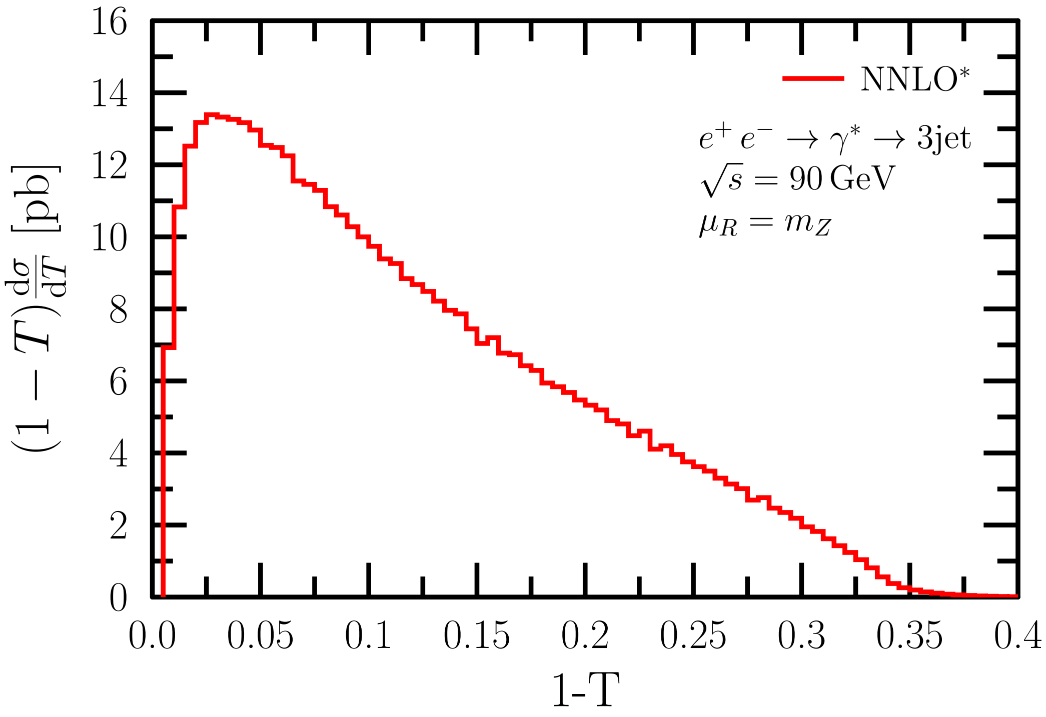


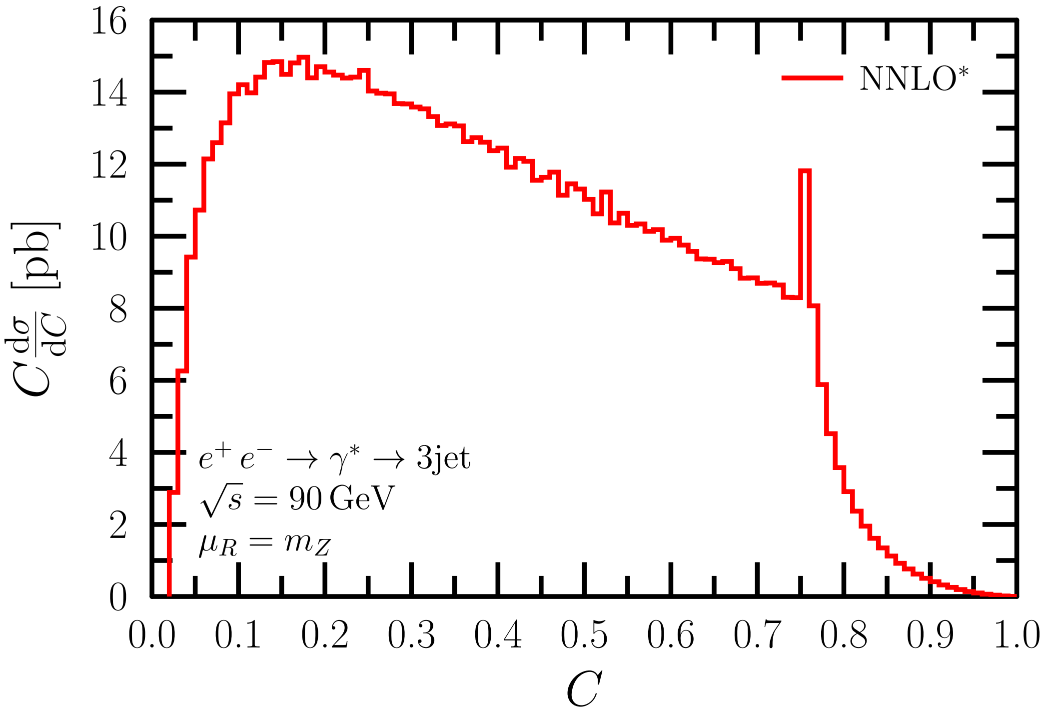


Preliminary

$e^+ e^- \rightarrow 3$ jets

The jet transition variable and large hemisphere invariant mass:



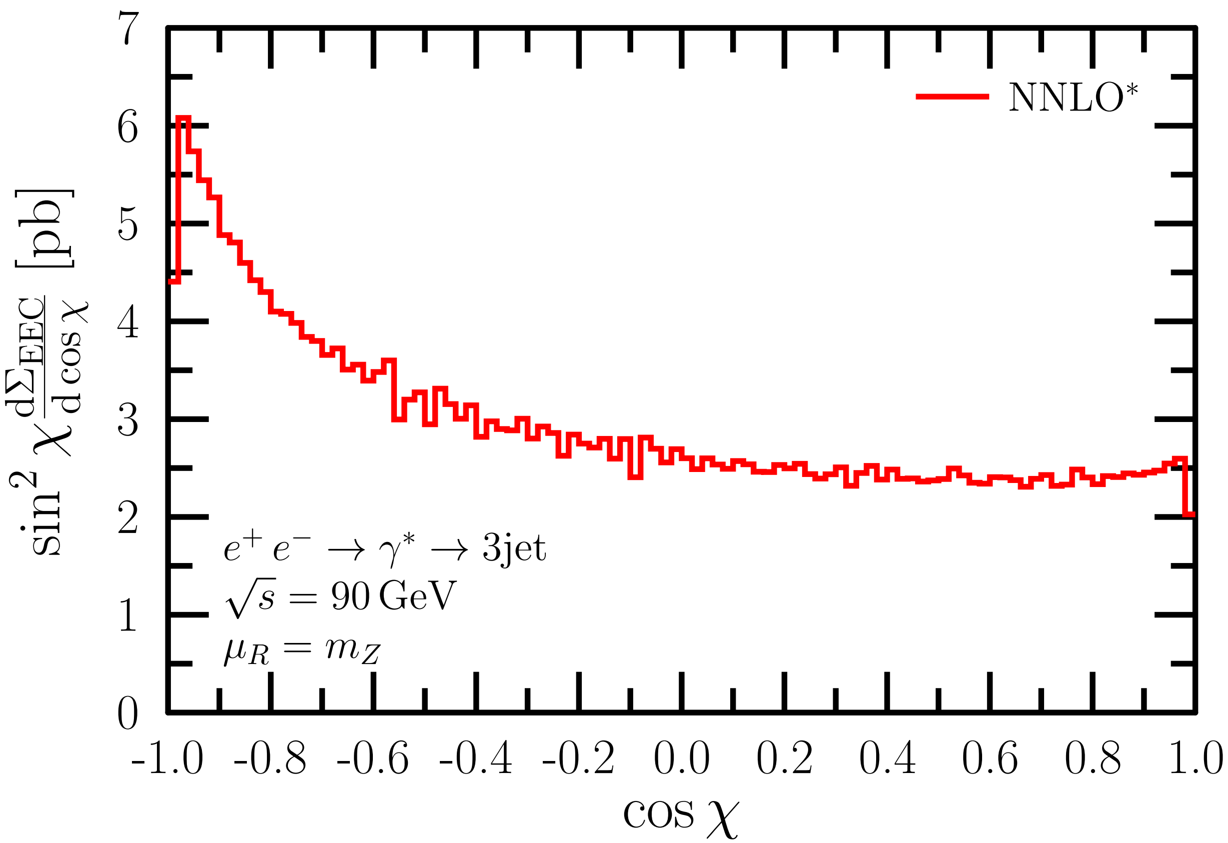


Preliminary

$e^+ e^- \rightarrow 3$ jets

… and even a completely new one, never appeared in the literature before:

Energy-energy correlation:



Preliminary

Calculation cost: 7 hours @ 300 cores.

Conclusions

Conclusions

- A general numerical NNLO framework is introduced to manage subtractions for the case of $e^+ e^-$ annihilation.
- It contains all subtraction terms, the user only has to provide his/her matrix elements (the I operators will also be included).
- The framework was applied to 3-jet production to obtain some key distributions.
- The inclusion of the I operators and the VV part for 3-jet production is in progress.

Thank you for your attention!