Higgs boson production through gluon fusion at N^{3}LO

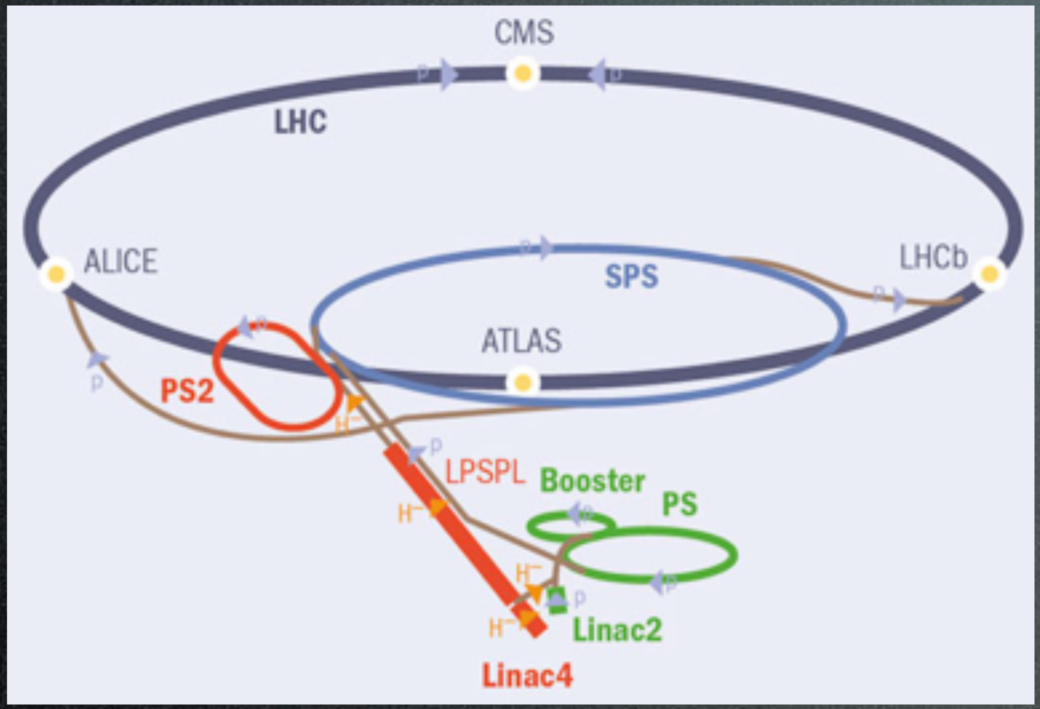
Elisabetta Furlan

Fermilab

Work in collaboration with
Babis Anastasiou, Claude Duhr, Falko Dulat, Franz Herzog, Thomas Gehrmann, Achilleas Lazopoulos, Bernhard Mistlberger

Motivation

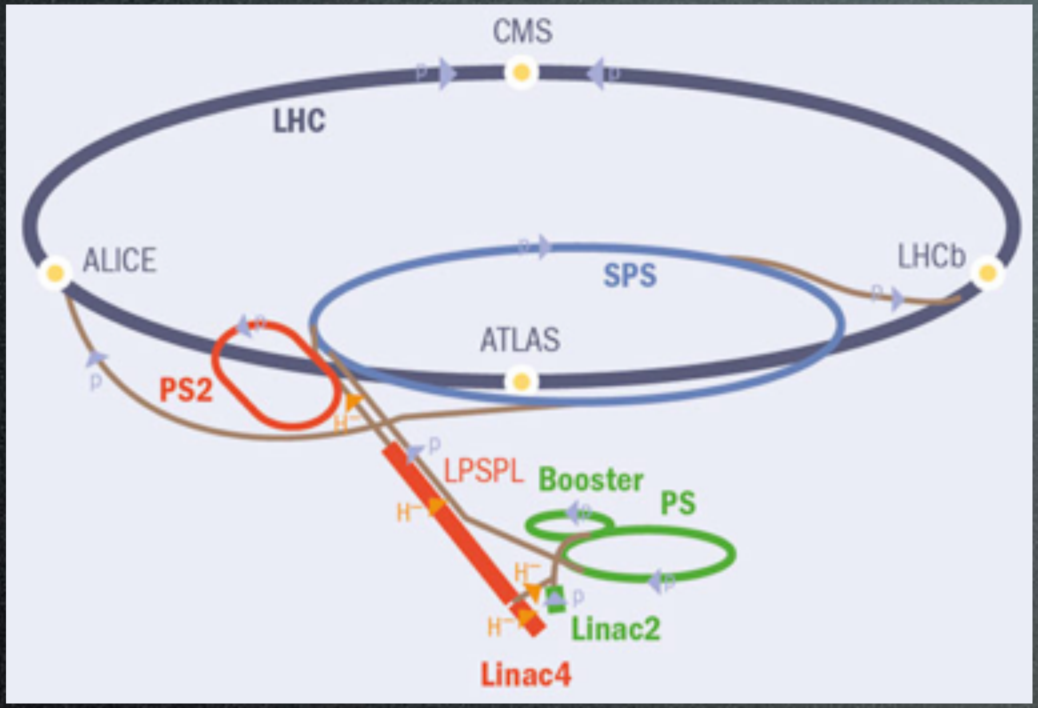
- The LHC is after a great *enterprise* ..



discover and *study* the Higgs boson

Motivation

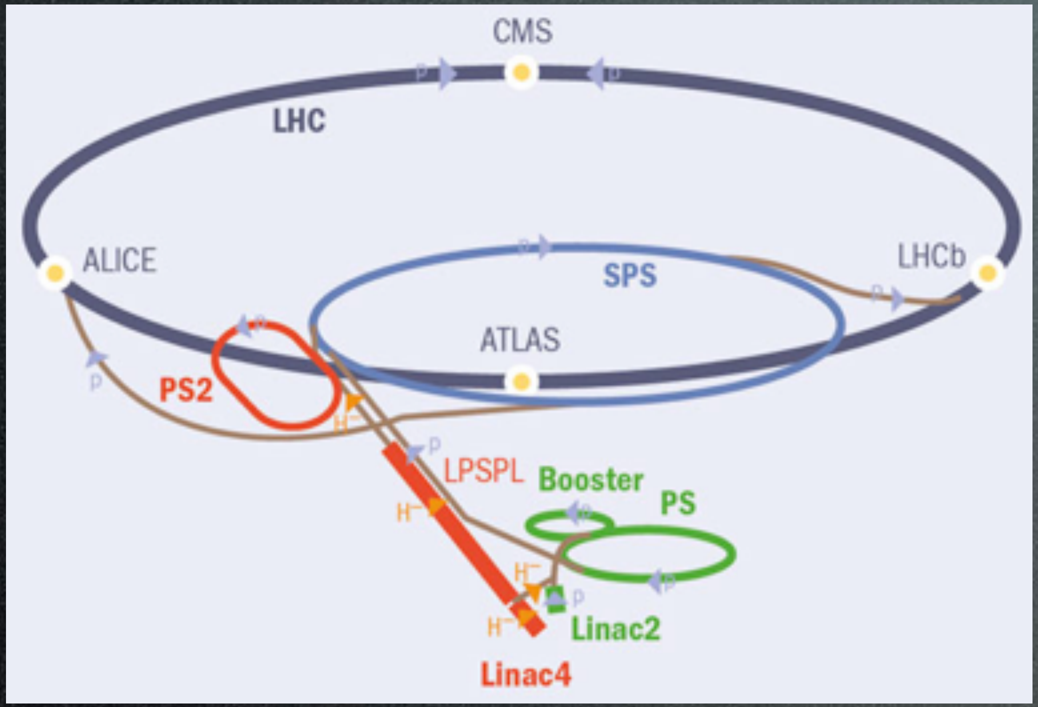
- The LHC is after a great *enterprise* ..



discover and *study* the Higgs boson

Motivation

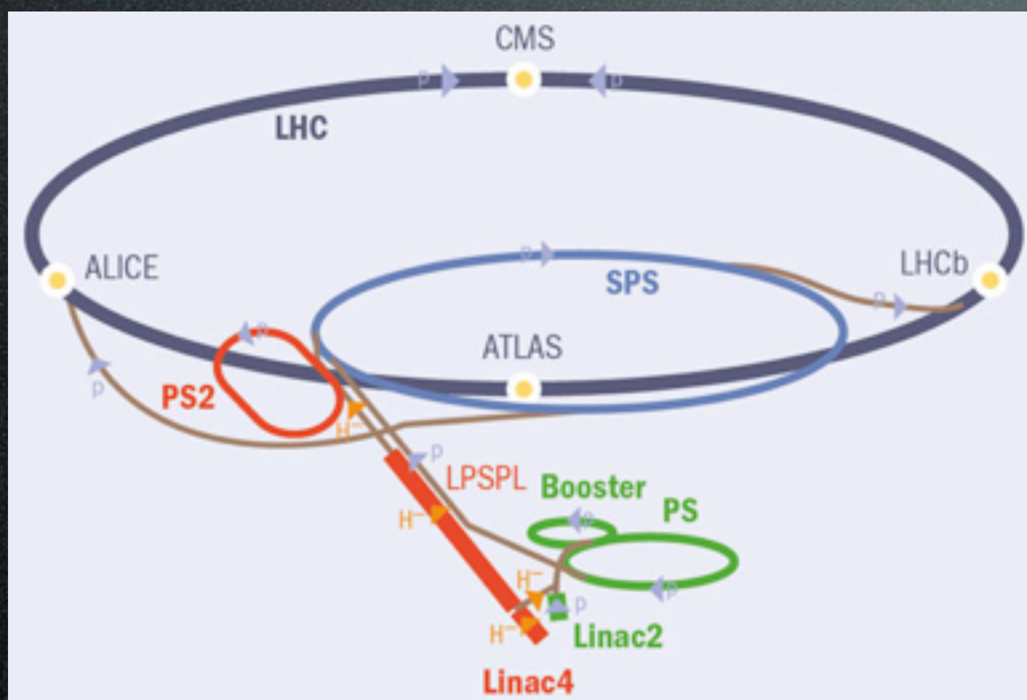
- The LHC is after a great *enterprise* ..

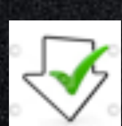
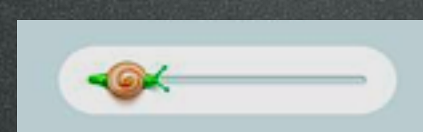


 *discover* and  *study* the Higgs boson

Motivation

- The LHC is after a great *enterprise* ..



 *discover* and  *study* the Higgs boson

validation of the Standard Model

probe of new physics

Motivation

- The Standard Model has so far been validated by a huge number of collider experiments; do we have the same kind of control in the Higgs sector?

- can we use the Higgs boson as a "window" on new physics?

$\longrightarrow$ deviations in Higgs couplings

Model	κ_V	κ_b	κ_γ
Singlet Mixing	$\sim 6\%$	$\sim 6\%$	$\sim 6\%$
2HDM	$\sim 1\%$	$\sim 10\%$	$\sim 1\%$
Decoupling MSSM	$\sim -0.0013\%$	$\sim 1.6\%$	$\sim -.4\%$
Composite	$\sim -3\%$	$\sim -(3-9)\%$	$\sim -9\%$
Top Partner	$\sim -2\%$	$\sim -2\%$	$\sim +1\%$

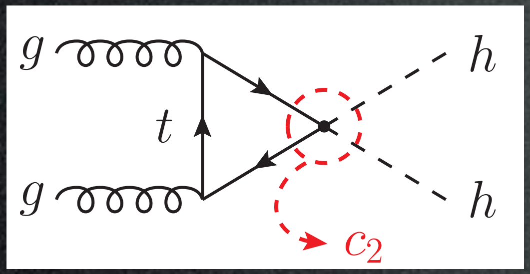
Dawson et al., Snowmass WG Report: Higgs Boson, arxiv:1310.8361

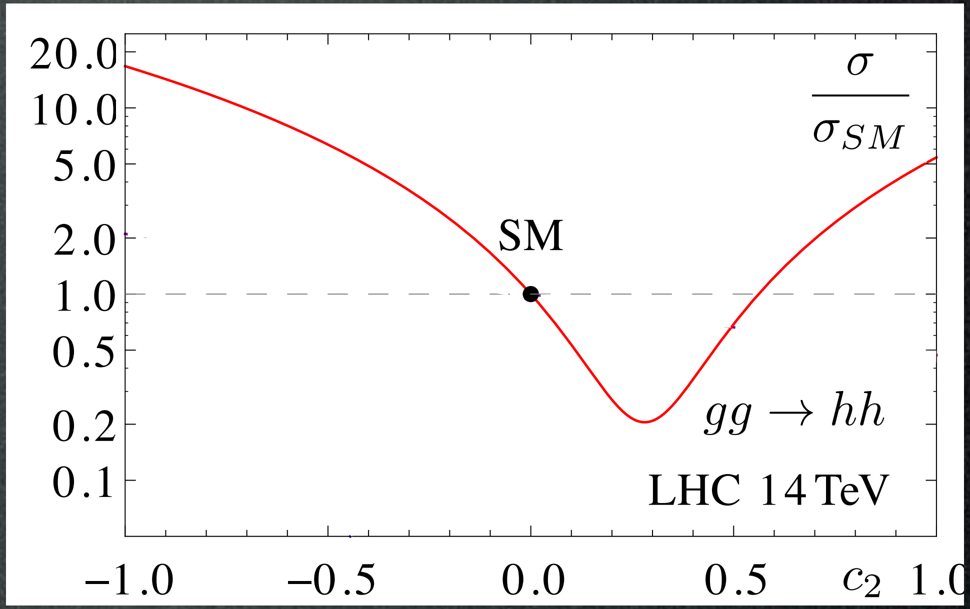
Motivation

- The Standard Model has so far been validated by a huge number of collider experiments; do we have the same kind of control in the Higgs sector?

- can we use the Higgs boson as a "window" on new physics?

  → new interactions → different phenomenology

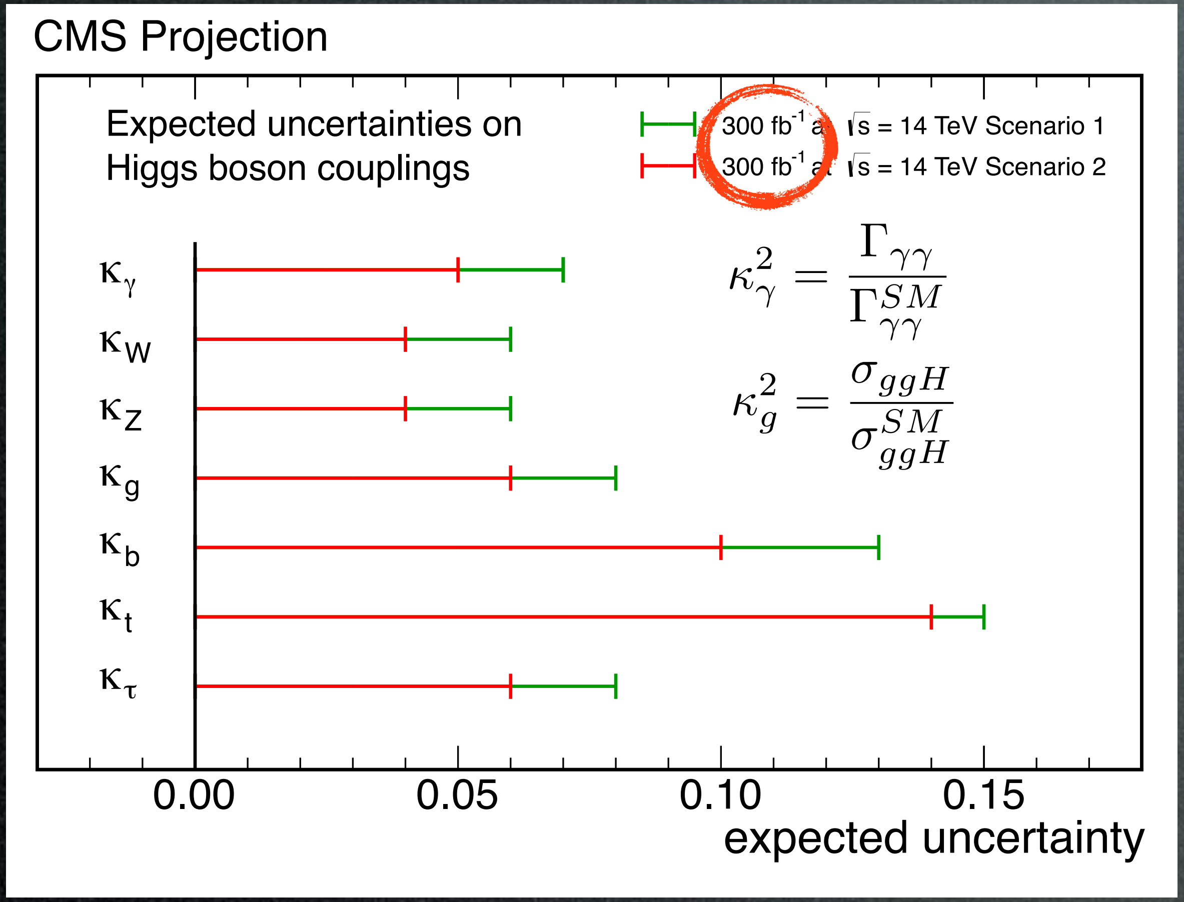


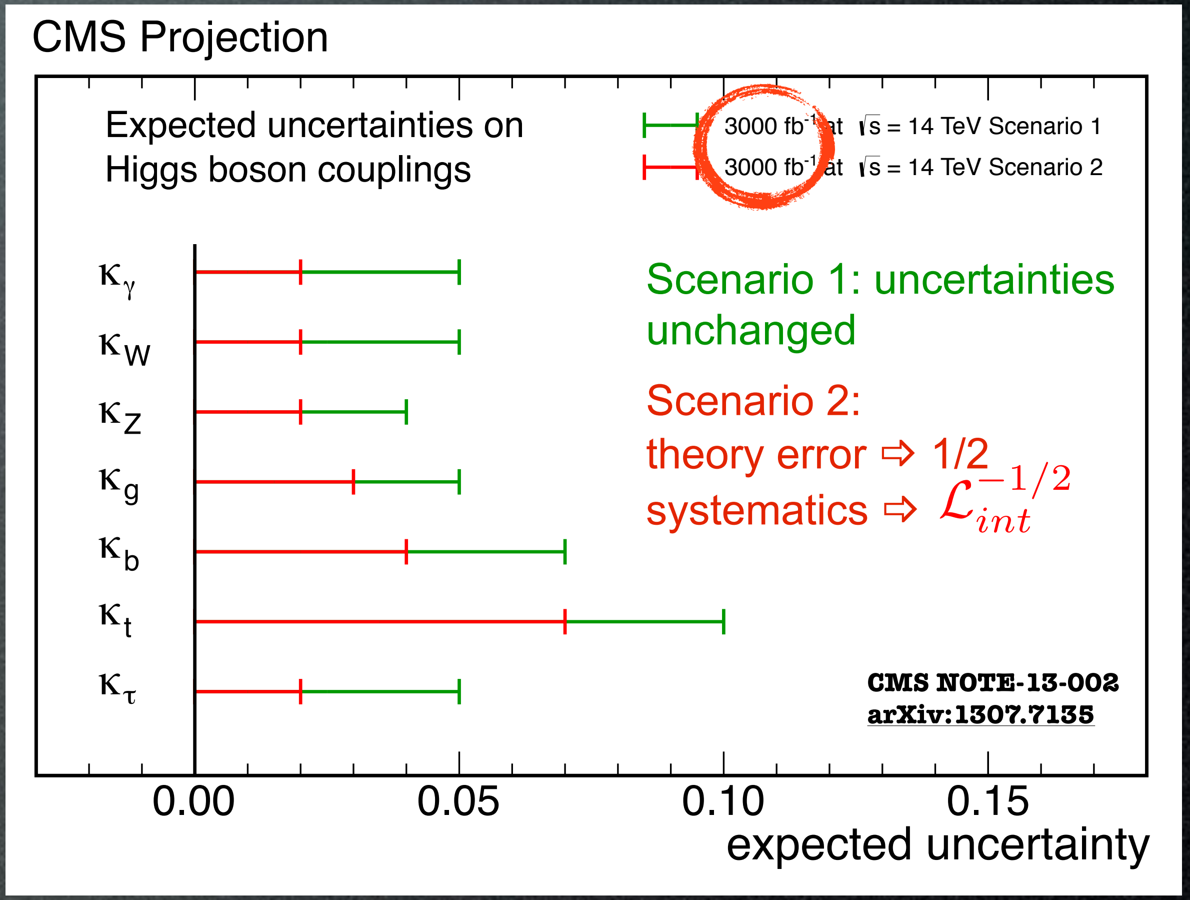


Contino et al., JHEP 1208, 154 (2012), arxiv:1205.5444

Precision Higgs physics

- How important is the theory error in these studies?





oct 21 2014
similar projections from atlas
http://cds.cern.ch/record/1956710/files/ATL-PHYS-PUB-2014-016.pdf,
ATL-PHYS-PUB-2014-016

Precision Higgs physics

- How important is the theory error in these studies?
  - ➡ the reduction of the theory error is crucial for improving the bounds on the Higgs couplings
  - ➡ gluon-fusion Higgs production is the one single process that drives the theory error

Precision Higgs physics

$\sqrt{s} = 13$ TeV	σ [pb]	$\delta\sigma^{theo}/\sigma$
ggH$^{\text{(NNLO)}}$	44	+7.4% -7.9%
VBF	3.7	+0.7% -0.7%
WH	1.4	+0.7% -1.5%
ZH	0.87	+3.8% -3.8%
⋮	⋮	⋮

LHC Higgs cross section WG recommendations, 2014

Precision Higgs physics

- How important is the theory error in these studies?
  - ➡ the reduction of the theory error is crucial for improving the bounds on the Higgs couplings
  - ➡ gluon-fusion Higgs production is the one single process that drives the theory error
  - ➡ the inclusion of the N^3LO corrections allows to match the experimental accuracy after Run 2

Precision Higgs physics

theory

	$\sigma^{13\,\mathrm{TeV}}[\mathrm{pb}]$	$\frac{\delta\sigma^{\mathrm{scale}}}{\sigma}$
LO	23.9	$\sim 16\%$
NLO	37.1	$\sim 14\%$
NNLO	43.6	$\sim\ 7\%$
$\mathrm{N^3LO}$ C. Anastasiou et al., PRL 114, 21 (2015)	44.3	$\sim\ 3\%$ (also, S. Buelher et al., JHEP 1310, 096 (2010))

$K_{NLO} \sim 1.6$

$K_{NNLO} \sim 1.9$

experiment

- current precision $\sim 30\%$
- end of Run 2 $\sim 10\%$

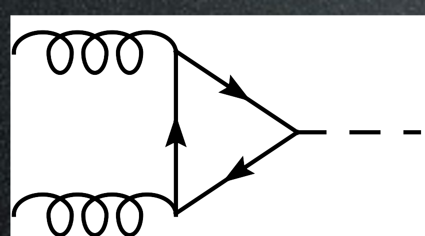
S. Dawson et al., *Higgs WG report of the Snowmass 2013 Community Planning Study*

Gluon fusion Higgs production

- Framework:

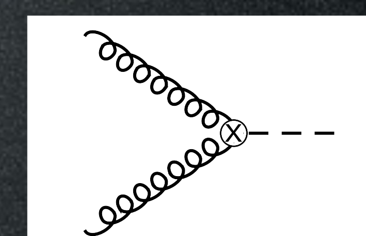
  (C. Duhr, Tuesday)

  - $m_t \gg \frac{m_H}{2} \Rightarrow$ heavy-quark effective theory



$$\mathcal{L}_{eff} = -\frac{\alpha_S}{4v}\, C_1\, H G^a_{\mu\nu} G^{a\mu\nu}$$

$C_1 \cdot$



  - threshold expansion around $z = \frac{m_H^2}{s} \simeq 1$

$$\hat{\sigma}(z) = \hat{\sigma}_{SV} + \hat{\sigma}^{(0)} + \sum_{n=1}^{N_{trunc}} \sigma^{(n)} (1-z)^n$$

Techniques

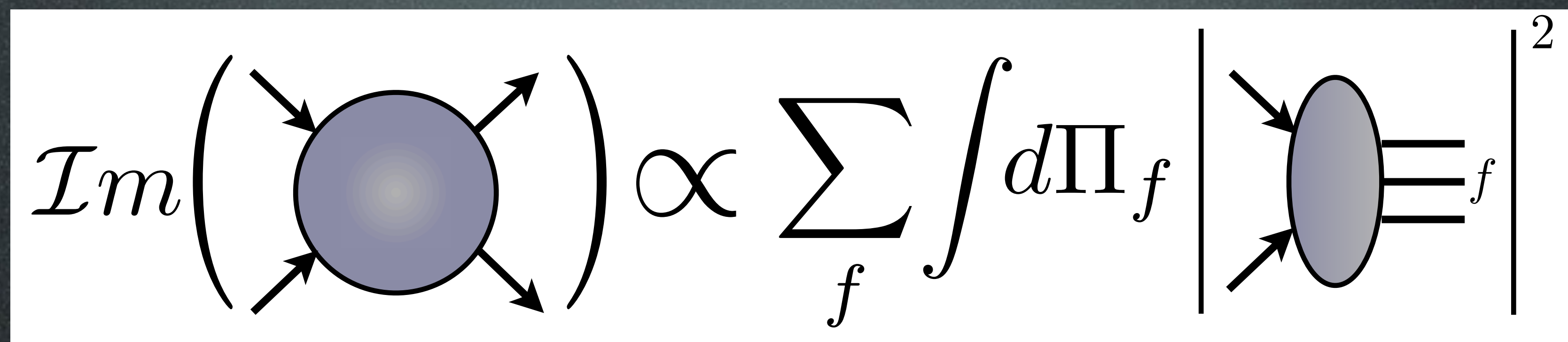
Reverse unitarity

- Allows to map phase space integrals into loop integrals with cut propagators

  ➡ apply "standard" techniques for the treatment of loop integrals also to phase space integrals

- How it works:
  By the optical theorem, "The imaginary part of the forward scattering amplitude is proportional to the total scattering cross section"

Reverse unitarity



unitarity methods

Bern, Dixon, Kosower, NPB 513, 3 (1998)
Britto, Cachazo, Feng, NPB 725, 275 (2005)
Ossola, Papadopoulos, Pittau, NPB 763, 147 (2007)

reverse unitarity

Anastasiou, Melnikov, NPB 646 (2002) 220;
Anastasiou, Dixon, Melnikov, Petriello, PRL 91 (2003) 182002

Reverse unitarity

➡ the on-shell condition on phase space integrals corresponds to cuts, i.e. discontinuities, of propagators

$$\delta_+(q^2) \to \left(\frac{1}{q^2}\right)_c = \frac{1}{2\pi i}\mathrm{Disc}\frac{1}{q^2}$$

Cutkosky, J. Math. Phys. 1, 429 (1960)

- Cut propagators can be differentiated with respect to their momenta as normal propagators, with the condition that

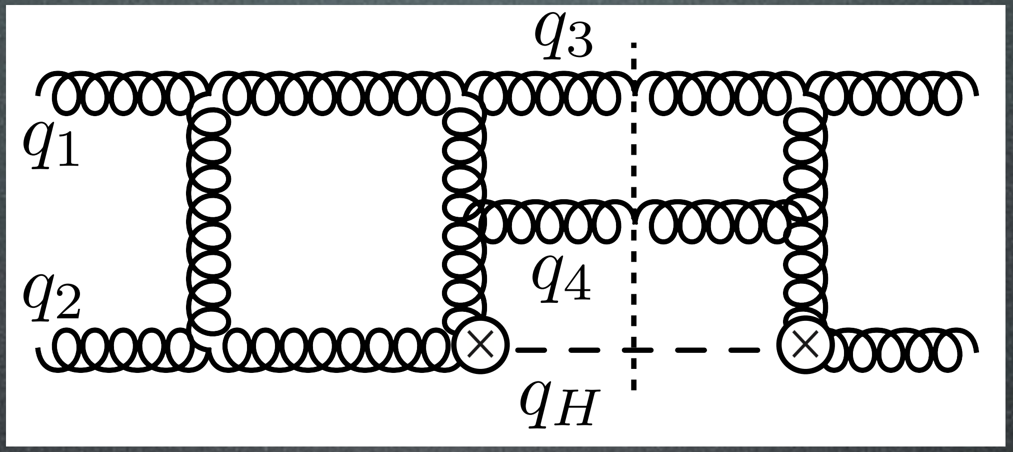
$$\left(\frac{1}{q^2}\right)_c^n (q^2)^m = \begin{cases} \left(\frac{1}{q^2}\right)_c^{n-m}, & \text{if } n > m \;, \\ 0\,, & \text{if } m \geq n \end{cases}$$

Reverse unitarity

- ➡ as regular propagators, cut propagators can also be Taylor-expanded in some small expansion parameter, or differentiated with respect to the loop momentum

Threshold expansion

- Example: double-real virtual contributions



- Aim: expand *amplitudes* and *phase-space measure* around $\bar{z} = 1 - z = 0$, i.e. for q_3, q_4 soft

➡ introduce the scaling properties

$$q_1, q_2 \to p_1, p_2$$

$$q_3, q_4 \to \bar{z}p_3, \bar{z}p_4$$

Phase-space measure

- How do we expand

$$\delta_+(p_H^2 - m_H^2) = \frac{1}{\bar{z}}\,\delta_+(s_{12} - 2\,p_{12}\cdot p_{34} + \bar{z}\,s_{34})$$

around $\bar{z} = 0$?

➡ using reverse unitarity!
   – map this on-shell condition into the corresponding cut propagator
   – Taylor expand it around $\bar{z} = 0$

Phase-space measure

➡ the on-shell condition for the Higgs boson assumes the form

$$\delta_+(s_{12} - 2\,p_{12} \cdot p_{34}) \left(\frac{1}{s_{12} - 2\,p_{12} \cdot p_{34}} \right)_c^n \bar{z}^n$$

Phase-space measure

➡ the on-shell condition for the Higgs boson assumes the form

$$\underbrace{\delta_+(s_{12} - 2\,p_{12} \cdot p_{34})} \left(\frac{1}{s_{12} - 2\,p_{12} \cdot p_{34}} \right)_c^n \bar{z}^n$$

enters the “soft” phase-space measure

$$d\Phi_3^S \sim \delta_+(s_{12} - 2p_{12} \cdot p_{34}) \prod_{i=3}^{4} \frac{d^D p_i}{(2\pi)^{D-1}} \delta_+(p_i^2)$$

Phase-space measure

➡ the on-shell condition for the Higgs boson assumes the form

$$\delta_+(s_{12} - 2\,p_{12}\cdot p_{34})\;\underbrace{\left(\frac{1}{s_{12} - 2\,p_{12}\cdot p_{34}}\right)^n_c}\;\bar{z}^n$$

powers of cut propagators

➡ can be reduced with the use of “integration by part” identities

Expansion by regions

- Integration over loop momenta and expansion in $\bar{z}$ do not commute

  ➡ we cannot perform a Taylor expansion of the integrand in $\bar{z}$ and then integrate over the loop momentum

- The correct approach is expand the integrand according to the scaling properties of the loop momentum in different regions and sum the contribution from all regions

Beneke, Smirnov, NPB 522 (1998) 321; Jantzen, A. V. Smirnov, V. A. Smirnov, Eur. Phys. J. C 72 (2012) 2139; Pak, Smirnov, Eur. Phys. J. C 71 (2011) 1626; Jantzen, JHEP 12 (2011) 076

Example

Smirnov, Beneke

- Consider the one - dimensional bubble

$$B = \int_0^\infty \mathrm{d}k \, \frac{1}{(k+m)(k+p)} \quad , \; p \ll m \, .$$

- Direct integration using partial fractioning yields

$$B = \frac{\log \frac{p}{m}}{p-m} \simeq -\frac{1}{m} \log \frac{p}{m} \left(1 + \frac{p}{m} + \frac{p^2}{m^2} + \ldots \right)$$

  - Can we obtain this expansion using the hierarchy among the physical scales involved?

    Not trivial: p acts as IR regulator, so *B* diverges for $p \to 0$!

Example

Smirnov, Beneke

- Introduce a regulator

$$B(\epsilon) = \int_0^\infty \mathrm{d}k \, \frac{k^{-\epsilon}}{(k+m)(k+p)}$$

- The loop momentum can be either of the order of p (i.e., soft) or m (i.e., hard)
- Expand in the corresponding small parameter in the two regions, integrate *over the entire range of the loop momentum*, and sum the two contributions

Example

Smirnov, Beneke

- hard region ($k \sim m \gg p$)

$$B(\epsilon)_h \simeq \int_0^\infty \mathrm{d}k \, \frac{k^{-\epsilon}}{(k+m)k} \left(1 - \frac{p}{k} + \frac{p^2}{k^2} + \ldots\right)$$

$$= -\frac{1}{m}\left(\frac{1}{\epsilon} - \log m\right)\left(1 + \frac{p}{m} + \frac{p^2}{m^2} + \ldots\right)$$

- soft region ($k \sim p \ll m$)

$$B(\epsilon)_s \simeq \frac{1}{m}\int_0^\infty \mathrm{d}k \, \frac{k^{-\epsilon}}{k+p} \left(1 - \frac{k}{m} + \frac{k^2}{m^2} + \ldots\right)$$

$$= \frac{1}{m}\left(\frac{1}{\epsilon} - \log p\right)\left(1 + \frac{p}{m} + \frac{p^2}{m^2} + \ldots\right)$$

Example

Smirnov, Beneke

- The poles cancel between the two region and

$$B(\epsilon)_h + B(\epsilon)_s = B(\epsilon)$$

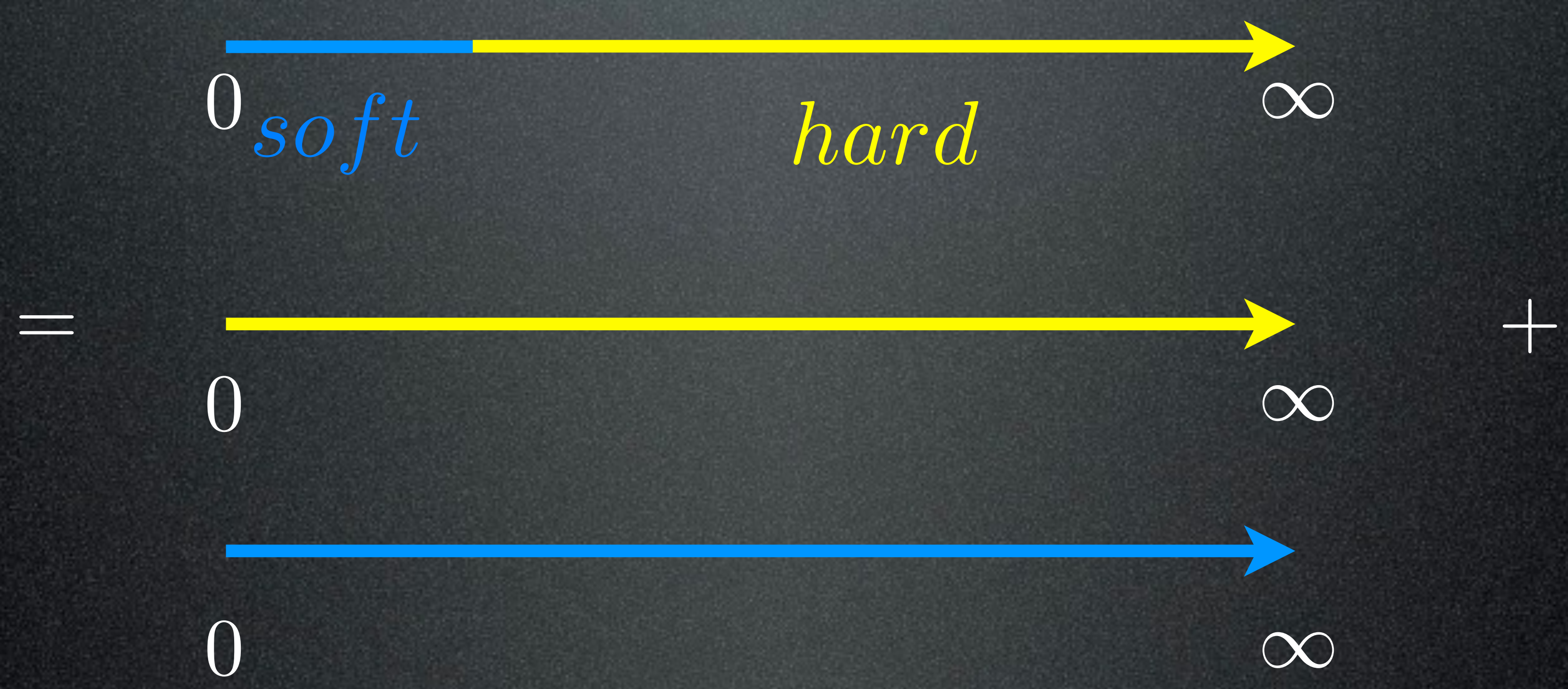
Example

Smirnov, Beneke

- The poles cancel between the two region and

$$B(\epsilon)_h + B(\epsilon)_s = B(\epsilon)$$

- Schematically,



where the overlaps cancel

Jantzen, JHEP 12 (2011) 076

Expansion by regions

- In our case, the loop momentum can be
  - ➡ hard, $k^2 \sim s$
  - ➡ collinear to p_1 , $k.p_1 \sim s$, $k.p_2 \sim \bar{z}s$, $k_\perp^2 \sim \bar{z}s$
  - ➡ collinear to p_2 , $k.p_1 \sim \bar{z}s$, $k.p_2 \sim s$, $k_\perp^2 \sim \bar{z}s$
  - ➡ soft, $k^2 \sim s\bar{z}^2$ (B. Mistlberger, Wednesday)
- In each region, expand the integrand according to these scaling properties

Integration by part identities

- They allow to write relations among the integrals that we need to compute

Tkachov, PLB100, 65 (1981); Chetyrkin, Tkachov, NPB192, 159 (1981); Gehrmann, Remiddi, NPB 580 (2000) 485

- The large system of equation generated can be solved in an automated way implementing the Laporta algorithm

Laporta, Int. J. Mod. Phys. A 15, 5087 (2000)

➡ in house software

➡ eg: for threshold Higgs production, reduce to the calculation of ~50 master integrals

Summary

- Gluon-fusion Higgs production can be studied in an heavy quark effective theory
  - ➡ integrate out the top quark and only work in massless QCD
- The Higgs is almost at threshold
  - ➡ to tackle the N^3LO, we can start from a threshold expansion
  - ➡ but we need a few terms in this expansion to get a reliable prediction

Summary

- We use reverse unitarity to map phase space integrals into (cut) loop integrals

  ➡ can apply IBP identities and reduce to masters

- Loop amplitudes are expanded around the Higgs threshold with the strategy of regions

  ➡ four regions: soft, collinear (1 & 2) and hard

- Alternatively, master integrals have been computed using the method of differential equations (⟶ Bernhard's talk yesterday)