The Drell-Yan process beyond threshold

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based on arXiv:1410.6406 and arXiv:1503.05156
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Outline

THRESHOLD EXPANSION

Soft gluons generate $\log(\xi)$ that spoil perturbation theory when $\xi \to 0$

$$\frac{d\sigma}{d\xi} = \sum_{n=0}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \left(a_{nm} \frac{\log^m(\xi)}{\xi} + b_{nm} \log^m(\xi) + \mathcal{O}(\xi) \right) \quad (1)$$

- a_{nm}: **LP** Logs $\mathcal{D}^i \to$ eikonal approximation
- b_{nm}: **NLP** Logs $L^i \to$ "next-to-eikonal" or "next-to-soft"

WHY WE NEED NEXT-TO-SOFT

- phenomenology
  - including recoil and spin interactions in soft emissions
  - precision (e.g. Higgs production)
- theory
  - next-to-soft theorems
  - understand infrared structure of gauge and gravity theories

A VERY RECENT STORY

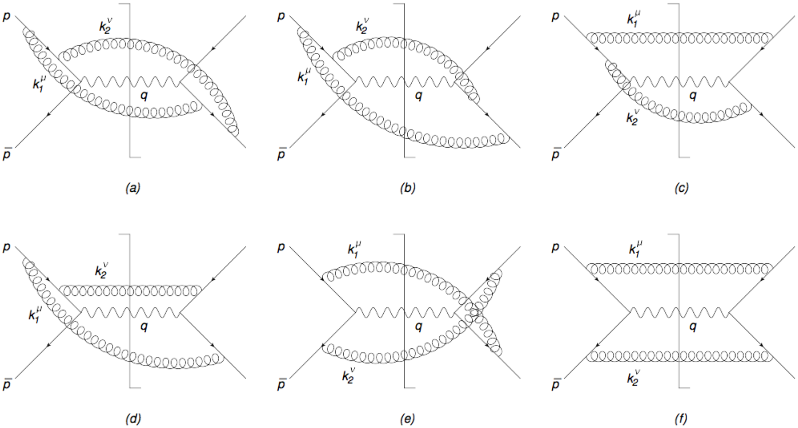
- next-to-soft theorems [Cachazo, Strominger 2014, Bern, Davis, Nohle 2014]
- SCET [Larkosky, Neill, Stewart 2014]
- N^3LO Higgs via threshold expansion [Anastasiou, Duhr, Dulat, Herzog, Mistlberger 2013]

WHY DY?

A bottom-up approach towards an understanding of all-orders properties of next-to-threshold corrections

- **simplest** application (we consider only C_F^2 terms)
- NNLO **known** for many years
- very similar to **Higgs** production via gluon fusion
- threshold radiation is forced to be **soft**
  (all Log(1-z) come from soft real radiation)
- interplay between soft and **collinear** effects
  (some Log(1-z) are affected by collinear virtual radiation)

DOUBLE-REAL (ABELIAN-LIKE) DY



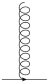
K factor from full-QCD

Result from full QCD for the K factor $K^{(2)}_{2r}(z)$ at next-to-threshold level for the double-real (abelian) DY:

$$K^{(2)}_{2r}(z) = \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{32 - 32\mathcal{D}_0(z)}{\epsilon^3} + \frac{80 + 128\mathcal{D}_1(z) - 128\log(1-z)}{\epsilon^2} + \frac{-256\mathcal{D}_2(z) - 320\log(1-z) + 256\log^2(1-z)}{\epsilon} + -\frac{1024}{3}\log^3(1-z) + 640\log^2(1-z) + 32 - \frac{1024\mathcal{D}_3(z)}{3}\right] \quad (2)$$

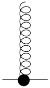
AN EFFECTIVE APPROACH

- Eikonal (E)

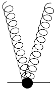


$$\frac{p^\mu}{p\cdot k} \tag{3}$$

- Next-to-Eikonal (NE)



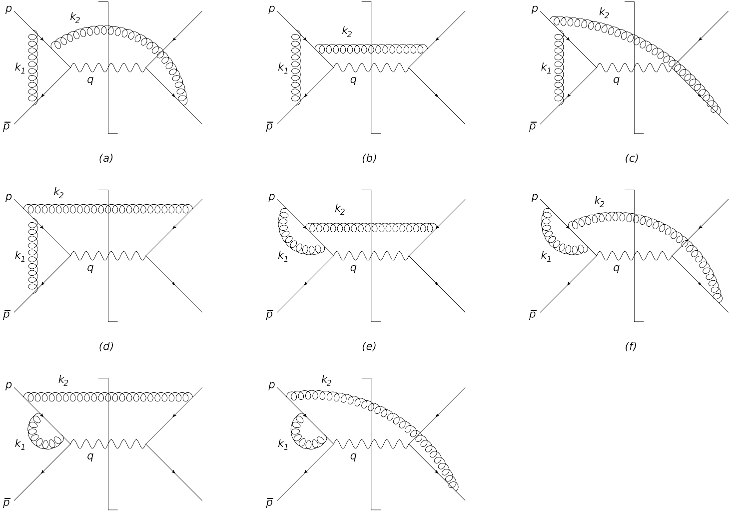
$$\frac{k^2\, p^\mu}{2(p\cdot k)^2} - \frac{\not{k}\gamma^\mu}{2p\cdot k} \tag{4}$$



$$\frac{(pk_2)p^\mu k_1^\nu + (pk_1)k_2^\mu p^\nu - (pk_1)(pk_2)g^{\mu\nu} - (k_1k_2)p^\mu p^\nu}{p\cdot(k_1+k_2)} \tag{5}$$

Both LP ($\mathcal{D}_i$) and NLP (L_i) are reproduced

1 REAL 1 VIRTUAL (ABELIAN-LIKE) DY



(a) (b) (c) (d) (e) (f) (g) (h)

Full QCD computation

Result from full QCD for the K factor $K^{(2)}_{1r1v}(z)$ at next-to-threshold level for the 1-real 1-virtual (abelian) DY:

$$\begin{aligned} K^{(2)}_{1r1v}(z) &= \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[\frac{2\mathcal{D}_0(z) - 2}{\epsilon^3} + \frac{-4\mathcal{D}_1(z) + 3\mathcal{D}_0(z) + 4\log(1-z) - 6}{\epsilon^2} \\ &+ \frac{16\mathcal{D}_2(z) - 24\mathcal{D}_1(z) + 32\mathcal{D}_0(z)}{4\epsilon} \\ &+ \frac{-16\log^2(1-z) + 52\log(1-z) - 49}{4\epsilon} \\ &- \frac{8\mathcal{D}_3(z)}{3} + 6\mathcal{D}_2(z) - 16\mathcal{D}_1(z) + 16\mathcal{D}_0(z) + \frac{8}{3}\log^3(1-z) \\ &- \frac{29}{2}\log^2(1-z) + \frac{103}{4}\log(1-z) - \frac{51}{2} \Bigg] \end{aligned} \quad (6)$$

LP Logs (all powers of $\mathcal{D}_i$) and the leading NLP (L^3) are reproduced

No matching for L^2 and L^1 !

Beyond the threshold limit, an effective field theory approach fails for 1r-1v, because the soft expansion of the real gluon's momentum neglect collinear effects of the virtual gluon. We need another strategy:

We tackle the problem in two different ways:

- Method of Regions
- Factorization Approach

The method of regions

The Method of Regions

A systematic procedure for expanding loop integrals about their singular regions. [Beneke, Smirnov 1998, Jantzen 2011]

We distinguish different regions for the momentum l^μ by the different scalings of its components

$$\begin{aligned} \text{Hard}: &\quad l \sim \sqrt{\hat{s}}\,(1,1,1) \\ \text{Soft}: &\quad l \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda^2,\lambda^2) \\ \text{Collinear}: &\quad l \sim \sqrt{\hat{s}}\,(1,\lambda,\lambda^2) \\ \text{Anticollinear}: &\quad l \sim \sqrt{\hat{s}}\,(\lambda^2,\lambda,1) \end{aligned} \tag{7}$$

DY via MoR: soft region

$$K^{(2)}_{\mathrm{E,\,s}}(z) = K^{(2)}_{\mathrm{NE,\,s}}(z) = 0 \quad (8)$$

DY via MoR: hard region

$$K^{(2)}_{\mathrm{E,h}}(z) = \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[\frac{2\mathcal{D}_0(z)}{\epsilon^3} + \frac{-4+3\mathcal{D}_0(z)-4\mathcal{D}_1(z)}{\epsilon^2} + \frac{-6+8\mathcal{D}_0(z)-6\mathcal{D}_1(z)+4\mathcal{D}_2(z)+8\log(1-z)}{\epsilon} - 16 + 16\mathcal{D}_0(z) - 16\mathcal{D}_1(z) + 6\mathcal{D}_2(z) - \frac{8\mathcal{D}_3(z)}{3} + 12\log(1-z) - 8\log^2(1-z)\Bigg] \tag{9}$$

$$K^{(2)}_{\mathrm{NE,h}}(z) = \left(\frac{\alpha_s}{\pi}\right)^2 \Bigg[-\frac{2}{\epsilon^3} + \frac{1+4\log(1-z)}{\epsilon^2} + \frac{-5+2\log(1-z)-4\log^2(1-z)}{\epsilon} - 8 + 10\log(1-z) - 2\log^2(1-z) + \frac{8}{3}\log^3(1-z)\Bigg] \tag{10}$$

DY VIA MOR: (ANTI)COLLINEAR REGION

Soft emission from triangle diagrams:

$$K^{(2)}_{\mathrm{NE,\,c+\bar{c}}}(z) = \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{1}{2\epsilon^2} + \frac{3\log(1-z)}{2\epsilon} + 1 - \frac{9}{4}\log^2(1-z) \right] \tag{11}$$

Soft emission from self energy diagrams:

$$K^{(2)}_{\mathrm{NE,\,c+\bar{c}}}(z) = \left(\frac{\alpha_s}{\pi}\right)^2 \left[-\frac{1}{2\epsilon^2} + \frac{-5+6\log(1-z)}{4\epsilon} - \frac{5}{2} + \frac{15}{4}\log(1-z) - \frac{9}{4}\log^2(1-z) \right] \tag{12}$$

Note it is only NE
Eikonal terms are present but they cancel

ABOUT THE ORDER OF EXPANSION

At amplitude level

$$\text{Hard}: \quad \frac{(2p\cdot\bar{p})^{-\epsilon}}{\epsilon^2}\left[\mathrm{E}+\mathrm{NE}+\ldots\right]+\mathcal{O}\left(\epsilon^{-1}\right) \tag{13}$$

$$\text{Collinear}: \quad \frac{(-2p\cdot k_2)^{-\epsilon}}{\epsilon}\left[\mathrm{NE}+\ldots\right]+\mathcal{O}\left(\epsilon^{0}\right) \tag{14}$$

$$(-2p\cdot k_2)^{-\epsilon} \sim (1-z)^{-\epsilon} \tag{15}$$

Such a factor would be absent if one performed the soft expansion before the dimensional regularization expansion

$$K^{(2)}_{soft}(z) + K^{(2)}_{hard}(z) + K^{(2)}_{coll}(z) + K^{(2)}_{anti-coll}(z) = K^{(2)}_{1r1v}(z) \qquad (16)$$

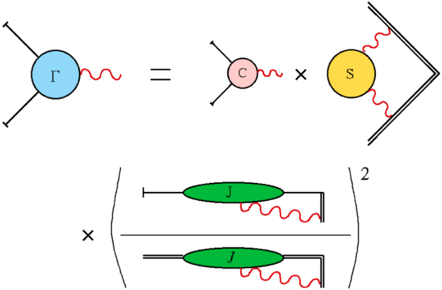
With the MoR we reproduced the entire next-to-threshold Log structure (both $\mathcal{D}^i$ and L^i) of the NNLO DY.

Very powerful, but no insight to higher orders. We need factorization approach

A factorization approach

Soft Collinear Factorization

[Dixon,Magnea,Sterman 0805.3515]



$$\textbf{Soft}\quad \mathcal{S}\left(\beta_1\cdot\beta_2, \alpha_s(\mu^2), \epsilon\right) = \langle 0|\Phi_{\beta_2}(\infty,0)\Phi_{\beta_1}(0,-\infty)|0\rangle$$

$$\textbf{Jet}\quad J\left(\frac{(p\cdot n)^2}{n^2\mu^2}, \alpha_s(\mu^2), \epsilon\right) u(p) = \langle 0|\Phi_n(\infty,0)\psi(0)|p\rangle$$

$$\textbf{Eik Jet}\quad \mathcal{J}\left(\frac{(\beta_1\cdot n)^2}{n^2\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \langle 0|\Phi_n(\infty,0)\Phi_{\beta_1}(0,-\infty)|0\rangle$$

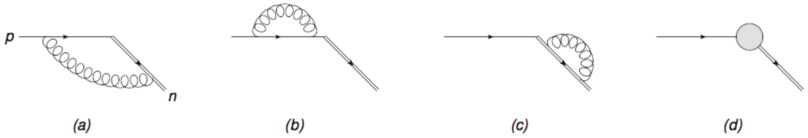
About the n dependence

At one loop this becomes

$$\Gamma = \mathcal{H} + \mathcal{S} + \sum_i (J_i - \mathcal{J}_i) \qquad (17)$$

- at pole level n (and β) dependence cancels among $\mathcal{S}$, J, $\mathcal{J}$
- $\mathcal{H}$ is a finite (process dependent) function *defined* to cancel the residual n dependence, by matching with the full 1-loop form factor.

The jet function with $n^2 = 0$



- (a) scaless integral (UV+IR=0)
- (b) standard QCD counterterm
- (c) 0
- (d) counterterm for a.

THE JET FUNCTION WITH $n^2 = 0$

$$J^{(1)}(n,p) = -\frac{\alpha_s}{4\pi}\left(-\frac{4\pi\mu^2 e^{-\gamma_E}}{2p\cdot n}\right)^{\epsilon}\left[\frac{2}{\epsilon^2}+\frac{3}{2\epsilon}+2+\frac{\pi^2}{6}\right] \tag{18}$$

Spurious collinear singularities are introduced by the choosing $n^2 = 0$, but this greatly simplify the calculation.

Moreover, at all orders

$$J(n,p)_{bare} = 1 \tag{19}$$

The Low-Burnett-Kroll theorem

How does a soft emission affect an amplitude? Low theorem: express a radiative amplitude in terms of the non radiative one.

- Low: scalar particles
- Burnett and Kroll: spinor particles
- Del Duca: theorem extended to the region $\frac{m^2}{Q^2} < E < m^2$ (collinear region). In Low original analysis gluon energy E is the smallest scale of the problem.

Del Duca modifications (NuclPhysB345 '90)

Del Duca generalized Low's original analysis attaching an extra soft gluon to the factorized amplitude (via Ward Identity) Introduce 2 polarization tensors:

$$K^{\nu\mu}(p,k) = \frac{k^\nu (2p+k)^\mu}{k^2 + 2p \cdot k} \tag{20}$$

$$G^{\nu\mu}(p,k) = g^{\mu\nu} - K^{\nu\mu} \tag{21}$$

Emission contributes with terms got via Ward identity

- $K + G$ emission from $\mathcal{H}$
- $K + G$ emission from $\mathcal{S}$
- K emission from J

and with a contribution excluded in Low original analysis

- G emission from J

A factorization formula

The ansatz proposed by Del Duca with modern definition of $\mathcal{H}$, $\mathcal{S}$ and J (i.e. taking special care of the auxiliary vector n) offers a **factorization formula** for next-to-threshold Logs at amplitude level:

$$\mathcal{A}^{\mu} = \sum_{i} \Bigg[q_i \left(\frac{(2p_i - k)^{\mu}}{2p_i \cdot k - k^2} - G^{\nu\mu} \frac{\partial}{\partial p_i^{\nu}} \right) \mathcal{A} + \mathcal{H}(p_i) \bar{\mathcal{S}}(\beta_i) G^{\nu\mu} \left(J^{\mu}(p_i, k) - q_i \frac{\partial}{\partial p_i^{\nu}} J(p_i, k) \right) \prod_{j \neq i} J(p_j) \Bigg] \quad (22)$$

A factorization formula

It is made out of 3 main ingredients

- external (scalar) emission
- derivative of the non radiative amplitude
- jet emission function J^{μ}

BARE VS RENORMALIZED

From RG arguments for the radiative and non-radiative amplitude, it follows that all the counterterms must vanish.

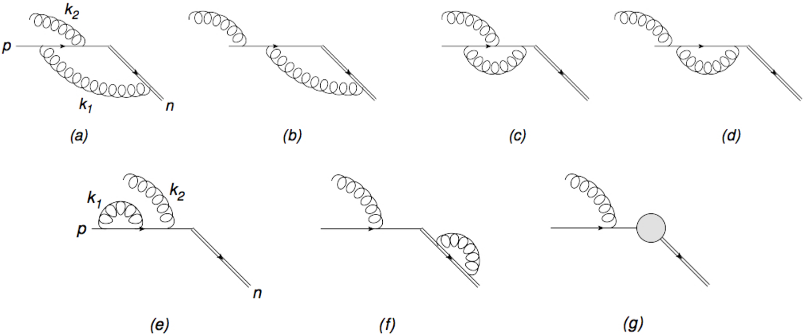
We can left the quantities unrenormalized, with great advantage. E.g.: if $J = 1$ then $\frac{\partial}{\partial_\mu} J = 0$

$$\mathcal{A}^\mu = \sum_i \left(q_i \frac{(2p_i - k)^\mu}{2p_i \cdot k - k^2} + q_i G^{\nu\mu} \frac{\partial}{\partial p_i^\nu} + J^\mu(p_i, k) \right) \mathcal{A} \tag{23}$$

In this way the comparison with the MoR (bare) is even stronger

THE JET EMISSION FUNCTION FOR $n^2 = 0$

$$J_\mu(p,n,k_2)\,u(p) = \left\langle 0 \left| \int d^d y e^{-i(p+k_2)\cdot y} \Phi_n(y,\infty)\psi(y) j_\mu(0) \right| p \right\rangle$$



RESULT

$$J^{\nu(1)}(p,n,k;\epsilon) = (2p\cdot k)^{-\epsilon}\Bigg[\left(\frac{2}{\epsilon}+4+8\epsilon\right)\left(\frac{n\cdot k}{p\cdot k}\frac{p^\nu}{p\cdot n}-\frac{n^\nu}{p\cdot n}\right) -(1+2\epsilon)\frac{\mathrm{i}\,k_\alpha[\gamma^\alpha\gamma^\nu]}{4p\cdot k}+\left(\frac{1}{\epsilon}-\frac{1}{2}-3\epsilon\right)\frac{k^\nu}{p\cdot k} +(1+3\epsilon)\left(\frac{\gamma^\nu\,\not{n}}{p\cdot n}-\frac{p^\nu\,\not{k}\,\not{n}}{p\cdot k\,p\cdot n}\right)\Bigg]+\mathcal{O}(\epsilon^2,k) \tag{24}$$

CONTRACTION WITH THE G TENSOR

The role of the G tensor is to project out the pure spin-dependent part of the jet emission function:

$$G^{\nu\mu}\left(-\frac{p_\nu}{p\cdot k_2}+\frac{\not{k}_2\gamma_\nu}{2p\cdot k_2}\right)=\frac{k_{2\,\nu}\left[\gamma^\nu,\gamma^\mu\right]}{4p\cdot k_2} \tag{25}$$

This term then represents the coupling of the soft gluon to the magnetic moment of the hard leg.

It combines with the scalar external contribution, to give a full external emission.

FACTORIZATION APPROACH: EXTERNAL EMISSION

$$K^{(2)}_{\rm ext}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \Bigg\{ \frac{32}{\epsilon^3}\Big[\mathcal{D}_0(z)-1\Big] + \frac{8}{\epsilon^2}\Big[-8\mathcal{D}_1(z)+6\mathcal{D}_0(z)+8L(z)-14\Big]$$
$$+\frac{16}{\epsilon}\Big[4\mathcal{D}_2(z)-6\mathcal{D}_1(z)+8\mathcal{D}_0(z)-4L^2(z)+14L(z)-14\Big]$$
$$-\frac{128}{3}\mathcal{D}_3(z)+96\mathcal{D}_2(z)-256\mathcal{D}_1(z)+256\mathcal{D}_0(z)$$
$$+\frac{128}{3}L^3(z)-224L^2(z)+448L(z)-512\Bigg\}. \quad (26)$$

This can be achieved via an effective field theory approach.

All $\mathcal{D}$'s come from this contribution

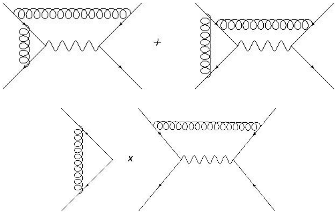
i.e. all Eikonal bits come from external emission

i.e. Eikonal emissions factorize

FACTORIZATION APPROACH: DERIVATIVE OF THE FULL FORM FACTOR

$$K^{(2)}_{\partial\mathcal{A}}(z) = \left(\frac{\alpha_s}{4\pi}C_F\right)^2 \left\{\frac{32}{\epsilon^2}+\frac{16}{\epsilon}\left[-4L(z)+3\right]+64L^2(z)-96L(z)+128\right\} \quad (27)$$

This cannot be caught by a factorization effective Feynman rule approach:



Factorization approach: the collinear bit

$$K^{(2)}_{\text{collinear}}(z) = \left(\frac{\alpha_s}{4\pi} C_F\right)^2 \left\{ -\frac{16}{\epsilon^2} + \frac{4}{\epsilon}\left[12L(z) - 5\right] - 72L^2(z) + 60L(z) - 24 \right\} \tag{28}$$

$$K^{(2)}_{\text{ext}}(z) + K^{(2)}_{\partial\mathcal{A}}(z) + K^{(2)}_{\text{collinear}}(z) = K^{(2)}_{1r1v} \tag{29}$$

All $\mathcal{D}^i$ and L^i and even the constant!

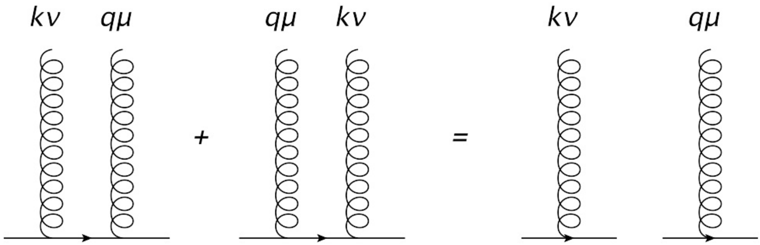
We reproduced the entire next-to-threshold Log structure of the NNLO DY
with a factorization approach

Conclusions

- problem
  - We don't know how to resum next-to-threshold Logs
  - Important for both theory and phenomenology
- proposal
  - A factorization formula has been proposed, based on universal functions
  - The Method of Regions is a powerful tool to understand the asymptotic behavior of those functions
  - Crucial to this analysis is a treatment of collinear region (jet emission function)
- outlook
  - from factorization to resummation
  - non abelian generalization and Higgs production
  - application to other processes (DIS, e^+e^-, HH)

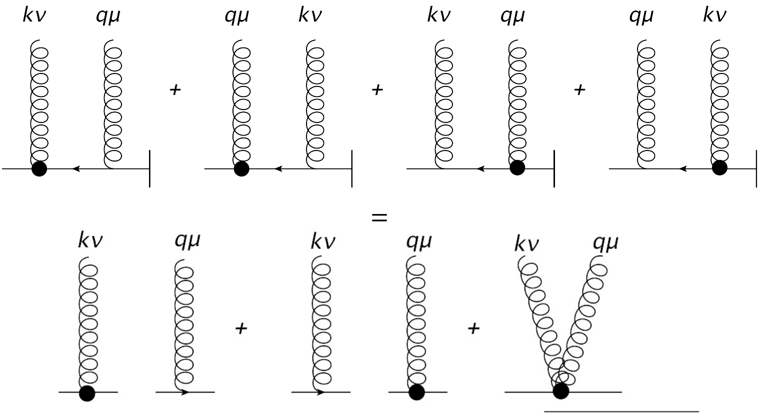
Thanks for your attention!

BACKUP SLIDES: EIKONAL IDENTITY



Factorization works very easily at eikonal level.

Backup slides: Next-to-Eikonal identity



There are correlations between pairs of gluons. Naive factorization is broken at NE level.

Backup slides: Null vs non-null

$$J^{\mu(1)}_{n^2\neq 0} = \left(-\frac{\not{k}_2\gamma^\mu}{2pk_2} + \frac{p^\mu}{pk_2}\right) J^{(1)}_{n^2\neq 0} \quad + J^\mu_{c+d}$$
$$+ 2\frac{k_2^\mu}{pk_2} + (2pk_2)^{-\epsilon}\left(\frac{1}{\epsilon}+2\right)\frac{\not{k}_2\gamma^\mu}{pk_2}$$
$$+ \frac{1}{pn}\left[2\left(\frac{nk_2}{pk_2}p^\mu - n^\mu\right)\log\left(\frac{n^2\,2pk_2}{(2pn)^2}\right) - \frac{\not{n}\not{k}_2}{pk_2}p^\mu + \not{n}\gamma^\mu\right] + \mathcal{O}(\epsilon) + \mathcal{O}(k_2) \tag{30}$$

$$J^{\mu(1)}_{n^2=0} = \left(-\frac{\not{k}_2\gamma^\mu}{2pk_2} + \frac{p^\mu}{pk_2}\right) J^{(1)}_{n^2=0} \quad + J^\mu_{c+d}$$
$$+ 2\frac{k_2^\mu}{pk_2} + (2pk_2)^{-\epsilon}\left(\frac{1}{\epsilon}+2\right)\frac{\not{k}_2\gamma^\mu}{pk_2}$$
$$+ \frac{1}{pn}\left[2\left(\frac{nk_2}{pk_2}p^\mu - n^\mu\right)\left(-\frac{1}{\epsilon} - 1 + \log(2pk_2)\right) - \frac{\not{n}\not{k}_2}{pk_2}p^\mu + \not{n}\gamma^\mu\right] + \mathcal{O}(\epsilon) + \mathcal{O}(k_2) \tag{31}$$

where

$$J^\mu_{c+d} = \frac{(2pk_2)^{-\epsilon}}{\epsilon}\left(\frac{\not{k}_2\gamma^\mu}{pk_2} + \frac{k_2^\mu}{pk_2}\right) + \frac{5}{2}\frac{\not{k}_2\gamma^\mu}{pk_2} + \frac{k_2^\mu}{pk_2} + \mathcal{O}(\epsilon) + \mathcal{O}(k_2) \tag{32}$$