Renormalization scheme dependence and $\mathcal{O}(\varepsilon)$ terms in MSSM Higgs-boson mass predictions

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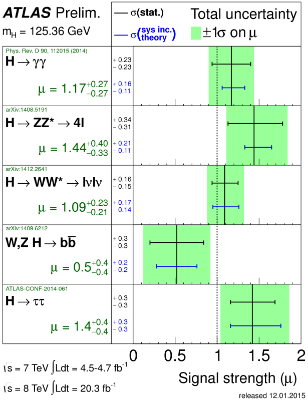
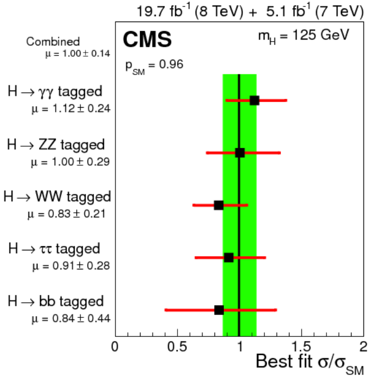
UCLA Radcor & LoopFest 2015, Los Angeles, June 17th, 2015

http://www.feynhiggs.de/ *http://secdec.hepforge.org/*

So far: discovery of one scalar particle at the LHC



ATLAS Jan '15



CMS Dec '14


What's
next
?

- Standard Model Higgs-boson only? More to follow?

MSSM: low-energy SUSY predictions for LHC

- Supersymmetry is still a prime candidate for extensions beyond the Standard Model
- Particles of the MSSM are within reach in run II of the LHC
- We want to be prepared for the day when supersymmetric partner particles are discovered
- Accurate predictions at scales currently accessible are necessary
- Assessment of theoretical uncertainties important

Higgs sector of the MSSM with real parameters

- The Higgs sector of the MSSM has two scalar doublets

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 - \mathrm{i}\chi_1^0) \\ -\phi_1^- \end{pmatrix} \quad H_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + \mathrm{i}\chi_2^0) \end{pmatrix}$$

$\Rightarrow$ 5 Higgs-bosons: h, H, A, $H^\pm$

- Potential of the Higgs sector (incl. soft SUSY breaking terms)

$$\begin{aligned} V =& m_1|H_1|^2 + m_2|H_2|^2 - m_{12}(\epsilon_{ab}H_1^a H_2^b + h.c.) \\ &+ \frac{1}{8}(g_1^2 + g_2^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2}g_2^2|H_1^\dagger H_2|^2 \end{aligned}$$

g_1, g_2: electro-weak gauge couplings,
v_1, v_2: the v.e.v.'s in $\tan\beta \equiv \frac{v_2}{v_1}$,
m_{12}: soft SUSY breaking term in $M_A^2 = m_{12}^2(\tan\beta + \cot\beta)$

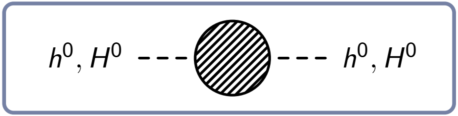
The neutral $\mathcal{CP}$-even Higgs-boson masses in the MSSM

- Feature in the MSSM: Light Higgs-boson mass can be predicted!
- The tree-level neutral $\mathcal{CP}$-even Higgs-boson masses

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

are limited to $m_h \leq \min(M_Z, M_A)\,|\cos(2\beta)|$

- Higher-order corrections shift the Higgs-boson masses considerably



- These lead to maximal values for $m_{h_{max}} \approx 135$ GeV

Status: Radiative corrections in the real MSSM

Higher-order corrections to the Higgs-boson mass in the rMSSM:

1-loop 2-loop 3-loop RGE approach

- Ellis, Ridolfi, Zwirner '91; Okada, Yamaguchi, Yanagida '91; Haber & Hempfling '91; Brignole '92; Chankowski, Pokorski, Rosiek '92 '94; Dabelstein '95
- Hempfling & Hoang '94; Carena et al. '95 '96; Espinosa et al. '95 '00 '01; Heinemeyer, Hollik, Weiglein et al. '98 '99 '99 '00; Zhang '99; Degrassi, Slavich et al. '01 '03; Brignole, Degrassi, Slavich, Zwirner '02; Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '05 '13; S. P. Martin '02 '03 '04 '05; SB, H^4 '14; Degrassi, Di Vita, Slavich '14; Hollik, Paßehr '15
- S.P. Martin '07; Harlander, Kant, Mihaila, Steinhauser '08 '10
- Binger '04; Giudice, Strumia '11; Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '13; Draper, Lee, Wagner '13; Bagnaschi, Giudice, Slavich, Strumia '14

Many more contributions...

Computation of the MSSM Higgs-boson masses

- Self-energy corrections are included in the inverse Higgs-boson propagator matrix

$$\Gamma \equiv \Delta_{\text{Higgs}}^{-1} = -\mathrm{i} \begin{pmatrix} p^2 - m_{\phi_1}^2 + \hat{\Sigma}_{\phi_1}(p^2) & -m_{\phi_1\phi_2}^2 + \hat{\Sigma}_{\phi_1\phi_2}(p^2) \\ -m_{\phi_1\phi_2}^2 + \hat{\Sigma}_{\phi_1\phi_2}(p^2) & p^2 - m_{\phi_2}^2 + \hat{\Sigma}_{\phi_2}(p^2) \end{pmatrix}$$

  with renormalized self-energies $\hat{\Sigma}$
- The neutral $\mathcal{CP}$-even masses are the real parts of the poles of the propagator matrix Δ_{Higgs}
- Strategy (in FeynHiggs): Find complex solutions to $\text{Det}(\Gamma) = 0$

Public codes implementing the MSSM corrections

FeynHiggs Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '00 '03 '07 '14
SoftSusy Allanach '02 SPheno Porod '03; Porod, Staub '11
CPsuperH Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '04 '09 '12
Suspect Djouadi, Kneur, Moultaka '07
H3m Kant, Harlander, Mihaila, Steinhauser '10

Summary of the implemented real MSSM self-energy corrections:

1-loop complete

2-loop $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_s\alpha_b)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ at $p^2 = 0$
$\mathcal{O}(\alpha_s\alpha_t)$ at $p^2 \neq 0$

3-loop $\mathcal{O}(\alpha_s^2\alpha_t)$ at $p^2 = 0$

Estimation of uncertainty of available corrections

1) gather the most dominant corrections:
1-loop $\mathcal{O}(\alpha_t)$ + 2-loop $\mathcal{O}(\alpha_s\alpha_t)$

2) pinpoint strong parameter dependences: α_s, m_t

$$m_t = 173.34 \pm 0.76\,\mathrm{GeV} \text{ (world av., Mar '14)}$$
$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0006 \text{ (world av., PDG '14)}$$

experimental uncertainty of m_t is still very large

- variation of the top-quark mass renormalization gives a good estimate of missing non-logarithmic higher-order corrections

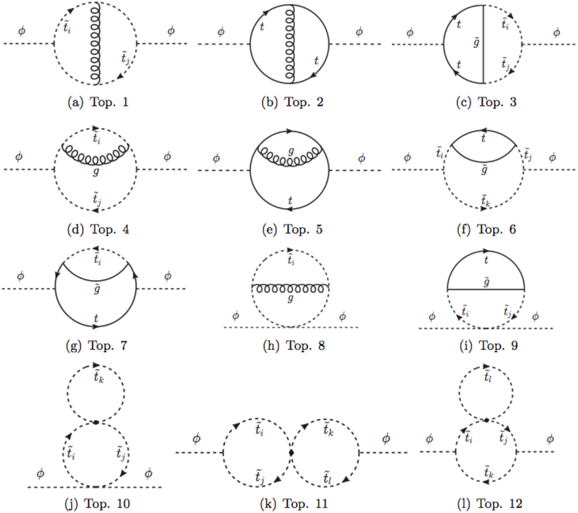
Higgs boson self-energy diagrams for $\mathcal{O}(\alpha_s\alpha_t)$

$p^2 = 0$ result: Heinemeyer, Hollik, G. Weiglein '98
$p^2 \neq 0$ result: SB, Hahn, Heinemeyer, Heinrich, Hollik Apr '14; Degrassi, Di Vita, Slavich Oct '14

- Tensor reduction with TwoCalc Weiglein et al. '93 & FormCalc Hahn et al. '99 '08
- Numerical evaluation with SecDec
  SB, Carter, Heinrich '12;
  SB, Heinrich '13;
  SB, Heinrich, Jones, Kerner, Schlenk, Zirke '15

  → see talk by Johannes Schlenk on Friday

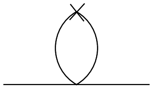
$$\phi = h, H, A$$



Renormalization at the two-loop level

Choosing BPHZ renormalization we have

- 1-loop subrenormalization:
  counterterm insertions enter 1-loop diagrams, are of 1-loop order and independent of p^2; field renormalizations (top, stops) cancel in the sum



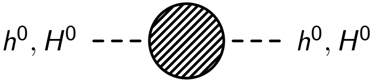
$$\delta A_t^{(1)},\ \delta m_{\tilde{t}_1}^{(1)},\ \delta m_{\tilde{t}_2}^{(1)},\ \delta m_t^{(1)}$$

- 2-loop counter terms:
  counterterm insertions are two-loop diagrams and independent of p^2; here: gaugeless limit

$$\delta M_A^{2(2)},\ \delta T_1^{(2)},\ \delta T_2^{(2)},\ \delta Z_{\mathcal{H}_1}^{(2)},\ \delta Z_{\mathcal{H}_2}^{(2)},\ \delta\tan\beta^{(2)}$$

$X_t = A_t - \mu\cot\beta$, with A_t a soft SUSY breaking parameter

Two-loop renormalization for neutral $\mathcal{CP}$-even Higgs-boson self-energies



- Renormalization procedure consistent with other higher-order corrections in FeynHiggs

  Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein ’06

- Parameter renormalization in the OS scheme:

$$\delta M_A^{2(2)},\ \delta T_1^{(2)},\ \delta T_2^{(2)},\ \delta m_{\tilde{t}_1}^{(1)},\ \delta m_{\tilde{t}_2}^{(1)},\ \delta A_t^{(1)}$$

- Field and $\tan\beta$ renormalization: only divergent terms (poles in ε)

$$\delta Z_{\mathcal{H}_1}^{(2)},\ \delta Z_{\mathcal{H}_2}^{(2)},\ \delta\tan\beta^{(2)} = \frac{1}{2}\left(\delta Z_{\mathcal{H}_2}^{(2)} - \delta Z_{\mathcal{H}_1}^{(2)}\right)$$

Renormalization scheme choice for the top mass

- When renormalizing the top-quark mass on-shell, we use

$$\delta m_t^{(1)} = \delta m_t^{\mathrm{OS}} = \frac{1}{\varepsilon}\,\delta m_t^{\mathrm{div}} + \delta m_t^{\mathrm{fin}} + \varepsilon\,\delta m_t^{\varepsilon} + \ldots \qquad (1)$$

- Alternative renormalization

$$\delta m_t^{\mathrm{FIN}} = \frac{1}{\varepsilon}\,\delta m_t^{\mathrm{div}} + \delta m_t^{\mathrm{fin}} \qquad (2)$$

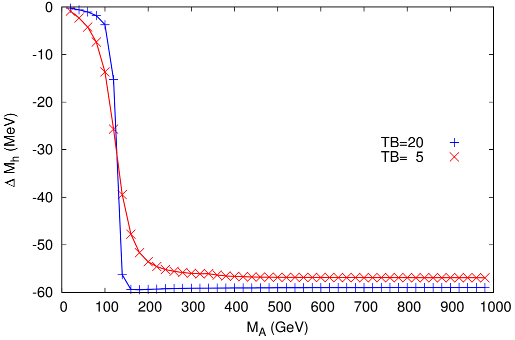
- Argument in favor of the first:
  - corresponds to the definition of the pole in the D-dimensional top-quark propagator when performing a complete expansion in the dimensional regulator ε in a D-dimensional calculation
- $\overline{\mathrm{DR}}$ renormalization

$$\delta m_t^{\overline{\mathrm{DR}}} = \frac{1}{\varepsilon}\,\delta m_t^{\mathrm{div}} \qquad (3)$$

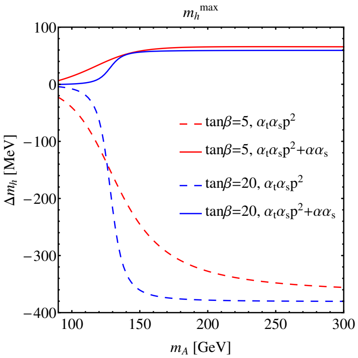
- All different scheme choices are valid

Impact of top mass renormalization scheme choice: OS scheme vs. FIN scheme

The two renormalization schemes yield different results for $p^2 \neq 0$:



SB, Hahn, Heinemeyer, Heinrich, Hollik Apr '14



Degrassi, Di Vita, Slavich Oct '14

Impact of top mass renormalization scheme choice: OS scheme vs. FIN scheme II

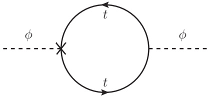
- Difference in the self-energies when performing the transition from the $\delta m_t^{\mathrm{FIN}}$ to δm_t^{OS} renormalization scheme

$$\Sigma_{\phi_{ij}}^{(2)\mathrm{OS}}(p^2) = \Sigma_{\phi_{ij}}^{(2)\,\mathrm{FIN}}(p^2) + \delta_{\Sigma_{ij}}(p^2)$$
$$\Sigma_{AA}^{(2)\mathrm{OS}}(p^2) = \Sigma_{AA}^{(2)\,\mathrm{FIN}}(p^2) + \delta_{A}(p^2)$$

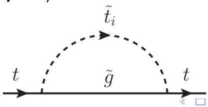
with

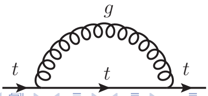
$$\delta_{\Sigma_{22}}(p^2) = \frac{3\alpha_t}{2\pi}\, p^2 \frac{\delta m_t^{\varepsilon}}{m_t} + \delta_{\Sigma_{22}}(0), \qquad \delta_{\Sigma_{11}}(p^2) = \delta_{\Sigma_{11}}(0)$$
$$\delta_{A}(p^2) = \frac{3\alpha_t}{2\pi}\, p^2 \cos^2\beta\, \frac{\delta m_t^{\varepsilon}}{m_t} + \delta_{A}(0), \qquad \delta_{\Sigma_{12}}(p^2) = \delta_{\Sigma_{12}}(0)$$

- Diagrams contributing to $p^2 \neq 0$ terms are



with





Vanishing of $\mathcal{O}(\varepsilon)$ terms in $p^2=0$ MSSM calculations

- Explicit calculation shows the vanishing of the $\mathcal{O}(\varepsilon)$ terms from δm_t^{OS} when $p^2 = 0$

$$\hat{\Sigma}^{(2)}_{\phi_1} :\ 0 = -\sin^2\beta\,\delta_A(0) - \frac{e}{2M_W s_{\mathrm{w}}}\,\cos^2\beta\sin\beta\,\delta_{T_2}$$

$$\hat{\Sigma}^{(2)}_{\phi_1\phi_2} :\ 0 = \sin\beta\cos\beta\,\delta_A(0) + \frac{e}{2M_W s_{\mathrm{w}}}\,\cos^3\beta\,\delta_{T_2}$$

$$\hat{\Sigma}^{(2)}_{\phi_2} :\ 0 = \delta_{\Sigma_{22}}(0) - \cos^2\beta\,\delta_A(0) + \frac{e}{2M_W s_{\mathrm{w}}}\,\sin\beta(1+\cos^2\beta)\,\delta_{T_2}$$

Non-vanishing $\mathcal{O}(\varepsilon)$ terms in $p^2 \neq 0$ calculations

- Lifting the $p^2 = 0$ restriction, momentum-dependent divergent and finite parts appear in the unrenormalized self-energies
- The non-vanishing p^2-dependent divergent terms are cancelled by the field renormalization constants $\delta Z^{(2)}_{\mathcal{H}_1}$ and $\delta Z^{(2)}_{\mathcal{H}_2}$
- Whether the additional p^2-dependent finite terms are cancelled depends on the renormalization scheme choice
- We make the choice

$$\delta Z^{(2)}_{\mathcal{H}_i} = \delta Z^{\delta m_t^{\mathrm{OS}}}_{\mathcal{H}_i}\Big|_{\mathrm{div}} = -\left[\mathrm{Re}\,\Sigma'^{(2)}_{\phi_i}\right]^{\mathrm{div}}_{|p^2=0},$$

$$\delta Z^{(2)}_{\mathcal{H}_2} = \delta Z^{\delta m_t^{\mathrm{OS}}}_{\mathcal{H}_2}\Big|_{\mathrm{div}} = \frac{\alpha_s \alpha_t}{2\pi^2}\left(\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\right) - \frac{1}{\varepsilon}\,\frac{3\alpha_t}{2\pi}\,\frac{\delta m_t^{\mathrm{fin}}}{m_t}$$

therefore the additional finite p^2-dependent terms in the unrenormalized self-energies do not cancel during renormalization

Analytically: Non-vanishing $\mathcal{O}(\varepsilon)$ terms for $p^2 \neq 0$

- For each self-energy non-vanishing $\mathcal{O}(\varepsilon)$ terms from δm_t^{OS} remain

$\hat{\Sigma}^{(2)}_{\phi_1}$:

$$-\sin^2\beta\,[\delta_A(M_A^2) - \delta_A(0)] = \frac{3\alpha_t}{2\pi}\left(-\cos^2\beta\,\sin^2\beta\,M_A^2\right)\frac{\delta m_t^\varepsilon}{m_t}$$

$\hat{\Sigma}^{(2)}_{\phi_1\phi_2}$:

$$\sin\beta\cos\beta\,[\delta_A(M_A^2) - \delta_A(0)] = \frac{3\alpha_t}{2\pi}\left(\cos^3\beta\,\sin\beta\,M_A^2\right)\frac{\delta m_t^\varepsilon}{m_t}$$

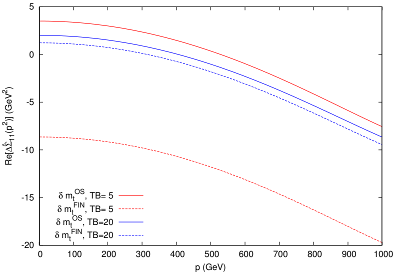
$\hat{\Sigma}^{(2)}_{\phi_2}$:

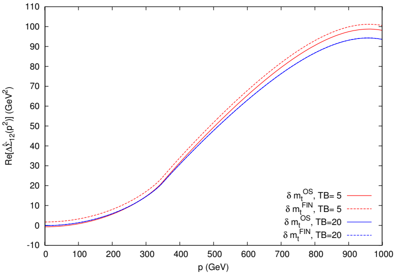
$$[\delta_{\Sigma_{22}}(p^2) - \delta_{\Sigma_{22}}(0)] - \cos^2\beta\,[\delta_A(M_A^2) - \delta_A(0)] = \frac{3\alpha_t}{2\pi}\left(p^2 - \cos^4\beta\,M_A^2\right)\frac{\delta m_t^\varepsilon}{m_t}$$

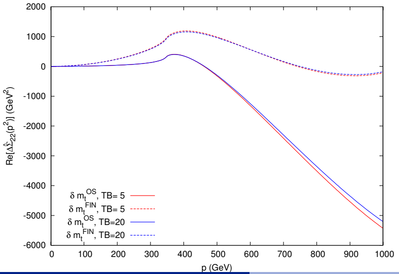
- $\delta_A(p^2)$ appears as shift in $\delta M_A^{2(2)\mathrm{OS}} = \delta M_A^{2(2)\,\mathrm{FIN}} + \delta_A(M_A^2)$

Numerically: Non-vanishing $\mathcal{O}(\varepsilon)$ terms for $p^2 \neq 0$

Numerical difference in the self-energies:

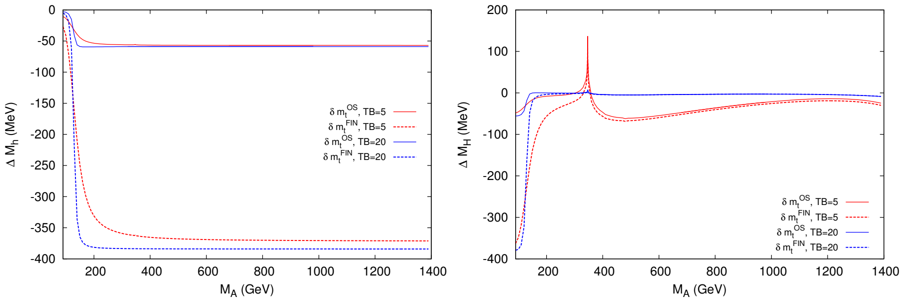






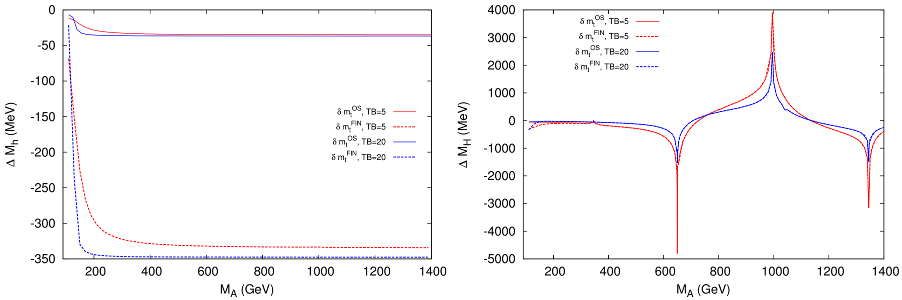
m_h^{max} scenario, shifts and momentum-dependence become apparent in the self-energies

m_h^{max} scenario: Variation of mass shifts with M_A



- light Higgs-boson mass: differences are large for higher values of M_A
- heavy Higgs-boson mass: large differences for low values of M_A
- Using $\delta m_t^{\mathrm{FIN}}$, the results of Degrassi, di Vita, Slavich '14 are reproduced

light-stop scenario: Variation of mass shifts with M_A



- light Higgs-boson mass: large differences for higher values of M_A
- heavy Higgs-boson mass: large differences for low values of M_A
- Using $\delta m_t^{\mathrm{FIN}}$, the results of Degrassi, di Vita, Slavich '14 are reproduced

Impact of top mass renormalization scheme choice: OS scheme vs. $\overline{\mathrm{DR}}$ scheme

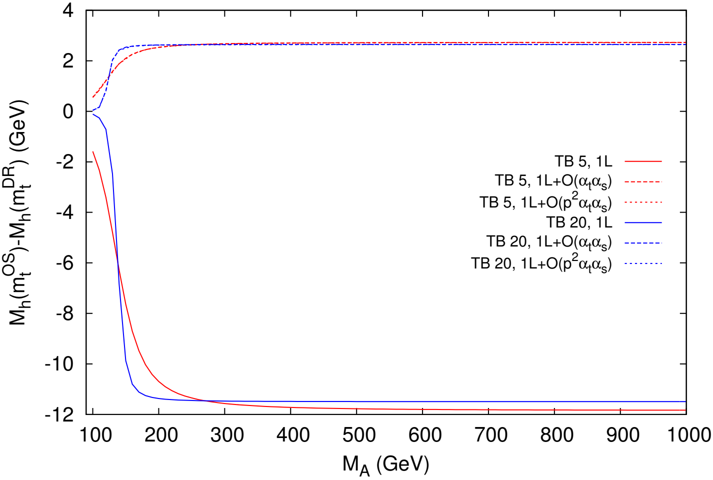
Numerical value for the $\overline{\mathrm{DR}}$ top-quark mass

$$m_t^{\overline{\mathrm{DR}}}(\mu) = m_t \cdot \left[1 + \frac{\delta m_t^{\mathrm{fin}}}{m_t} + \mathcal{O}\left(\left(\alpha_s^{\overline{\mathrm{DR}}}\right)^2\right)\right]$$

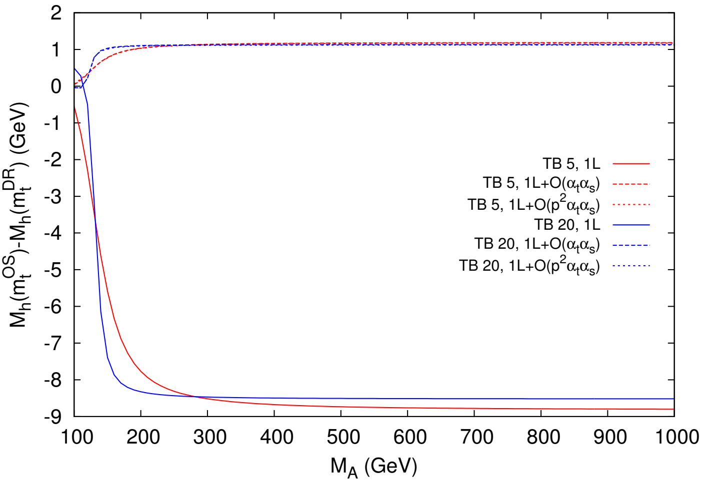
with

$$\begin{aligned}\frac{\delta m_t^{\mathrm{fin}}}{m_t} =& \alpha_s^{\overline{\mathrm{DR}}}(\mu) \Bigg(-\frac{5}{3\pi} + \frac{1}{\pi}\log(m_t^2/\mu^2) + \frac{m_{\tilde{g}}^2}{3m_t^2\pi}\left(-1 + \log(m_{\tilde{g}}^2/\mu^2)\right) \\ &+ \frac{1}{6m_t^2\pi}\Big(m_{\tilde{t}_1}^2\left(1 - \log(m_{\tilde{t}_1}^2/\mu^2)\right) + m_{\tilde{t}_2}^2\left(1 - \log(m_{\tilde{t}_2}^2/\mu^2)\right) \\ &+ \left(m_{\tilde{g}}^2 + m_t^2 - m_{\tilde{t}_1}^2 - 2m_{\tilde{g}} m_t \sin(2\theta_t)\right) \mathrm{Re}[B_0^{\mathrm{fin}}(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_1}^2)] \\ &+ \left(m_{\tilde{g}}^2 + m_t^2 - m_{\tilde{t}_2}^2 + 2m_{\tilde{g}} m_t \sin(2\theta_t)\right) \mathrm{Re}[B_0^{\mathrm{fin}}(m_t^2, m_{\tilde{g}}^2, m_{\tilde{t}_2}^2)]\Big)\Bigg)\end{aligned}$$

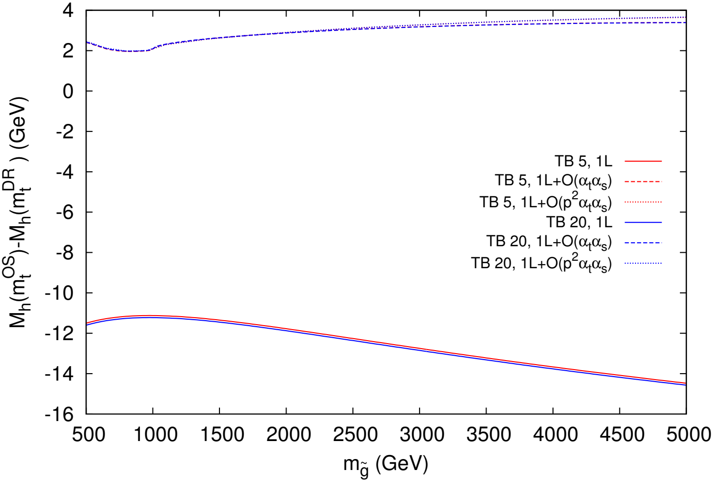
M_A dependence in m_h^{max} scenario



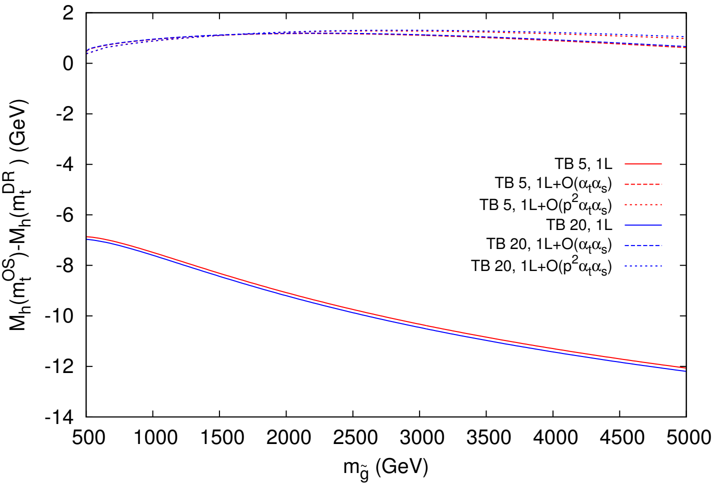
M_A dependence in light-stop scenario



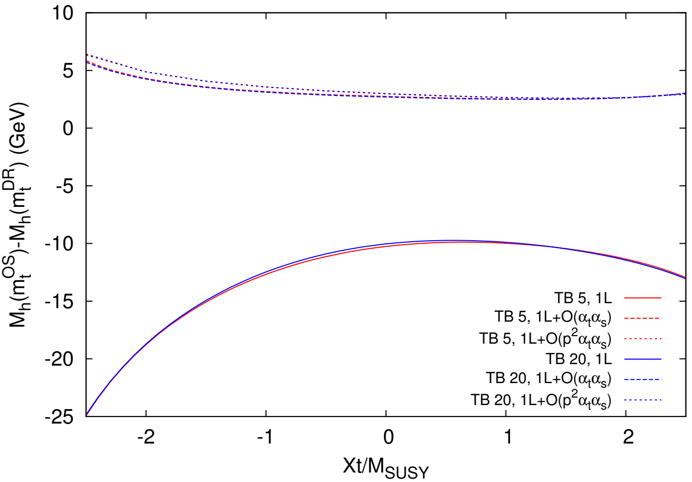
$m_{\tilde{g}}$ dependence in m_h^{max} scenario



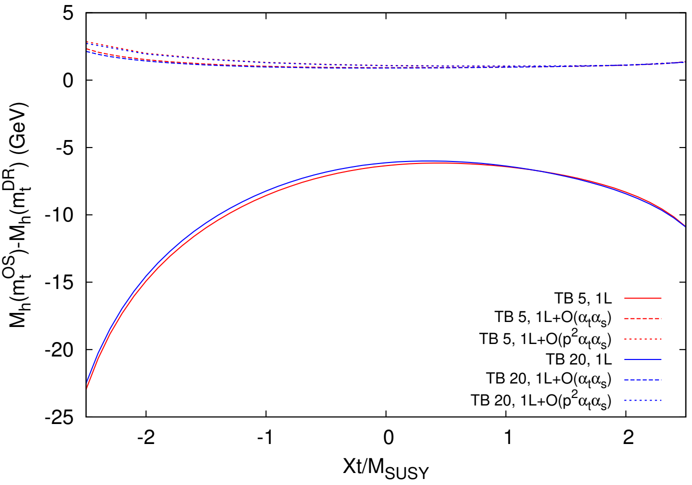
$m_{\tilde{g}}$ dependence in light-stop scenario



X_t dependence in m_h^{max} scenario



X_t dependence in light-stop scenario



Summary and Outlook

Summary

- Variation of top mass renormalization gives good estimate of missing non-logarithmic higher-order corrections
- Differences in the predicted Higgs masses with m_t^{OS} vs. $m_t^{\overline{\mathrm{DR}}}$ decrease drastically with the inclusion of two-loop corrections
- $\overline{\mathrm{DR}}$ calculation included in FeynHiggs2.11.1
- Cutting the top-quark renormalization constant at the $\mathcal{O}(\varepsilon^0)$ coefficient is also a valid choice
- Differences between using δm_t^{OS} and $\delta m_t^{\mathrm{FIN}}$ point towards the size of missing higher-order momentum-dependent corrections

Outlook

- Improve on the theoretical uncertainty stemming from the non-logarithmic higher-order corrections

Backup

Evaluation of neutral $\mathcal{CP}$-even MSSM Higgs-boson masses in $\overline{DR}$ scheme

$$\left[p^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_{hh}(p^2)\right]\left[p^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_{HH}(p^2)\right] - \left[\hat{\Sigma}_{hH}(p^2)\right]^2 = 0$$

Three steps:

1. Compute $M_{h,0}$ and $M_{H,0}$ from the 1-loop + 2-loop $\mathcal{O}(\alpha_s\alpha_t)$ self-energies
2. Compute momentum-dependent renormalized $\mathcal{O}(\alpha_s\alpha_t)$ self-energies for $p^2 = M^2_{h,0}$ and $p^2 = M^2_{H,0}$
3. Include new self-energy contributions as constant shifts into FeynHiggs and find poles M_h and M_H

- corrections available since FeynHiggs 2.10.1

The mass shifts are $\Delta M_{\{h,H\}} = M_{\{h,H\}} - M_{\{\{h,0\},\{H,0\}\}}$

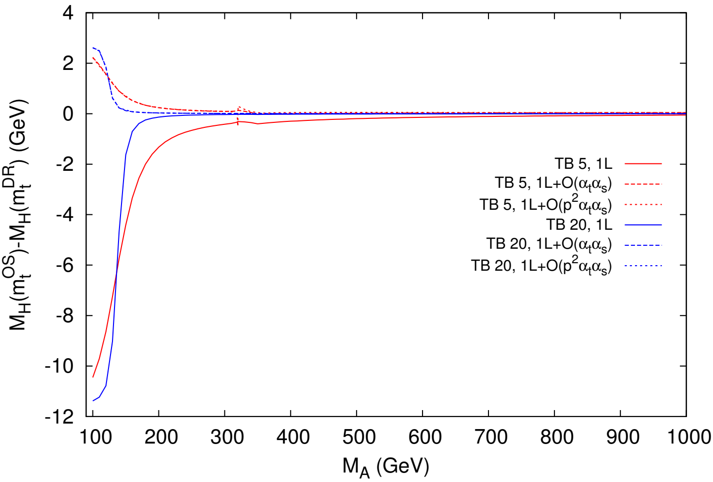
Analysis of 2 benchmark scenarios

Carena, Heinemeyer, Stal, Wagner, Weiglein '13,

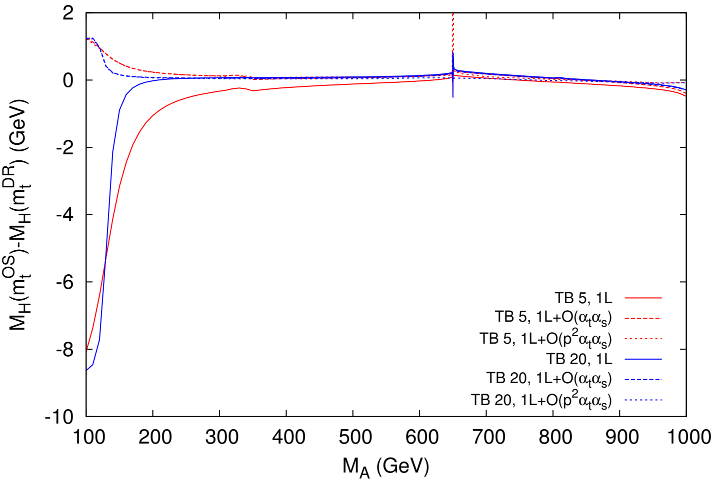
Bagnaschi, Harlander, Liebler, Mantler, Slavich, Vicini '14

m_h^{max} **scenario**	**light stop scenario** (updated)
$m_t = 173.2$ GeV	$m_t = 173.2$ GeV
$\mu = 200$ GeV	$\mu = 400$ GeV
$X_t = 2\ M_{SUSY}$	$X_t = 2\ M_{SUSY}$
$M_{SUSY} = 1$ TeV	$M_{SUSY} = 0.5$ TeV
$\rightarrow m_{\tilde{t}_1} = 826.8$ GeV,	$\rightarrow m_{\tilde{t}_1} = 326.8$ GeV,
$m_{\tilde{t}_2} = 1173.2$ GeV	$m_{\tilde{t}_2} = 673.2$ GeV
$m_{\tilde{g}} = 1.5$ TeV	$m_{\tilde{g}} = 1.5$ TeV
$m_A = 250$ GeV	$m_A = 250$ GeV
$\tan\beta = 5, 20$	$\tan\beta = 5, 20$

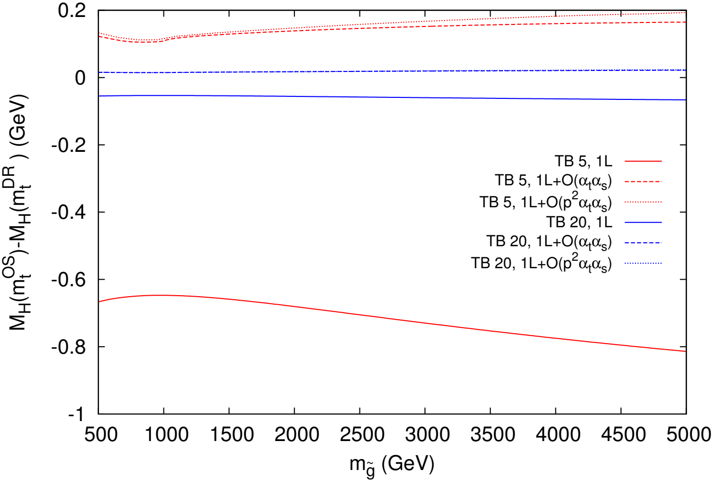
M_A dependence in m_h^{max} scenario



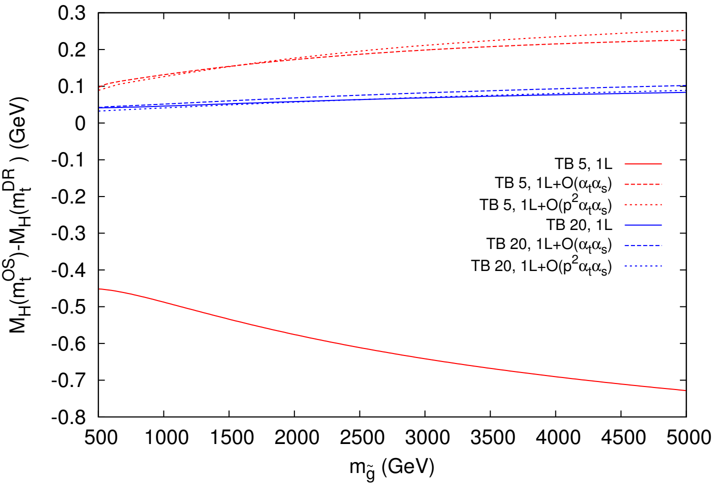
M_A dependence in light-stop scenario



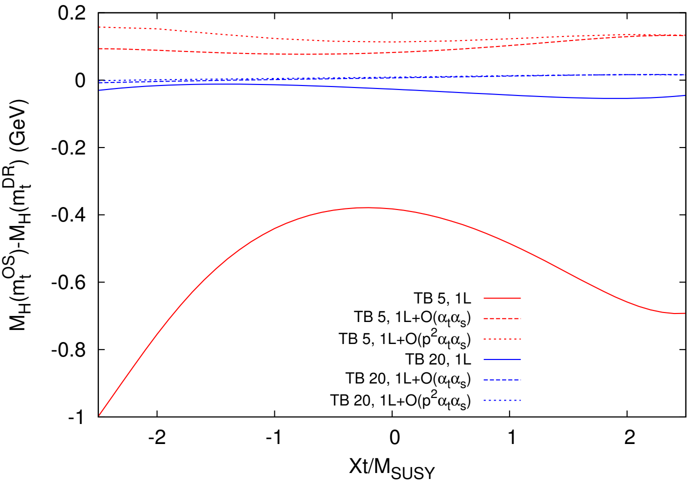
$m_{\tilde{g}}$ dependence in m_h^{max} scenario



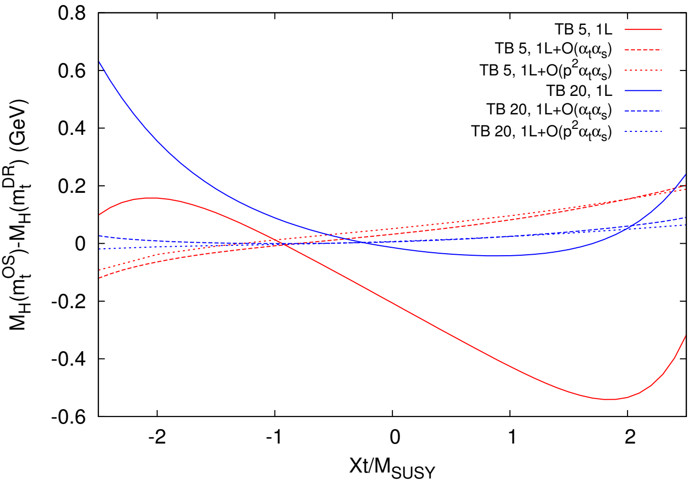
$m_{\tilde{g}}$ dependence in light-stop scenario



X_t dependence in m_h^{max} scenario

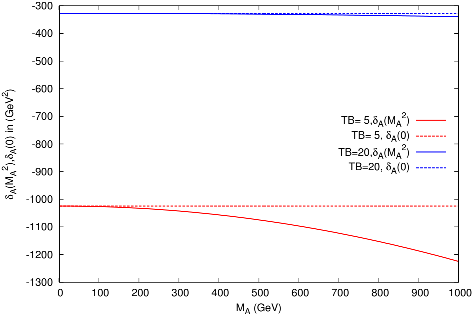


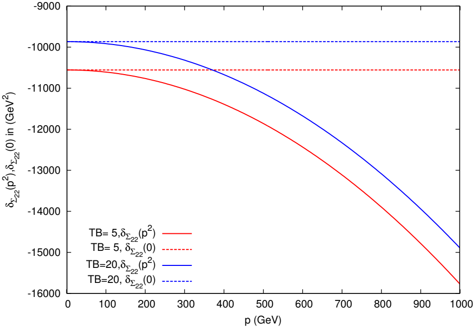
X_t dependence in light-stop scenario



Numerical verification

- Additional finite parts from $\delta_A(M_A^2)$ and $\delta_{\Sigma_{22}}(p^2)$





Renormalized two-loop self-energies

Renormalized self-energies in unphysical ϕ_1-ϕ_2 basis

$$\hat{\Sigma}^{(i)}_{\phi_1}(p^2) = \Sigma^{(i)}_{\phi_1}(p^2) + \delta Z^{(i)}_{\phi_1}\,(p^2 - m^2_{\phi_1}) - \delta m^{2(i)}_{\phi_1}\,, \tag{4}$$

$$\hat{\Sigma}^{(i)}_{\phi_1\phi_2}(p^2) = \Sigma^{(i)}_{\phi_1\phi_2}(p^2) - \delta Z^{(i)}_{\phi_1\phi_2}\, m^2_{\phi_1\phi_2} - \delta m^{2(i)}_{\phi_1\phi_2}\,, \tag{5}$$

$$\hat{\Sigma}^{(i)}_{\phi_2}(p^2) = \Sigma^{(i)}_{\phi_2}(p^2) + \delta Z^{(i)}_{\phi_2}\,(p^2 - m^2_{\phi_2}) - \delta m^{2(i)}_{\phi_2}\,. \tag{6}$$

Rotation of the unphysical $\hat{\Sigma}^{(2)}_{\phi_i\phi_j}$ self-energies into the physical $h - H$ basis

$$\hat{\Sigma}^{(2)}_{H} = \cos^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_1\phi_1} + \sin^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_2\phi_2} + 2\sin\alpha\cos\alpha\hat{\Sigma}^{(2)}_{\phi_1\phi_2}$$

$$\hat{\Sigma}^{(2)}_{h} = \sin^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_1\phi_1} + \cos^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_2\phi_2} - 2\sin\alpha\cos\alpha\hat{\Sigma}^{(2)}_{\phi_1\phi_2}$$

$$\hat{\Sigma}^{(2)}_{hH} = -\sin\alpha\cos\alpha\,(\hat{\Sigma}^{(2)}_{\phi_1\phi_1} - \hat{\Sigma}^{(2)}_{\phi_2\phi_2}) + (\cos^2\alpha - \sin^2\alpha)\,\hat{\Sigma}^{(2)}_{\phi_1\phi_2}$$

Two-loop parameter renormalization

$$\delta m^{2(2)}_{\phi_1} = \delta M^{2(2)}_Z \cos^2\beta + \delta M^{2(2)}_A \sin^2\beta - \delta T^{(2)}_1 \frac{e}{2M_W s_\mathrm{w}} \cos\beta(1+\sin^2\beta) + \delta T^{(2)}_2 \frac{e}{2M_W s_\mathrm{w}} \cos^2\beta\sin\beta + 2\,\delta\tan\beta^{(2)} \cos^2\beta\sin^2\beta\,(M_A^2 - M_Z^2)\,, \tag{7}$$

$$\delta m^{2(2)}_{\phi_1\phi_2} = -(\delta M^{2(2)}_Z + \delta M^{2(2)}_A)\sin\beta\cos\beta - \delta T^{(2)}_1 \frac{e}{2M_W s_\mathrm{w}} \sin^3\beta - \delta T^{(2)}_2 \frac{e}{2M_W s_\mathrm{w}} \cos^3\beta - \delta\tan\beta^{(2)} \cos\beta\sin\beta\cos 2\beta\,(M_A^2 + M_Z^2)\,, \tag{8}$$

$$\delta m^{2(2)}_{\phi_2} = \delta M^{2(2)}_Z \sin^2\beta + \delta M^{2(2)}_A \cos^2\beta + \delta T^{(2)}_1 \frac{e}{2M_W s_\mathrm{w}} \sin^2\beta\cos\beta - \delta T^{(2)}_2 \frac{e}{2M_W s_\mathrm{w}} \sin\beta(1+\cos^2\beta) - 2\,\delta\tan\beta^{(2)} \cos^2\beta\sin^2\beta\,(M_A^2 - M_Z^2)\,. \tag{9}$$

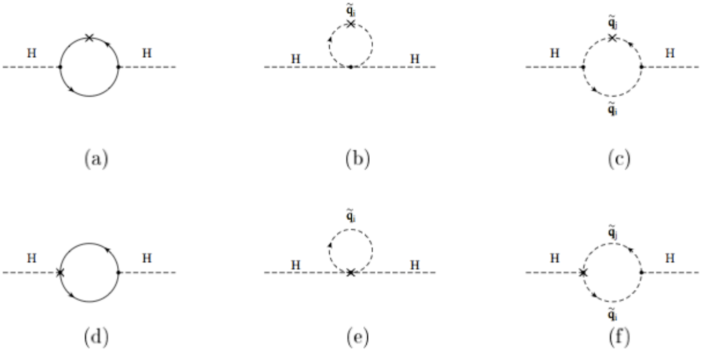
Two-loop counterterm of the A-boson mass

The two-loop renormalization constant of the A-boson mass needs to be defined as

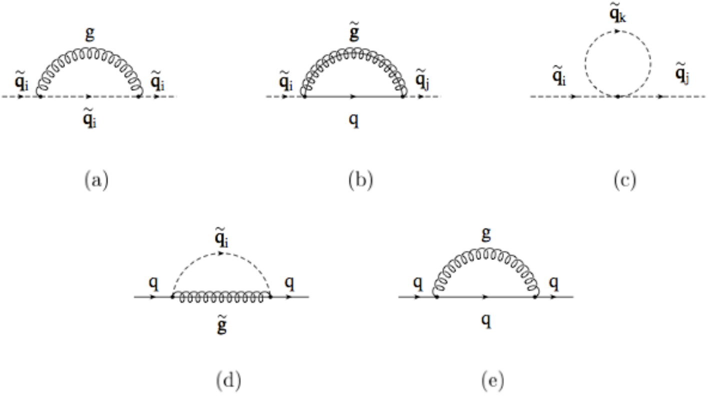
$$\delta M_A^{2(2)} = \mathrm{Re}\,\Sigma_{AA}^{(2)}(M_A^2), \qquad (10)$$

in order to cancel divergences entering with the terms proportional to the $\delta \tan \beta^{(2)}$ renormalization constant (see previous slide)

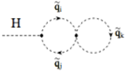
Diagrams for sub-loop renormalization



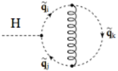
Counter term insertions for sub-loop renormalization



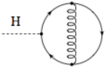
Tadpole diagrams needed in the renormalization



(a)

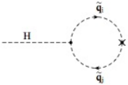


(b)



(c)

(a)



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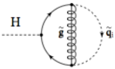

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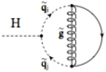

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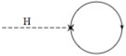
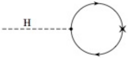


(c)



(d)