Mixed QCD–electroweak $\mathcal{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region

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in collaboration with
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MC@NNLO

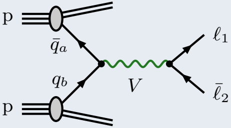
based on Nucl. Phys. B**885** (2014) 318 [arXiv:1403.3216 [hep-ph]]

Outline

1. Motivation and Introduction
2. Pole Approximation @ NNLO $\mathcal{O}(\alpha_{\rm s}\alpha)$
3. Conclusions and Outlook

Motivation and Introduction

$\mathrm{W}^\pm$ and Z production at the LHC



$$q\,\bar{q} \to \mathrm{Z}/\gamma \to \ell^-\,\ell^+ \qquad \text{“neutral current”}$$

$$\left.\begin{array}{l} u\,\bar{d} \to \mathrm{W}^+ \to \nu_\ell\,\ell^+ \\ d\,\bar{u} \to \mathrm{W}^- \to \ell^-\,\bar{\nu}_\ell \end{array}\right\} \text{“charged current”}$$

Clean experimental signature & Large cross section

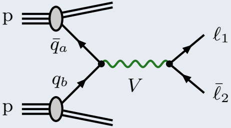
⇒ One of the most precise probes to test the Standard Model!

- important “standard candles” at the LHC
  ↪ detector calibration, luminosity monitor, quark PDFs,. . .
- searches for physics beyond the Standard Model
  ↪ background in Z', W' searches (high $M_{\ell\ell}$, $M_{\mathrm{T},\nu\ell}$ tails)
- **precision measurements**
  ↪ $\boldsymbol{M}_{\mathbf{W}}$, $\sin^2\theta^{\mathrm{lept}}_{\mathrm{eff}}$

Motivation and Introduction

$\mathrm{W}^{\pm}$ and Z production at the LHC

$14\ \mathrm{TeV}\ (\mathcal{L} = 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1})$



$q\,\bar{q} \rightarrow \mathrm{Z}/\gamma \rightarrow \ell^-\,\ell^+$ $\sim$ **20 events/sec**

$u\,\bar{d} \rightarrow \mathrm{W}^+ \rightarrow \nu_\ell\,\ell^+$

$d\,\bar{u} \rightarrow \mathrm{W}^- \rightarrow \ell^-\,\bar{\nu}_\ell$

$\Big\}$ $\sim$ **200 events/sec**

(per lepton flavour)

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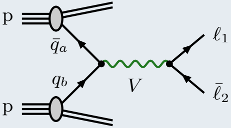


$q\,\bar{q} \to Z/\gamma \to \ell^-\,\ell^+$ ~ **20** events/sec

$u\,\bar{d} \to W^+ \to \nu_\ell\,\ell^+$

$d\,\bar{u} \to W^- \to \ell^-\,\bar{\nu}_\ell$

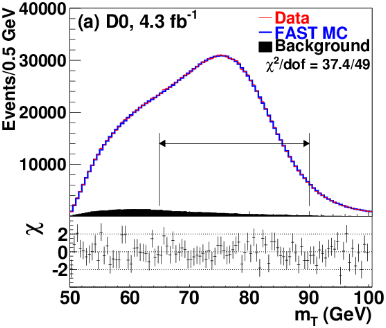
} ~ **200** events/sec

(per lepton flavour)

Clean experimental signature & Large cross section

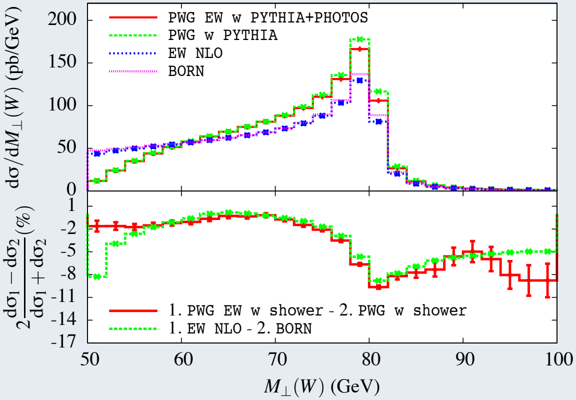
- **Tevatron**: (most precise measurement)
  $M_W = 80.387 \pm 0.016\ \mathrm{GeV}$
- **LHC**: aimed precision of
  $\Delta M_W \lesssim 7\ \mathrm{MeV}$
  [Besson et al. ’08] [Baak et al. ’13]
- fits to kinematic distributions
  $M_{T,\nu\ell} \in [65, 90]\ \mathrm{GeV}$

[Abazov et al. ’12]



Motivation: M_W measurement

Largest missing piece: Mixed QCD×EW corrections [Barzè, et al. '12]



NLO (QCD + EW) ⊗ (QCD + QED) PS
v.s. NLO (QCD) ⊗ (QCD) PS

NLO (EW)
v.s. LO

estimate of missing $\mathcal{O}(\alpha_s\alpha)$ corrections

Difference ∼ %-level!

Theoretical Status

QCD NNLO $\mathcal{O}(\alpha_\mathrm{s}^2)$ .... [Hamberg et al. '91 / Harlander, Kilgore '02]
(differential) [Anastasiou et al. '04] [Melnikov, Petriello '06] [Catani et al. '09]
resummation .. [Balazs, Yuan '97] [Bozzi et al. '10] [Becher et al. '11] [Banfi et al. '12] [Guzzi et al. '13]
PS matching .... [Frixione Webber '06] [Alioli et al. '08] [Hamilton, Richardson, Tully '08]
(matching@NNLO) [Höche, Li, Prestel '14] [Karlberg, Re, Zanderighi '14]

EW NLO $\mathcal{O}(\alpha)$
W .... [Dittmaier, Krämer '02] [Baur, Wackeroth '04] [Carloni Calame et al. '06] [Arbuzov et al. '06]
Z .... [Baur et al. '02] [Carloni Calame, Montagna, Nicrosini, Vicini '07] [Dittmaier, Huber '10]
multi-photon radiation .... [Baur, Stelzer '00] [Placzek, Jadach '03] [Calame et al. '04]

\+ much more...

Approaches to Combination

- Additive/multiplicative combinations .... [Cao, Yuan '04] [Li, Petriello '12] [Richardson et al. '12]
- NLO (EW+QCD) + PS matching .... [Bernaciak, Wackeroth '12] [Barzè et al. '12, '13]

Steps towards NNLO QCD×EW $\mathcal{O}(\alpha_\mathrm{s}\alpha)$

- NLO EW $\mathcal{O}(\alpha)$ to $V+$ jet .... (off-shell + decay) [Denner, Dittmaier, Kasprzik, Mück '09, '11]
- Decay widths .... [Czarnecki, Kühn '96](Z) [Kara '13](W)
- 2-loop $\mathrm{Z}\bar{f}f$ vertex .... [Kotikov, Kühn, Veretin '08]
- 2-loop on-shell V production .... [Bonciani '11]
- 2-loop QCD×QED .... [Kilgore, Sturm '12]
- Pole expansion around the resonance .... [Dittmaier, AH, Schwinn '14]

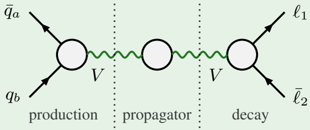
Pole Approximation

[Stuart '91] [H.Veltman '94] [Aeppli, v.Oldenborgh, Wyler '94]

Aim: Improve the theoretical prediction in resonance region
↪ Expansion about complex pole $\mu_V^2 = M_V^2 - \mathrm{i}M_V\Gamma_V$ ⇝ leading: $(p_V^2 - \mu_V^2)^{-1}$

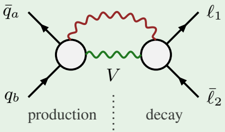
Leading pole approximation (PA)

Factorizable corrections:



- on-shell production & decay
  ↪ fact. ini $(2 \to 1)$
  ↪ fact. fin $(1 \to 2)$
  ↪ propagator $(1 \to 1)$
  (taken care by on-shell scheme)

Non-factorizable corrections:



- connect production & decay
  resonant contribution
  ↔ only soft-photon exchange
  ↪ non-fact. $(2 \to 2)$

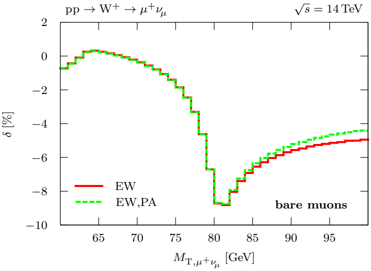
⇒ Simplifications compared to the full off-shell calculation $(2 \to 2)$ [Dittmaier, Krämer '01], etc.

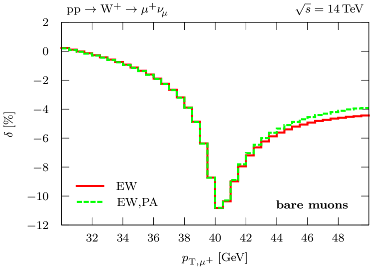
PA @ NLO

— full $\mathcal{O}(\alpha)$ corrections

- - - PA

- ⋯ fact. ini.
- ⋯ fact. fin.
- ⋯ non-fact



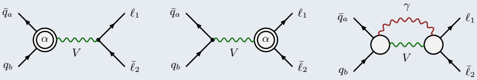


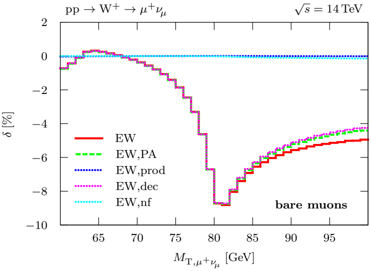
- full calculation vs. PA: agreement $\lesssim 1\%$ around resonance
- PA for $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$ corrections expected to be sufficient
- dominant contribution: factorizable corrections to the decay

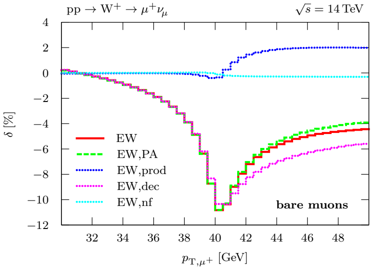
PA @ NLO

🟥 full $\mathcal{O}(\alpha)$ corrections

🟩 PA
- fact. ini.
- fact. fin.
- non-fact







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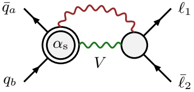
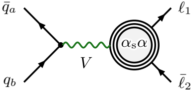
Pole Approximation @ NNLO $\mathcal{O}(\alpha_s\alpha)$—Contributions



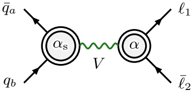
Non-factorizable corrections* ✓ [Dittmaier, AH, Schwinn '14]

- QCD corrections to production
  × soft-photon exchange: production & decay



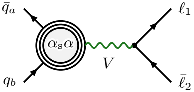
Factorizable final–final corrections ✓

- only a constant $\mathcal{O}(\alpha_s\alpha)$ counterterm



Factorizable initial–final corrections* ✓

- QCD corrections to production
  × EW corrections to decay
- large corrections & shape distortion expected



Factorizable initial–initial corrections*

- $\mathcal{O}(\alpha_s\alpha)$ corrections to on-shell V production
- no significant shape distortion expected
- no $\mathcal{O}(\alpha_s\alpha)$ PDFs

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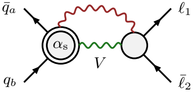
* only virtual contributions indicated ⇝ also real-, double-real emission, interferences,. . .

Pole Approximation @ NNLO $\mathcal{O}(\alpha_s\alpha)$—Contributions



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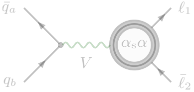

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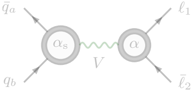

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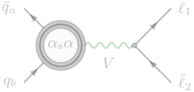

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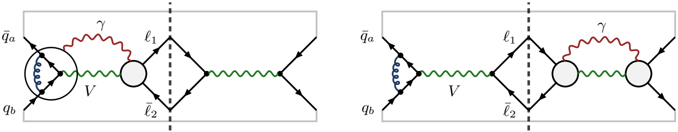
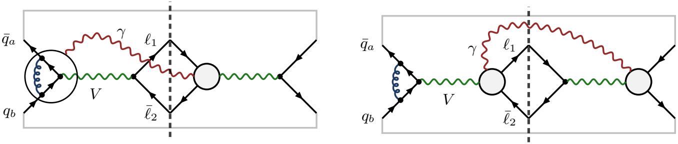
Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections: Ingredients

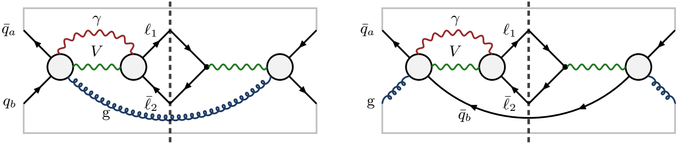
(Virtual QCD)
× (Virtual EW)



(Virtual QCD)
× (Real EW)



(Real QCD)
× (Virtual EW)



(Real QCD)
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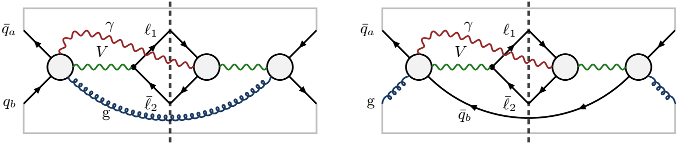

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

- issue of overlapping IR singularities
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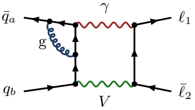
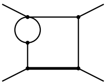
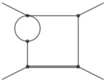
  - effective field theory for unstable particles [Beneke et al. '03,'04]
  - generalization of [Yennie, Frautschi, Suura '61]

↪ factorized structure: (NNLO **QCD**×**EW**)$_{\text{nf}}$ ∼ (NLO **QCD**) × δ_{nf}



$$\sim -\frac{C_F\alpha_s}{2\pi}\,\frac{Q_qQ_l\alpha}{2\pi}\,\mathcal{M}^0\,(1-\epsilon)\,(-\hat{t})\,(\mu_V^2-\hat{s}^2)\quad\times$$





$$= \frac{(4\pi)^{2\epsilon}\Gamma^2(1+\epsilon)}{(\mu_V^2-\hat{s})(-\hat{t})}\left(\frac{\mu_V^2-\hat{s}}{M_V^2}\right)^{-3\epsilon}\left(\frac{-\hat{t}}{\mu^2}\right)^{-2\epsilon}\left\{\frac{1}{2\epsilon^3}+\frac{1}{\epsilon^2}+\frac{1}{\epsilon}\left[2+\frac{5\pi^2}{12}+\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right]\right.$$

$$+\,2\,\mathrm{Li}_3\left(\frac{-\hat{t}}{M_V^2}\right)+\mathrm{Li}_3\left(1+\frac{\hat{t}}{M_V^2}\right)-6\zeta(3)-2\ln\left(\frac{-\hat{t}}{M_V^2}\right)\left[\frac{\pi^2}{6}-\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)\right]$$

$$\left.+\,\ln^2\left(\frac{-\hat{t}}{M_V^2}\right)\ln\left(1+\frac{\hat{t}}{M_V^2}\right)+\frac{5\pi^2}{6}+2\,\mathrm{Li}_2\left(1+\frac{\hat{t}}{M_V^2}\right)+4+\mathcal{O}(\epsilon)+\mathcal{O}\left(\hat{s}-\mu_V^2\right)\right\}$$

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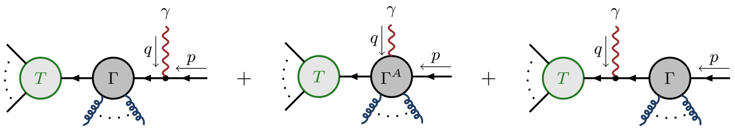

- soft photon insertion along quark line
- hard interaction T
- gluons: virtual/real, hard/soft

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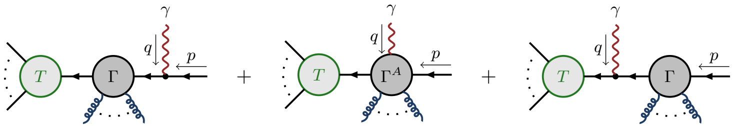
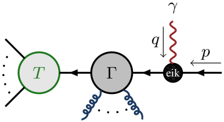
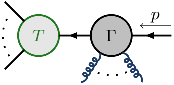


$\underset{q\to 0}{\sim}$



$$\equiv \; eQ_q \, \frac{2p_\nu + q_\nu}{2(p\cdot q) + q^2}$$



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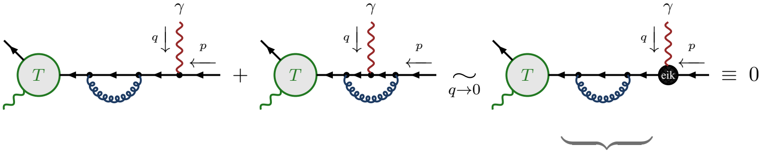
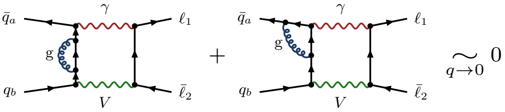


(scale-less integral) $\rightarrow 0$



Non-factorizable $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$ corrections

$$
\begin{aligned}
\hat{\sigma}_{\mathsf{nf}}^{\mathsf{QCD}\otimes\mathsf{EW}} = {} & \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\ \delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\mathsf{nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\ \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\mathsf{nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\ \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\mathsf{nf}} \\
& + \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\ 2\,\mathrm{Re}\left\{\delta^{2\to3}_{\mathrm{V_{ew}},\mathsf{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\ 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\mathsf{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\ 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\mathsf{nf}}\right\}
\end{aligned}
$$

Infrared singularities—QCD corrections: dipole subtraction formalism

$$
\begin{aligned}
\hat{\sigma}^{\mathsf{QCD}} &= \int_3 \mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}} \\
&= \int_3 \left[\left(\mathrm{d}\sigma^{\mathrm{R_s}}\right)_{\epsilon=0} - \left(\mathrm{d}\sigma^{\mathrm{A_s}}\right)_{\epsilon=0}\right] + \int_2 \left[\mathrm{d}\sigma^{\mathrm{V_s}} + \mathrm{d}\sigma^{\mathrm{C_s}} + \int_1 \mathrm{d}\sigma^{\mathrm{A_s}}\right]_{\epsilon=0}
\end{aligned}
$$

Infrared singularities—EW corrections: phase-space slicing method $\Delta E \ll \Gamma_V$

$$
\begin{aligned}
\int_\gamma \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathsf{QCD}}\ \delta^{\gamma}_{\mathrm{R_{ew}},\mathsf{nf}} &= \int_{E_\gamma<\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathsf{QCD}}\ \delta^{\gamma}_{\mathrm{R_{ew}},\mathsf{nf}} + \int_{E_\gamma>\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathsf{QCD}}\ \delta^{\gamma}_{\mathrm{R_{ew}},\mathsf{nf}} \\
&= \underbrace{\int_{E_\gamma<\Delta E} \mathrm{d}\Phi_\gamma\ \delta^{\gamma}_{\mathsf{eik}}\ \mathrm{d}\sigma^{\mathsf{QCD}}}_{=\ \delta_{\mathsf{soft}}(\Delta E)} + \int_{E_\gamma>\Delta E} \mathrm{d}\Phi_\gamma\, \mathrm{d}\sigma^{\mathsf{QCD}}\ \delta^{\gamma}_{\mathrm{R_{ew}},\mathsf{nf}}
\end{aligned}
$$

Non-factorizable $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$ corrections

$$\hat{\sigma}_{\mathsf{nf}}^{\mathsf{QCD}\otimes\mathsf{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\ \delta_{\mathrm{Rew},\mathsf{nf}}^{2\to3+\gamma} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\ \delta_{\mathrm{Rew},\mathsf{nf}}^{2\to2+\gamma} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\ \delta_{\mathrm{Rew},\mathsf{nf}}^{2\to2+\gamma}$$

$$+ \int_{3} \mathrm{d}\sigma^{\mathrm{R_s}}\ 2\,\mathrm{Re}\left\{\delta_{\mathrm{Vew},\mathsf{nf}}^{2\to3}\right\} + \int_{2} \mathrm{d}\sigma^{\mathrm{V_s}}\ 2\,\mathrm{Re}\left\{\delta_{\mathrm{Vew},\mathsf{nf}}^{2\to2}\right\} + \int_{2} \mathrm{d}\sigma^{\mathrm{C_s}}\ 2\,\mathrm{Re}\left\{\delta_{\mathrm{Vew},\mathsf{nf}}^{2\to2}\right\}$$

Example: double-real corrections

$$\widetilde{\sigma}_{\mathsf{nf}}^{\mathrm{R_s}\otimes\mathrm{Rew}} = \iint_{\substack{3+\gamma \\ E_\gamma > \Delta E}} \left\{ \mathrm{d}\sigma^{\mathrm{R_s}}\ \delta_{\mathrm{Rew},\mathsf{nf}}^{2\to3+\gamma}(\Phi_{3+\gamma}) - \sum_{\mathsf{dipoles}} \mathrm{d}\sigma^{0}\ \delta_{\mathrm{Rew},\mathsf{nf}}^{2\to2+\gamma}(\widetilde{\Phi}_{2+\gamma}) \otimes \mathrm{d}V_{\mathsf{dip}} \right\}$$

- $\delta_{\mathrm{Rew},\mathsf{nf}}^{2\to2+\gamma}(\widetilde{\Phi}_{2+\gamma})$
  ↪ analytic integration over unresolved phase space
- cut $E_\gamma > \Delta E$ frame dependent ⇝ partonic CMS of $\Phi_{3+\gamma}$
  ↪ $\widetilde{\Phi}_{2+\gamma}$ boosted in beam direction

Non-factorizable $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\;\delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\;\delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\;\delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}$$
$$+ \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\;2\,\mathrm{Re}\left\{\delta^{2\to3}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\;2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\;2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\}$$

Example: double-real corrections

$$\tilde{\sigma}_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} = \iint_{\substack{3+\gamma\\ E_\gamma>\Delta E}} \left\{\mathrm{d}\sigma^{\mathrm{R_s}}\;\delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}}(\Phi_{3+\gamma}) - \sum_{\text{dipoles}} \mathrm{d}\sigma^{0}\;\delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}(\widetilde{\Phi}_{2+\gamma})\otimes\mathrm{d}V_{\text{dip}}\right\}$$

- $\delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}(\widetilde{\Phi}_{2+\gamma})$
  $\hookrightarrow$ analytic integration over unresolved phase space
- cut $E_\gamma > \Delta E$ frame dependent $\rightsquigarrow$ partonic CMS of $\Phi_{3+\gamma}$
  $\hookrightarrow$ $\widetilde{\Phi}_{2+\gamma}$ boosted in beam direction

Non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat{\sigma}_{\text{nf}}^{\text{QCD}\otimes\text{EW}} = \iint\limits_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}}\, \delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{V_s}}\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}}\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}$$

$$+ \int_3 \mathrm{d}\sigma^{\mathrm{R_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to3}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{V_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\} + \int_2 \mathrm{d}\sigma^{\mathrm{C_s}}\, 2\,\mathrm{Re}\left\{\delta^{2\to2}_{\mathrm{V_{ew}},\text{nf}}\right\}$$

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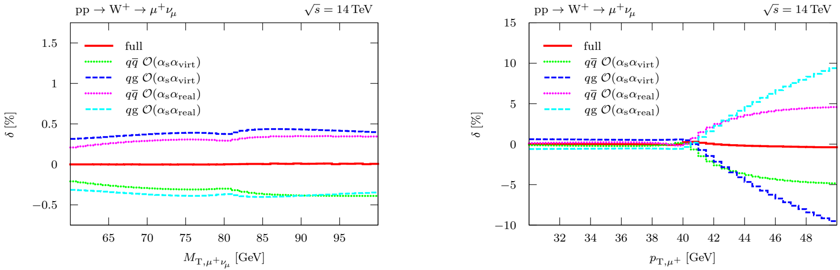
$$\tilde{\sigma}_{\text{nf}}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}} = \iint\limits_{\substack{3+\gamma\\ E_\gamma > \Delta E}} \left\{\mathrm{d}\sigma^{\mathrm{R_s}}\, \delta^{2\to3+\gamma}_{\mathrm{R_{ew}},\text{nf}}(\Phi_{3+\gamma}) - \sum_{\text{dipoles}} \mathrm{d}\sigma^0\, \delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}(\widetilde{\Phi}_{2+\gamma}) \otimes \mathrm{d}V_{\text{dip}}\right\}$$

- $\delta^{2\to2+\gamma}_{\mathrm{R_{ew}},\text{nf}}(\widetilde{\Phi}_{2+\gamma})$
  ↪ analytic integration over unresolved phase space
- cut $E_\gamma > \Delta E$ frame dependent ⇝ partonic CMS of $\Phi_{3+\gamma}$
  ↪ $\widetilde{\Phi}_{2+\gamma}$ boosted in beam direction

W^+ distributions @ NNLO $\mathcal{O}(\alpha_s\alpha)$ (non-factorizable)

⋯⋯ / - - - } $\mathcal{O}(\alpha_s\alpha_{\text{virt}})$: including soft photon emission $E_\gamma < \Delta E$

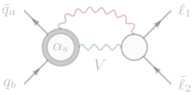
⋯⋯ / - - - } $\mathcal{O}(\alpha_s\alpha_{\text{real}})$: with cut $E_\gamma > \Delta E$ $(\Delta E \ll \Gamma_V)$



- almost perfect cancellation between different contributions
- tiny & flat corrections!

$\Rightarrow$ dominant contributions at $\mathcal{O}(\alpha_s\alpha)$ from the factorizable corrections!

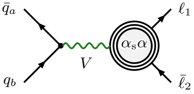
Pole Approximation @ NNLO $\mathcal{O}(\alpha_s\alpha)$—Contributions



Non-factorizable corrections* ✓

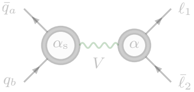
[Dittmaier, AH, Schwinn '14]

- QCD corrections to production
  × soft-photon exchange: production & decay
- phenomenologically negligible



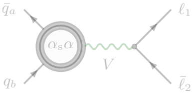
Factorizable final–final corrections ✓

- only a constant $\mathcal{O}(\alpha_s\alpha)$ counterterm



Factorizable initial–final corrections* ✓

- QCD corrections to production
  × EW corrections to decay
- large corrections & shape distortion expected



Factorizable initial–initial corrections*

- $\mathcal{O}(\alpha_s\alpha)$ corrections to on-shell V production
- no significant shape distortion expected
- no $\mathcal{O}(\alpha_s\alpha)$ PDFs

---

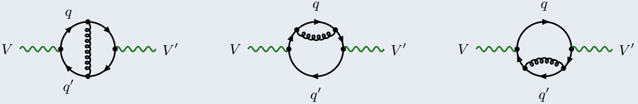
* only virtual contributions indicated ⇝ also real-, double-real emission, interferences,...

Factorizable final–final $\mathcal{O}(\alpha_s\alpha)$ corrections

- pure $V\bar{\ell}_1\ell_2$-vertex counterterm $\rightsquigarrow$ finite

$$\delta\mathcal{M}^{\text{QCD}\otimes\text{EW}}_{\text{fact. fin-fin}} = \delta^{\text{ct},(\alpha_s\alpha)}_{V\ell_1\bar{\ell}_2}\ \mathcal{M}^0_{\text{PA}}$$

- $\mathcal{O}(\alpha_s\alpha)$: only vector boson self-energies [Djouadi, Gambino '94]



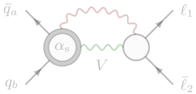
↪ **small** $\mathcal{O}(\lesssim 0.1\%)$ and **flat** corrections

....

G_μ input parameter scheme

$$\alpha_{G_\mu} = \frac{\sqrt{2}G_\mu M_W^2}{\pi}\left(1 - \frac{M_W^2}{M_Z^2}\right), \qquad \delta Z_e^{G_\mu} = \delta Z_e^{\alpha(0)} - \frac{1}{2}\,\Delta r$$

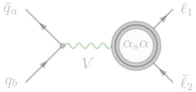
Pole Approximation @ NNLO $\mathcal{O}(\alpha_s\alpha)$—Contributions



Non-factorizable corrections* ✓

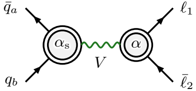
[Dittmaier, AH, Schwinn '14]

- QCD corrections to production
  × soft-photon exchange: production & decay
- phenomenologically negligible



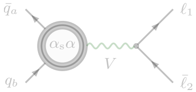
Factorizable final–final corrections ✓

- only a constant $\mathcal{O}(\alpha_s\alpha)$ counterterm
- small & no impact on shape



Factorizable initial–final corrections* ✓

- QCD corrections to production
  × EW corrections to decay
- large corrections & shape distortion expected



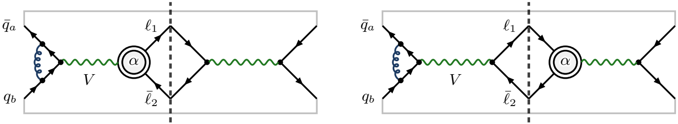
Factorizable initial–initial corrections*

- $\mathcal{O}(\alpha_s\alpha)$ corrections to on-shell V production
- no significant shape distortion expected
- no $\mathcal{O}(\alpha_s\alpha)$ PDFs

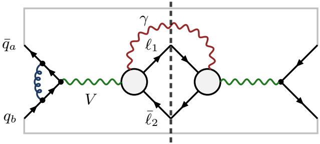
* only virtual contributions indicated ⇝ also real-, double-real emission, interferences,. . .

Factorizable initial–final $\mathcal{O}(\alpha_s\alpha)$ corrections: Ingredients

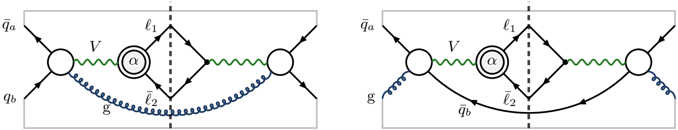
(Virtual QCD)
× (Virtual EW)



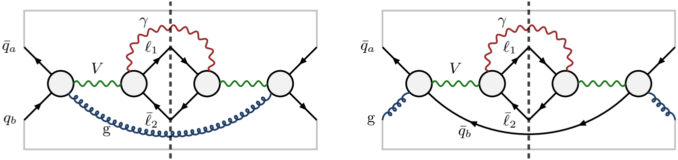
(Virtual QCD)
× (Real EW)



(Real QCD)
× (Virtual EW)



(Real QCD)
× (Real EW)



Factorizable initial–final $\mathcal{O}(\alpha_s\alpha)$ corrections

- **QCD** corrections confined to production sub-process
- **EW** corrections confined to decay sub-process

$\Rightarrow$ reducible structure*: (NLO **QCD**) × (NLO **EW**)

*exception: double-real corrections

$$\hat{\sigma}^{\text{QCD}\otimes\text{EW}}_{\text{fact. ini-fin}} = \int_3 2\,\mathrm{Re}\Big\{\delta^{V\,\bar{\ell}_1\,\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 4\,\mathrm{Re}\Big\{\delta^{V\,\bar{\ell}_1\,\ell_2}_{\mathrm{V_{ew}}}\Big\}\;\mathrm{Re}\Big\{\delta^{V\,\bar{q}_a\,q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{0} + \int_2 2\,\mathrm{Re}\Big\{\delta^{V\,\bar{\ell}_1\,\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{C_s}}$$
$$+ \iint\limits_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}} + \iint\limits_{2+\gamma} 2\,\mathrm{Re}\Big\{\delta^{V\,\bar{q}_a\,q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}}$$

Treatment of infrared (IR) singularities

composition of two NLO methods:

- **QCD**: dipole subtraction formalism
- **EW**: dipole subtraction formalism

Factorizable initial–final $\mathcal{O}(\alpha_s\alpha)$ corrections

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- **EW** corrections confined to decay sub-process

⇒ reducible structure*: (NLO **QCD**) × (NLO **EW**)

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$$\hat{\sigma}^{\text{QCD}\otimes\text{EW}}_{\text{fact. ini-fin}} = \int_3 2\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 4\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\;\mathrm{Re}\Big\{\delta^{V\bar{q}_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{0} + \int_2 2\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{C_s}}$$

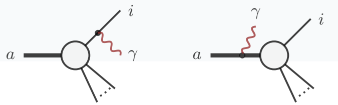
$$+ \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}} + \iint_{2+\gamma} 2\,\mathrm{Re}\Big\{\delta^{V\bar{q}_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}}$$

Treatment of infrared (IR) singularities

composition of two NLO methods:

- **QCD**: dipole subtraction formalism
- **EW**: dipole subtraction formalism ⇝

extension to cover decays:



Factorizable initial–final $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$ corrections

$$\hat{\sigma}^{\text{QCD}\otimes\text{EW}}_{\text{fact. ini-fin}} = \int_3 2\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 4\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\;\mathrm{Re}\Big\{\delta^{V\bar{q}_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{0} + \int_2 2\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{C_s}}$$
$$+ \iint\limits_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}} + \iint\limits_{2+\gamma} 2\,\mathrm{Re}\Big\{\delta^{V\bar{q}_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_{ew}}} + \iint\limits_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}}$$

Example: double-real corrections

$$\widetilde{\sigma}^{\mathrm{R_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}} = \iint\limits_{3+\gamma} \Bigg\{ \mathrm{d}\sigma^{\mathrm{R_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}} - \sum_{\substack{\text{QCD}\\\text{dipoles}}} \mathrm{d}\sigma^{\mathrm{R_{ew}}}_{\text{dec}} \otimes \mathrm{d}V_{\text{dip}} - \sum_{\substack{I,J\\I\neq J}} \mathrm{d}\sigma^{\mathrm{R_s}}_{\text{PA}} \otimes \mathrm{d}V^{\mathrm{ew}}_{\text{dip},IJ}$$
$$+ \sum_{\substack{\text{QCD}\\\text{dipoles}}} \sum_{\substack{I,J\\I\neq J}} \mathrm{d}\sigma^{0}_{\text{PA}} \otimes \mathrm{d}V_{\text{dip}} \otimes \mathrm{d}V^{\mathrm{ew}}_{\text{dip},IJ} \Bigg\}$$

- QCD subtraction term for $V+\gamma$
- EW subtraction term for $V+$jet
- compensate over-subtraction in double-unresolved limit

Factorizable initial–final $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$ corrections

$$\hat{\sigma}^{\text{QCD}\otimes\text{EW}}_{\text{fact. ini-fin}} = \int_3 2\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 4\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{Re}\Big\{\delta^{V\bar{q}_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{0} + \int_2 2\,\mathrm{Re}\Big\{\delta^{V\bar{\ell}_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{C_s}}$$
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$$+ \sum_{\substack{\text{QCD}\\ \text{dipoles}}} \sum_{\substack{I,J\\ I\neq J}} \mathrm{d}\sigma^{0}_{\text{PA}} \otimes \mathrm{d}V_{\text{dip}} \otimes \mathrm{d}V^{\text{ew}}_{\text{dip},\,IJ} \Bigg\}$$

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Factorizable initial–final $\mathcal{O}(\alpha_s\alpha)$ corrections

$$\hat\sigma^{\text{QCD}\otimes\text{EW}}_{\text{fact. ini-fin}} = \int_3 2\,\mathrm{Re}\Big\{\delta^{V\bar\ell_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_s}} + \int_2 4\,\mathrm{Re}\Big\{\delta^{V\bar\ell_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{Re}\Big\{\delta^{V\bar q_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{0} + \int_2 2\,\mathrm{Re}\Big\{\delta^{V\bar\ell_1\ell_2}_{\mathrm{V_{ew}}}\Big\}\,\mathrm{d}\sigma^{\mathrm{C_s}}$$
$$+ \iint_{3+\gamma} \mathrm{d}\sigma^{\mathrm{R_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}} + \iint_{2+\gamma} 2\,\mathrm{Re}\Big\{\delta^{V\bar q_a q_b}_{\mathrm{V_s}}\Big\}\,\mathrm{d}\sigma^{\mathrm{R_{ew}}} + \iint_{2+\gamma} \mathrm{d}\sigma^{\mathrm{C_s}\otimes\mathrm{R_{ew}}}_{\text{fact. ini-fin}}$$

Example: double-real corrections

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- compensate over-subtraction in double-unresolved limit

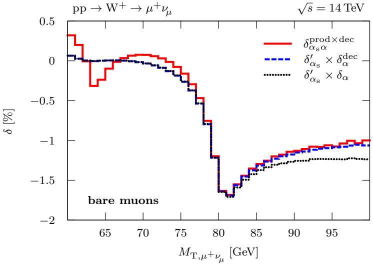
W^+ distributions @ NNLO $\mathcal{O}(\alpha_\mathrm{s}\alpha)$ (factorizable initial–final)

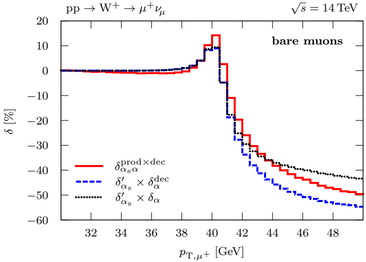
— best prediction: $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_\mathrm{s}\alpha}$

- - - naive product: (NLO QCD) × (NLO EW)$_{\mathrm{dec}}$

⋯ naive product: (NLO QCD) × (NLO EW)

} Δ ∼ missing $\mathcal{O}(\alpha_\mathrm{s}\alpha)$ terms





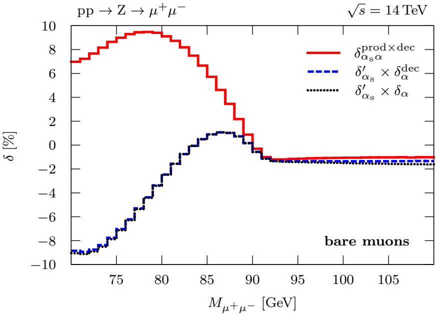
- $\delta^{\mathrm{prod}\times\mathrm{dec}}_{\alpha_\mathrm{s}\alpha}$ well approximated by naive products
- $M_{\mathrm{T},\ell\nu}$ insensitive to ISR effects
  $\hookrightarrow$ small differences in naive products

- enhancement from large QCD corrections
- sensitive to ISR
  $\hookrightarrow$ naive factorization fails

(naive sum $\rightsquigarrow$ 0)

Z distributions @ NNLO $\mathcal{O}(\alpha_s\alpha)$ (factorizable initial–final)

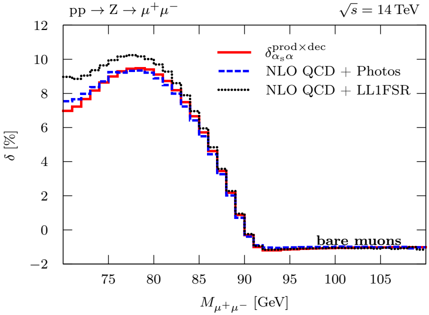
observable insensitive to ISR $\Rightarrow$ naive products work? $\rightsquigarrow$ **No!**



- $M_{\ell\ell}$ unaffected by ISR ↔ naive products practically identical
- naive products completely fail already a little away from the resonance
- large corrections below resonance
  $\hookrightarrow$ reconstructed $M_{\ell\ell}$ shifted to lower values by final-state radiation

Z distributions @ NNLO $\mathcal{O}(\alpha_s\alpha)$ (factorizable initial–final)

observable insensitive to ISR $\Rightarrow$ naive products work? $\rightsquigarrow$ **No!**



- **comparison with MC approaches:** (preliminary)

⋯⋯ “(NLO QCD) $\otimes$ LL1FSR”

structure function: $\Gamma_{\ell\ell}^{\mathrm{LL},1}(z) = \frac{\alpha}{2\pi}\left(\frac{1+z^2}{1-z}\right)_+\left[\ln\left(\frac{Q^2}{m_\ell^2}\right)-1\right]$

- - - “(NLO QCD) $\otimes$ Photos”

γ shower (restricted to at most one emission)

Impact on M_{W} extraction

- bin-by-bin χ^2 fit

$$\chi^2 = \sum_{i \in \text{bins}} \frac{\left[\sigma_i^{\text{data}}(M) - \sigma_i^{\text{template}}(M + \Delta M)\right]^2}{2\Delta\sigma_i^2}$$

- “templates”: LO with
  $M_{\mathrm{W}} = 80.085 \ldots 80.785\ \mathrm{GeV}$
- “pseudo-data”: HO with
  $M_{\mathrm{W}} = 80.385\ \mathrm{GeV}$
- distributions normalized in fit interval $M_{\mathrm{T},\ell\nu} \in [65, 90]\ \mathrm{GeV}$

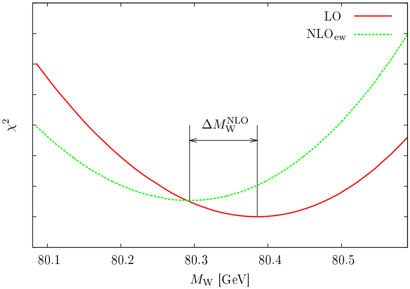
LO $\rightarrow$ NLO(EW) *

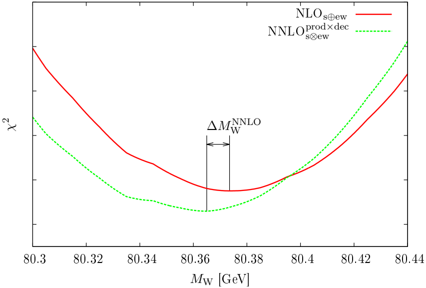
$\Delta M_{\mathrm{W}} \approx -90\ \mathrm{MeV}$

NLO(EW+QCD) $\rightarrow$ NNLO(EW$\times$QCD) *

$\Delta M_{\mathrm{W}} \approx -10\ \mathrm{MeV}$

* bare muons





1. Motivation and Introduction

2. Pole Approximation @ NNLO $\mathcal{O}(\alpha_{\mathrm{s}}\alpha)$

3. Conclusions and Outlook

Conclusions

Largest theoretical unknown in Drell–Yan processes: $\mathcal{O}(\alpha_s\alpha)$
Important around resonance $\rightsquigarrow$ Pole approximation

Pole approximation @ $\mathcal{O}(\alpha_s\alpha)$

- calculation of non-factorizable corrections $\rightsquigarrow$ negligible
  $\hookrightarrow$ only factorizable corrections are relevant at $\mathcal{O}(\alpha_s\alpha)$
- $\mathcal{O}(\alpha_s\alpha)$ corrections to $V \to \ell_1\bar{\ell}_2$ decay
  $\hookrightarrow$ only a small constant off-set $\rightsquigarrow$ irrelevant for resonance shape
- (QCD to $\bar{q}_a q_b \to V$) × (EW to $V \to \ell_1\bar{\ell}_2$)
  $\hookrightarrow$ expected to be the dominant contributions
  $\hookrightarrow$ mass shift $\approx 10\,\mathrm{MeV}$

todo: $\mathcal{O}(\alpha_s\alpha)$ corrections to $\bar{q}_a q_b \to V$ production
$\hookrightarrow$ no significant shape distortion expected

Conclusions

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Thank you

Backup Slides

Calculational setup

LHC @ 14 TeV

NNPDF2.3 (QED) [Ball, et al. ’13]

$$\mathrm{p}+\mathrm{p} \to \mathrm{W}^+ \to \nu_\mu + \mu^+$$

$$\mathrm{p}+\mathrm{p} \to \mathrm{Z}\,(\gamma^*) \to \mu^- + \mu^+$$

Event selection cuts

$$p_{\mathrm{T},\ell^\pm} > 25\ \mathrm{GeV}$$

$$|\eta_{\ell^\pm}| < 2.5$$

$$E_\mathrm{T}^\text{miss} > 25\ \mathrm{GeV}$$ (charged-current DY)

$$M_{\ell\ell} > 50\ \mathrm{GeV}$$ (neutral-current DY)

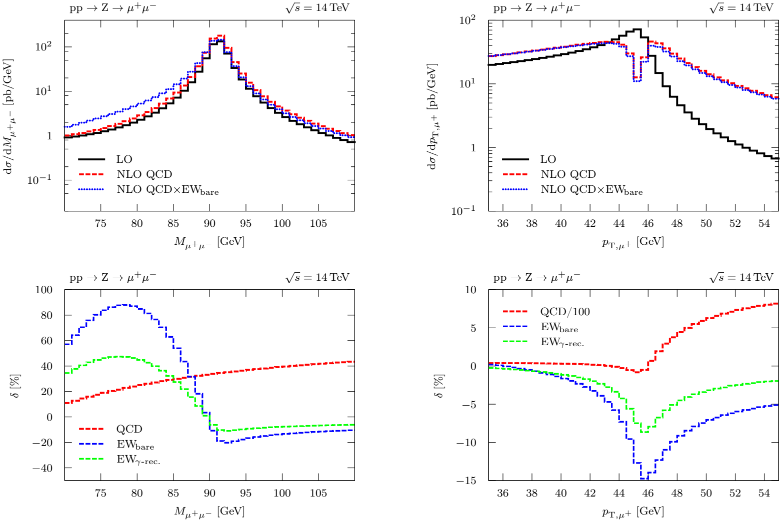
Photon recombination ⇝ “dressed” leptons

Merge photons “collinear” to the charged leptons:

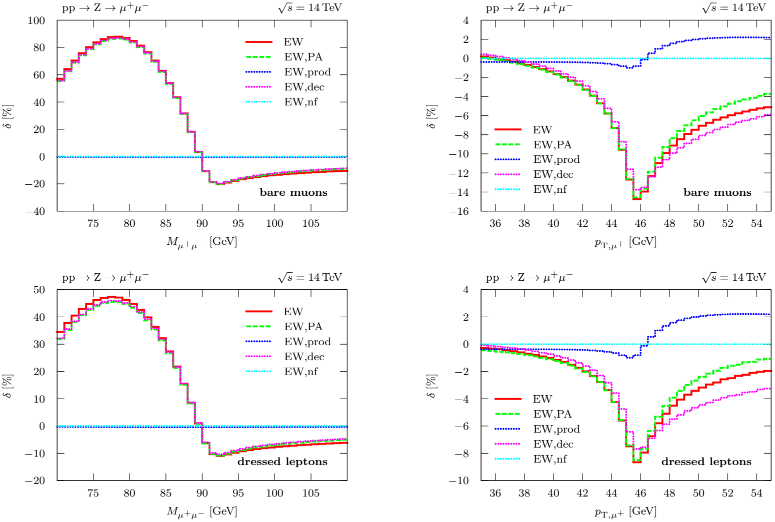
$$\Delta R_{\ell\gamma} < 0.1, \qquad R_{\ell\gamma}=\sqrt{(\eta_\ell-\eta_\gamma)^2+(\phi_\ell-\phi_\gamma)^2}$$

↪ corrections independent of the lepton flavour

Z distributions @ NLO & PA $\mathcal{O}(\alpha)$



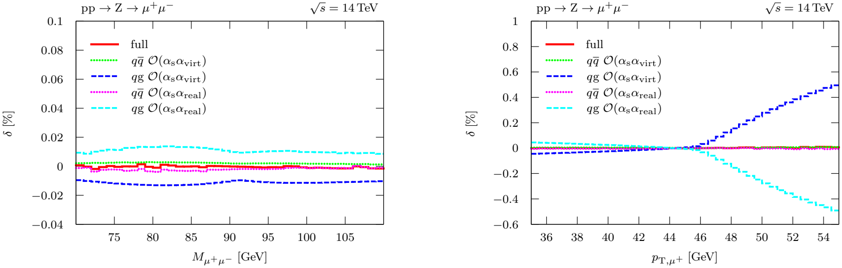
Z distributions @ NLO & PA $\mathcal{O}(\alpha)$



Z distributions @ NNLO $\mathcal{O}(\alpha_s\alpha)$ (non-factorizable)

⋯⋯ / – – – } $\boldsymbol{\mathcal{O}(\alpha_s\alpha_{\text{virt}})}$: including soft photon emission $E_\gamma < \Delta E$

⋯⋯ / – – – } $\boldsymbol{\mathcal{O}(\alpha_s\alpha_{\text{real}})}$: with cut $E_\gamma > \Delta E$ $(\Delta E \ll \Gamma_V)$



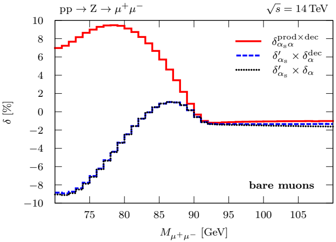
- almost perfect cancellation between different contributions
- tiny & flat corrections!

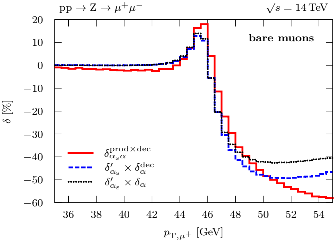
Z: even smaller: δ_{nf} anti-symmetric ⇝ suppression

⇒ dominant contributions at $\mathcal{O}(\alpha_s\alpha)$ from the factorizable corrections!

Initial–final factorizable corrections

bare muons





dressed leptons