Higher-order QCD corrections to the Higgs-boson $qT$ distribution

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Motivation

- Transverse momentum distribution of the Higgs boson in gluon fusion: probe of standard model EW dynamics, test of QCD factorization

- Differentiate different origins of $ggH$ effective couplings

- Probe production mechanism through QCD color structures

- Ideal for testing QCD factorization in gluon fusion process

Harlander, Neumann, 1308.2225

Balazs et al., hep-ph/0403052
Experimental precision at LHC run 1 is limited by statistics but could be largely improved at run 2 or high luminosity LHC; current uncertainties on theoretical predictions are not small
Theoretical predictions on SM Higgs boson qT distribution in different kinematic region: 1, non-perturbative region; 2, small-qT; 3, intermediate qT; 4, large qT; 5, Large-qT
Resummation of small-qT logarithms ($\log(qT/Q)$) to all-order in QCD in impact parameter space, first developed by Collins, Soper and Sterman (CSS, 1985)

Recent CSS application on Higgs boson production:

Florian et al., 2011, NNLL+NLO [HqT/HRes]

Wang et al., 2012, NNLL+NLO [Resbos]

\[
\left[ \frac{d\sigma_F}{dQ^2 \, dq_{\perp}^2} \right]_{\text{res.}} = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty db \frac{b}{2} J_0(bq_\perp) f_a/h_1(x_1, b_0^2/b^2) f_b/h_2(x_2, b_0^2/b^2) \\
\cdot W_{ab}^F(x_1, x_2; s, Q, b) . \\
W_{ab}^F(s; Q, b) = \sum_c \int_0^1 dz_1 \int_0^1 dz_2 C_{ca}(\alpha_S(b_0^2/b^2), z_1) C_{cb}(\alpha_S(b_0^2/b^2), z_2) \delta(Q^2 - z_1z_2s) \\
\cdot \sigma_{cc}^F(Q^2, \alpha_S(Q^2)) S_c(Q, b) .
\]
Higgs qT resummation based on Soft-Collinear effective theory [note the different convention in counting of matching order]


NNLL+NLO: Becher, Neubert, 2010, Becher et al., 2013

NNLL+NNLO: Chiu et al., 2012, Neill et al., 2015

NNLL: Echevarria et al., 2015

Fixed-order calculations on Higgs qT spectrum, NLO/NNLO QCD in heavy top-quark limit, NLO QCD with approx. top mass dependence

NLO: Florian et al., 1999, Ravindran et al., 2002, Glosser, Schmidt, 2002

NNLO: Boughezal et al., 2013, Chen et al., 2014, Boughezal et al., 2015

LO: Ellis et al., 1988, Baur, Glover, 1990

NLO (approx.): Dawson et al., 2014
small-qT factorization and resummation in SCET [following the scheme by Becher, Neubert, 2010]

\[ d\sigma = \sigma_0(\mu) C_t^2(m_t^2, \mu) |C_S(-m_H^2, \mu)|^2 \frac{m_H^2}{\tau s} dy \frac{d^2q_\perp}{(2\pi)^2} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \]

\[ \times 2B^{\mu\nu}_c(\xi_1, x_\perp, \mu) B_{\bar{c}, \mu\nu}(\xi_2, x_\perp, \mu) S(x_\perp, \mu) \]

Collinear anomaly and refactorization, qT<<m_H

\[ d\sigma = \sigma_0(\mu) C_t^2(m_t^2, \mu) |C_S(-m_H^2, \mu)|^2 \frac{m_H^2}{\tau s} dy \frac{d^2q_\perp}{(2\pi)^2} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \]

\[ \times 2 \left( \frac{x_T m_H^2}{b_0^2} \right)^{-F_{gg}(x_T^2, \mu)} B^{\mu\nu}_g(\xi_1, x_\perp, \mu) B^{\rho\sigma}_g(\xi_2, x_\perp, \mu) \]
small-qT factorization and resummation in SCET [following the scheme by Becher, Neubert, 2010]

Matching on to standard collinear PDFs, qT>>ΛQCD

\[
\frac{d^2\sigma}{dq_T^2 \, dy} = \sigma_0(\mu) \left[ C_t^2(m_t^2, \mu) \right] \left| C_S(-m_H^2, \mu) \right|^2 \sum_{i,j=g,q,\bar{q}} \int_{\xi_1}^{1} \frac{dz_1}{z_1} \int_{\xi_2}^{1} \frac{dz_2}{z_2} \times \bar{C}_{gg\leftarrow ij}(z_1, z_2, q_T^2, m_H^2, \mu) \phi_{i/P}(\xi_1/z_1, \mu) \phi_{j/P}(\xi_2/z_2, \mu)
\]

\[
\bar{C}_{gg\leftarrow ij}(z_1, z_2, q_T^2, m_H^2, \mu) = \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \left( \frac{x_\perp^2 m_H^2}{b_0^2} \right)^{-F_{gg}(L_\perp, a_s)} \times \sum_{n=1,2} I_{g\leftarrow i}^{(n)}(z_1, L_\perp, a_s) I_{g\leftarrow j}^{(n)}(z_2, L_\perp, a_s)
\]

\[L_\perp = \ln \frac{x_T^2 \mu^2}{b_0^2}\]

with qT>>ΛQCD all colored objects can be calculated perturbatively; scale dependence are organized by RG equation with anomalous dimensions
Resummation of Sudakov logarithms is accomplished by evolving hard functions from canonical scales to a proper scale at which the expansion of remaining kernel $C$ can be perturbatively controlled.

**RG evolution of the hard function**

$$C_t(m_t^2, \mu_f^2) = \frac{\beta(\alpha_s(\mu_f^2))/\alpha_s^2(\mu_f^2)}{\beta(\alpha_s(\mu_t^2))/\alpha_s^2(\mu_t^2)} C_t(m_t^2, \mu_t^2)$$

$$C_S(-m_H^2-i\epsilon, \mu_f^2) = \exp \left[ 2S(\mu_h^2, \mu_f^2) - a_\Gamma(\mu_h^2, \mu_f^2) \ln \frac{-m_H^2 - i\epsilon}{\mu_h^2} - a_\gamma s(\mu_h^2, \mu_f^2) \right] C_S(-m_H^2-i\epsilon, \mu_h^2)$$

**proper scale (mu=<x_T>^{-1}) ~q^*+qT, q^* ~8 GeV**

$$q_* = m_H \exp \left( -\frac{2\pi}{\Gamma_0^A \alpha_s(q_*)} \right)$$

**Improved expansion in momentum space, Lperp \sim 1/as^{1/2}**

$$\bar{C}_{gg \leftarrow ij}(z_1, z_2, q_T^2, m_H^2, \mu) = \frac{1}{2} \int_0^\infty dx_T x_T J_0(x_T q_T) \exp \left[ g_A(\eta, L_\perp, a_s) \right]$$

$$\times \sum_{n=1,2} \bar{I}_{g \leftarrow i}^{(n)}(z_1, L_\perp, a_s) \bar{I}_{g \leftarrow j}^{(n)}(z_2, L_\perp, a_s) ,$$

$$g_A(\eta, L_\perp, a_s) = -[\eta L_\perp]_{\epsilon^{-1/2}} - \left[ a_s \left( \Gamma_0^A + \eta \beta_0 \right) L_\perp^2 \right]_{\epsilon^{1/2}} +$$
power counting and perturbative ingredients needed for the N3LL resummation

- QCD beta function to 4-loop [Ritbergen et al., 1997], Cusp anomalous dimension to 4-loop [3-loop, Moch et al., 2004], quark and gluon anomalous dimension to 3-loop [Becher, Neubert, 2009], Ct and Cs to 2-loop [Ahrens et al, 2009] at matching scales


Pade approx. for cusp at 4-loop, effects are small as in many other studies will vary 3-loop collinear anomaly (d3) in a reasonable range
Fixed-order matching (non-singular/power corrections) is important even in peak region for Higgs case, NNLO/N3LO [Glosser , Schmidt, 2002, Boughezal et al., 2015]

\[
\left[ \frac{d\hat{\sigma}^{(\text{fin.})}_{F_{ab}}}{dq_T^2} \right]_{\text{f.o.}} = \left[ \frac{d\hat{\sigma}_{F_{ab}}}{dq_T^2} \right]_{\text{f.o.}} - \left[ \frac{d\hat{\sigma}^{(\text{res.})}_{F_{ab}}}{dq_T^2} \right]_{\text{f.o.}} \sim 10\% \text{ in peak region}
\]

We adopt the order matching, NLL+NLO, NNLL+NNLO, N3LL+N3LO [also require 3-loop DGLAP kernel]; will vary the canonical scale choices/hard functions in non-singular piece to further investigate the perturbative convergence

Long-distance effects are found to be small in peak region of the Higgs qT [Becher et al., 2013, Echevarria et al., 2015, Florian et al., 2011]

\[
B_g^{(n)}(\xi, x_T^2, \mu) = f_{\text{hadr}}(x_T \Lambda_{\text{NP}}) B_g^{(n)\text{pert}}(\xi, x_T^2, \mu)
\]

\[
f_{\text{hadr}}^{\text{gauss}}(x_T \Lambda_{\text{NP}}) = \exp \left( -\Lambda_{\text{NP}}^2 x_T^2 \right)
\]

\[
f_{\text{hadr}}^{\text{pole}}(x_T \Lambda_{\text{NP}}) = \frac{1}{1 + \Lambda_{\text{NP}}^2 x_T^2}
\]
Numerical results [preliminary]

- resummed component and scale variation [two matching scales +resummation scale, factorization scale], \( qT \) distribution

resummed cross sections can be divided into two separate scale invariant parts to have a better gauge on missing higher-order contributions [see Becher et al., 2013]
resummed component and scale variation [two matching scales +resummation scale, factorization scale],

\[ \text{intercept} \]

\begin{align*}
\text{no scale separation} & \\
\text{with scale separation} & 
\end{align*}

\begin{align*}
\text{pp} \rightarrow H, 125 \text{ GeV} & \\
\text{at LHC 13 TeV} & \\
d_3 = -1000 & 
\end{align*}

resummed cross sections can be divided into two separate scale invariant parts to have a better gauge on missing higher-order contributions [see Becher et al., 2013]
resummed component and variation with d3 [variation with cusp at 4-loop is negligible]

\[ F_{gg}(\perp, \alpha_s) = \sum_{n=1}^{\infty} d_n^g(\perp) \left( \frac{\alpha_s}{4\pi} \right)^n \]

\[ \frac{d_2^g}{C_F} = \frac{d_2^g}{C_A} = C_A \left( \frac{808}{27} - 28\zeta_3 \right) - \frac{224}{27} T_F n_f \]

being the scale independent part in collinear anomaly exponent, d1=0, d2=-95.8, d3=?
non-singular component in conventional scheme, canonical \( \mu = m_H \), no RG improvement [restrict to NNLO here]

\[
q_T^2 \frac{d\sigma_{\sin}^{(n)}}{dq_T^2} \sim \alpha_s^n \ln^{2n-1} \left( \frac{q_T^2}{m_H^2} \right)
\]

\[
q_T^2 \frac{d\sigma_{\text{ns}}^{(n)}}{dq_T^2} \sim \alpha_s^n \frac{q_T^2}{m_H^2} \ln^{2n-1} \left( \frac{q_T^2}{m_H^2} \right)
\]

from left to right: vanishing of non-singular piece when \( q_T \to 0 \); non-singular piece dominate in FO when \( q_T \sim 60 \) GeV; turn-off of resummation at high \( q_T \)
non-singular component in conventional/modified scheme, canonical mu=m_H, no/with RG improvement [restrict to NNLO here]

non-singular contributions are sensitive to the matching scheme used including choice of the canonical scale; uncertainties reduced from NLO to NNLO; N3LO must needed to match the precision in N3LL resummation
non-singular component in conventional/modified scheme, canonical \( \mu = m_H/q^* + q_T \), no RG improvement [restrict to NNLO here]

\[
\mu = m_H(\text{ns}) \\
\mu = q^* + q_T(\text{ns})
\]

non-singular contributions are sensitive to the matching scheme used including choice of the canonical scale; uncertainties reduced from NLO to NNLO; N3LO must needed to match the precision in N3LL resummation
non-singular component in conventional/modified scheme, \( \mu = m_H / (q^* + q_T) \), no/with RG improvement [restrict to NNLO here]

non-singular contributions are sensitive to the matching scheme used including choice of the canonical scale; uncertainties reduced from NLO to NNLO; N3LO must needed to match the precision in N3LL resummation
uncertainties in the combined spectrum, for NNLL+NNLO and N3LL+N3LO [preliminary]

with N3LL+N3LO accuracy we expect to bring down the perturbative uncertainties to within 10% from peak region to large qT (not include uncertainties from unknown d3)
Summary

- We report progress in a N3LL+N3LO calculation of qT spectrum of the SM Higgs boson in gluon fusion production based on SCET.

- N3LL resummation reduce the scale uncertainties significantly and the resummation show good convergence.

- Non-singular component (FO matching) is important for predictions even around the peak region; N3LO contributions are needed.

- With all available perturbative inputs a reduction of perturbative uncertainties to within 10% are expected for the spectrum from small-qT to ~mH.
non-singular component in conventional/modified scheme, canonical $\mu = m_H/q^*+q_T$, with RG improvement [restrict to NNLO here]

\[ \mu = m_H(\text{ns}) \]

\[ \mu = q^*+q_T(\text{ns}) \]

non-singular contributions are sensitive to the matching scheme used including choice of the canonical scale; uncertainties reduced from NLO to NNLO; N3LO must needed to match the precision in N3LL resummation