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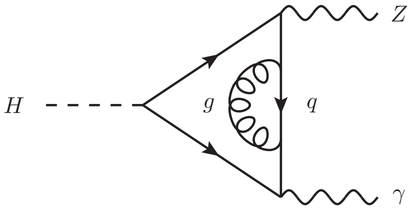
UCLA Radcor-Loopfest 2015

The rare decay $H \to Z\gamma$ in perturbative QCD

[arXiv: hep-ph/1505.00561]

Thomas Gehrmann, Sam Guns & Dominik Kara | June 15, 2015

RADCOR 2015 AND LOOPFEST XIV - UNIVERSITY OF CALIFORNIA, LOS ANGELES



Contents

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Motivation: Experiment

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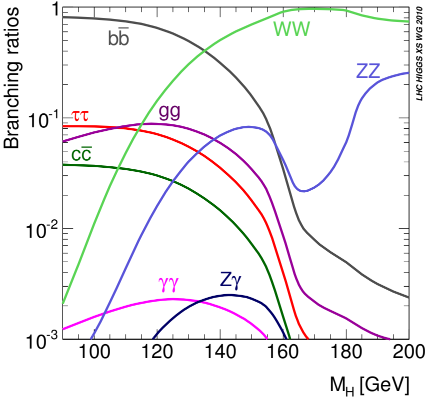
$H \to \gamma\gamma$

- loop-mediated
  $\Rightarrow$ sensitive to new physics
- clean signature
  $\Rightarrow$ 20% relative precision

$H \to Z\gamma$

- more background
- smaller branching ratio
- spin-dependent particle correlations through $Z \to ll$



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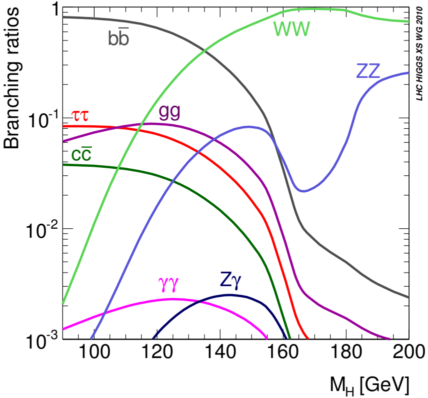

$\Rightarrow$ **broader spectrum of observables: more info on H couplings**

Motivation: Theory

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$H \to Z\gamma$

- Analytical LO result available [Cahn, Chanowitz, Fleishon (1979)] [Bergstrom, Huth (1985)]
- Numerical NLO result in QCD available [Spira, Djouadi, Zerwas (1992)]

$\Rightarrow$ **Analytical NLO result: independent check**

$H + j$ production

- NLO result in QCD available for $m_t \to \infty$ [Schmidt (1997)] [Glosser, Schmidt (2002)] [de Florian, Grazzini, Kunszt (1999)] [Ravindran, Smith, van Neerven (2002)]
- NNLO calculation in QCD well-advanced for $m_t \to \infty$ [Gehrmann, Glover, Jaquier, Koukoutsakis (2012)] [Chen, Gehrmann, Glover, Jaquier (2012)] [Boughezal, Caola, Melnikov, Petriello, Schulze (2013, 2015)] [Boughezal, Focke, Giele, Liu, Petriello (2015)]

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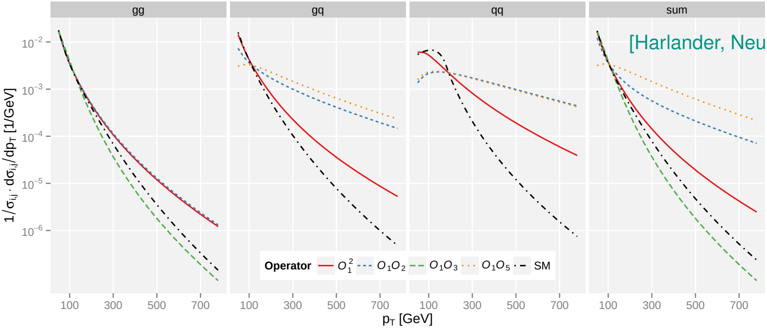


[Harlander, Neumann (2013)]

$m_t \to \infty$

SM

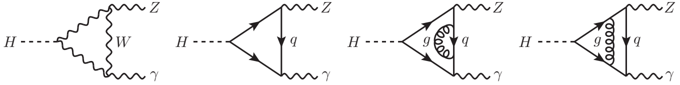
$H + j$ production

- EFT is likely to break down at high p_T
- High-priority aim: NLO QCD corrections with full m_t dependence

$\Rightarrow$ **Two-loop integrals for $H \to Z\gamma$ pave the way**

Outline of the calculation

QGRAF ■ Generate Feynman diagrams for process $H(q) \to Z(p_1)\gamma(p_2)$



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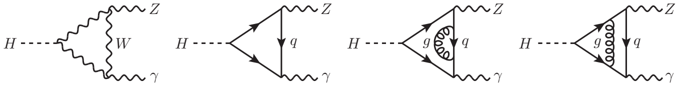

FORM

- Project relevant Feynman diagrams onto tensor structure

$$\mathcal{M} = A \; \epsilon_{1,\mu}(p_1, \lambda_1)\, \epsilon_{2,\nu}(p_2, \lambda_2)\, \frac{P^{\mu\nu}}{P^2}$$

with projector

$$P^{\mu\nu} = p_2^{\mu} p_1^{\nu} - (p_1 \cdot p_2) g^{\mu\nu}$$

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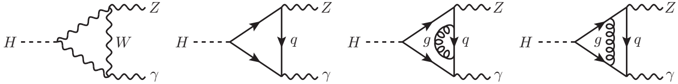

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- Reduce Feynman Integrals to set of Master Integrals (MIs)
  - Integration-by-parts identities (IBPs)
  - Laporta algorithm

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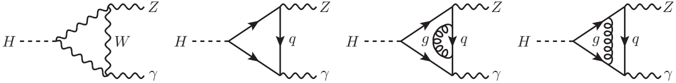

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REDUZE

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The amplitude

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can be further decomposed

$$A = c_W A_W + \sum_q c_q A_q$$

with

$$c_W = \frac{\cos\theta_w}{\sin\theta_w}, \qquad c_q = N_c \frac{2\,Q_q\left(I_q^3 - 2\,Q_q \sin^2\theta_w\right)}{\sin\theta_w \cos\theta_w}$$

- Born-level contribution:

$$A^{(1)} = c_W A_W^{(1)} + c_t A_t^{(1)} + c_b A_b^{(1)}$$

- NLO QCD corrections:

$$A_q(m_H, m_Z, m_q, \alpha_s, \mu) = A_q^{(1)}(m_H, m_Z, m_q) + \frac{\alpha_s(\mu)}{\pi} A_q^{(2)}(m_H, m_Z, m_q, \mu)$$

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The decay width

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is obtained from the amplitude as

$$\Gamma = \frac{\pi\, G_F\, \alpha^2}{4\sqrt{2}\, m_H^3\, (m_H^2 - m_Z^2)} \boxed{|A|^2}$$

$\Rightarrow$ We are left with computation of MIs

Parametrization

Use Landau-type variables to absorb natural roots

$$m_H^2 = -m_q^2 \frac{(1-h)^2}{h}, \qquad m_Z^2 = -m_q^2 \frac{(1-z)^2}{z}$$

$$\Rightarrow \sqrt{1 - 4\frac{m_q^2}{m_H^2}} \to \frac{|h+1|}{|h-1|}, \quad \sqrt{1 - 4\frac{m_q^2}{m_Z^2}} \to \frac{|z+1|}{|z-1|}$$

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Differential equations

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1. Choose MI
2. Compute derivative of integrand with respect to internal mass and external invariants
3. Use IBPs to relate resulting integrals to original MI

[Kotikov (1991)]
[Remiddi (1997)]
[Gehrmann, Remiddi (2000)]

Full system takes form of total differential

$$\mathrm{d}\vec{I}(h,z) = \sum_{k=1}^{N} R_k(\epsilon)\,\mathrm{d}\log(d_k)\,\vec{I}(h,z)$$

$d_1 = z$	$d_5 = h+1$	$d_9 = h^2 - hz - h + 1$
$d_2 = z+1$	$d_6 = h-1$	$d_{10} = h^2 z - hz - h + z$
$d_3 = z-1$	$d_7 = h-z$	$d_{11} = z^2 - hz - z + 1$
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Differential equations

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Canonical form

[Henn (2013)]

$$\mathrm{d}\vec{M}(h,z) = \boxed{\epsilon} \sum_{k=1}^{12} R_k \,\mathrm{d}\log(d_k)\, \vec{M}(h,z)$$

- ✓ reduced number of polynomials d_k
- ✓ can be integrated in terms of GHPLs:

$$G\left(w_1,\ldots,w_n;x\right) \equiv \int_0^x \mathrm{d}t\,\frac{1}{t-w_1}\,G\left(w_2,\ldots,w_n;t\right)$$

$$G\left(\vec{0}_n;x\right) \equiv \frac{\log^n x}{n!}$$

- ✓ leads to linear combinations of GHPLs of homogeneous weight

$\Rightarrow$ Change basis from Laporta integrals $\vec{I}$ to canonical integrals $\vec{M}$

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$\Rightarrow$ **Change basis from Laporta integrals $\vec{I}$ to canonical integrals $\vec{M}$**

Integral basis

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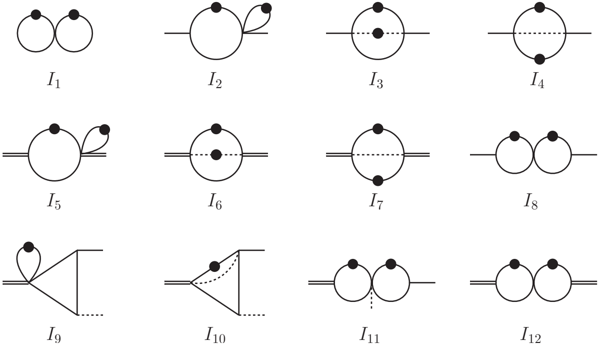
1. Start with Laporta basis: [Gehrmann, von Manteuffel, Tancredi, Weihs (2014)]
   Topology under consideration must
   - ✓ have at most linear dependence on ϵ
   - ✓ be triangular in $D = 4$ dimensions



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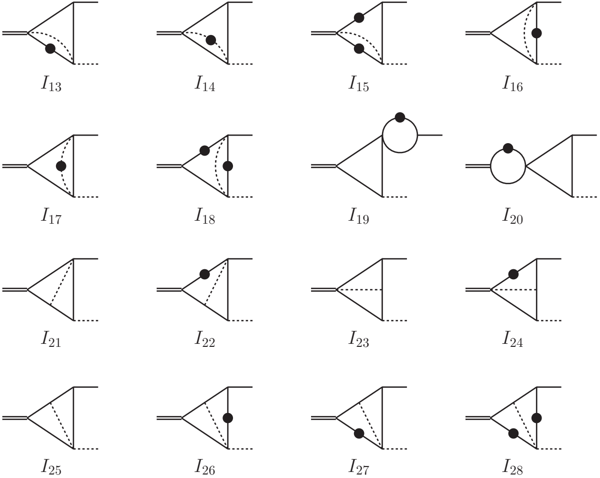

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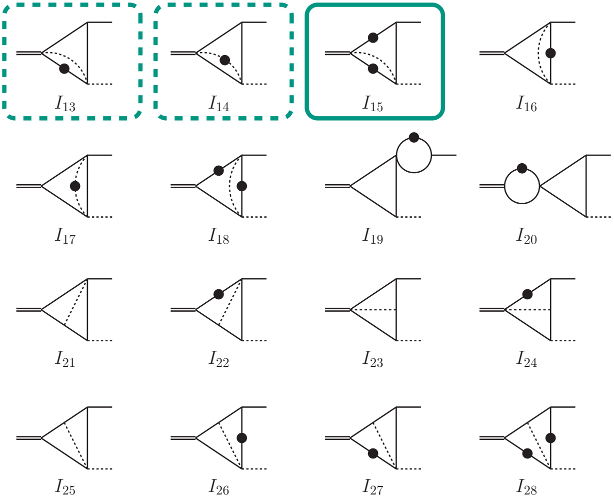

Integral basis

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2. Perform transformation to canonical basis by integrating out homogeneous parts in $D = 4$

$$\frac{\mathrm{d}I_{15}}{\mathrm{d}z} = \frac{h\,z}{d_1\,d_2\,d_3\,d_4\,d_9\,d_{10}\,\left(2\,z\,(h^2+1) - h\,(z+1)^2\right)} \cdot \Big\{ \frac{3}{2}\,h\,z\,(h-1)^2\,\left((h^2+1)\,(z^2+1) - h\,(z+1)^2\right) I_7$$
$$- h\left[h^4\,z\,(z^2+1) - h\,(h^2+1)\,(z^4+z^3+4z^2+z+1) + h^2\,(z^4+4z^3+2z^2+4z+1) + z\,(z^2+1)\right](2\,I_{13}+I_{14})$$
$$- \frac{1}{z}\left[2\,h^6\,z^2\,(z^2+1) - 2\,h\,z\,(h^4+1)\,(z+1)^2\,(2\,z^2-z+2) + h^2\,(h^2+1)\,(z^6+8\,z^5+17\,z^4+8\,z^3+17\,z^2+8\,z+1) + 2\,z^2\,(z^2+1)\right] I_{15}\Big\}$$

$$\frac{\mathrm{d}I_{15}}{\mathrm{d}h} = \frac{h\,z}{d_1\,d_4\,d_9\,d_{10}\,\left(2\,z\,(h^2+1) - h\,(z+1)^2\right)} \cdot \Big\{ -\frac{3}{2}\,z^2\,(h-1)^3\,(h+1)\,I_7 + z\,(h^2-1)\,\left(z\,(h^2+1) - h\,(z^2+1)\right)(2\,I_{13}+I_{14})$$
$$- \frac{h^2-1}{h}\left[2z^2\,(h^4+1) - 2hz\,(z+1)^2\,(h^2+1) + h^2\,(z^4+2z^3+6z^2+2z+1)\right] I_{15}\Big\}$$

Integral basis

University of Zurich[UZH]

2. Perform transformation to canonical basis by integrating out homogeneous parts in $D = 4$

$$M_{15} = \frac{z\,(h^2+1) - h\,(z+1)}{2\,z\,(h^2+1) - h\,(z+1)^2} \left[\frac{3}{2}\,\frac{(h-1)^2}{h} I_7 - \frac{(h-z)(hz-1)}{hz}\,(2\,I_{13} + I_{14}) - \frac{(z^2-1)(h^2+1-h\,(z+1))}{hz}\,I_{15} \right]$$

Integral basis

University of Zurich[UZH]

2. Perform transformation to canonical basis by integrating out homogeneous parts in $D = 4$

$$
\begin{aligned}
\mathrm{d}\boldsymbol{M_{15}} = \epsilon \Big[& -\left(M_2 + \frac{3}{2} M_7 + 5\, M_{13} + M_{14} - 4\, \boldsymbol{M_{15}} \right) \mathrm{d}\log(d_1) \\
& + \left(\frac{3}{2} M_7 + 2\, M_{13} + M_{14} - 2\, \boldsymbol{M_{15}} \right) \mathrm{d}\log(d_2) \\
& - \left(M_2 + \frac{3}{2} M_7 + M_{13} - M_{14} - \boldsymbol{M_{15}} \right) \mathrm{d}\log(d_3) \\
& - \left(M_2 + 2\, M_6 + \frac{5}{2} M_7 + M_{13} - M_{14} - \boldsymbol{M_{15}} \right) \mathrm{d}\log(d_4) + 3\, M_7\, \mathrm{d}\log(d_6) \\
& + (M_2 + M_6 + 2\, M_7 + 3\, M_{13} - 2\, \boldsymbol{M_{15}})\, \mathrm{d}\log(d_7) \\
& + (M_2 - M_6 + M_7 + 3\, M_{13} - 2\, \boldsymbol{M_{15}})\, \mathrm{d}\log(d_8) \\
& - \left(\frac{3}{2} M_7 + 2\, M_{13} + M_{14} - \boldsymbol{M_{15}} \right) \mathrm{d}\log(d_9) \Big]
\end{aligned}
$$

Integration

- Integrate differential equation in h or z up to constant $C(z)$ or $C(h)$
- Use boundary conditions:

$$
\begin{aligned}
h &= 1 &\leftrightarrow&& m_H^2 &= 0\\
z &= 1 &\leftrightarrow&& m_Z^2 &= 0\\
h &= z &\leftrightarrow&& m_H^2 &= m_Z^2\\
h &= \frac{1}{z} &\leftrightarrow&& m_H^2 &= m_Z^2
\end{aligned}
$$

- Perform transformations with the help of symbol and coproduct:

$$
\begin{aligned}
&G\left(w_1(x), \ldots, w_n(x); x\right) \to G\left(a_1, \ldots, a_n; x\right)\\
&G\left(w_1(x), \ldots, w_n(x); y\right) \to G\left(c_1(y), \ldots, c_n(y); x\right)
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Integration

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Results in terms of GHPLs up to weight four

$$G(a_1,\ldots,a_n;h) \quad \text{with} \quad a_i \in \left\{0,\pm1,z,\frac{1}{z},J_z,\frac{1}{J_z},K_z^\pm,L_z^\pm\right\}$$

$$G(b_1,\ldots,b_n;z) \quad \text{with} \quad b_i \in \{0,\pm1,c,\bar{c}\}$$

$$c = \frac{1}{2}\left(1+i\sqrt{3}\right) \qquad K_z^\pm = \frac{1}{2}\left(1+z\pm\sqrt{-3+2z+z^2}\right)$$

$$J_z = \frac{z}{1-z+z^2} \qquad L_z^\pm = \frac{1}{2z}\left(1+z\pm\sqrt{1+2z-3z^2}\right)$$

- ✓ verified through differential equation in other variable
- ✓ checked numerically against SecDec

[Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke (2015)]

Integration

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Results in terms of GHPLs up to weight four

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$$c = \frac{1}{2}\left(1 + i\sqrt{3}\right) \qquad K_z^{\pm} = \frac{1}{2}\left(1 + z \pm \sqrt{-3 + 2\,z + z^2}\right)$$

$$J_z = \frac{z}{1 - z + z^2} \qquad L_z^{\pm} = \frac{1}{2\,z}\left(1 + z \pm \sqrt{1 + 2\,z - 3\,z^2}\right)$$

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Integration

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Example

$$
\begin{aligned}
M_{15} = \epsilon^2 \Big[& -\frac{\pi^2}{6} - G(\frac{1}{z},0;h) + G(z,0;h) - G(0;z)G(\frac{1}{z};h) - G(0;z)G(z;h) + G(0;z)G(0;h) \\
& -2G(0,0;h) + G(1,0;z)\Big] \\
+ \epsilon^3 \Big[& 5\zeta_3 + 2G(\frac{1}{z},\frac{1}{z},0;h) - 2G(\frac{1}{z},z,0;h) + \frac{2\pi^2}{3}G(\frac{1}{z};h) + 6G(\frac{1}{z},-1,0;h) \\
& - 3G(\frac{1}{z},0,0;h) + 2G(\frac{1}{z},1,0;h) + 2G(z,\frac{1}{z},0;h) - 2G(z,z,0;h) + \frac{\pi^2}{3}G(z;h) \\
& - 6G(z,-1,0;h) + 5G(z,0,0;h) - 2G(z,1,0;h) - G(K_z^-,\frac{1}{z},0;h) + G(K_z^-,z,0;h) \\
& - \frac{\pi^2}{6}G(K_z^-;h) + 2G(K_z^-,0,0;h) - G(K_z^+,\frac{1}{z},0;h) + G(K_z^+,z,0;h) - \frac{\pi^2}{6}G(K_z^+;h) \\
& + 2G(K_z^+,0,0;h) + \frac{\pi^2}{3}G(-1;z) + 2G(-1,0;z)G(\frac{1}{z};h) + 2G(-1,0;z)G(z;h) \\
& - 2G(-1,0;z)G(0;h) + G(-1,0,0;z) - 2G(-1,1,0;z) - G(0,\frac{1}{z},0;h) - \frac{\pi^2}{2}G(0;z)
\end{aligned}
$$

Integration

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Example

$$
\begin{aligned}
&+2G(0;z)G(\frac{1}{z},\frac{1}{z};h)+2G(0;z)G(\frac{1}{z},z;h)-2G(0;z)G(\frac{1}{z},0;h)+2G(0;z)G(z,\frac{1}{z};h)\\
&+2G(0;z)G(z,z;h)-2G(0;z)G(z,0;h)-G(0;z)G(K_z^-,\frac{1}{z};h)-G(0;z)G(K_z^-,z;h)\\
&+G(0;z)G(K_z^-,0;h)-G(0;z)G(K_z^+,\frac{1}{z};h)-G(0;z)G(K_z^+,z;h)+G(0;z)G(K_z^+,0;h)\\
&-G(0;z)G(0,\frac{1}{z};h)-G(0;z)G(0,z;h)+G(0,z,0;h)+G(0;h)G(1,0;z)+12G(0,-1,0;h)\\
&-G(0,0;z)G(\frac{1}{z};h)-G(0,0;z)G(z;h)-G(0,0;z)G(K_z^-;h)-G(0,0;z)G(K_z^+;h)\\
&+2G(0,0;z)G(0;h)+G(0,0;h)G(0;z)-G(0,0,0;z)-8G(0,0,0;h)+2G(0,1,0;z)\\
&+4G(0,1,0;h)-\frac{\pi^2}{3}G(1;z)-2G(1,-1,0;z)-2G(1,0;z)G(\frac{1}{z};h)-2G(1,0;z)G(z;h)\\
&+G(1,0;z)G(K_z^-;h)+G(1,0;z)G(K_z^+;h)+2G(1,0,0;z)-6G(1,0,0;h)+G(1,1,0;z)\Big]\\
&+\mathcal{O}\left(\epsilon^4\right)
\end{aligned}
$$

Renormalization

University of Zurich[UZH]

(a) quark mass M_q and Yukawa coupling Y_q in OS scheme

$$A_q^{(2,a)}(m_H, m_Z, M_q) = A_{q,\mathrm{bare}}^{(2)} + Z_{\mathrm{OS}}\, A_q^{(1)} + \frac{\delta m_{\mathrm{OS}}}{m_q}\, C_q^{(1)}$$

- Renormalization constants

$$Z_{\mathrm{OS}} = \frac{\alpha_s(\mu)}{\pi} 16\, i\, \pi^2\, S_\epsilon\, \frac{C_F}{4}\, \frac{3-2\epsilon}{\epsilon\,(1-2\epsilon)}$$

$$\delta m_{\mathrm{OS}} = m_q\, Z_{\mathrm{OS}}$$

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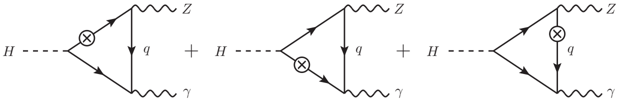
- Counterterms

$C_q^{(1)} =$



Renormalization

University of Zurich[UZH]

(b) quark mass M_q in OS, Yukawa coupling $\overline{y}_q$ in $\overline{\mathrm{MS}}$ scheme

$$A_q^{(2,b)}(m_H, m_Z, \overline{m}_q, \mu) = A_q^{(2,a)}(m_H, m_Z, \overline{m}_q(\mu)) + \Delta \cdot A_q^{(1)}(m_H, m_Z, \overline{m}_q(\mu))$$

The finite shift in the amplitude is induced by expressing the OS quantities in terms of $\overline{\mathrm{MS}}$ quantities:

$$M_q = \overline{m}_q(\mu)\ (1 + \Delta)$$

$$Y_q = \overline{y}_q(\mu)\ (1 + \Delta)$$

$$\Delta = \frac{\alpha_s(\mu)}{\pi}\, C_F \left(1 + \frac{3}{4}\log\frac{\mu^2}{\overline{m}_q^2(\mu)}\right)$$

Renormalization

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(c) quark mass $\overline{m}_q$ and Yukawa coupling $\overline{y}_q$ in $\overline{\mathrm{MS}}$ scheme

$$A_q^{(2,c)}(m_H, m_Z, \overline{m}_q, \mu) = A_q^{(2,b)}(m_H, m_Z, \overline{m}_q(\mu)) + \overline{\Delta} \cdot \left. \frac{\partial A_q^{(1)}(m_H, m_Z, M_q)}{\partial M_q} \right|_{M_q = \overline{m}_q(\mu)}$$

$\overline{\Delta}$ is defined through the following replacements in $A_q^{(1)}$:

$$h = \bar{h} - 2\,\Delta\,\bar{h}\,\frac{\bar{h}-1}{\bar{h}+1}$$

$$z = \bar{z} - 2\,\Delta\,\bar{z}\,\frac{\bar{z}-1}{\bar{z}+1}$$

Numerical results

Analytic continuation from Euclidean to Minkowski region

- Region I: $m_Z^2 < m_H^2 < 4\,m_q^2$ $\Rightarrow$ top quark amplitude
- ~~Region II: $m_Z^2 < 4\,m_q^2 < m_H^2$~~ $\Rightarrow$ not needed
- Region III: $4\,m_q^2 < m_Z^2 < m_H^2$ $\Rightarrow$ bottom quark amplitude

NLO decay width $\Gamma^{(2)}$ in renormalization schemes (a), (b) and (c)

$$\Gamma^{(2,a)} = \left[7.04500 + 0.42617\,\frac{\alpha_s(\mu)}{\pi}\right]\mathrm{keV} \stackrel{\mu=m_H}{=} 7.06033\,\mathrm{keV} \qquad 2‰$$

$$\Gamma^{(2,b)} = \left[7.06369 + \frac{\alpha_s(\mu)}{\pi}\left(-0.53038 - 0.76333\,\log\frac{\mu^2}{\overline{m}_t^2(\mu)} + 0.01224\,\log\frac{\mu^2}{\overline{m}_b^2(\mu)}\right)\right]\mathrm{keV}$$

$$\stackrel{\mu=m_H}{=} 7.06363\,\mathrm{keV} \qquad < 10^{-5}$$

$$\Gamma^{(2,c)} = \left[7.02908 + \frac{\alpha_s(\mu)}{\pi}\left(0.64310 + 0.10551\,\log\frac{\mu^2}{\overline{m}_t^2(\mu)} + 0.01446\,\log\frac{\mu^2}{\overline{m}_b^2(\mu)}\right)\right]\mathrm{keV}$$

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University of Zurich[UZH]

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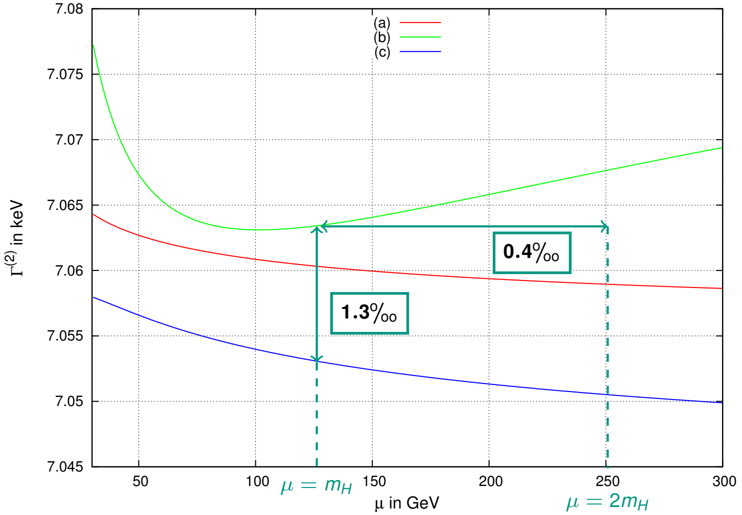
Numerical results

University of Zurich[UZH]

Partial width	(a)	(b)	(c)
$\Gamma^{(1)}_{WW}$	7.83473859	7.83473859	7.83473859
$\Gamma^{(1)}_{Wt}$	−0.83278001	−0.80390896	−0.83655274
$\Gamma^{(1)}_{Wb}$	0.02206641	0.01288843	0.00904595
$\Gamma^{(1)}_{tt}$	0.02212973	0.02062193	0.02233069
$\Gamma^{(1)}_{tb}$	−0.00117276	−0.00066123	−0.00048294
$\Gamma^{(1)}_{bb}$	0.00002094	0.00000714	0.00000324
$\Gamma^{(1)}$	7.04500291	7.06368591	7.02908279
$\Gamma^{(2)}_{Wt}$	0.02203714	−0.00078280	0.02457012
$\Gamma^{(2)}_{Wb}$	−0.00586227	0.00072730	0.00175365
$\Gamma^{(2)}_{tt}$	−0.00117120	0.00004016	−0.00131174
$\Gamma^{(2)}_{tb}$	0.00031156	−0.00003731	−0.00009362
$\Gamma^{(2)}_{bt}$	0.00003103	−0.00000064	0.00001418
$\Gamma^{(2)}_{bb}$	−0.00001585	−0.00000081	0.00000078
$\Gamma^{(2)}$	7.06033332	7.06368591	7.05401616

Numerical results

University of Zurich[UZH]



Conclusions

Renormalization

- OS scheme
- $\overline{\mathrm{MS}}$ scheme
- hybrid scheme with OS mass and $\overline{\mathrm{MS}}$ Yukawa coupling

Numerical results

- Corrections in sub-per-cent range and consistent with each other
- Residual QCD uncertainty: 1.7‰

Checks

- Confirmation of previously available numerical OS result [Spira, Djouadi, Zerwas (1992)]
- Agreement with independent calculation [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov (2015)]

Master Integrals

- Two-loop three-point integrals with two different external legs and one internal mass derived analytically using differential equations
- Important ingredient to two-loop amplitudes of $H + j$ production

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