The rare decay $H \rightarrow Z \gamma$ in perturbative QCD


Thomas Gehrmann, Sam Guns & Dominik Kara | June 15, 2015
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Motivation: Experiment

$H \rightarrow \gamma\gamma$
- loop-mediated
  $\Rightarrow$ sensitive to new physics
- clean signature
  $\Rightarrow$ 20% relative precision

$H \rightarrow Z\gamma$
- more background
- smaller branching ratio
- spin-dependent particle correlations through $Z \rightarrow ll$
Motivation: Experiment

$H \rightarrow \gamma\gamma$
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- more background
- smaller branching ratio
- spin-dependent particle correlations through $Z \rightarrow ll$

⇒ broader spectrum of observables: more info on $H$ couplings
**Motivation: Theory**

\[ H \rightarrow Z\gamma \]

- Analytical LO result available  
  - [Cahn, Chanowitz, Fleishon (1979)]  
  - [Bergstrom, Huth (1985)]

- Numerical NLO result in QCD available  
  - [Spira, Djouadi, Zerwas (1992)]

\[ \Rightarrow \text{Analytical NLO result: independent check} \]

\[ H + j \text{ production} \]

- NLO result in QCD available for \( m_t \rightarrow \infty \)  
  - [Schmidt (1997)]  
  - [Glosser, Schmidt (2002)]  
  - [de Florian, Grazzini, Kunszt (1999)]  
  - [Ravindran, Smith, van Neerven (2002)]

- NNLO calculation in QCD well-advanced for \( m_t \rightarrow \infty \)  
  - [Gehrmann, Glover, Jaquier, Koukoutsakis (2012)]  
  - [Chen, Gehrmann, Glover, Jaquier (2012)]  
  - [Boughezal, Caola, Melnikov, Petriello, Schulze (2013, 2015)]  
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**H + j production**

- EFT is likely to break down at high $p_T$
- High-priority aim: NLO QCD corrections with full $m_t$ dependence

$\Rightarrow$ Two-loop integrals for $H \rightarrow Z\gamma$ pave the way

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- EFT is likely to break down at high $p_T$
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Outline of the calculation

QGRAF ■ Generate Feynman diagrams for process $H(q) \rightarrow Z(p_1)\gamma(p_2)$

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Outline of the calculation

QGRAF  
- Generate Feynman diagrams for process $H(q) \rightarrow Z(p_1)\gamma(p_2)$

FORM  
- Project relevant Feynman diagrams onto tensor structure

$$\mathcal{M} = A \epsilon_{1,\mu}(p_1, \lambda_1) \epsilon_{2,\nu}(p_2, \lambda_2) \frac{P_{\mu\nu}}{P^2}$$

with projector

$$P_{\mu\nu} = p_2^{\mu} p_1^{\nu} - (p_1 \cdot p_2) g^{\mu\nu}$$

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**REDUCE**
- Reduce Feynman Integrals to set of Master Integrals (MIs)
  - Integration-by-parts identities (IBPs)
  - Laporta algorithm

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**REDUCE**
- Reduce Feynman Integrals to set of Master Integrals (MIs)
  - Integration-by-parts identities (IBPs)
  - Laporta algorithm
The amplitude can be further decomposed

\[ A = c_W A_W + \sum_q c_q A_q \]

with

\[ c_W = \frac{\cos \theta_w}{\sin \theta_w}, \quad c_q = N_c \frac{2 Q_q \left( l_q^3 - 2 Q_q \sin^2 \theta_w \right)}{\sin \theta_w \cos \theta_w} \]

- Born-level contribution:
  \[ A^{(1)} = c_W A_W^{(1)} + c_t A_t^{(1)} + c_b A_b^{(1)} \]

- NLO QCD corrections:
  \[ A_q(m_H, m_Z, m_q, \alpha_s, \mu) = A_q^{(1)}(m_H, m_Z, m_q) + \frac{\alpha_s(\mu)}{\pi} A_q^{(2)}(m_H, m_Z, m_q, \mu) \]
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The decay width

is obtained from the amplitude as

$$\Gamma = \frac{\pi G_F \alpha^2}{4\sqrt{2} m^3_H (m^2_H - m^2_Z)} |A|^2$$

$$\Rightarrow$$ We are left with computation of MIs

Parametrization

Use Landau-type variables to absorb natural roots

$$m^2_H = -m^2_q \frac{(1 - h)^2}{h}, \quad m^2_Z = -m^2_q \frac{(1 - z)^2}{z}$$

$$\Rightarrow \sqrt{1 - 4 \frac{m^2_q}{m^2_H}} \rightarrow \frac{|h + 1|}{|h - 1|}, \quad \sqrt{1 - 4 \frac{m^2_q}{m^2_Z}} \rightarrow \frac{|z + 1|}{|z - 1|}$$
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\[ \Gamma = \frac{\pi G_F \alpha^2}{4\sqrt{2} m_H^3 (m_H^2 - m_Z^2)} |A|^2 \]

⇒ We are left with computation of MIs

Parametrization

Use Landau-type variables to absorb natural roots

\[ m_H^2 = -m_q^2 \frac{(1 - h)^2}{h}, \quad m_Z^2 = -m_q^2 \frac{(1 - z)^2}{z} \]

⇒ \[ \sqrt{1 - 4 \frac{m_q^2}{m_H^2}} \to \frac{|h + 1|}{|h - 1|}, \quad \sqrt{1 - 4 \frac{m_q^2}{m_Z^2}} \to \frac{|z + 1|}{|z - 1|} \]
Differential equations

1. Choose MI
2. Compute derivative of integrand with respect to internal mass and external invariants
3. Use IBPs to relate resulting integrals to original MI

Full system takes form of total differential

\[ d\vec{I}(h, z) = \sum_{k=1}^{N} R_k(\epsilon) \, d \log(d_k) \, \vec{I}(h, z) \]

<table>
<thead>
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<th>(d_1)</th>
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Full system takes form of total differential

\[ d\vec{l}(h, z) = \sum_{k=1}^{N} R_k(\epsilon) \, d\log(d_k) \, \vec{l}(h, z) \]

\[
\begin{align*}
d_1 &= z & d_5 &= h + 1 & d_9 &= h^2 - hz - h + 1 \\
d_2 &= z + 1 & d_6 &= h - 1 & d_{10} &= h^2z - hz - h + z \\
d_3 &= z - 1 & d_7 &= h - z & d_{11} &= z^2 - hz - z + 1 \\
d_4 &= h & d_8 &= hz - 1 & d_{12} &= z^2h - hz - z + h \ldots & d_N
\end{align*}
\]
Differential equations

1. Choose MI
2. Compute derivative of integrand with respect to internal mass and external invariants
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Full system takes form of total differential

\[ d\vec{I}(h, z) = \sum_{k=1}^{N} R_k \left( \epsilon \right) d\log(d_k) \vec{I}(h, z) \]

\[
\begin{align*}
d_1 &= z \\
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d_{12} &= z^2 h - hz - z + h \\ &\ldots \\
d_N
\end{align*}
\]
Differential equations

Canonical form

\[ \frac{d\tilde{M}(h, z)}{\epsilon} = \sum_{k=1}^{12} R_k \, d \log(d_k) \, \tilde{M}(h, z) \]

- reduced number of polynomials \( d_k \)
- can be integrated in terms of GHPLs:
  \[ G(w_1, \ldots, w_n; x) \equiv \int_0^x dt \, \frac{1}{t - w_1} \, G(w_2, \ldots, w_n; t) \]
  \[ G(\tilde{0}_n; x) \equiv \frac{\log^n x}{n!} \]
- leads to linear combinations of GHPLs of homogeneous weight

\[ \Rightarrow \text{Change basis from Laporta integrals } \tilde{I} \text{ to canonical integrals } \tilde{M} \]

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### Differential equations

#### Canonical form

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  $$G(\vec{0}_n; x) \equiv \frac{\log^n x}{n!}$$

- Leads to linear combinations of GHPLs of homogeneous weight

⇒ Change basis from Laporta integrals $\vec{I}$ to canonical integrals $\vec{M}$

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Integral basis

1. Start with Laporta basis: [Gehrmann, von Manteuffel, Tancredi, Weihs (2014)]

Topology under consideration must

✓ have at most linear dependence on $\epsilon$
✓ be triangular in $D = 4$ dimensions

$I_1$ $I_2$ $I_3$ $I_4$

$I_5$ $I_6$ $I_7$ $I_8$

$I_9$ $I_{10}$ $I_{11}$ $I_{12}$
Integral basis

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Start with Laporta basis:

[Gehrmann, von Manteuffel, Tancredi, Weihs (2014)]
Integral basis

Perform transformation to canonical basis by integrating out homogeneous parts in $D = 4$

\[
\frac{dh_{15}}{dz} = \frac{hz}{d_1 d_2 d_3 d_4 d_9 d_{10} (2z (h^2 + 1) - h(z + 1)^2)}.
\]

\[
\left\{ \begin{array}{l}
\frac{3}{2} h z (h - 1)^2 \left( (h^2 + 1) (z^2 + 1) - h (z + 1)^2 \right) h_7 \\
- h \left[ h^4 z (z^2 + 1) - h (h^2 + 1) (z^4 + z^3 + 4z^2 + z + 1) \\
+ h^2 (z^4 + 4z^3 + 2z^2 + 4z + 1) + z (z^2 + 1) \right] (2l_{13} + l_{14}) \\
- \frac{1}{z} \left[ 2h^6 z^2 (z^2 + 1) - 2hz (h^4 + 1) (z + 1)^2 (2z^2 - z + 2) \\
+ h^2 (h^2 + 1) (z^6 + 8z^5 + 17z^4 + 8z^3 + 17z^2 + 8z + 1) + 2z^2 (z^2 + 1) \right] l_{15} \end{array} \right\}
\]

\[
\frac{dh_{15}}{dh} = \frac{hz}{d_1 d_4 d_9 d_{10} (2z (h^2 + 1) - h(z + 1)^2)}.
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\[
\left\{ \begin{array}{l}
- \frac{3}{2} z^2 (h - 1)^3 (h + 1) h_7 + z (h^2 - 1) (z (h^2 + 1) - h (z^2 + 1)) (2l_{13} + l_{14}) \\
- \frac{h^2 - 1}{h} \left[ 2z^2 (h^4 + 1) - 2hz(z + 1)^2 (h^2 + 1) + h^2 (z^4 + 2z^3 + 6z^2 + 2z + 1) \right] l_{15} \end{array} \right\}
\]
Integral basis

2 Perform transformation to canonical basis by integrating out homogeneous parts in $D = 4$

\[ M_{15} = \frac{z(h^2 + 1) - h(z + 1)}{2z(h^2 + 1) - h(z + 1)^2} \left[ \frac{3}{2} \left( \frac{h - 1)^2}{h} I_7 \right. \
\left. - \frac{(h - z)(hz - 1)}{hz} (2I_{13} + I_{14}) \right. \
\left. - \frac{(z^2 - 1)(h^2 + 1 - h(z + 1))}{hz} I_{15} \right] \]
Perform transformation to canonical basis by integrating out homogeneous parts in $D = 4$

$$dM_{15} = \epsilon \left[ - \left( M_2 + \frac{3}{2} M_7 + 5 M_{13} + M_{14} - 4 M_{15} \right) d \log(d_1) ight. $$

$$+ \left( \frac{3}{2} M_7 + 2 M_{13} + M_{14} - 2 M_{15} \right) d \log(d_2) $$

$$- \left( M_2 + \frac{3}{2} M_7 + M_{13} - M_{14} - M_{15} \right) d \log(d_3) $$

$$- \left( M_2 + 2 M_6 + \frac{5}{2} M_7 + M_{13} - M_{14} - M_{15} \right) d \log(d_4) + 3 M_7 d \log(d_6) $$

$$+ (M_2 + M_6 + 2 M_7 + 3 M_{13} - 2 M_{15}) d \log(d_7) $$

$$+ (M_2 - M_6 + M_7 + 3 M_{13} - 2 M_{15}) d \log(d_8) $$

$$- \left( \frac{3}{2} M_7 + 2 M_{13} + M_{14} - M_{15} \right) d \log(d_9) \right] $$
Integrate differential equation in $h$ or $z$ up to constant $C(z)$ or $C(h)$

Use boundary conditions:

\[
\begin{align*}
  h = 1 & \iff m_H^2 = 0 \\
  z = 1 & \iff m_Z^2 = 0 \\
  h = z & \iff m_H^2 = m_Z^2 \\
  h = \frac{1}{z} & \iff m_H^2 = m_Z^2
\end{align*}
\]

Perform transformations with the help of symbol and coproduct:

\[
\begin{align*}
  G(w_1(x), \ldots, w_n(x); x) & \rightarrow G(a_1, \ldots, a_n; x) \\
  G(w_1(x), \ldots, w_n(x); y) & \rightarrow G(c_1(y), \ldots, c_n(y); x)
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Integration

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Integration

Results in terms of GHPLs up to weight four

\[ G(a_1, \ldots, a_n; h) \quad \text{with} \quad a_i \in \{0, \pm 1, z, \frac{1}{z}, J_z, \frac{1}{J_z}, K_z^\pm, L_z^\pm\} \]

\[ G(b_1, \ldots, b_n; z) \quad \text{with} \quad b_i \in \{0, \pm 1, c, \bar{c}\} \]

\[ c = \frac{1}{2} \left(1 + i\sqrt{3}\right) \]

\[ K_z^\pm = \frac{1}{2} \left(1 + z \pm \sqrt{-3 + 2z + z^2}\right) \]

\[ J_z = \frac{z}{1 - z + z^2} \]

\[ L_z^\pm = \frac{1}{2z} \left(1 + z \pm \sqrt{1 + 2z - 3z^2}\right) \]

✓ verified through differential equation in other variable
✓ checked numerically against SecDec

[Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke (2015)]

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\[ G(b_1, \ldots, b_n; z) \quad \text{with} \quad b_i \in \{0, \pm 1, c, \bar{c}\} \]

\[ c = \frac{1}{2} \left(1 + i \sqrt{3}\right) \quad K_z^\pm = \frac{1}{2} \left(1 + z \pm \sqrt{-3 + 2z + z^2}\right) \]

\[ J_z = \frac{z}{1 - z + z^2} \quad L_z^\pm = \frac{1}{2z} \left(1 + z \pm \sqrt{1 + 2z - 3z^2}\right) \]

✓ verified through differential equation in other variable
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[Borowka, Heinrich, Jones, Kerner, Schlenk, Zirke (2015)]
Integration

Example

\[ M_{15} = \epsilon^2 \left[ -\frac{\pi^2}{6} - G\left(\frac{1}{z}, 0; h\right) + G(z, 0; h) - G(0; z) G\left(\frac{1}{z}; h\right) - G(0; z) G(z; h) + G(0; z) G(0; h) \\
- 2G(0, 0; h) + G(1, 0; z) \right] \\
+ \epsilon^3 \left[ 5\zeta_3 + 2G\left(\frac{1}{z}, \frac{1}{z}, 0; h\right) - 2G\left(\frac{1}{z}, z, 0; h\right) + \frac{2\pi^2}{3} G\left(\frac{1}{z}; h\right) + 6G\left(\frac{1}{z}, -1, 0; h\right) \\
- 3G\left(\frac{1}{z}, 0, 0; h\right) + 2G\left(\frac{1}{z}, 1, 0; h\right) + 2G(z, \frac{1}{z}, 0; h) - 2G(z, z, 0; h) + \frac{\pi^2}{3} G(z; h) \\
- 6G(z, -1, 0; h) + 5G(z, 0, 0; h) - 2G(z, 1, 0; h) - G(K_z^-, \frac{1}{z}, 0; h) + G(K_z^-, z, 0; h) \\
- \frac{\pi^2}{6} G(K_z^-; h) + 2G(K_z^-, 0, 0; h) - G(K_z^+, \frac{1}{z}, 0; h) + G(K_z^+, z, 0; h) - \frac{\pi^2}{6} G(K_z^+; h) \\
+ 2G(K_z^+, 0, 0; h) + \frac{\pi^2}{3} G(-1; z) + 2G(-1, 0; z) G\left(\frac{1}{z}; h\right) + 2G(-1, 0; z) G(z; h) \\
- 2G(-1, 0; z) G(0; h) + G(-1, 0, 0; z) - 2G(-1, 1, 0; z) - G(0, \frac{1}{z}, 0; h) - \frac{\pi^2}{2} G(0; z) \right] \]
\[
\begin{align*}
&+ 2G(0; z)G\left(\frac{1}{z}, \frac{1}{z}; h\right) + 2G(0; z)G\left(\frac{1}{z}, z; h\right) - 2G(0; z)G\left(\frac{1}{z}, 0; h\right) + 2G(0; z)G(z, \frac{1}{z}; h) \\
&+ 2G(0; z)G(z, z; h) - 2G(0; z)G(z, 0; h) - G(0; z)G(K_0^-, \frac{1}{z}; h) - G(0; z)G(K_0^-, z; h) \\
&+ G(0; z)G(K_0^-, 0; h) - G(0; z)G(K_0^+, \frac{1}{z}; h) - G(0; z)G(K_0^+, z; h) + G(0; z)G(K_0^+, 0; h) \\
&- G(0; z)G(0, \frac{1}{z}; h) - G(0; z)G(0, z; h) + G(0, z, 0; h) + G(0; h)G(1, 0; z) + 12G(0, -1, 0; h) \\
&- G(0, 0; z)G\left(\frac{1}{z}; h\right) - G(0, 0; z)G(z; h) - G(0, 0; z)G(K_0^-; h) - G(0, 0; z)G(K_0^+; h) \\
&+ 2G(0, 0; z)G(0; h) + G(0, 0; h)G(0; z) - G(0, 0; 0; z) - 8G(0, 0, 0; h) + 2G(0, 1, 0; z) \\
&+ 4G(0, 1, 0; h) - \frac{\pi^2}{3}G(1; z) - 2G(1, -1, 0; z) - 2G(1, 0; z)G\left(\frac{1}{z}; h\right) - 2G(1, 0; z)G(z; h) \\
&+ G(1, 0; z)G(K_0^-; h) + G(1, 0; z)G(K_0^+; h) + 2G(1, 0, 0; z) - 6G(1, 0, 0; h) + G(1, 1, 0; z)] \\
&+ \mathcal{O}\left(\epsilon^4\right)
\end{align*}
\]
(a) quark mass $M_q$ and Yukawa coupling $Y_q$ in OS scheme

\[ A_q^{(2,a)}(m_H, m_Z, M_q) = A_q^{(2)}_{\text{bare}} + Z_{\text{OS}} A_q^{(1)} + \frac{\delta m_{\text{OS}}}{m_q} C_q^{(1)} \]

Renormalization constants

\[ Z_{\text{OS}} = \frac{\alpha_s(\mu)}{\pi} 16 i \pi^2 S_\epsilon \frac{C_F}{4} \frac{3 - 2\epsilon}{\epsilon(1 - 2\epsilon)} \]

\[ \delta m_{\text{OS}} = m_q Z_{\text{OS}} \]
(a) quark mass $M_q$ and Yukawa coupling $Y_q$ in OS scheme

$$A_{q}^{(2,a)}(m_H, m_Z, M_q) = A_{q,bare}^{(2)} + Z_{OS} A_{q}^{(1)} + \frac{\delta m_{OS}}{m_q} C_q^{(1)}$$

Renormalization constants

$$Z_{OS} = \frac{\alpha_s(\mu)}{\pi} 16 i \pi^2 S_\epsilon \frac{C_F}{4} \frac{3 - 2\epsilon}{\epsilon (1 - 2\epsilon)}$$

$$\delta m_{OS} = m_q Z_{OS}$$
**Renormalization**

(a) quark mass $M_q$ and Yukawa coupling $Y_q$ in OS scheme

\[ A^{(2,a)}_q(m_H, m_Z, M_q) = A^{(2)}_{q,bare} + Z_{OS} A^{(1)}_q + \frac{\delta m_{OS}}{m_q} C^{(1)}_q \]

- Renormalization constants

\[ Z_{OS} = \frac{\alpha_s(\mu)}{\pi} 16 i \pi^2 S_\epsilon \frac{C_F}{4} \frac{3 - 2\epsilon}{\epsilon (1 - 2\epsilon)} \]

\[ \delta m_{OS} = m_q Z_{OS} \]

- Counterterms

\[ C^{(1)}_q = \]

---

**Introduction**  |  The decay in the Standard Model  |  Calculation of the two-loop amplitude  |  Numerical results  |  Conclusions
---|---|---|---|---

Thomas Gehrmann, Sam Guns & Dominik Kara – The rare decay $H \rightarrow Z\gamma$ in perturbative QCD

June 15, 2015  |  14/20
Renormalization

(b) quark mass $M_q$ in OS, Yukawa coupling $\bar{y}_q$ in $\overline{\text{MS}}$ scheme

$$A_{q}^{(2,b)}(m_H, m_Z, \bar{m}_q, \mu) = A_{q}^{(2,a)}(m_H, m_Z, \bar{m}_q(\mu)) + \Delta \cdot A_{q}^{(1)}(m_H, m_Z, \bar{m}_q(\mu))$$

The finite shift in the amplitude is induced by expressing the OS quantities in terms of $\overline{\text{MS}}$ quantities:

$$M_q = \bar{m}_q(\mu) \left(1 + \Delta\right)$$
$$Y_q = \bar{y}_q(\mu) \left(1 + \Delta\right)$$

$$\Delta = \frac{\alpha_s(\mu)}{\pi} C_F \left(1 + \frac{3}{4} \log \frac{\mu^2}{\bar{m}_q^2(\mu)}\right)$$
Renormalization

\( A^{(2, c)}_{q}(m_H, m_Z, \overline{m}_q, \mu) = A^{(2, b)}_{q}(m_H, m_Z, \overline{m}_q(\mu)) \)

\[ + \Delta \cdot \frac{\partial A^{(1)}_{q}(m_H, m_Z, M_q)}{\partial M_q} \bigg|_{M_q = \overline{m}_q(\mu)} \]

\( \overline{\Delta} \) is defined through the following replacements in \( A^{(1)}_{q} \):

\[ h = \overline{h} - 2 \Delta \overline{h} \frac{\overline{h} - 1}{\overline{h} + 1} \]

\[ z = \overline{z} - 2 \Delta \overline{z} \frac{\overline{z} - 1}{\overline{z} + 1} \]
Numerical results

Analytic continuation from Euclidean to Minkowski region

- Region I: \( m_Z^2 < m_H^2 < 4m_q^2 \) \( \Rightarrow \) top quark amplitude
- Region II: \( m_Z^2 < 4m_q^2 < m_H^2 \) \( \Rightarrow \) not needed
- Region III: \( 4m_q^2 < m_Z^2 < m_H^2 \) \( \Rightarrow \) bottom quark amplitude

NLO decay width \( \Gamma^{(2)} \) in renormalization schemes (a), (b) and (c)

\[
\Gamma^{(2,a)} = \left[ 7.04500 + 0.42617 \frac{\alpha_s(\mu)}{\pi} \right] \text{keV} \quad \mu = m_H = 7.06033 \text{keV} \\
\Gamma^{(2,b)} = \left[ 7.06369 + \frac{\alpha_s(\mu)}{\pi} \left( -0.53038 - 0.76333 \log \frac{\mu^2}{m_t^2(\mu)} + 0.01224 \log \frac{\mu^2}{m_b^2(\mu)} \right) \right] \text{keV} \\
\mu = m_H = 7.06363 \text{keV} < 10^{-5} \\
\Gamma^{(2,c)} = \left[ 7.02908 + \frac{\alpha_s(\mu)}{\pi} \left( 0.64310 + 0.10551 \log \frac{\mu^2}{m_t^2(\mu)} + 0.01446 \log \frac{\mu^2}{m_b^2(\mu)} \right) \right] \text{keV} \\
\mu = m_H = 7.05402 \text{keV} \quad 3\% \text{oo}
\]
Numerical results

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NLO decay width \( \Gamma^{(2)} \) in renormalization schemes (a), (b) and (c)

\[
\Gamma^{(2,a)} = \left[ 7.04500 + 0.42617 \frac{\alpha_s(\mu)}{\pi} \right] \text{ keV} \quad \mu = m_H = 7.06033 \text{ keV} \quad 2\% \\
\Gamma^{(2,b)} = \left[ 7.06369 + \frac{\alpha_s(\mu)}{\pi} \left( -0.53038 - 0.76333 \log \frac{\mu^2}{m_t^2(\mu)} + 0.01224 \log \frac{\mu^2}{m_b^2(\mu)} \right) \right] \text{ keV} \quad \mu = m_H = 7.06363 \text{ keV} \quad < 10^{-5} \\
\Gamma^{(2,c)} = \left[ 7.02908 + \frac{\alpha_s(\mu)}{\pi} \left( 0.64310 + 0.10551 \log \frac{\mu^2}{m_t^2(\mu)} + 0.01446 \log \frac{\mu^2}{m_b^2(\mu)} \right) \right] \text{ keV} \quad \mu = m_H = 7.05402 \text{ keV} \quad 3\% 
\]
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<th>(c)</th>
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<td>7.06368591</td>
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</tr>
</tbody>
</table>
Numerical results

\[ \Gamma^{(2)} \text{ in keV} \]
\[ \mu \text{ in GeV} \]

(a) \( \mu = m_H \)
(b) \( \mu = 2m_H \)

- 0.4% \( \mu = m_H \)
- 1.3% \( \mu = 2m_H \)

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- $\overline{MS}$ scheme
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- Corrections in sub-per-cent range and consistent with each other
- Residual QCD uncertainty: 1.7\%

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  [Spira, Djouadi, Zerwas (1992)]
- Agreement with independent calculation
  [Bonciani, Del Duca, Frellesvig, Henn, Moriello, Smirnov (2015)]

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- Two-loop three-point integrals with two different external legs and one internal mass derived analytically using differential equations
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