Importance of radiative corrections for π^0 decays

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Outline:

- Introduction
- Leading logs
- processes:
  - $\pi^0 \to \gamma\gamma$
  - Dalitz decay
  - $\pi^0 \to e^+e^-$
- summary

Overview of low energy QCD

- history: current algebra
- modern form: ChPT [Weinberg'79], [Gasser,Leutwyler'84-'85]
- clash between the order of perturbation theory (number of LECs) and precision
- present status: ChPT for 2 and 3-flavours up to NNLO for the even sector, and up to NLO for the odd sector (chiral anomaly)
- number of parameters for 3-flavour ChPT: 2, 10, 90 for LO, NLO, NNLO, resp.
- ChPT described dynamics of Goldstone bosons (pions, kaons, eta)
- can be extended systematically by other particles (photon, resonances, etc.)
- first two-loop calculation: $\gamma\gamma \to \pi^0\pi^0$ [Bellucci, Gasser, Sainio '94]

Leading logarithms

Renormalizable theories

- we calculate e.g. $F(M)$:

$$F = F_0 + F_1^1 L + F_0^1 + F_2^2 L^2 + F_1^2 L + F_0^2 + \dots$$
$$= \alpha + \alpha^2 f_1^1 L + \alpha^2 f_0^1 + \alpha^3 f_2^2 L^2 + \alpha^3 f_1^2 L + \alpha^3 f_0^2 + \dots$$

- where we have defined $L \equiv \log(\mu/M)$
- renormalization condition $\mu \frac{dF}{d\mu} = 0$
- non-trivial dependence on α

$$\mu \frac{d\alpha}{d\mu} = \beta_0 \alpha^2 + \beta_1 \alpha^3 + \dots$$

- β_0 obtained from 1-loop diagrams
- renormalization condition $\Rightarrow$

$$f_1^1 = -\beta_0,\ f_2^2 = \beta_0^2,\ f_3^3 = -\beta_0^3 \quad \Rightarrow \quad F\Big|_{LL} = \frac{\alpha}{1 + \alpha\beta_0 L}$$

Non-renormalizable theories

What are the Leading Logarithms (LL)?

- We calculate e.g. $F(M)$:

$$F = F_0 + \underline{F_1^1 L} + F_0^1 + \underline{F_2^2 L^2} + F_1^2 L + F_0^2 + \dots$$

- where we have defined $L \equiv \log(\mu/M)$

Why they are special?

- they are parameter-free
- to all orders from one-loop diagrams only (based on [Weinberg '79], [Büchler, Colangelo'03])

$O(N)$ sigma model

- $O(N+1)/O(N)$ nonlinear sigma model

$$\mathcal{L}_{n\sigma} = \frac{F^2}{2}\partial_\mu \Phi^T \partial^\mu \Phi + F^2 \chi^T \Phi \,.$$

- explicit + spontaneous symmetry breaking

$$\langle \Phi^T \rangle = (1\;0\ldots0) \qquad \chi^T = (M^2\;0\ldots0)$$

- we have N Goldstone bosons: ϕ
- $N=3$ equivalent to two-flavour ChPT

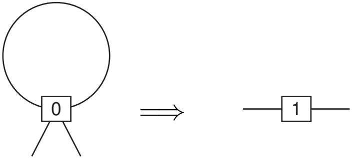
$O(N)$ sigma model: physical mass M_π

LL for the physical mass [Bijnens, Carloni '10], [Bijnens,KK,Lanz '12]

- $\mathcal{L}_{n\sigma} \Rightarrow \boxed{0}$
- mass: two-point function
- schematically at LO:

$$\Leftrightarrow \quad M_\pi = M$$

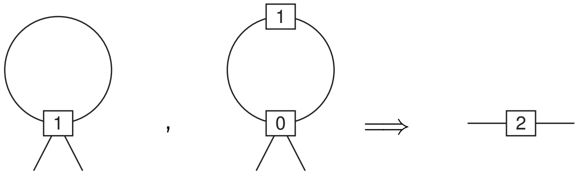
- NLO



$O(N)$ sigma model: physical mass M_π

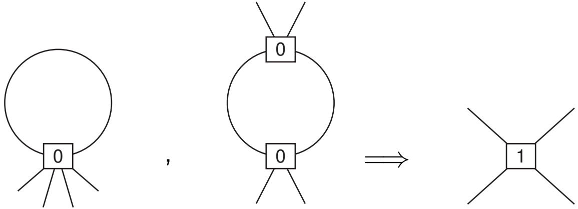
LL for physical mass [Bijnens, Carloni '10], [Bijnens,KK,Lanz '12]

- NNLO



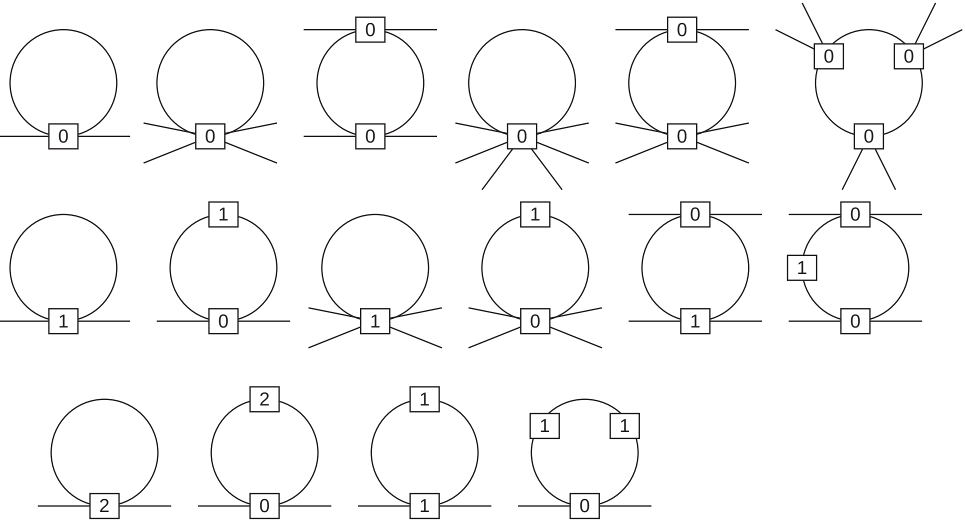
(1)

- we have to calculate first:

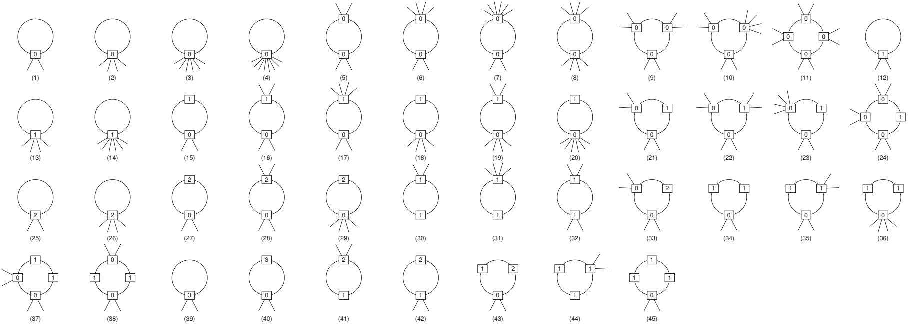


(2)

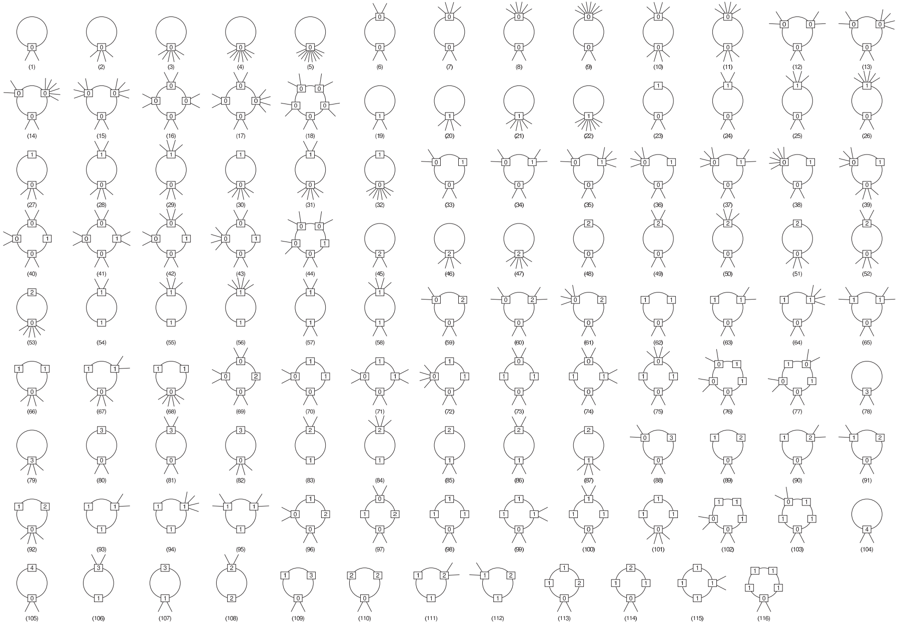
mass up-to 3-loop order



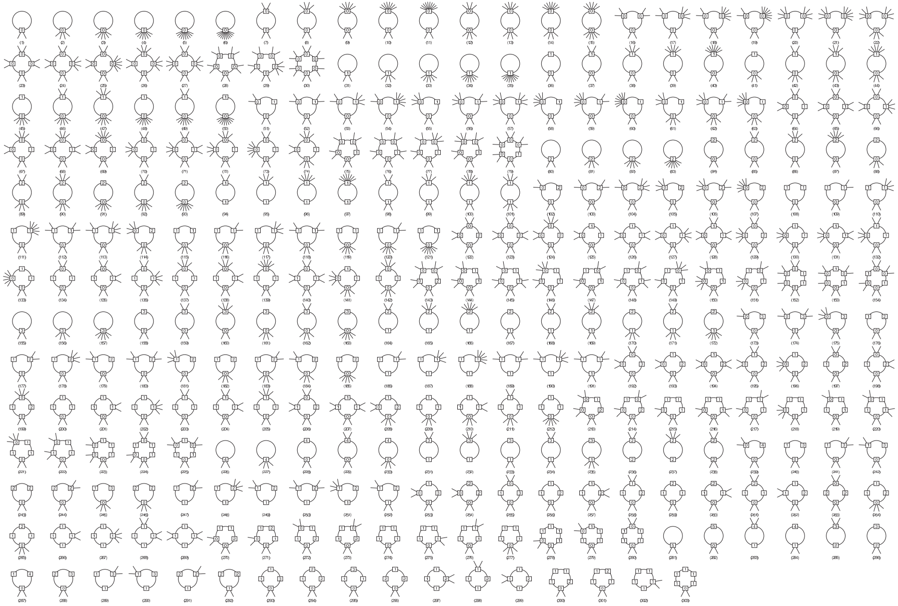
mass up-to 4-loop order



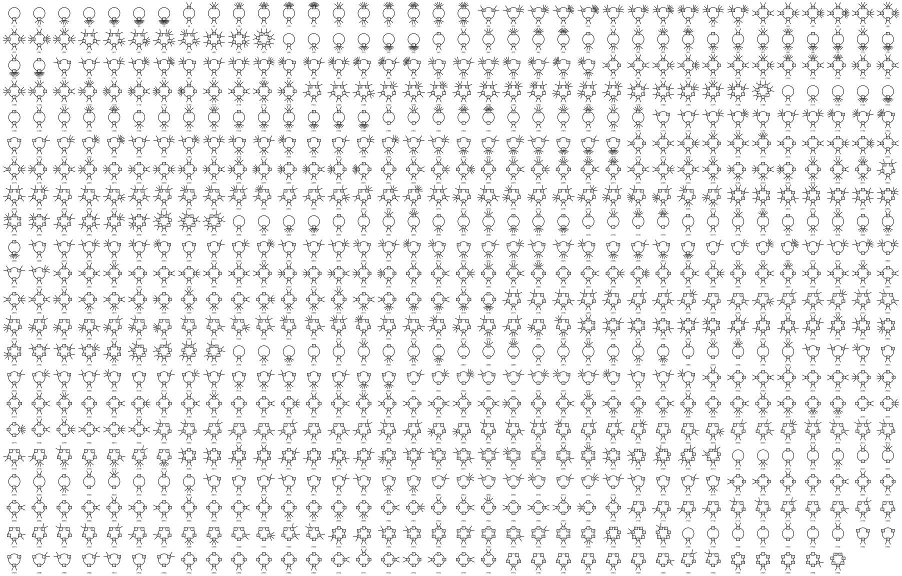
mass up-to 5-loop order



mass up-to 6-loop order



mass up-to 7-loop order



$O(N)$ sigma model: physical mass M_π, results

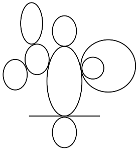
LL for physical mass [Bijnens, Carloni '10], [Bijnens,KK,Lanz '12]

- # of diagrams: $1, 5, 16, 45, 116, 303, 790, \ldots$
- $M_\pi^2 = M^2(1 + a_1 L_M + a_2 L_M^2 + \ldots)$
  $L_M = M^2/(16\pi^2 F^2)\log(\mu^2/M^2)$

i	a_i for $N=3$	a_i for general N
1	$-1/2$	$1 - 1/2\ N$
2	17/8	$7/4 - 7/4\ N + 5/8\ N^2$
3	$-103/24$	$37/12 - 113/24\ N + 15/4\ N^2 - N^3$
4	24367/1152	$839/144 - 1601/144\ N + 695/48\ N^2 - 135/16\ N^3 + 231/128\ N^4$
5	$-8821/144$	$33661/2400 - 1151407/43200\ N + 197587/4320\ N^2 - 12709/300\ N^3$ $+6271/320\ N^4 - 7/2\ N^5$
6	$\frac{1922964667}{6220800}$	$158393809/3888000 - 182792131/2592000\ N + 1046805817/7776000\ N^2$ $-17241967/103680\ N^3 + 70046633/576000\ N^4 - 23775/512\ N^5$ $+7293/1024\ N^6$
7	$-\frac{1804453729667}{1714608000}$	$1098817478897/8573040000 - 286907006651/1428840000\ N$ $+4533157401977/11430720000\ N^2 - 1986536871797/3429216000\ N^3$ $+436238667943/762048000\ N^4 - 7266210703/21168000\ N^5$ $+99977/896\ N^6 - 15\ N^7$

Leading logs

- brutal force
- however, now, fully automatic
- can be used, in principle, up to all orders
- limited by computers and time constraints, up to 7th-8th loop order
- smarter ways for simplified cases
  - massless case [M.Polyakov et al.'08,'09,'10]
  - Large N limit (so-called cactus diagrams) [Bijnens,Carloni '10,'11]

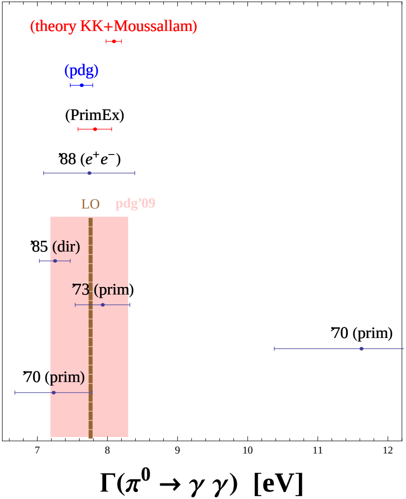


- anomalous sector of two-flavour QCD, $\pi^0 \to \gamma\gamma$ and $\pi\gamma \to \pi\pi$ [Bijnens,KK,Lanz'12]
- N-flavour ChPT, $\pi^0 \to \gamma\gamma$ and $\pi\gamma \to \pi\pi$ [Bijnens,KK,Lanz'13], unfortunately no educated guess for large N

$\pi^0 \to \gamma\gamma$

- one of the most important processes for theory of particle physics
- π^0 lightest hadron $\Rightarrow$ dominant decay mode $\pi^0 \to \gamma\gamma$ (br=98.82%)
- non-existence of logarithmic correction to the current algebra result at NLO
- connection with the non-renormalization theorem ?
- new experimental activities
- theory – NNLO calculation: [KK,Moussallam'09] $\to$ see next

π^0 life time



π^0 mean life, PDG history:

1985 $(8.4 \pm 0.6) \times 10^{-17}$ s

...

2009 $(8.4 \pm 0.6) \times 10^{-17}$ s

2010 $(8.4 \pm 0.5) \times 10^{-17}$ s

2011 $(8.4 \pm 0.4) \times 10^{-17}$ s

2012 $(8.52 \pm 0.18) \times 10^{-17}$ s $\leftarrow$ PrimEx col.

...

today $(8.52 \pm 0.18) \times 10^{-17}$ s

theory: [KK,Moussallam] $(8.04 \pm 0.11) \times 10^{-17}$ s

$\pi^0 \to \gamma\gamma$: chiral expansion

- in chiral limit exact due to QCD axial anomaly:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{m_{\pi^0}^3}{64\pi}\left(\frac{\alpha N_C}{3\pi F_\pi}\right)^2 = 7.73\, eV$$

Correction to the current algebra prediction:

- using [Pagels and Zepeda '72] sum rules in [Kitazawa '85]
- NLO corrections are hidden in $F \to F_{\pi^0}$ and $O(p^6)$ LECs [Donoghue, Holstein, Lin '85] [Bijnens, Bramon, Cornet '88]
- in 3-flavour case we can study π^0, η, η' mixing, resulting to [Goity, Bernstein, Holstein '02]:

$$\Gamma^{\text{NLO}} = 8.1 \pm 0.08\,\text{eV}$$

  in 2-flavour case EM corrections [Ananth., Moussallam '02]:

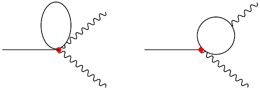
$$\Gamma^{\text{NLO}} = 8.06 \pm 0.02 \pm 0.06\,\text{eV}$$

- Quite recently another study based on dispersion relations, QCD sum rules, using only the value $\Gamma(\eta \to \gamma\gamma)$ gives [Ioffe, Oganesian '07]:

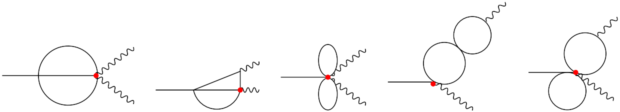
$$\Gamma^{\text{NLO}} = 7.93 \pm 0.11\,\text{eV}$$

$\pi^0 \to \gamma\gamma$ at NNLO in 2 flavour ChPT: technical part

- NLO: a) One-loop diagrams with one vertex from $\mathcal{L}^{WZ}$, b) tree diagrams with one vertex from $\mathcal{L}^{WZ}$ and one vertex from $O(p^4)$ Lagrangian, c) tree diagrams with one vertex from $O(p^6)$ anomalous-parity sector
- $O(p^6)$ anomalous-parity sector from [Bijnens, Girlanda, Talavera '02]
- representation of chiral field: $U = \sigma + i\frac{\tau.\pi}{F}$, $\sigma = \sqrt{1 - \vec{\pi}^2/F^2}$ (no $\gamma 4\pi$ vertex at LO)
- one-loop

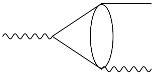


- two-loop



- verification of Z-factor, F_π/F [Bürgi '96], [Bijnens, Colangelo, Ecker, Gasser, Sainio '02]
- double log checked by Weinberg consistency rel. [Colangelo '95]

$\pi^0 \to \gamma\gamma$ technical part: some details on two-loop calculation



$$\mathcal{T} \sim \epsilon^{\mu\nu\rho\sigma} e^1_\alpha \, k^2_\rho \, e^2_\sigma \, p_\lambda$$
$$\times \int \frac{d^d l_1}{i(2\pi)^d} \frac{d^d l_2}{i(2\pi)^d} \frac{l_1^\lambda \, l_1^\mu \, l_2^\alpha \, l_2^\nu}{(l_1^2 - M^2)(l_2^2 - M^2)[(l_2+k_1)^2 - M^2][(l_1+l_2-k_2)^2 - M^2]}$$

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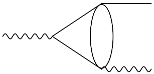

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$$J(t) = \int \frac{d^d l_1}{i(2\pi)^d} \frac{l_1^\lambda\, l_1^\mu}{(l_1^2 - M^2)((l_1+t)^2 - M^2)} \quad \to \quad \frac{g_{\lambda\mu}}{4(2w+3)} \int_{4M^2}^\infty \frac{[d\sigma]}{\sigma - t^2}(4M^2 - \sigma)$$

$$\Rightarrow R^{\alpha\nu} = \frac{1}{4(2w+3)} \int \frac{d^d l_2}{i(2\pi)^d} \frac{l_2^\alpha\, l_2^\nu}{(l_2^2 - M^2)((l_2+k_1)^2 - M^2)} \int_{4M^2}^\infty \frac{[d\sigma](4M^2 - \sigma)}{\sigma - t^2}$$

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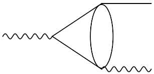

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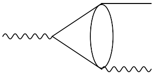
Divergences can be separated by Taylor expansion around $\sigma = \infty$

$$\int_{4M^2}^\infty [d\sigma](4M^2-\sigma)\sigma^l = -\frac{2(2w+3)}{(4\pi)^{2+\omega}}\Gamma(-2-w-l)\frac{\Gamma(-l)}{\Gamma(-2l)}(M^2)^{w+2+l}, \qquad \text{conv. for } l < -2$$

$\pi^0 \to \gamma\gamma$ technical part: some details on two-loop calculation



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so far one would obtain:

$$R = \frac{3581}{8064} + \frac{\pi^2}{24} + R_1 + R_2\,, \qquad \text{with} \qquad R_1 = -\frac{1}{84}\int_4^\infty ds \sqrt{\frac{(s-4)^3}{s}}(\log(s) pol_1 + pol_2)$$

and R_2 can be expressed as double integral, where one integral comes from

$$I = \frac{M^6}{4}\int dxdydz \int \frac{d^d l}{i(2\pi)^d} \frac{60\, x^3 y^2 z^3 (1-z)^4\, l^2}{[A_z - x(1-y)B_z - l^2]^6} \quad \text{with} \quad A_z = z\sigma + (1-z)M^2, \quad B_z = z(1-z)M^2$$

n.b. possible expansion in $B_z/A_z \le 1/9$

$\pi^0 \to \gamma\gamma$ at NNLO, result

$$
\begin{aligned}
A_{NNLO} = \frac{e^2}{F_\pi}\Big\{ & \frac{1}{4\pi^2} \\
& + \frac{16}{3} m_\pi^2 (-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr}) + \frac{64}{9} B(m_d - m_u)(5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr}) \\
& - \frac{M^4}{24\pi^2 F^4}\Big(\frac{1}{16\pi^2} L_\pi\Big)^2 + \frac{M^4}{16\pi^2 F^4} L_\pi \Big[\frac{3}{256\pi^4} + \frac{32F^2}{3}\big(2c_2^{Wr} + 4c_3^{Wr} + 2c_6^{Wr} + 4c_7^{Wr} - c_{11}^{Wr}\big)\Big] \\
& + \frac{32M^2 B(m_d - m_u)}{48\pi^2 F^4} L_\pi \left[-6c_2^{Wr} - 11c_3^{Wr} + 6c_4^{Wr} - 12c_5^{Wr} - c_7^{Wr} - 2c_8^{Wr}\right] \\
& + \frac{M^4}{F^4}\lambda_+ + \frac{M^2 B(m_d - m_u)}{F^4}\lambda_- + \frac{B^2(m_d - m_u)^2}{F^4}\lambda_{--}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\lambda_+ &= \frac{1}{\pi^2}\left[-\frac{2}{3} d_+^{Wr}(\mu) - 8c_6^r - \frac{1}{4}(l_4^r)^2 + \frac{1}{512\pi^4}\left(-\frac{983}{288} - \frac{4}{3}\zeta(3) + 3\sqrt{3}\,\mathrm{Cl}_2(\pi/3)\right)\right] \\
&\quad + \frac{16}{3} F^2 \left[8 l_3^r (c_3^{Wr} + c_7^{Wr}) + l_4^r(-4c_3^{Wr} - 4c_7^{Wr} + c_{11}^{Wr})\right] \\
\lambda_- &= \frac{64}{9}\left[d_-^{Wr}(\mu) + F^2 l_4^r (5c_3^{Wr} + c_7^{Wr} + 2c_8^{Wr})\right] \\
\lambda_{--} &= d_{--}^{Wr}(\mu) - 128 F^2 l_7 (c_3^{Wr} + c_7^{Wr}) \, .
\end{aligned}
$$

- 4 LECs in 2 combinations of NLO
- additional 4 LECs in 3 combinations of NNLO

Is it at all possible to make some reliable prediction?

$\pi^0 \to \gamma\gamma$: modified counting

- Use of $SU(3)$ phenomenology via $c_i^{Wr} \leftrightarrow C_i^{Wr}$ connection (based on [Gasser, Haefeli, Ivanov, Schmid '07,'08])

$$c_i^{Wr} = \frac{\alpha_i}{m_s} + \left(\beta_i + \gamma_{ij} C_j^{Wr} + \delta_i \ln \frac{B_0 m_s}{\mu^2}\right) + O(m_s)$$

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- implementation of modified counting

$$m_u, m_d \sim O(p^2) \quad \text{and} \quad m_s \sim O(p)$$

Result:

$$A_{NNLO}^{mod} = \frac{e^2}{F_\pi}\Bigg\{ \frac{1}{4\pi^2} - \frac{64}{3} m_\pi^2 C_7^{Wr} + \frac{1}{16\pi^2}\frac{m_d - m_u}{m_s}\Big[1 - \frac{3}{2}\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\Big] + 32B(m_d - m_u)\Bigg[\frac{4}{3}C_7^{Wr} + 4C_8^{Wr}\Big(1 - 3\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\Big) - \frac{1}{16\pi^2 F_\pi^2}\Big(3L_7^r + L_8^r - \frac{1}{512\pi^2}(L_K + \frac{2}{3}L_\eta)\Big)\Bigg] - \frac{1}{24\pi^2}\left(\frac{m_\pi^2}{16\pi^2 F_\pi^2} L_\pi\right)^2 \Bigg\}$$

$\pi \to \gamma\gamma$: Phenomenology

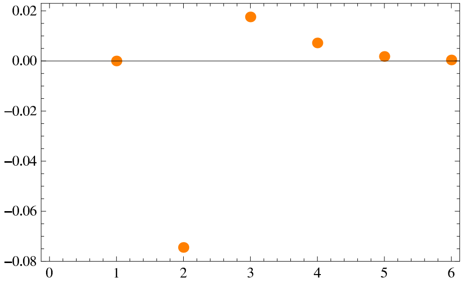
- $F_\pi = 92.22 \pm 0.07$ MeV (using updated value of V_{ud} [Towner, Hardy'08]).
  rem.: if SM violated: $F_\pi \to \hat{F}_\pi$ [Bernard, Oertel, Passemar, Stern '08]

  using quark mass ratio (from lattice), pseudo-scalar meson masses, R from $\eta \to 3\pi$ (cf. [KK,Knecht,Novotny,Zdrahal'11])
- $\frac{m_d - m_u}{m_s} = (2.29 \pm 0.23)\, 10^{-2}$
- $B(m_d - m_u) = (0.32 \pm 0.03)\, M_{\pi^0}^2$
- $3L_7 + L_8^r(\mu) = (0.10 \pm 0.06)\, 10^{-3} \quad (\mu = M_\eta)$ (from pseudo-scalar meson masses formula [Gasser, Leutwyler '85])
- $C_7^W = 0$ (more precisely $C_7^W \ll C_8^W$, motivated by simple resonance saturation)
- $C_8^W = (0.58 \pm 0.2) 10^{-3}$GeV^{-2} (from $\eta \to 2\gamma$)

result $\Gamma_{\pi^0 \to 2\gamma} = (8.09 \pm 0.11)$eV

$\pi \to \gamma\gamma$: leading logs

Leading logarithm contribution of individual orders in percent of the leading order:



Adler-Lee-Treiman-Zee-Terentev theorem on triangle and box anomaly

$$F^{3\pi}(0,0,0) = \frac{1}{eF_\pi^2} F_{\pi\gamma\gamma}(0,0)$$

is valid up to 2-loop order for LL beyond the soft-photon limit

$\pi^0 \to e^+e^-$

- first studied by [S. Drell '59]
- radiative corrections: [L.Bergström '83]
- most recent experiment: KTeV E799-II [Abouzaid'07]
- radiative corrections play important role

$\pi^0 \to e^+e^-$

KTeV's measurement:

$$\frac{\Gamma(\pi^0 \to e^+e^-,\, x > 0.95)}{\Gamma(\pi^0 \to e^+e^-\gamma,\, x > 0.232)} = (1.685 \pm 0.064 \pm 0.027) \times 10^{-4}\,.$$

by extrapolating the Dalitz branching ratio to the full range of x

$$B(\pi^0 \to e^+e^-(\gamma),\, x > 0.95) = (6.44 \pm 0.25 \pm 0.22) \times 10^{-8}\,.$$

Extrapolating the radiative tail using Bergström:

$$B^{\text{no-rad}}_{\text{KTeV}}(\pi^0 \to e^+e^-) = (7.48 \pm 0.29 \pm 0.25) \times 10^{-8}\,.$$

Theoretical prediction [Dorokhov, Ivanov '07, '10]

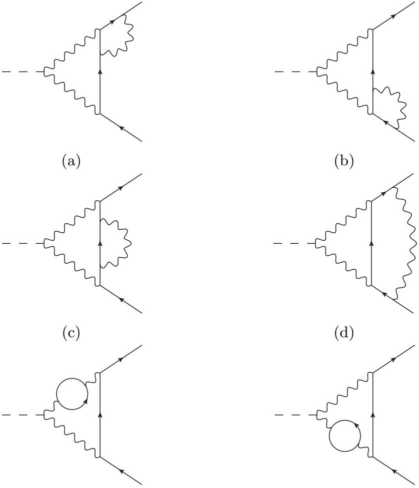
$$B^{\text{no-rad}}_{\text{SM}}(\pi^0 \to e^+e^-) = (6.23 \pm 0.09) \times 10^{-8}\,. \qquad (3)$$

3.3 $\sigma \Rightarrow$ New physics?

In any case, radiative corrections play an important role in the analysis

$\pi^0 \to e^+e^-$

Radiative corrections $\to$ two-loop graphs



(a) (b) (c) (d) (e) (f)

$\pi^0 \to e^+e^-$

- two-loop contributions, together with Bremsstrahlung (= Dalitz) [Dorokhov et al. '08], [Vasko,Novotny '11], [Husek,KK,Novotny'14]
- counter-term chiral Lagrangian for $\pi^0 l\bar{l}$ [Savage et al'92]
- modelled using the resonances [Knecht '99]

$$\chi^{(r)}_{\mathsf{LMD}}(M_\rho) = 2.2 \pm 0.9$$

- KTeV implies [Husek,KK,Novotny'14]

$$\chi^{(r)}_{\mathsf{KTeV}}(M_\rho) = 4.5 \pm 1.0$$

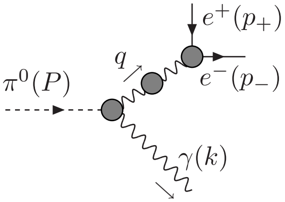
- original discrepancy down to 2 σ level

Dalitz decay

[K.K., Knecht, Novotný '06]

History

- First calculated by [Dalitz '51].
- Radiative corrections studied by [Joseph '60], [Lautrup, Smith'71], [Mikaelian, Smith'72]
- and during the 1980s by Tupper, Grose, Samuel, Lambin, Pestieau, Roberts...



$$x = m_{ee}^2/M_\pi^2, \quad y = \frac{E_+ - E_-}{E_\gamma}\Big|_{\pi^0\to 0}$$

NLO studied via $\delta(x,y)$ and $\delta(x)$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}y} = \delta(x,y)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y}, \quad \frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \delta(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}.$$

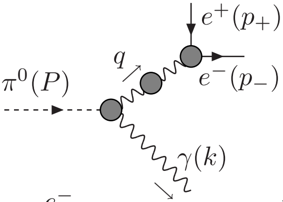
with (point-like pion)

$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x\mathrm{d}y} = \frac{\alpha^3}{(4\pi)^4}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\,[M_{\pi^0}^2 x(1+y^2) + 4m^2],$$

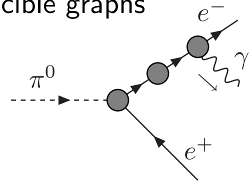
$$\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} = \frac{\alpha^3}{(4\pi)^4}\frac{8}{3}\frac{M_{\pi^0}}{F_\pi^2}\frac{(1-x)^3}{x^2}\,(xM_{\pi^0}^2 + 2m^2).$$

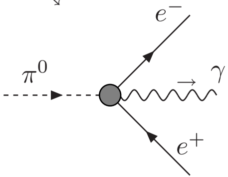
Dalitz decay: Anatomy of the amplitude

- one-photon reducible graphs: electron-positron pair is produced by a single photon (Dalitz pair)

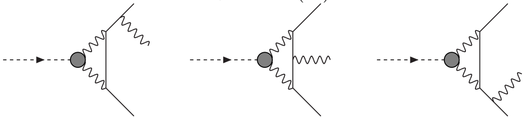


- one-photon irreducible graphs

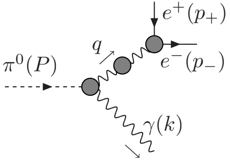




LO, order $O(e^5)$



Dalitz decay: slope parameter



$$\Gamma_\mu^{1\gamma R}(p_+, p_-, k) = \mathrm{i}e^2 \varepsilon_\mu{}^{\nu\alpha\beta} q_\alpha k_\beta \, \mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \, \mathrm{i}D^T_{\nu\rho}(q)(-\mathrm{i}e)\Lambda^\rho$$

$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2)$ is related to the doubly off-shell form factor $\mathcal{A}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

$$\int d^4x \, e^{il\cdot x} \langle 0|T(j^\mu(x) j^\nu(0)|\pi^0(P)\rangle = -\mathrm{i}\varepsilon^{\mu\nu\alpha\beta} l_\alpha P_\beta \, \mathcal{A}_{\pi^0\gamma^*\gamma^*}(l^2, (P-l)^2)$$

One can define a slope parameter a_π

$$\mathcal{F}_{\pi^0\gamma\gamma^*}(q^2) \;=\; \mathcal{F}_{\pi^0\gamma\gamma^*}(0) \left[1 \,+\, a_\pi \, \frac{q^2}{M^2_{\pi^0}} \,+\cdots\right],$$

$$\frac{\mathrm{d}\Gamma^{exp}}{\mathrm{d}x} - \delta_{QED}(x)\,\frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x} \;=\; \frac{\mathrm{d}\Gamma^{LO}}{\mathrm{d}x}\,[1 + 2x\, a_\pi].$$

Dalitz decay: Low's theorem

Soft limit due to the Low's theorem, naively:

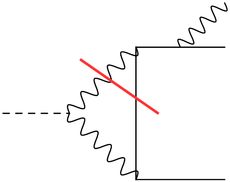
$$\mathcal{M}_{\pi^0\to e^+e^-\gamma} = \mathcal{M}^{\text{Low}} + O(k)$$

$$\mathcal{M}^{\text{Low}} = (s^{(0)} + s^{(1)})P_{\pi^0 e^- e^+}$$

or equivalently (the LO is of the order $O(k)$)

$$\delta^{1\gamma IR}(x,y) = \delta^{\text{Low}}(x,y) + O(1)$$

However, one should be careful. Due to the non-analyticity (the branch-cut of the intermediate $e\gamma$ state starts at m^2)



one should expect the logarithms i.e. →

Dalitz decay: Low's theorem

Soft limit due to the Low's theorem, properly:

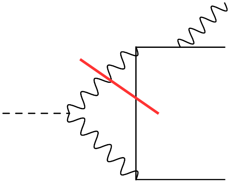
$$\mathcal{M}_{\pi^0\to e^+e^-\gamma} = \mathcal{M}^{\text{Low}} + O(k\ln k) + O(k)$$

$$\mathcal{M}^{\text{Low}} = (s^{(0)} + s^{(1)})P_{\pi^0 e^- e^+}$$

or equivalently (the LO is of the order $O(k)$)

$$\delta^{1\gamma IR}(x,y) = \delta^{\text{Low}}(x,y) + O(\ln(1-x)) + O(1)$$

However, one should be careful. Due to the non-analyticity (the branch-cut of the intermediate $e\gamma$ state starts at m^2)



one should expect the logarithms

for details see [KK,Knecht,Novotny '06]

Dalitz decay: summary of [KK,Knecht,Novotny'06] & [Husek,KK,Novotny'15]

- Our works provide a detailed analysis of NLO radiative corrections to the Dalitz decay amplitude.
- The off-shell pion-photon transition form factor was included:
  this requires a treatment of non perturbative strong interaction effects
- The one-photon irreducible contributions, which had been usually neglected, were included.
  We have shown that, although these contributions are negligible as far as the corrections to the total decay rate are concerned, they are however sizeable in regions of the Dalitz plot which are relevant for the determination of the slope parameter a_π of the pion-photon transition form factor.
- Our prediction for the slope parameter $a_\pi = 0.029 \pm 0.005$ is in good agreement with the determinations obtained from the (model dependent) extrapolation of the CELLO and CLEO data.
  Unfortunately, the experimental error bars on the latest values of a_π extracted from the Dalitz decay are still too large
- used in NA48 analysis for the search of dark photon [1504.00607]

Summary

Radiative and quantum corrections play an essential role for the π^0 processes. We have discussed

- leading logs
- $\pi^0 \to \gamma\gamma$
- $\pi^0 \to e^+e^-$
- $\pi^0 \to e^+e^-\gamma$

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Thank you.