The NNLO subtraction scheme STRIPPER: implementation and application

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collaboration with P. Fiedler, A. van Hameren, D. Heymes, A. Mitov

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The time of NNLO calculations

Antenna subtraction Gehrmann-De Ridder, Gerhmann, Glover ’05

First applied to e^+e^- -> 3jets Gerhmann-De Ridder, Gehrmann, Glover, Heinrich ’07

Subsequent successes:

- Dijet production
  - Leading color, only gluons Gehrmann-De Ridder, Gehrmann, Glover, Pires ’13
  - Full color, only gluons Currie, Gehrmann-De Ridder, Gerhmann, Glover, Pires ’13
  - Quark-anti-quark at leading color Currie, Glover, Wells ’13
- Higgs + jet (gluons) Chen, Gehrmann, Glover, Jacquier ’14
- Top-pair production (only quark scattering with approximations)
  Abelof, Gerhmann-De Ridder ‘14

The time of NNLO calculations

q_T subtraction Catani, Grazzini '07

Originally introduced for Higgs boson production
Based on transverse momentum resummation

Recentlly computed processes:

- Wγ, Zγ Grazzini, Kallweit, Rathlev '15
- W^+W^- Gehrmann, Grazzini, Kallweit, Maierhoeffer,von Manteuffel,Pozzorini, Rathlev, Tancredi '14
- ZH Ferrara, Grazzini, Tramontano '14
- ZZ Cascioli, Gehrmann, Grazzini, Kallweit, Maierhoeffer,von Manteuffel,Pozzorini, Rathlev, Tancredi, Weihs '14
- Zγ Grazzini, Kallweit, Rathlev, Torre '13
- γγ Catani, Cieri, de Florian, Ferrara, Grazzini '11
- WH Ferrara, Grazzini, Tramontano '11 '13

The time of NNLO calculations

"Colorful subtraction" del Duca, Somogyi, Trocsanyi '05

Applied to Higgs boson decay to b-quarks
Del Duca, Duhr, Somogyi, Tramontano, Trocsanyi '15

N-jettiness subtraction

Introduced in Boughezal, Focke, Liu, Petriello '15
Also discussed in Gaunt, Stahlhofen, Tackmann, Walsh '15

Applied to W + jet and Higgs + jet Boughezal, Focke, Giele, Liu, Petriello '15

The time of NNLO calculations

SecToR ImProved Phase spacE for real Radiation (STRIPPER)

Also known as Sector Improved Residue Subtraction Scheme

Invented: MC ’10 First applied: MC ’11 (top-quarks)

Subsequently applied by others to several non-trivial problems:

- Z -> e^+e^- (as a warmup) Boughezal, Melnikov, Petriello ’11
- top quark decay Brucherseifer, Caola, Melnikov ’13
- b -> X_ueν Brucherseifer, Caola, Melnikov ‘13
- Higgs + jet (gluons only) Boughezal, Caola, Melnikov, Petriello, Schulze ’13
- Muon decay spin asymmetry Caola, Czarnecki, Liang, Melnikov, Szafron ’14
- Single top-quark production Brucherseifer, Caola, Melnikov ’14
- Higgs + jet (full) Boughezal, Caola, Melnikov, Petriello, Schulze ‘15

And by the inventor:

- Total cross sections for top pair production Baernreuther, MC, Fiedler, Mitov 12’, 13’
- Differential top pair production MC, Fiedler, Mitov ’14

Four-dimensional formulation: MC, D. Heymes ’14

Inspiration

I. Frixione, Kunszt, Signer ‘96 (FKS)

BUT:

1) No analytic integration over subtraction terms
2) Solution to overlapping singularities not present at NLO

II. Binoth, Heinrich ‘00 (Sector decomposition)
Anastasiou, Melnikov, Petriello ‘04 (in the context of phase spaces)

BUT:

1) Process independent
2) Decomposition driven by physical singularities

Outline of the scheme

I. D-dimensional formulation

1) Phase space decomposition
2) Phase space parameterization
3) Generation of subtraction and integrated subtraction terms

II. 4-dimensional formulation

1) Average over azimuthal angles in integrated contributions
2) Separation of finite contributions
3) ‘t Hooft-Veltman regularization of separately finite contributions

III. Result: local 4-dimensional subtraction scheme exploiting all information on factorization at amplitude level

Phase space decomposition

Goal: split the phase space into sectors with a controllable number of singularities

NLO

$$\sum_{ik} \mathcal{S}_{i,k} = 1 \qquad \mathcal{S}_{i,k} = \frac{1}{D_1\, d_{i,k}}, \qquad D_1 = \sum_{ik} \frac{1}{d_{i,k}} \qquad d_{i,k} = \left(\frac{E_i}{\sqrt{\hat{s}}}\right)^{\alpha} (1-\cos\theta_{ik})^{\beta}$$

Patterns:

(g,g), (g,q), $(g,\bar{q})$, (q,g), $(\bar{q},g)$, (q,q), $(\bar{q},\bar{q})$ — Initial state reference

(g,g), (g,q), $(g,\bar{q})$, $(q,\bar{q})$ — final state reference

NNLO

$$\sum_{ij}\left[\sum_k \mathcal{S}_{ij,k} + \sum_{kl} \mathcal{S}_{i,k;j,l}\right] = 1$$

$$\mathcal{S}_{ij,k} = \frac{1}{D_2\, d_{ij,k}}, \qquad \mathcal{S}_{i,k;j,l} = \frac{1}{D_2\, d_{i,k} d_{j,l}}$$

$$D_2 = \sum_{ii}\left[\sum_k \frac{1}{d_{ij,k}} + \sum_{kl} \frac{1}{d_{i,k} d_{j,l}}\right].$$

$$d_{ij,k} = \left(\frac{E_i}{\sqrt{\hat{s}}}\right)^{\alpha_i} \left(\frac{E_j}{\sqrt{\hat{s}}}\right)^{\alpha_j} \left[(1-\cos\theta_{ij})(1-\cos\theta_{ik})(1-\cos\theta_{jk})\right]^{\beta}$$

(g,g,g), (g,g,q), $(g,g,\bar{q})$, (g,q,g), $(g,\bar{q},g)$, (g,q,q), $(g,\bar{q},\bar{q})$, $(q,\bar{q},g)$, $(q,\bar{q},q')$, (q',q,q), $(q',\bar{q},\bar{q})$. — Initial state reference

(g,g,g), (g,g,q), $(g,g,\bar{q})$, $(g,q,\bar{q})$, $(q,\bar{q},g)$, $(q,\bar{q},q')$ — final state reference

Phase space parameterization

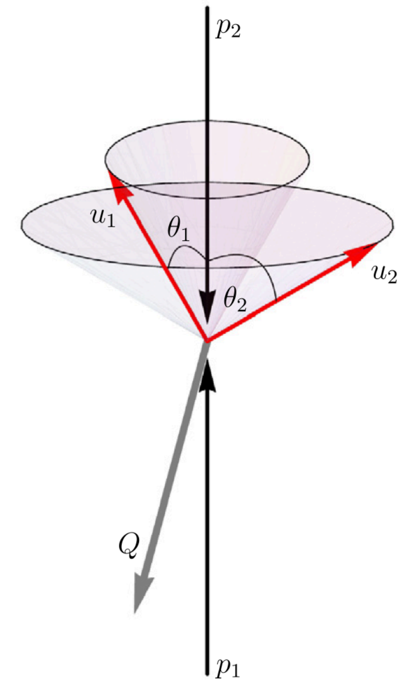
Example: triple-collinear sector parameterization

$$r^\mu = r^0 \hat{r}^\mu = r^0 \begin{pmatrix} 1 \\ \hat{\boldsymbol{r}} \end{pmatrix}, \qquad u_1^\mu = u_1^0 \hat{u}_1^\mu = u_1^0 \begin{pmatrix} 1 \\ \hat{\boldsymbol{u}}_1 \end{pmatrix}, \qquad u_2^\mu = u_2^0 \hat{u}_2^\mu = u_2^0 \begin{pmatrix} 1 \\ \hat{\boldsymbol{u}}_2 \end{pmatrix}$$

$$\begin{aligned}
\hat{\boldsymbol{r}} &= \hat{\boldsymbol{n}}^{(3-2\epsilon)}(\alpha_1, \alpha_2, \ldots), \\
\hat{\boldsymbol{u}}_1 &= \boldsymbol{R}_1^{(3-2\epsilon)}(\alpha_1, \alpha_2, \ldots)\hat{\boldsymbol{n}}^{(3-2\epsilon)}(\theta_1, \phi_1, \rho_1, \rho_2, \ldots), \\
\hat{\boldsymbol{u}}_2 &= \boldsymbol{R}_1^{(3-2\epsilon)}(\alpha_1, \alpha_2, \ldots)\boldsymbol{R}_2^{(3-2\epsilon)}(\phi_1, \rho_1, \rho_2, \ldots)\hat{\boldsymbol{n}}^{(3-2\epsilon)}(\theta_2, \phi_2, \sigma_1, \sigma_2, \ldots)
\end{aligned}$$

$$\int \mathrm{d}\boldsymbol{\Phi}_{n+2} = \int \mathrm{d}\boldsymbol{\Phi}_{\text{unresolved}} \int \mathrm{d}\boldsymbol{\Phi}_n(p_1 + p_2 - u_1 - u_2)$$

$$\int \mathrm{d}\boldsymbol{\Phi}_{\text{unresolved}} = \int \mathrm{d}\boldsymbol{\Phi}_{\text{unresolved}}\left(\theta\left(u_1^0 - u_2^0\right) + \theta\left(u_2^0 - u_1^0\right)\right)$$



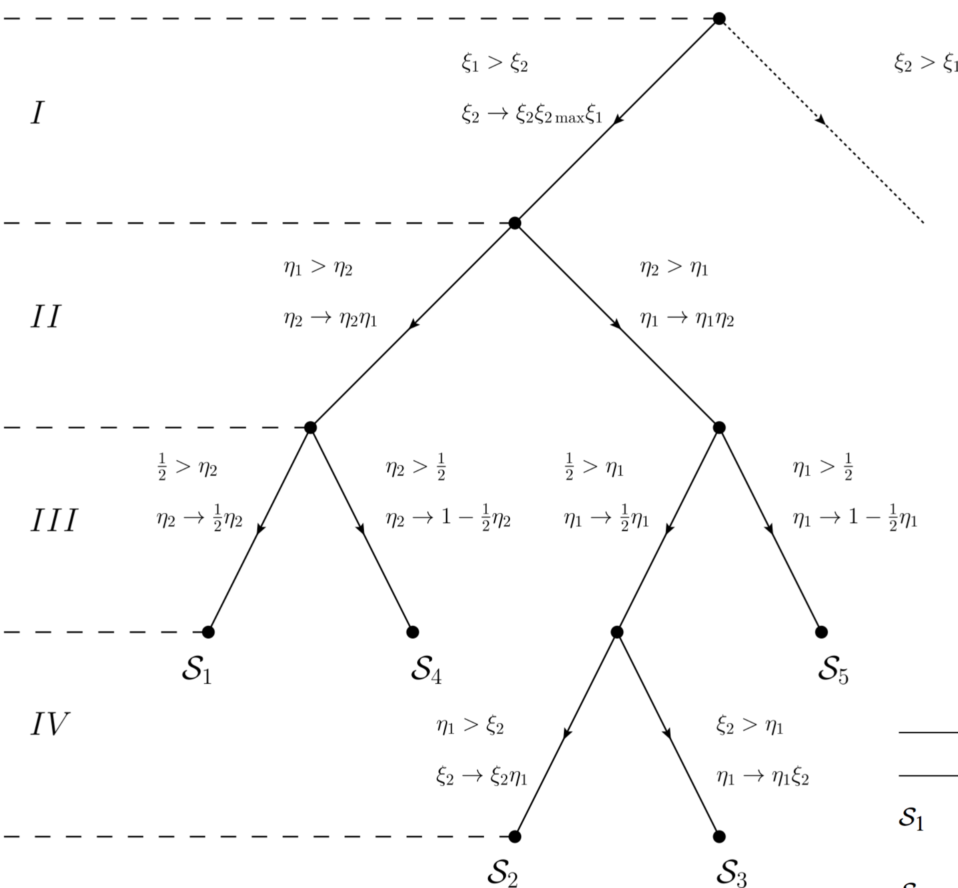
$$u_1^0 = E_{\max}\hat{\xi}_1, \qquad u_2^0 = E_{\max}\hat{\xi}_2,$$

$$\cos\theta_1 = 1 - 2\hat{\eta}_1, \qquad \cos\theta_2 = 1 - 2\hat{\eta}_2, \qquad \cos\phi_2 = \frac{1 - 2\eta_3 - (1 - 2\hat{\eta}_1)(1 - 2\hat{\eta}_2)}{4\sqrt{(1 - \hat{\eta}_1)\hat{\eta}_1(1 - \hat{\eta}_2)\hat{\eta}_2}}$$

$$\eta_3 = \frac{\hat{u}_1 \cdot \hat{u}_2}{2} = \frac{1 - \cos\theta_{12}}{2} = \frac{(\hat{\eta}_1 - \hat{\eta}_2)^2}{\hat{\eta}_1 + \hat{\eta}_2 - 2\hat{\eta}_1\hat{\eta}_2 - 2(1 - 2\zeta)\sqrt{\hat{\eta}_1(1 - \hat{\eta}_1)\hat{\eta}_2(1 - \hat{\eta}_2)}}.$$

Phase space parameterization

Triple unresolved parameterization



$$\int \mathrm{d}\boldsymbol{\Phi}_{\text{unresolved}}\, \theta\left(u_1^0 - u_2^0\right)$$

$$= \frac{E_{\max}^4}{(2\pi)^6} \left(\frac{\pi \mu_R^2 e^{\gamma_{\mathrm{E}}}}{8 E_{\max}^2}\right)^{2\epsilon} \int\limits_{\mathcal{S}_1^{1-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\phi_1, \rho_1, \ldots) \int\limits_{\mathcal{S}_1^{-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\sigma_1, \sigma_2, \ldots)$$

$$\times \int_0^1 \mathrm{d}\zeta \left(\zeta(1-\zeta)\right)^{-\frac{1}{2}-\epsilon} \iiiint_0^1 \mathrm{d}\eta_1 \mathrm{d}\eta_2 \mathrm{d}\xi_1 \mathrm{d}\xi_2 \sum_{i=1}^{5} \mu_{\mathcal{S}_i},$$

	$\mu_{\mathcal{S}_i}$
$\mathcal{S}_1$	$\eta_1^{1-2\epsilon} \eta_2^{-\epsilon} \xi_1^{3-4\epsilon} \xi_2^{1-2\epsilon} \left((1-\eta_1)(2-\eta_1\eta_2)\right)^{-\epsilon} \left(\frac{\eta_{31}(\eta_1,\eta_2)}{2-\eta_2}\right)^{1-2\epsilon} \xi_{2\,\max}^{2-2\epsilon}$
$\mathcal{S}_2$	$\eta_1^{2-3\epsilon} \eta_2^{1-2\epsilon} \xi_1^{3-4\epsilon} \xi_2^{1-2\epsilon} \left((1-\eta_2)(2-\eta_1\eta_2)\right)^{-\epsilon} \left(\frac{\eta_{31}(\eta_2,\eta_1)}{2-\eta_1}\right)^{1-2\epsilon} \xi_{2\,\max}^{2-2\epsilon}$
$\mathcal{S}_3$	$\eta_1^{-\epsilon} \eta_2^{1-2\epsilon} \xi_1^{3-4\epsilon} \xi_2^{2-3\epsilon} \left((1-\eta_2)(2-\eta_1\eta_2\xi_2)\right)^{-\epsilon} \left(\frac{\eta_{31}(\eta_2,\eta_1\xi_2)}{2-\eta_1\xi_2}\right)^{1-2\epsilon} \xi_{2\,\max}^{2-2\epsilon}$
$\mathcal{S}_4$	$\eta_1^{1-2\epsilon} \eta_2^{1-2\epsilon} \xi_1^{3-4\epsilon} \xi_2^{1-2\epsilon} \left((1-\eta_1)(2-\eta_2)(2-\eta_1(2-\eta_2))\right)^{-\epsilon} \eta_{32}^{1-2\epsilon}(\eta_1,\eta_2)\, \xi_{2\,\max}^{2-2\epsilon}$
$\mathcal{S}_5$	$\eta_1^{1-2\epsilon} \eta_2^{1-2\epsilon} \xi_1^{3-4\epsilon} \xi_2^{1-2\epsilon} \left((1-\eta_2)(2-\eta_1)(2-\eta_2(2-\eta_1))\right)^{-\epsilon} \eta_{32}^{1-2\epsilon}(\eta_2,\eta_1)\, \xi_{2\,\max}^{2-2\epsilon}$

Generation of subtraction terms

Example: real radiation at NLO

Behavior of the phase space at the boundaries: $\iint_0^1 \mathrm{d}\eta\, \mathrm{d}\xi\, \eta^{-\epsilon}\, \xi^{1-2\epsilon}$

Worst case behavior of the matrix element: $\frac{1}{\eta}\frac{1}{\xi^2}$

Write cross section as:

$$\hat{\sigma}^{\mathrm{R}} = \sum_{ik} \iint_0^1 \frac{\mathrm{d}\eta}{\eta^{1+\epsilon}} \frac{\mathrm{d}\xi}{\xi^{1+2\epsilon}} f_{i,k}(\eta,\xi)$$

$$f_{i,k}(\eta,\xi) = \frac{E_{\max}^2}{16\pi^3 \hat{s} N_{ab}} \left(\frac{\pi \mu_R^2 e^{\gamma_E}}{4E_{\max}^2(1-\eta)}\right)^{\epsilon} \int_{\mathcal{S}_1^{1-2\epsilon}} \mathrm{d}\boldsymbol{\Omega}(\phi, \rho_1, \ldots)$$

$$\times \int \mathrm{d}\boldsymbol{\Phi}_n(p_1+p_2-u)\, \mathcal{S}_{i,k}\left[\eta\,\xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle\right] \mathrm{F}_{n+1}$$

Apply formula:

$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon}\delta(x) + \left[\frac{1}{x^{1+a\epsilon}}\right]_+ \qquad \int_0^1 \mathrm{d}x \left[\frac{1}{x^{1+a\epsilon}}\right]_+ f(x) = \int_0^1 \mathrm{d}x\, \frac{f(x)-f(0)}{x^{1+a\epsilon}}$$

Calculate limits:

$$\lim_{\eta\to 0}\left[\eta\,\xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle\right], \qquad \lim_{\xi\to 0}\left[\eta\,\xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle\right],$$

$$\lim_{\eta\to 0}\lim_{\xi\to 0}\left[\eta\,\xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle\right].$$

Separation of finite contributions

At NLO (just the real and virtual corrections)

$$\hat{\sigma}^{\mathrm{R}} = \hat{\sigma}^{\mathrm{R}}_{\mathrm{F}} + \hat{\sigma}^{\mathrm{R}}_{\mathrm{U}}, \qquad \hat{\sigma}^{\mathrm{V}} = \hat{\sigma}^{\mathrm{V}}_{\mathrm{F}} + \hat{\sigma}^{\mathrm{V}}_{\mathrm{U}}$$

$$\hat{\sigma}^{\mathrm{R}}_{\mathrm{F}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_{n+1}\left[\langle\mathcal{M}^{(0)}_{n+1}|\mathcal{M}^{(0)}_{n+1}\rangle \mathrm{F}_{n+1} + \text{subtraction terms}\right]$$

$$\hat{\sigma}^{\mathrm{V}}_{\mathrm{F}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_{n}\, 2\,\mathrm{Re}\langle\mathcal{M}^{(0)}_{n}|\mathcal{F}^{(1)}_{n}\rangle \mathrm{F}_{n},$$

$$\hat{\sigma}^{\mathrm{V}}_{\mathrm{U}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_{n}\, 2\,\mathrm{Re}\langle\mathcal{M}^{(0)}_{n}|\mathbf{Z}^{(1)}|\mathcal{M}^{(0)}_{n}\rangle \mathrm{F}_{n}$$

At NNLO (only double real radiation)

$$\hat{\sigma}^{\mathrm{RR}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{F}} + \hat{\sigma}^{\mathrm{RR}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{RR}}_{\mathrm{DU}}$$

$$\hat{\sigma}^{\mathrm{RR}}_{\mathrm{F}} = \frac{1}{2\hat{s}}\frac{1}{N}\int \mathrm{d}\boldsymbol{\Phi}_{n+2}\left[\langle\mathcal{M}^{(0)}_{n+2}|\mathcal{M}^{(0)}_{n+2}\rangle \mathrm{F}_{n+2} + \text{subtraction terms}\right]$$

Non-trivial for

$$\hat{\sigma}^{\mathrm{RR}}_{\mathrm{SU}} \qquad \hat{\sigma}^{\mathrm{RR}}_{\mathrm{DU}}$$

Complete list

LO	$\hat{\sigma}^{B}$
NLO	$\hat{\sigma}^{\mathrm{R}}_{\mathrm{F}}$, $\hat{\sigma}^{\mathrm{V}}_{\mathrm{F}}$, $\hat{\sigma}_{\mathrm{U}} = \hat{\sigma}^{\mathrm{R}}_{\mathrm{U}} + \hat{\sigma}^{\mathrm{V}}_{\mathrm{U}} + \hat{\sigma}^{\mathrm{C}}$
NNLO	$\hat{\sigma}^{\mathrm{RR}}_{\mathrm{F}}$, $\hat{\sigma}^{\mathrm{RV}}_{\mathrm{F}}$, $\hat{\sigma}^{\mathrm{VV}}_{\mathrm{F}}$, $\hat{\sigma}_{\mathrm{FR}} = \hat{\sigma}^{\mathrm{RV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{FR}} + \hat{\sigma}^{\mathrm{C2}}_{\mathrm{FR}}$, $\hat{\sigma}_{\mathrm{SU}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{RV}}_{\mathrm{SU}} + \hat{\sigma}^{\mathrm{C1}}_{\mathrm{SU}}$,
	$\hat{\sigma}_{\mathrm{DU}} = \hat{\sigma}^{\mathrm{RR}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{RV}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{VV}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{C1}}_{\mathrm{DU}} + \hat{\sigma}^{\mathrm{C2}}_{\mathrm{DU}}$

‘t Hooft-Veltman regularization

Facts:

1) The finiteness does not depend on the functional form of the matrix elements

2) The integral over epsilon-dimensional volume of a finite function must be of order epsilon

Consequences:

1) Drop all epsilon expansion terms of the matrix elements

2) Restrict the phase space with delta-functions

Current project

C++ software with a complete implementation of the scheme

Born level amplitudes contained thanks to a FORTRAN95 library
by Andreas van Hameren

The user must provide the one-loop amplitudes

- (n+1)-point function
- n-point function spin and color correlated
- n-point function squared (note below)
- on the technical side:
    1) scale dependence evaluated separately
    2) everything as finite remainders

Would be a dream to have a standard software built in, but beware of high requirements on stability (much higher than at NLO) of the (n+1)-point function

The user may provide the two-loop finite remainder
However: the two-loop amplitude and the one-loop squared have no kinematic singularities at NNLO, therefore can be treated separately of our software

Current project

Goodies already available:

- Decays, lepton colliders, lepton-hadron colliders, hadron-hadron colliders
- Simultaneous evaluation for arbitrary PDFs and scales (fixed, dynamic)
- 1d, 2d, variable bin size histograms with/without gaussian edge smearing
- Monte Carlo over processes
- Monte Carlo over polarizations
    - the scheme uses factorization of polarized amplitudes
- Improvements of convergence
    - in particular missed binning avoidance in integrated subtraction terms

Planned:

- Narrow width approximation decays of top quarks
    - Interface partly built-in already

To be made publicly available next year!!!

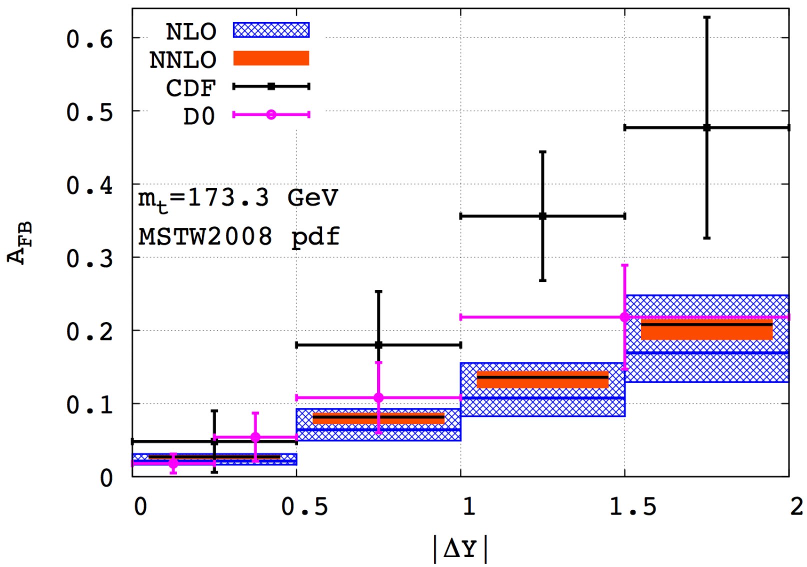
Most recent applications

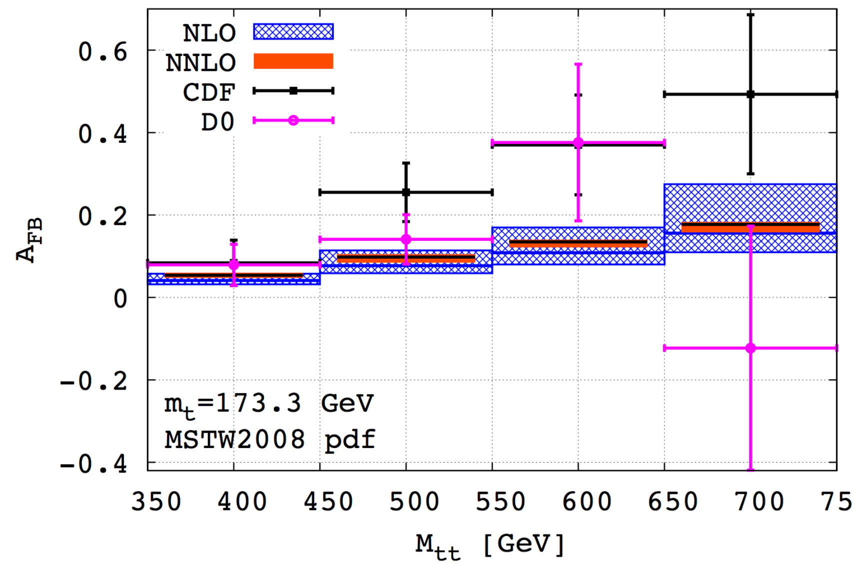
Resolving the Tevatron top quark forward-backward asymmetry puzzle

Michal Czakon,[1] Paul Fiedler,[1] and Alexander Mitov[2]

[1] *Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University, D-52056 Aachen, Germany*

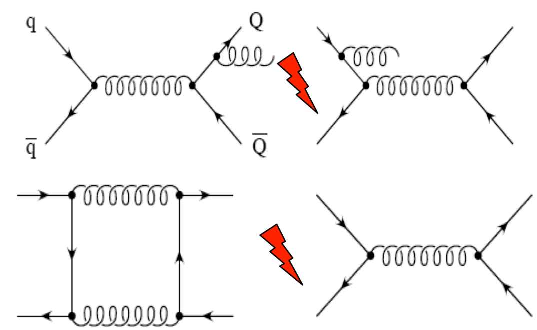
[2] *Cavendish Laboratory, University of Cambridge, Cambridge CB3 0HE, UK*



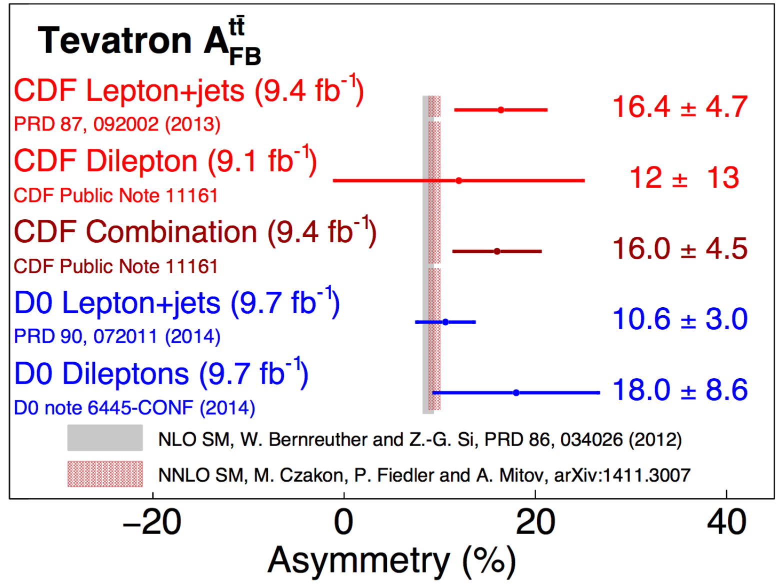


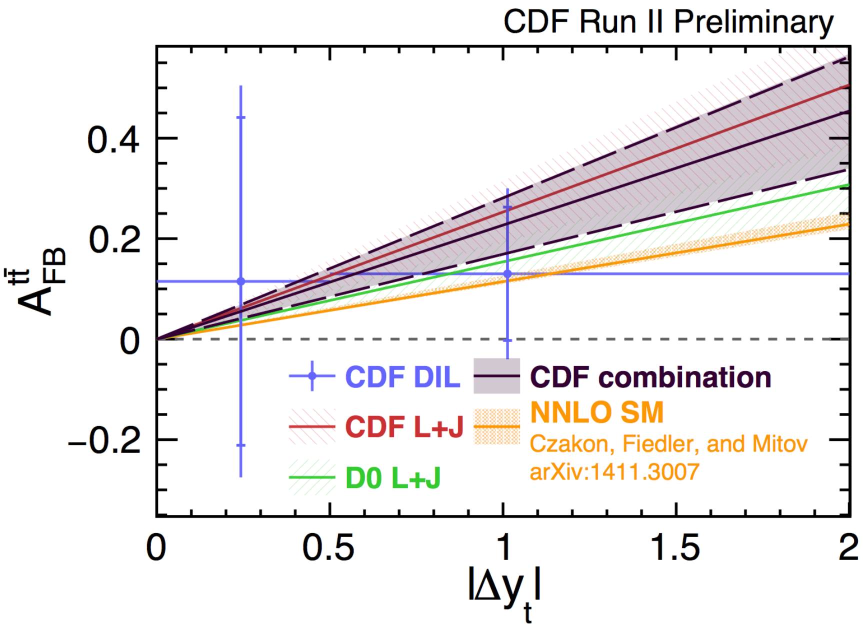
$$A^{t\bar{t}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}$$

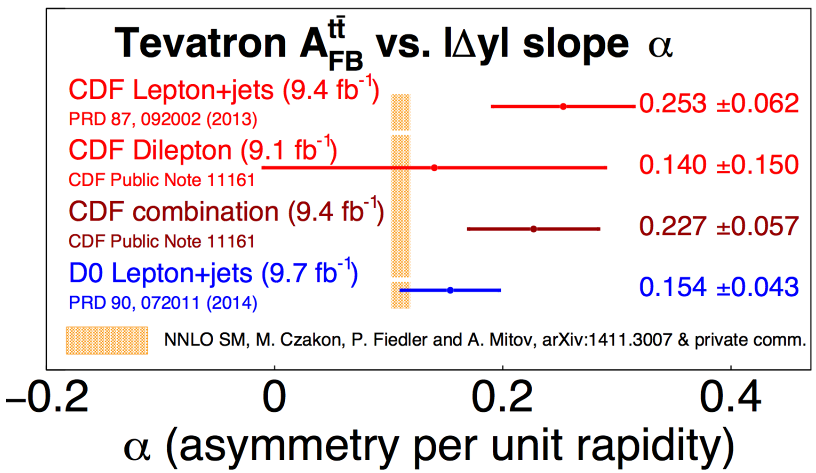
$$\Delta y = y_t - y_{\bar{t}}$$



Most recent applications

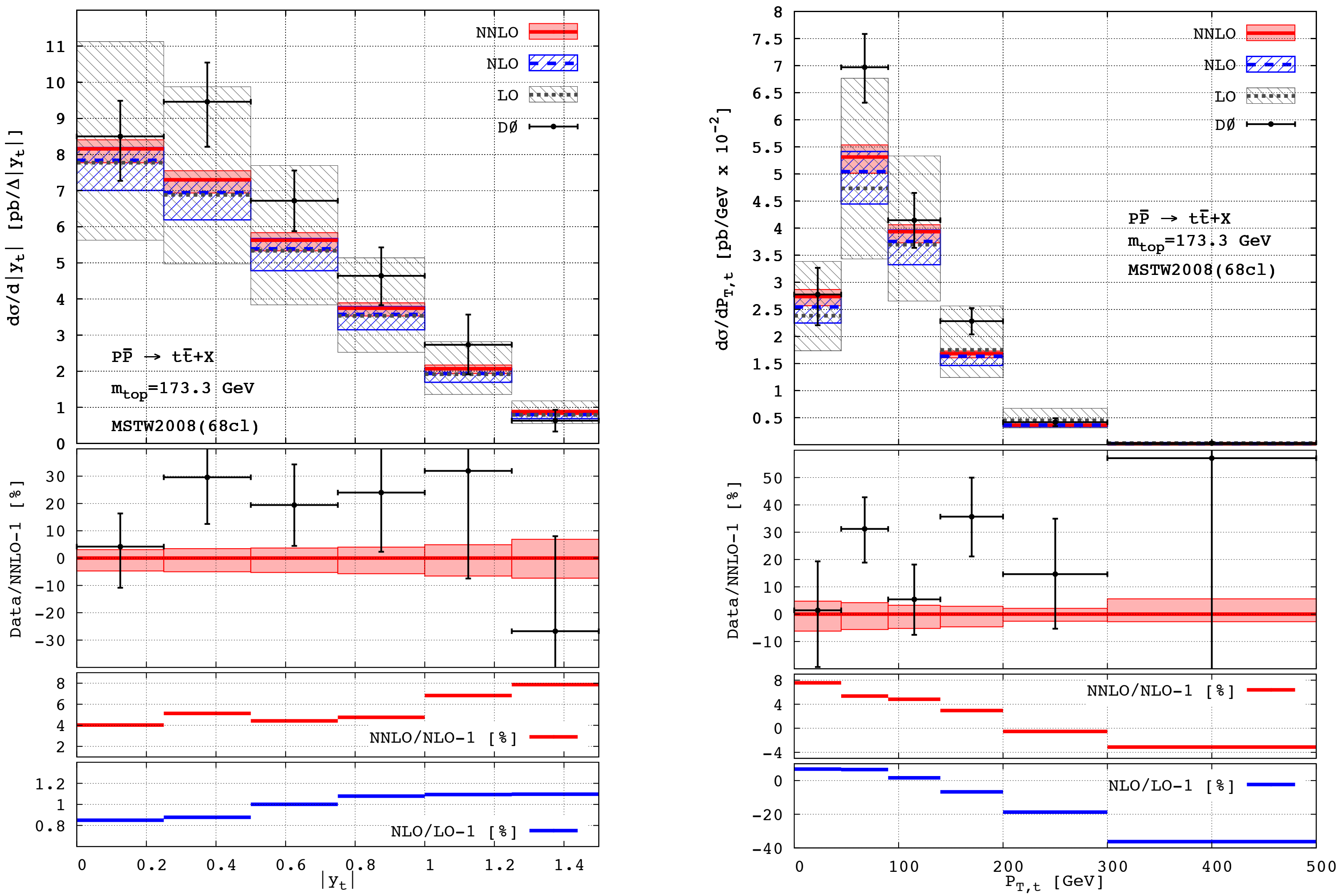




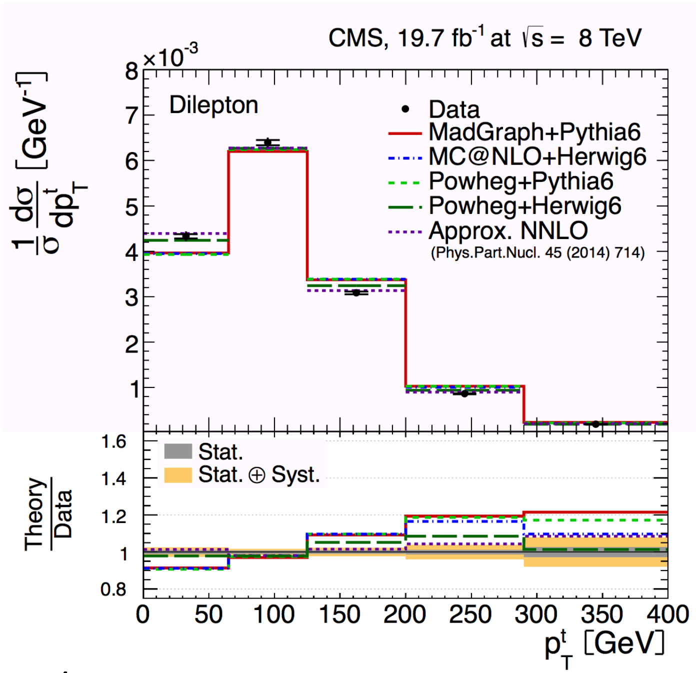


Most recent applications

MC, Fiedler, Mitov, preliminary



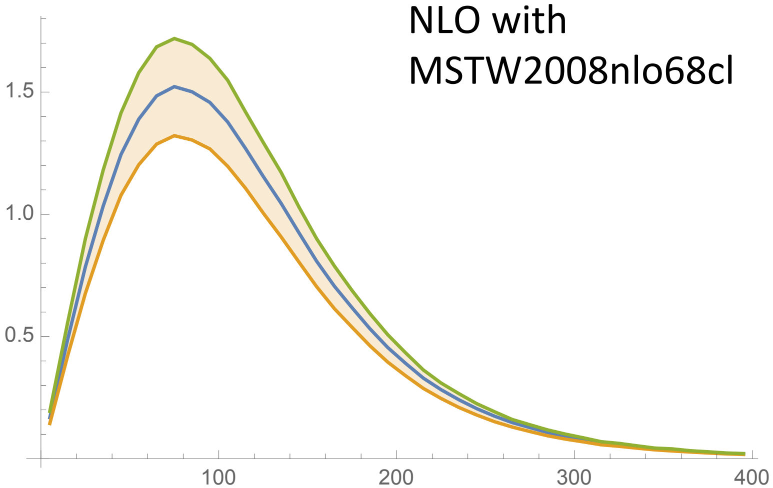
Most recent applications



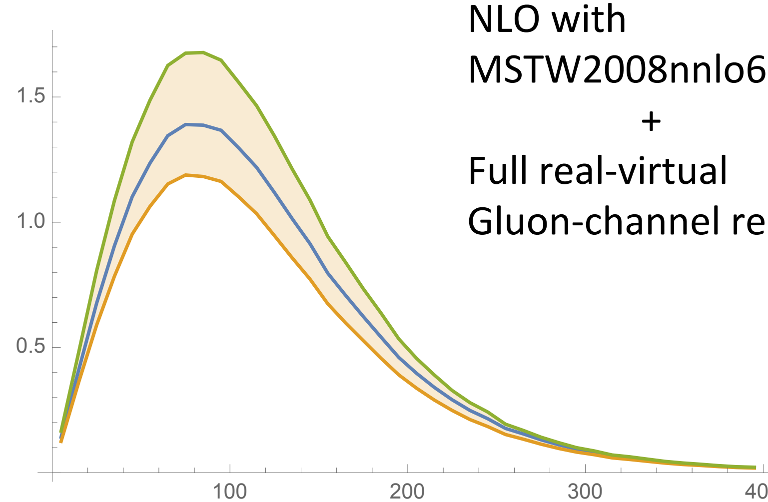
- 7 scales at once $\mu_R \neq \mu_F$
- 4 PDF sets at once
- 8 hours* for real-virtual (2 x 10^8 points) (amplitudes by Stefan Dittmaier)
- 2 hours* for double-real (2 x 10^9 points)

* 800 cores on a cluster

Pb/GeV



NLO with MSTW2008nlo68cl



NLO with MSTW2008nnlo68cl + Full real-virtual Gluon-channel real-real

Commercial

STRIPPER:

The first next-to-next-to-leading order subtraction scheme complete not only on paper but also in software