DIFFERENTIAL TOP PAIR PRODUCTION AT NNLO

Gabriel Abelof

NORTHWESTERN UNIVERSITY

Argonne NATIONAL LABORATORY

Based on work done in collaboration with
R. Bonciani, O. Dekkers, A. Gehrmann-De Ridder,
P. Maierhöfer, I. Majer, A. v. Manteuffel and S. Pozzorini

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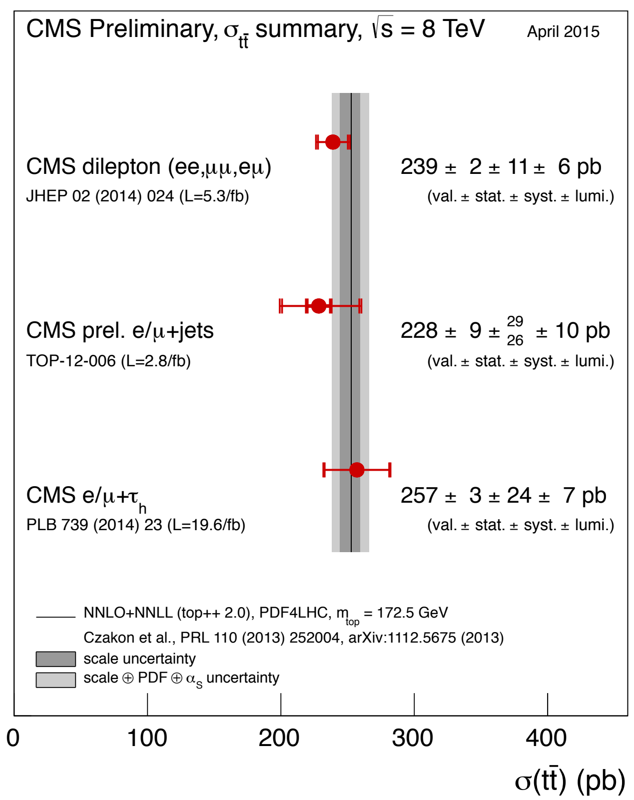
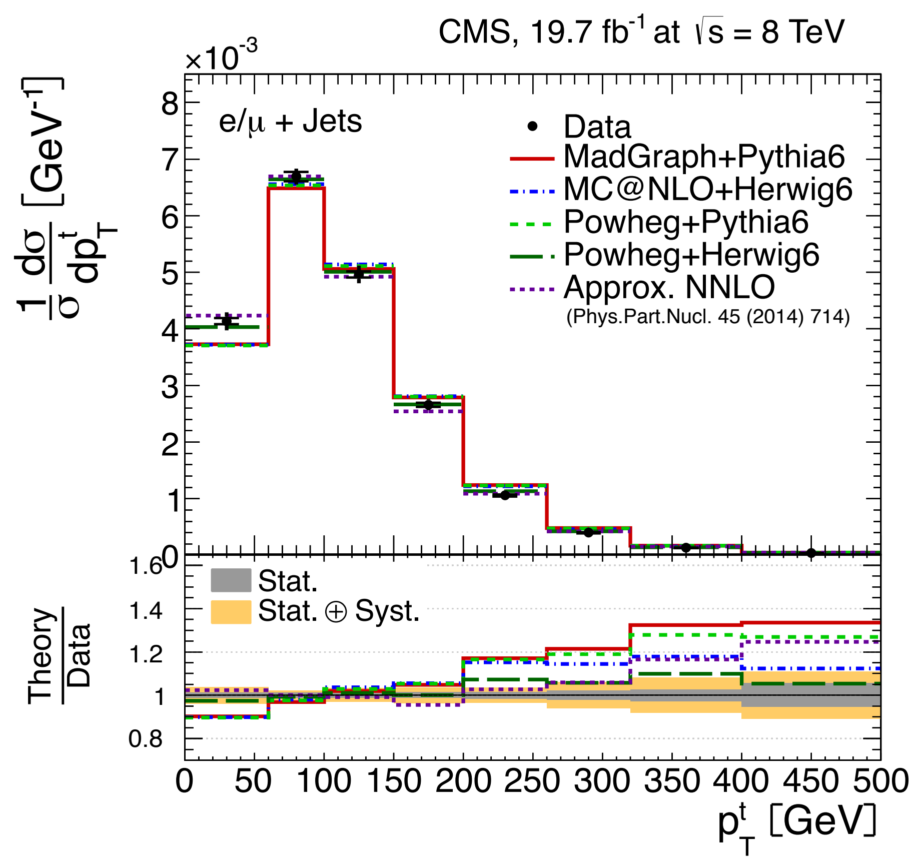
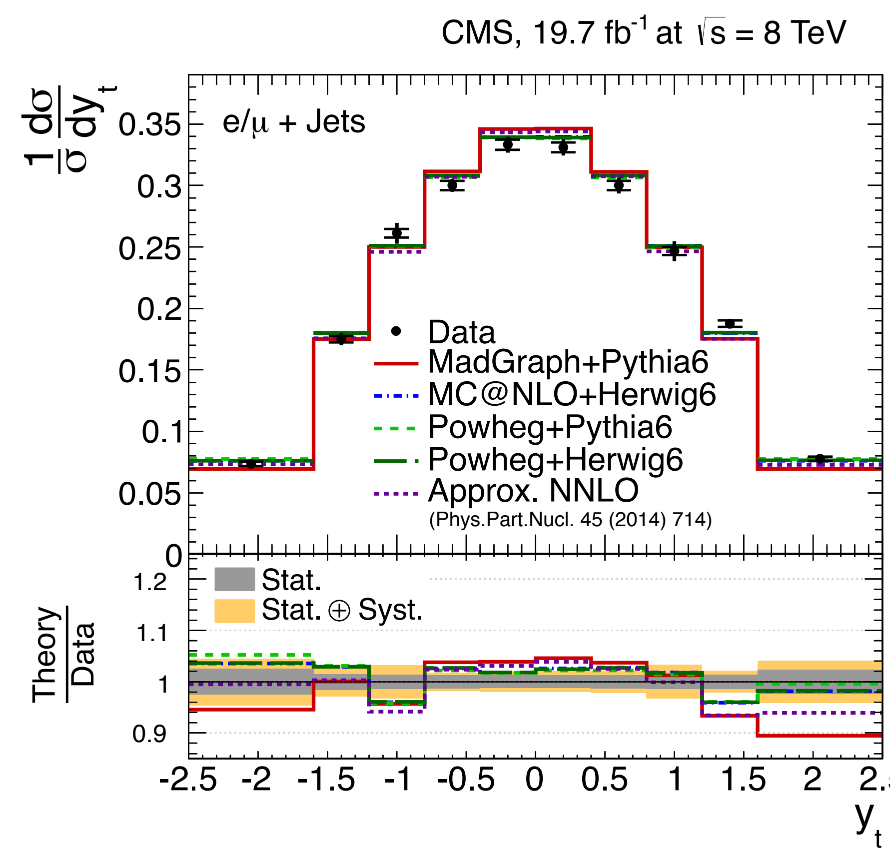
Motivation

- Percent level experimental accuracy in top pair production is a reality at the LHC
  - At inclusive level
  - In differential distributions







- Precise data provides an accurate probe of the $t\bar{t}$ production mechanism. Beneficial for
  - New physics searches
  - More pedestrian purposes: extraction of top quark pole mass, high x gluon PDF, …

Motivation

* To reliably interpret percent level experimental data, we need percent level theory predictions

* The combination of everything that has been known for a while

  * NLO QCD corrections: Ellis, Dawson, Nason; Beenakker, Kuijf, van Neerven, Smith '89

  * NLO EW corrections: Beenakker, Bernreuther, Denner, Fuecker, Hollik, Kao, Kollar, Kühn, Ladinsky, Mertig, Moretti, Nolten, Ross, Sack, Scharf, Si, Uwer, Wackenroth, Yuan

  * Threshold resummation and Coulomb corrections: Ahrens, Banfi, Berger, Bonciani, Catani, Contopanagos, Czakon, Ferroglia, Frixione, Kidonakis, Kiyo, Kühn, Laenen, Mangano, Mitov, Moch, Nason, Neubert, Pecjak Ridolfi, Steinhauser, Sterman, Uwer, Vogt, Yang

Yields a theoretical uncertainty ~10%

> To match theory and experimental accuracies at the LHC, cross sections for top pair production must be calculated through NNLO in pQCD

Top Pair Production At NNLO

* State of the art
  * Total NNLO cross section known (exact, all channels included) [Czakon, Fiedler, Mitov '13]
    * Applied to constraining high x gluon distribution [Czakon, Mangano, Mitov, Rojo '13]
  * Inclusive and differential Tevatron A_{FB} at NNLO [Czakon, Fiedler, Mitov '14]

* Our Goal: fully differential NNLO parton-level event generator that can efficiently compute all differential distributions for hadron colliders

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} \qquad X = p_T^t,\, p_T^{t^*},\, p_T^{t1},\, p_T^{t2},\, y^t,\, p_T^{t\bar{t}},\, m_{t\bar{t}},\, y^{t\bar{t}},\, \Delta\phi_{t\bar{t}}$$

* This talk: NNLO differential $t\bar{t}$ production in the $q\bar{q}$ channel (leading-color + fermionic)
  * Light quark (N_l) contributions computed in [GA, Gehrmann-De Ridder '14]
  * Leading-color. New [GA, Gehrmann-De Ridder, Majer '15]

Differential Top Pair Production In The $q\bar{q}$ Channel

* Cross section can be decomposed into color factors:

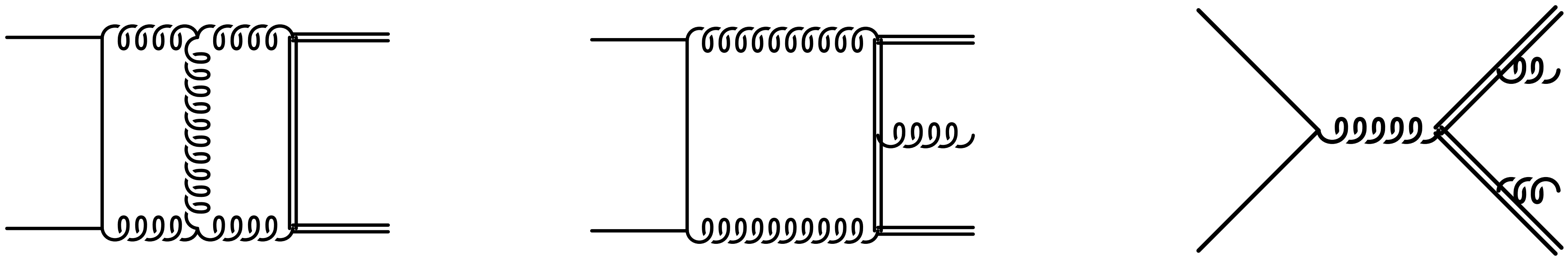
$$d\hat{\sigma}_{q\bar{q},\mathrm{NNLO}} = (N_c^2-1)\Big[N_c^2\,A + N_c\,B + C + \frac{D}{N_c} + N_l\left(N_c\,F_l + \frac{G_l}{N_c}\right) + \frac{E}{N_c^2} + N_h\left(N_c\,F_h + \frac{G_h}{N_c}\right) + N_l^2\,H_l + N_l\,N_h\,H_{lh} + N_h^2\,H_h\Big]$$

  * A : leading-color coefficient. New [GA, Gehrmann-De Ridder, Majer '15]
  * $F_l,\ G_l$: light quark contributions [GA, Gehrmann-De Ridder '14]
  * $F_h,\ G_h$: heavy quark contributions. In progress.
  * $B,\ D$: identical quark contributions. Only enter at RR level. In progress.
  * $H_l,\ H_h,\ H_{lh}$: IR finite. Only enter at VV level. New [GA, Gehrmann-De Ridder, Majer '15]
  * $C,\ E$: sub-leading-color terms.

* In this talk "leading-color (LC) + fermionic" means $A,\ F_l,\ G_l,\ H_l,\ H_h,\ H_{lh}$

Differential Top Pair Production In The $q\bar{q}$ Channel

* Analytic expressions for scattering amplitudes



  * $2\,\mathrm{Re}\left(\mathcal{M}^{2}_{q_1\bar{q}_2\to t_3\bar{t}_4}\mathcal{M}^{0\,\dagger}_{q_1\bar{q}_2\to t_3\bar{t}_4}\right)$ [Bonciani, Ferroglia, Gehrmann, Maître, v. Manteuffel, Studerus]
  * $|\mathcal{M}^{1}_{q_1\bar{q}_2\to t_3\bar{t}_4}|^2$
  * $2\,\mathrm{Re}\left(\mathcal{M}^{1}_{q_1\bar{q}_2\to t_3\bar{t}_4 g_5}\mathcal{M}^{0\,\dagger}_{q_1\bar{q}_2\to t_3\bar{t}_4 g_5}\right)$ obtained from OpenLoops [Cascioli, Maierhöfer,Pozzorini]
  * $|\mathcal{M}^{0}_{q_1\bar{q}_2\to t_3\bar{t}_4 g_5 g_6}|^2$, $|\mathcal{M}^{0}_{q_1\bar{q}_2\to t_3\bar{t}_4 q'_5\bar{q}'_6}|^2$

* Method needed to extract and cancel infrared divergences that plague partonic cross sections

$$\mathrm{d}\hat{\sigma}_{\mathrm{NNLO}} = \int_{\Phi_4}\mathrm{d}\hat{\sigma}^{\mathrm{RR}}_{\mathrm{NNLO}} + \int_{\Phi_3}\left(\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}} + \mathrm{d}\hat{\sigma}^{\mathrm{MF,1}}_{\mathrm{NNLO}}\right) + \int_{\Phi_2}\left(\mathrm{d}\hat{\sigma}^{\mathrm{VV}}_{\mathrm{NNLO}} + \mathrm{d}\hat{\sigma}^{\mathrm{MF,2}}_{\mathrm{NNLO}}\right)$$

  * Explicit poles from loop integration
  * Implicit singularities from phase space integration over single and double unresolved real emissions

Antenna Subtraction At NNLO

* Construct counter-terms $d\hat{\sigma}^{S}_{NNLO}$, $d\hat{\sigma}^{T}_{NNLO}$ and $d\hat{\sigma}^{U}_{NNLO}$

$$\begin{aligned} d\hat{\sigma}_{NNLO} &= \int_{\Phi_4} \left[d\hat{\sigma}^{RR}_{NNLO} - d\hat{\sigma}^{S}_{NNLO} \right] \\ &+ \int_{\Phi_3} \left[d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T}_{NNLO} \right] \\ &+ \int_{\Phi_2} \left[d\hat{\sigma}^{VV}_{NNLO} - d\hat{\sigma}^{U}_{NNLO} \right] \end{aligned}$$

  * $d\hat{\sigma}^{S}_{NNLO}$, $d\hat{\sigma}^{T}_{NNLO}$ approximate matrix elements in unresolved limits

$$d\hat{\sigma}^{S,T}_{NNLO} \xrightarrow{\forall \{j,k\},\{j\} \text{ unresolved}} d\hat{\sigma}^{RR,RV}_{NNLO}$$

  * All explicit poles are cancelled analytically

$$\mathcal{P}oles\left(d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T}_{NNLO} \right) = 0$$

$$\mathcal{P}oles\left(d\hat{\sigma}^{VV}_{NNLO} - d\hat{\sigma}^{U}_{NNLO} \right) = 0$$

* Content of square brackets is finite and regular. Phase space integration can be done numerically in d=4.

Antenna Subtraction At NNLO

- Building blocks for subtraction terms:
  - Antenna functions X_3^0, X_4^0, X_3^1
    - Constructed from ratios of physical matrix elements
    - Smoothly interpolate all unresolved limits of a cluster of color-connected partons

$$X_3^0(i,k,l) \xrightarrow{p_j \to 0} \mathcal{S}(i,j,k) \qquad X_3^0(i,j,k) \xrightarrow{p_i || p_j} \frac{1}{s_{ij}} P_{ij\to l}(z) \qquad X_3^0(i,j,k) \xrightarrow{p_j || p_k} \frac{1}{s_{kj}} P_{jk\to l}(z)$$

  - $3 \to 2$ and $4 \to 2$ on-shell phase space mappings for reduced matrix elements
  - Phase space factorizations to define integrated subtraction terms

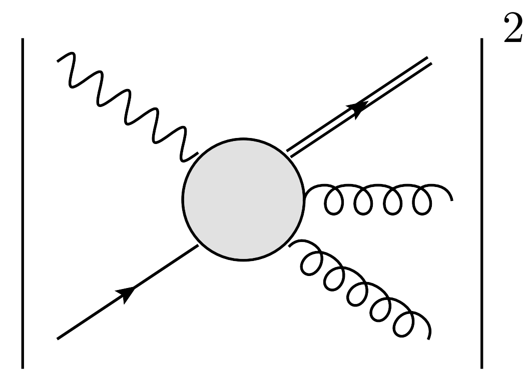
- Initial-state colored particles $\Longrightarrow$ Final-final, initial-final, initial-initial antennae, mappings and phase space factorizations

- Challenge: extend NNLO antenna subtraction method to treat massive quarks.
  - Generalize phase space mappings and factorizations [G.A., Gehrmann-De Ridder '11]
  - Compute and integrate massive antennae and convolutions. For $q\bar{q} \to t\bar{t} + X$
    - X_3^0, $\mathcal{X}_3^0$ [Gehrmann-De Ridder, Ritzmann '09; GA, Gehrmann-De Ridder '11]
    - $B^0_{Qq\bar{q}\bar{Q}}$, $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}$ [Bernreuther, Bogner, Dekkers '11]
    - $B^0_{q,Qq'\bar{q}'}$, $\mathcal{B}^0_{q,Qq'\bar{q}'}$ [GA, Dekkers, Gehrmann-De Ridder '12]
    - $A^0_{q,Qgg}$, $\mathcal{A}^0_{q,Qgg}$, $A^1_{q,Qg}$, $\mathcal{A}^1_{q,Qg}$, $[\Gamma^1_{qq} \otimes \mathcal{A}^0_{q,Qg}]$ New [GA, Gehrmann-De Ridder, Majer '15]

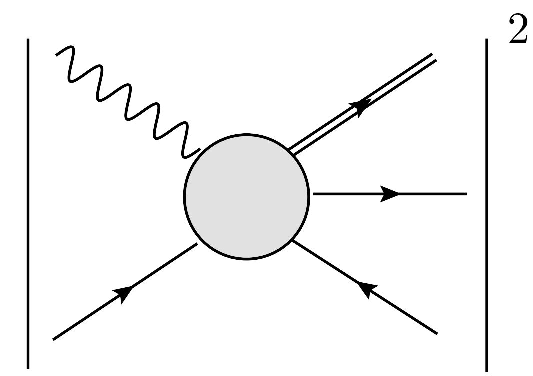
Initial-Final NNLO Massive Antennae

- Subtraction terms for $q\bar{q} \to t\bar{t} + X$ only require NNLO quark-antiquark antennae.
  - Derived from (crossed) matrix elements for processes $\gamma^* \to q\bar{q} +$ partons
- In particular, we need initial-final antennae with one massive final-state quark

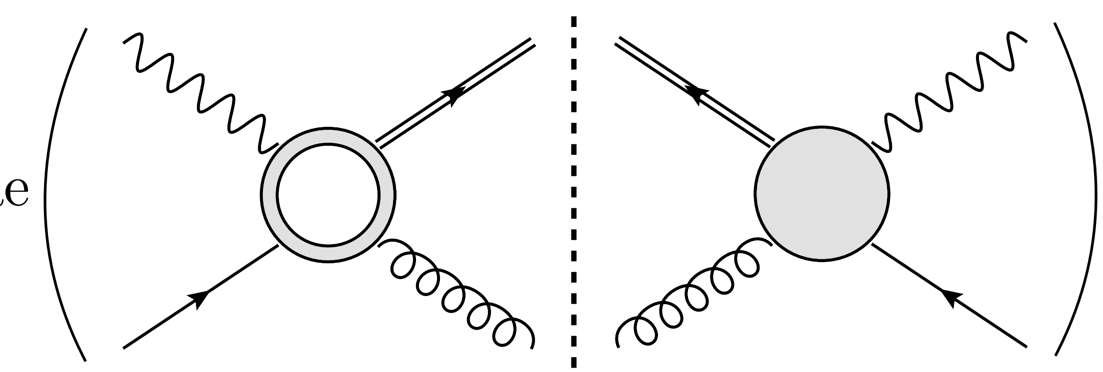
Four-parton tree-level $A^0_{q,Qgg} \sim$



$B^0_{q,Qq'\bar{q}'} \sim$



Three-parton one-loop $A^1_{q,Qg} \sim \mathrm{Re}$



(Note flavor-violating vertex $Q\gamma^*\bar{q}$)

Integrated Initial-Final NNLO Massive Antennae

Integrated initial-final antennae defined as inclusive phase space integrals:

$$\mathcal{X}^0_{i,jkl} = \frac{1}{C(\epsilon)} \frac{(Q^2+m_Q^2)}{2\pi} \int \mathrm{d}\Phi_3(p_j,p_k,p_l;p_i,q) X^0_{i,jkl}$$

$$\mathcal{X}^1_{i,jk} = \frac{1}{C(\epsilon)} \frac{(Q^2+m_Q^2)}{2\pi} \int \mathrm{d}\Phi_2(p_j,p_k;p_i,q) X^1_{i,jk}$$

- DIS-like $2 \to 2\,(3)$ kinematics with a massive final-state particle

$$p_i + q \to p_j + p_k(+p_l) \qquad p_i^2 = p_j^2 (= p_l^2) = 0 \quad p_k^2 = m_Q^2 \quad q^2 = -Q^2 < 0$$

- Three-scale problem: Q^2, m_Q^2, $p_i \cdot q$
  - Trade dependence on m_Q^2, $p_i \cdot q$ for dimensionless x_0, x

$$\underbrace{x_0 = \frac{Q^2}{Q^2+m_Q^2}}_{\text{Parametrizes mass dependence}} \qquad \underbrace{x = \frac{Q^2+m_Q^2}{2p_i \cdot q}}_{\text{Bjorken x}} \qquad \Longrightarrow \qquad \mathcal{X} = \mathcal{X}(Q^2, x, x_0, \epsilon)$$

Integrated Initial-Final NNLO Massive Antennae

- Integrated integrals computed analytically using reverse unitarity:
  - Express phase space integrals as cuts of two-loop four point functions in forward scattering kinematics with two off-shell legs. Reduce to master integrals.
- Singular factors $(1-x)^{m-n\epsilon}$ kept unexpanded in masters integrals

$$I_\alpha(x,x_0,\epsilon) = \sum_n (1-x)^{m-n\epsilon} \underbrace{R_\alpha^{(n)}(x,x_0,\epsilon)}_{\text{Regular as } x\to 1}$$

  - Single power of $(1-x)$ in pure phase space integrals for $\mathcal{X}^0_{i,jkl}$
  - In general, multiple powers of $(1-x)$ in mixed loop and phase space integrals for $\mathcal{X}^1_{i,jk}$
- Integrated antennae take the form

$$\mathcal{X}(x,x_0,\epsilon) = \sum_n (1-x)^{-1-n\epsilon} \underbrace{\mathcal{R}_{\mathcal{X}}^{(n)}(x,x_0,\epsilon)}_{\text{Regular as } x\to 1}$$

- Singular factors expanded in distributions

$$(1-x)^{-1-n\epsilon} = -\frac{\delta(1-x)}{n\epsilon} + \sum_{m=0}^{\infty} \frac{(-n\epsilon)^m}{m!}\mathcal{D}_m(x) \qquad \mathcal{D}_m(x) = \left(\frac{\ln(1-x)}{1-x}\right)_+$$

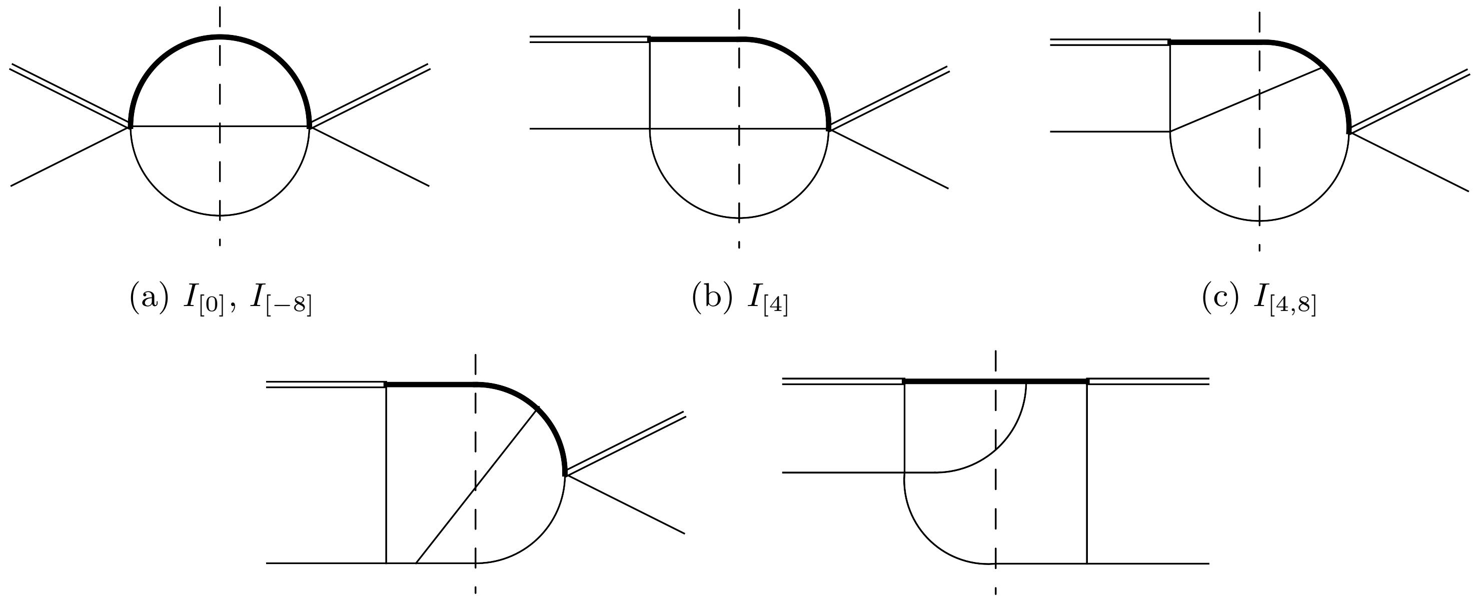
Integrated Initial-Final NNLO Massive Antennae

- Integrated NNLO massive initial-final antennae are distributions in x
- Poles starting at $1/\epsilon^4$
- Expressed in terms of HPLs and genuine GPLs with trascendentality up to 3 and 4 respectively. Re-written in terms of Logs and Li_n (n=2,3,4) for numerical implementation (no Li_{22} needed)

$$
\begin{aligned}
\mathcal{A}^0_{q,Qgg}(\epsilon, Q^2+m_Q^2, x, x_0) &= (Q^2+m_Q^2)^{-2\epsilon} \\
&\times\left\{ \frac{1}{4\epsilon^4}\delta(1-x) + \frac{1}{2\epsilon^3}\left[1+x+\delta(1-x)\left(\frac{35}{24}+G(1;x_0)\right) - 2\mathcal{D}_0(x)\right]\right. \\
&+\frac{1}{\epsilon^2}\left[\frac{(11x_0^2x^3+59x_0^2x^2-22x_0x^2-118x_0x+2x+68)}{24(1-x_0x)^2} - \frac{(9+11x^2)}{8(1-x)}G(0;x)\right. \\
&-2(1+x)G(1;x) + \frac{(7-x^2)}{4(1-x)}G(1;x_0) + \frac{3(1+x^2)}{4(1-x)}G\left(\frac{1}{x};x_0\right) \\
&+\delta(1-x)\left(\frac{331}{144}-\frac{13\pi^2}{48}+\frac{35}{24}G(1;x_0)+G(1,1;x_0)\right) - \mathcal{D}_0(x)\left(\frac{35}{12}+2G(1;x_0)\right) \\
&\left.\left.+4\mathcal{D}_1(x)\right] + \mathcal{O}(\epsilon^{-1})\right\}.
\end{aligned}
$$

Integrated Initial-Final NNLO Massive Antennae

Master integrals for $\mathcal{A}^0_{q,Qgg'}$, $\mathcal{B}^0_{q,Qq'\bar{q}'}$



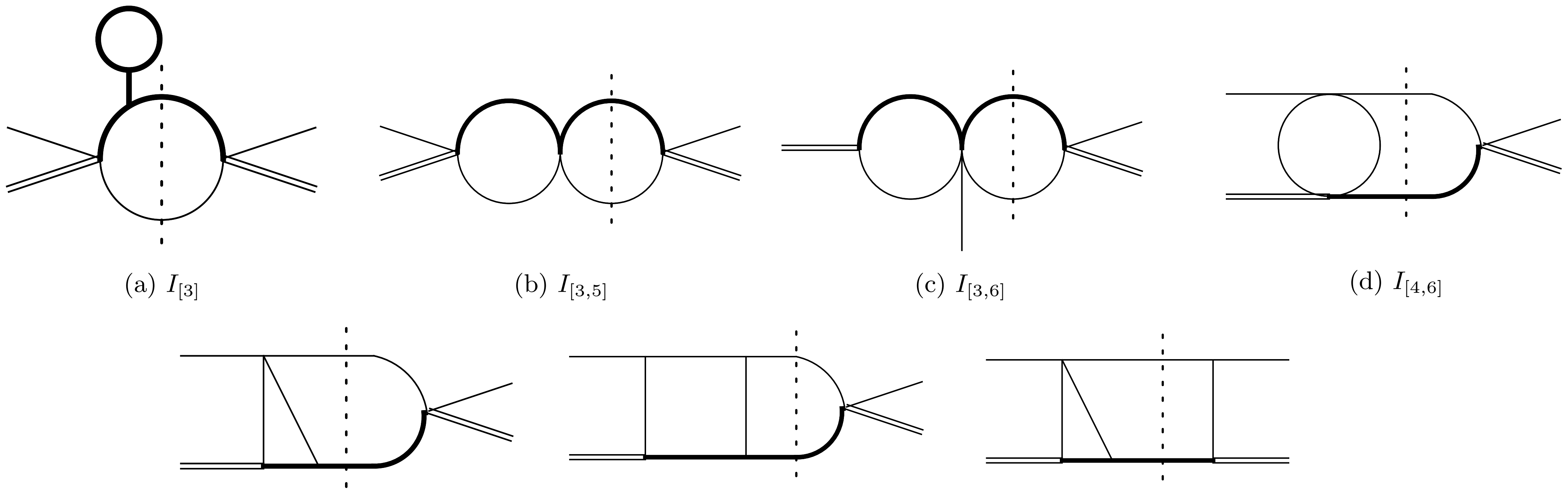
(a) $I_{[0]}$, $I_{[-8]}$ (b) $I_{[4]}$ (c) $I_{[4,8]}$

(d) $I_{[4,5,8]}$ (e) $I_{[4,7,8]}$

- $I_{[0]}$, $I_{[-8]}$, $I_{[4]}$ known [Gehrmann-De Ridder, Ritzmann '09; GA, Dekkers, Gehrmann-De Ridder '12]
- $I_{[4,8]}$, $I_{[4,5,8]}$, $I_{[4,7,8]}$ new. Computed with diff. eqs. [GA, Gehrmann-De Ridder, Majer '15]
  - Diff. eqs. only decouple order by order in ϵ
  - Integration constants fixed by
    - Demanding regularity of $I_{[4,8]}$, $I_{[4,7,8]}$ in soft limit $x \to 1$ after factoring out singular factor $(1-x)^{-4\epsilon}$ coming from phase space. Need to go one order higher in ϵ
    - Direct all-order evaluation of $I_{[4,7,8]}$ in the soft limit. New parametrization of DIS-like phase space with a massive final-state particle

Integrated Initial-Final NNLO Massive Antennae

Master integrals for $\mathcal{A}^1_{q,Qg}$. All new [GA, Gehrmann-De Ridder, Majer '15]



(a) $I_{[3]}$ (b) $I_{[3,5]}$ (c) $I_{[3,6]}$ (d) $I_{[4,6]}$

(e) $I_{[3,4,6]}$ (f) $I_{[3,4,5,6]}$ (g) $I_{[3,4,6,7]}$

- $I_{[3]}$, $I_{[3,5]}$, $I_{[3,6]}$, $I_{[4,6]}$ evaluated directly using all-order expressions of underlying loop-integrals and phase space parametrization in [GA, Gehrmann-De Ridder '11]
- $I_{[3,4,6]}$, $I_{[3,4,5,6]}$, $I_{[3,4,6,7]}$ computed with differential equations.
  - Integration constants fixed with independently derived soft limits. Need soft limits of underlying loop integrals
    - $I_{[3,4,5,6]}$ soft limit of box integral known [Brucherseifer, Caola, Melnikov '13]
    - $I_{[3,4,6]}$, $I_{[3,4,6,7]}$ soft limit of one-loop triangle derived with Mellin-Barnes expansion

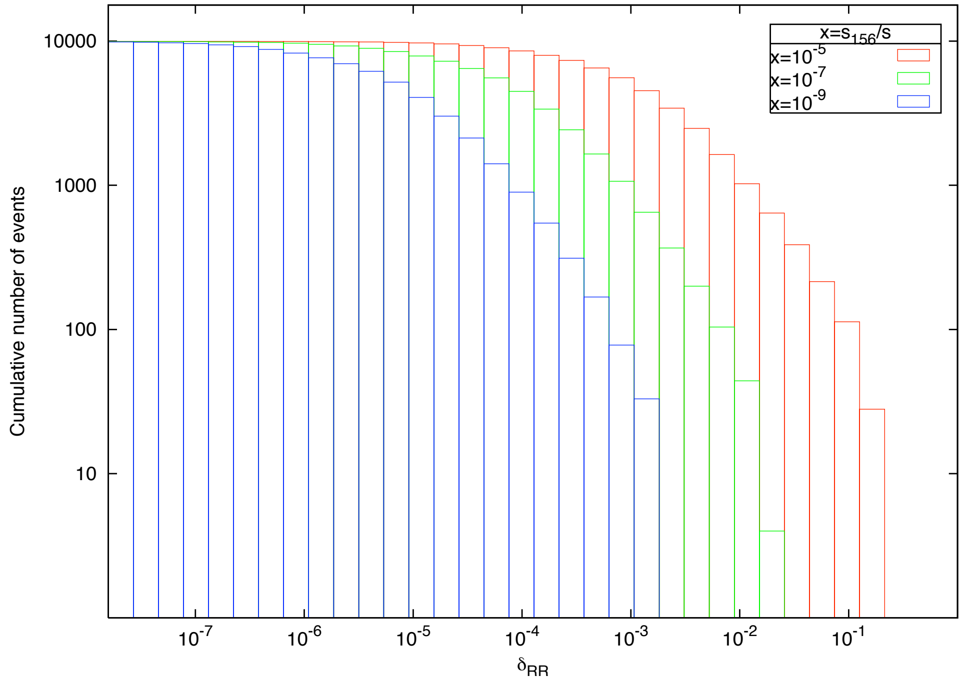
Implementation Checks

* Check of convergence [GA, Gehrmann-De Ridder, Maierhöfer, Pozzorini '14]
  * Generate events near every singular region of $\mathrm{d}\hat{\sigma}^{\mathrm{RR}}_{\mathrm{NNLO}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}}$
  * Control proximity to singularities with a control variable x (specific to each limit)
  * For each event, compute

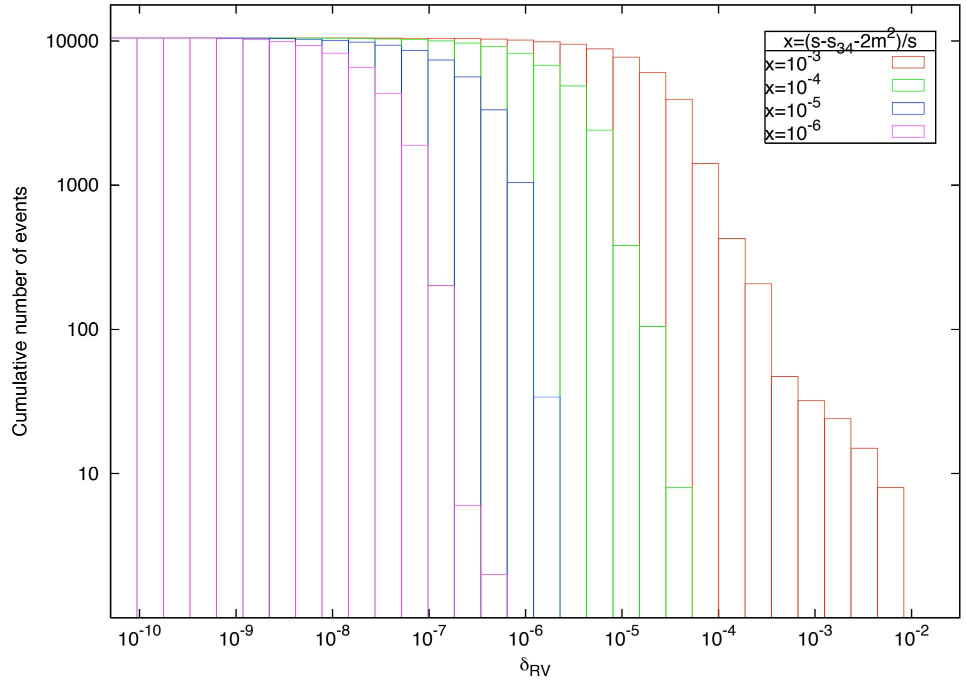
$$\delta_{\mathrm{RR}} = \left| \frac{\mathrm{d}\hat{\sigma}^{\mathrm{RR}}_{\mathrm{NNLO}}}{\mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NNLO}}} - 1 \right| \qquad \delta_{\mathrm{RV}} = \left| \frac{\mathcal{F}inite(\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}})}{\mathcal{F}inite(\mathrm{d}\hat{\sigma}^{\mathrm{T}}_{\mathrm{NNLO}})} - 1 \right|$$

  * Good convergence of $\mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NNLO}}$ ($\mathrm{d}\hat{\sigma}^{\mathrm{T}}_{\mathrm{NNLO}}$) to $\mathrm{d}\hat{\sigma}^{\mathrm{RR}}_{\mathrm{NNLO}}$ ($\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}}$) observed in cumulative histograms in δ_{RR} (δ_{RV})

Triple collinear limit RR (LC)



Single soft limit RV (LC)

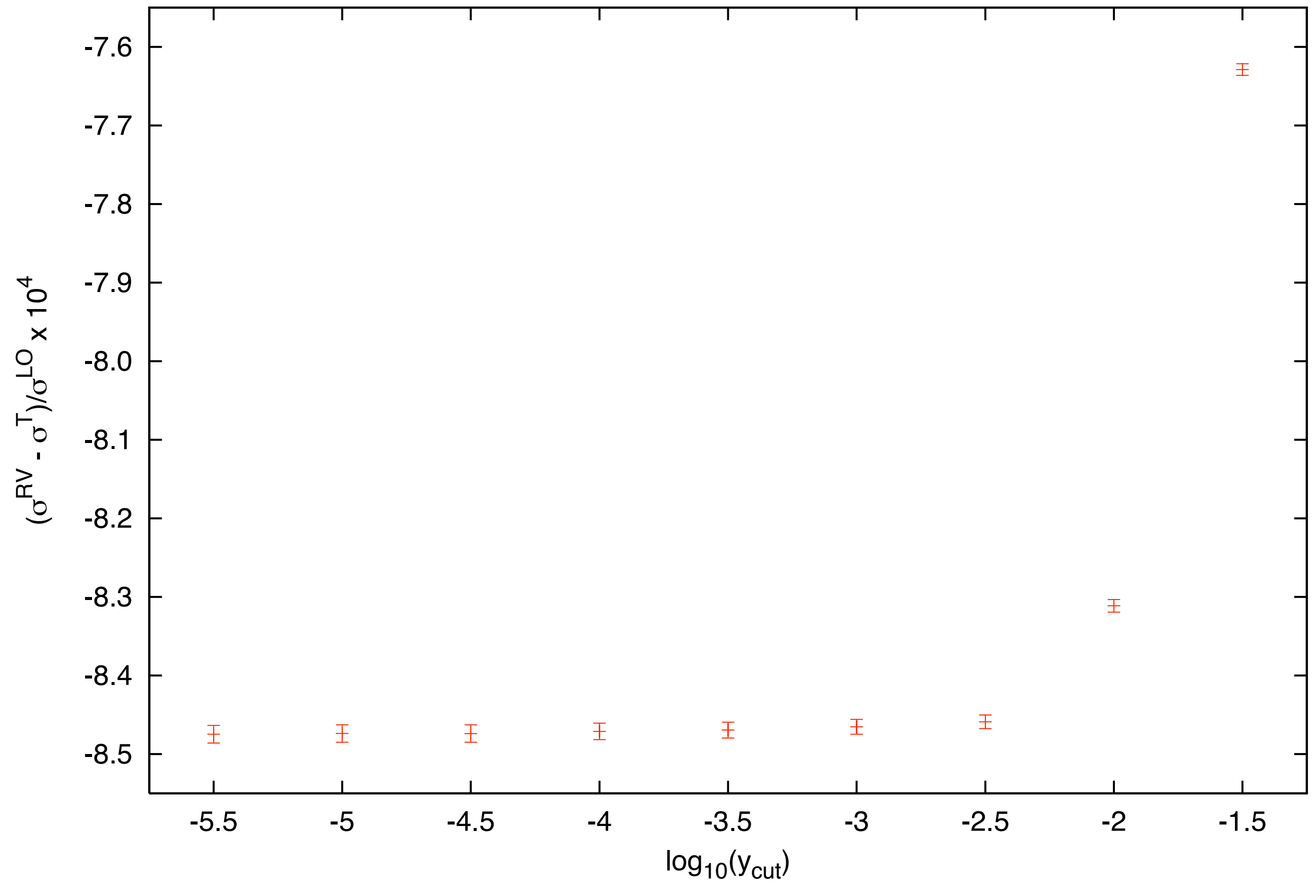


Implementation Checks

- Precision test in real-virtual contributions [GA, Gehrmann-De Ridder, Maierhöfer, Pozzorini '14]
  - Only "bad points" are (re)evaluated by OpenLoops in quadruple precision
    - Fraction of quadruple precision evaluations in $\int_{d\Phi_3}\left(d\hat{\sigma}^{RV}_{NNLO} - d\hat{\sigma}^{T}_{NNLO}\right)$?
    - Is the integration stable?

Check: Evaluate $R = \left(\sigma^{RV}_{NNLO} - \sigma^{T}_{NNLO}\right)/\sigma_{LO}$ as a function of $y_{cut} = p_T^g/\sqrt{\hat{s}}$

- Integration is stable
- R has a plateau for $y_{cut} < y_{cut}^{max} \sim 10^{-3}$
- Strong check of our subtraction terms
- We can run with $y_{cut} \sim 10^{-4}$. Only ~0.01% points require quadruple precision.
- Efficient evaluation in double precision for the vast majority of points



Implementation Checks

* Pointwise cancellation of explicit IR poles check analytically for leading-color and light-quark contributions [GA, Gehrmann-De Ridder '14; GA, Gehrmann-De Ridder, Majer '15]

$$\boxed{\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}} - \mathrm{d}\hat{\sigma}^{\mathrm{T}}_{\mathrm{NNLO}}\right) = 0}$$

$$\boxed{\mathcal{P}oles\left(\mathrm{d}\hat{\sigma}^{\mathrm{VV}}_{\mathrm{NNLO}} - \mathrm{d}\hat{\sigma}^{\mathrm{U}}_{\mathrm{NNLO}}\right) = 0}$$

```
Pole1LC =
 Simplify[
  Simplify[Coefficient[TwoLoopPolesLC, ep, -1] Delta[1 - x1] Delta[1 - x2] + Coefficient[OneTimesOneLoopPolesLC, ep, -1] Delta[1 - x1] Delta[1 - x2] -
     Coefficient[SubTermLC, ep, -1]] //. RepsLogsLC]
0
```

* Non-trivial check on new integrated massive antennae
* Proves applicability of NNLO antenna subtraction to reactions with massive fermions

(Note importance of analytic expressions for matrix elements in this check)

Numerical Implementation

Fully differential event generator written in Fortran

* LO, NLO: all channels, all color factors
* NNLO: so far only $q\bar{q}$ channel, leading-color + fermionic contributions

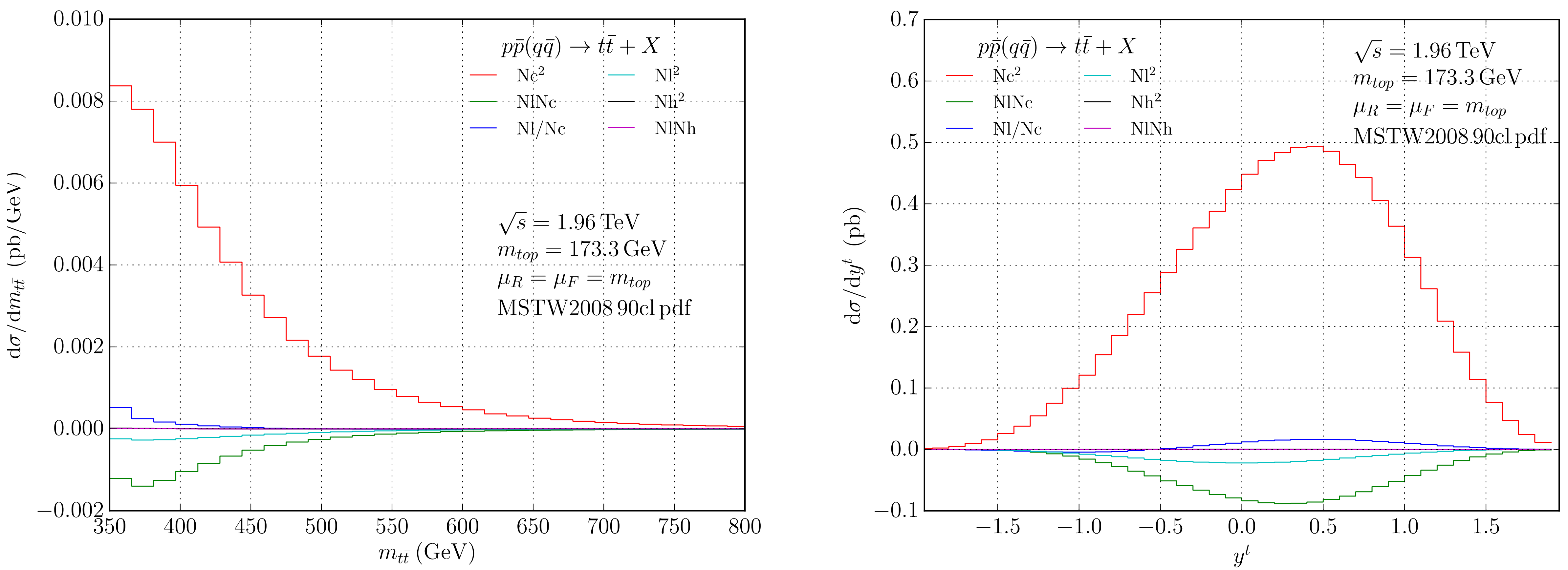
Runtime for NNLO contributions on 176 cores per choice of $\{\sqrt{s},\, m_{top},\, \mu, \text{PDF set}\}$

	Warm up (single core)	Production run (parallel)
$\int_{\Phi_4}\left(\mathrm{d}\hat{\sigma}^{\mathrm{RR}}_{\mathrm{NNLO}} - \mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NNLO}}\right)$	4 hours	5 hours
$\int_{\Phi_3}\left(\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}} - \mathrm{d}\hat{\sigma}^{\mathrm{T}}_{\mathrm{NNLO}}\right)$	7.5 hours	6 hours
$\int_{\Phi_2}\mathrm{d}\hat{\sigma}^{\mathrm{VV}}_{\mathrm{NNLO}}$	10 hours	4 hours
$\int_{\Phi_2}\mathrm{d}\hat{\sigma}^{\mathrm{U}}_{\mathrm{NNLO}}$	4 hours	3 hours

A couple of (unrelated) remarks:

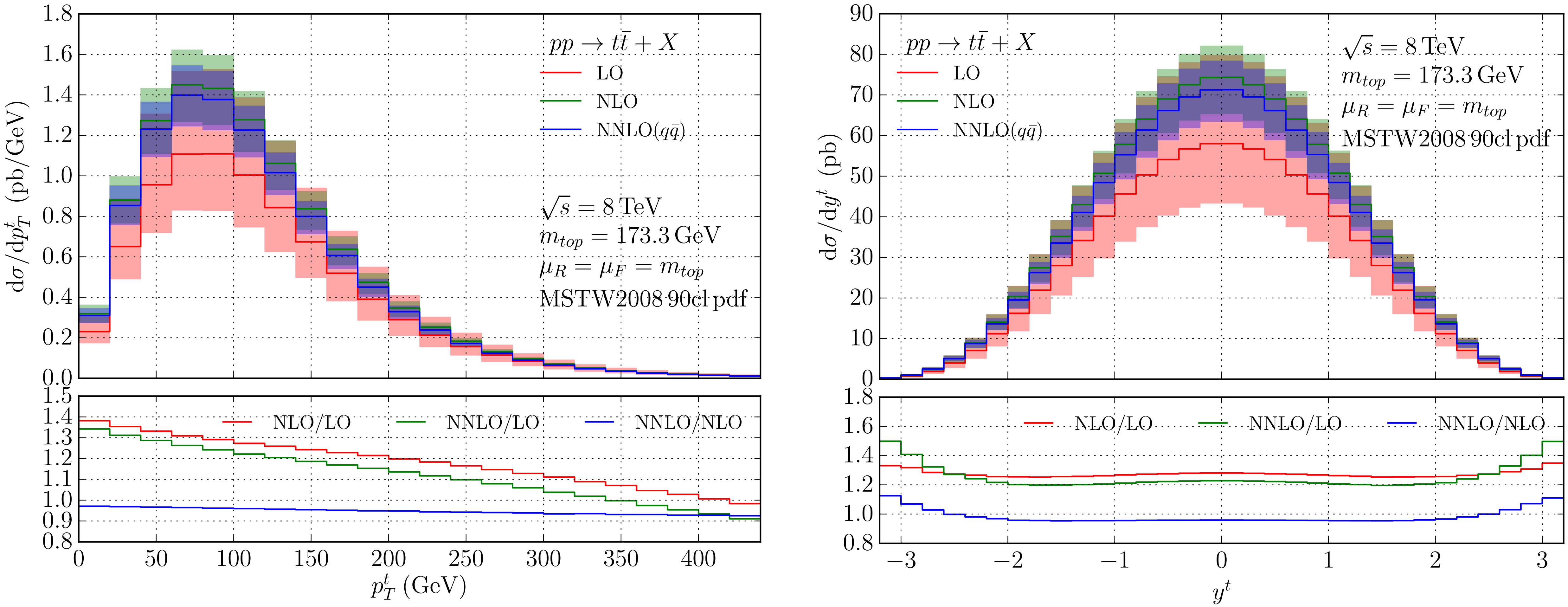
* All distributions computed in a single run
* Improved numerical stability and performance in $\mathrm{d}\hat{\sigma}^{\mathrm{VV}}_{\mathrm{NNLO}}$ with threshold expansion of matrix elements

Results: Breakdown Of NNLO Corrections Into Color Factors



Contributions to NNLO corrections from different color factors included in our calculation

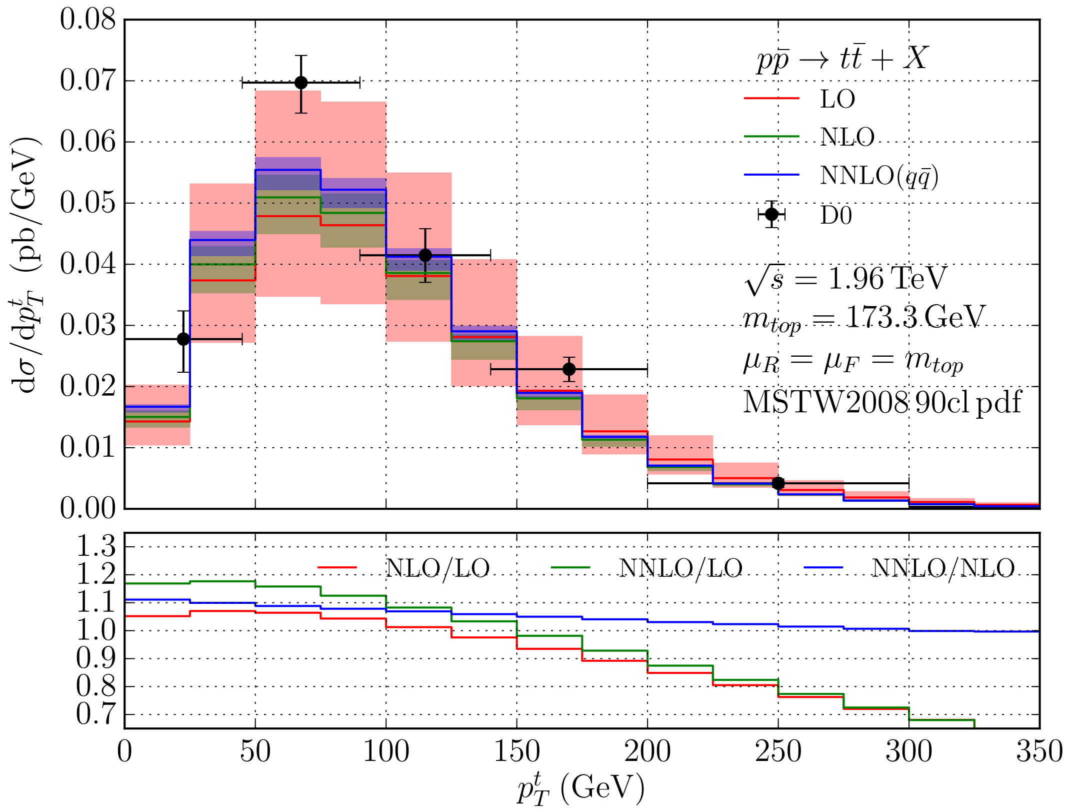
Results: LHC 8 TeV

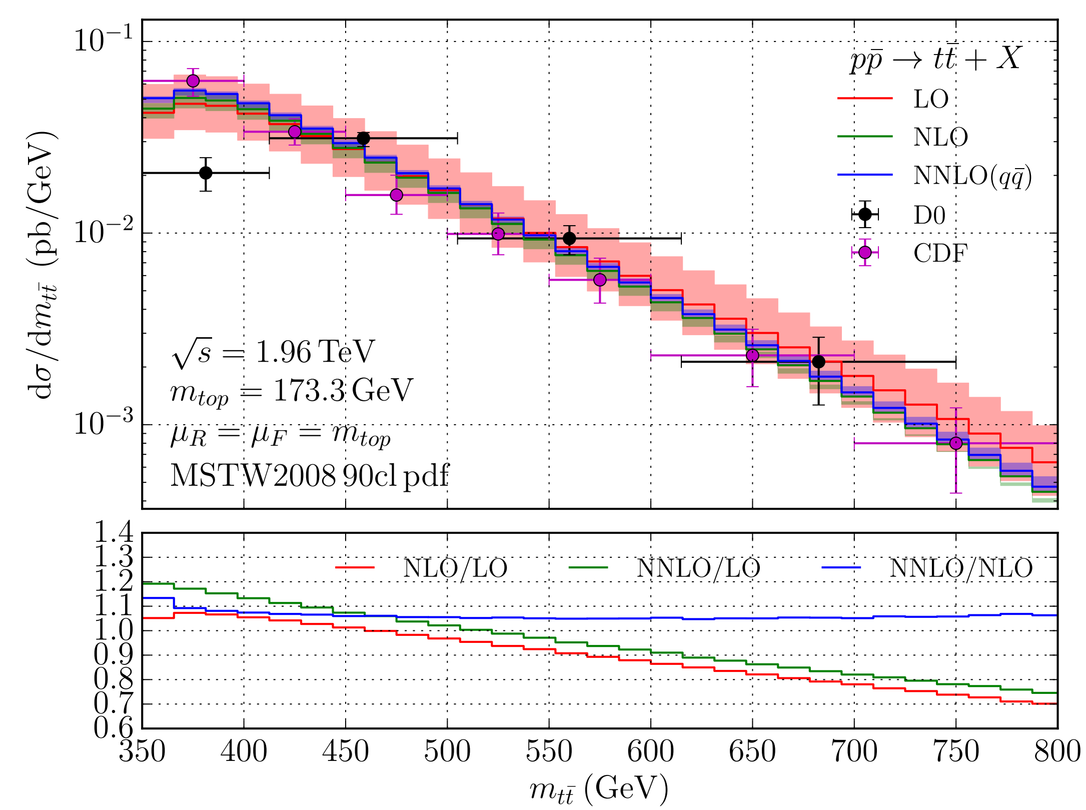


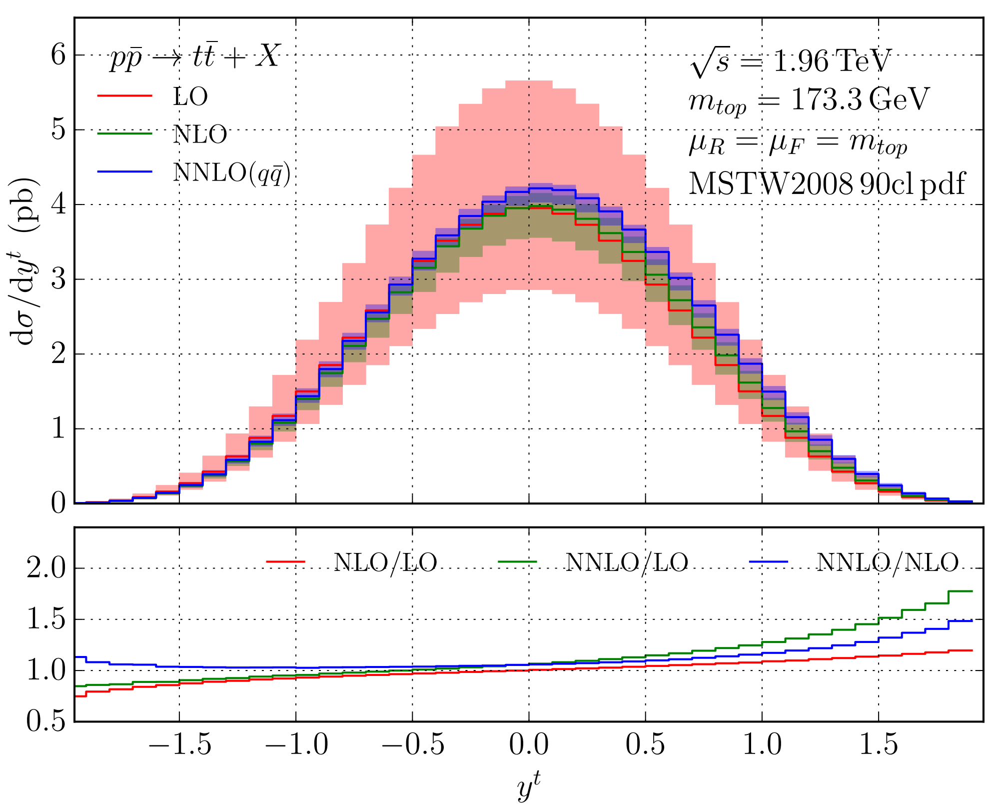
* As expected, mild impact of NNLO corrections to the $q\bar{q}$ channel (leading-color + fermionic), except in very forward and backward regions of the rapidity spectrum

* Slight reduction in scale dependence

Results: Tevatron







- More pronounced impact of NNLO corrections to the $q\bar{q}$ channel (leading + fermionic) in all distributions
- Substantial reduction in scale dependence

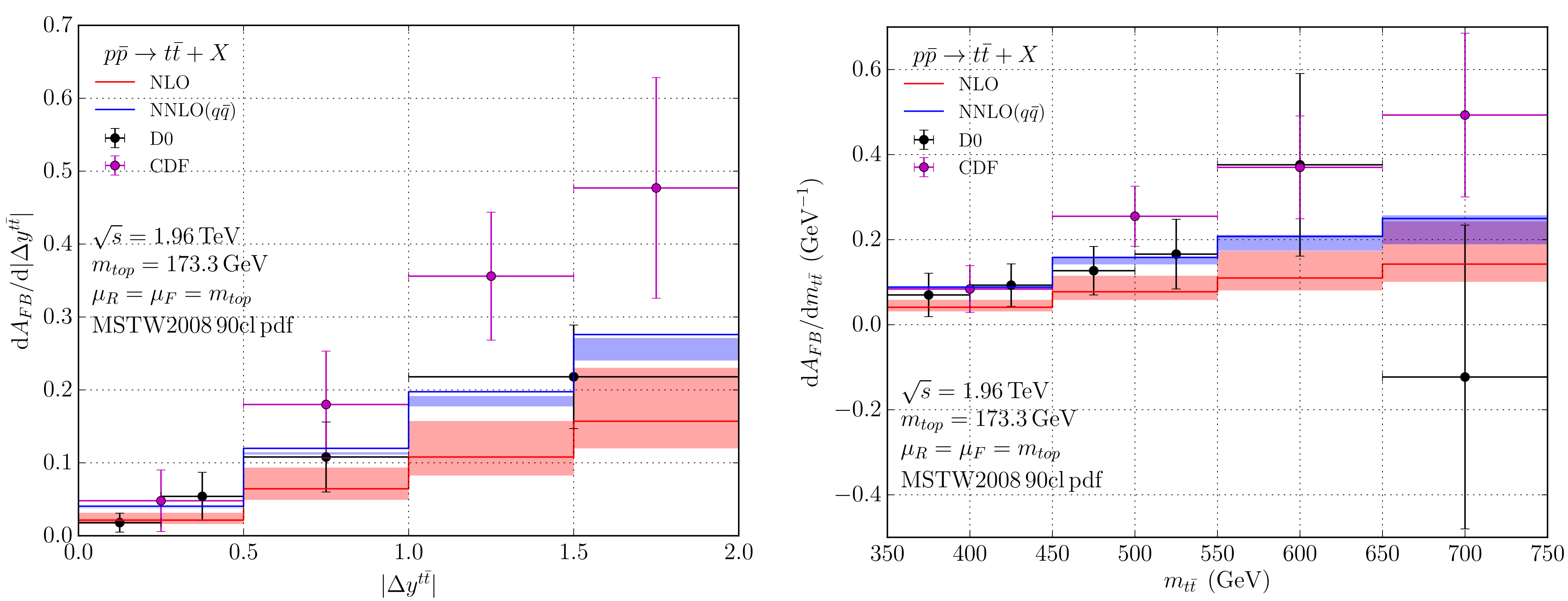
Results: Differential A_{FB} At Tevatron

Radiative corrections in the $q\bar{q}$ and qg channels induce an asymmetry between the number of top quarks produced forwards and backwards in $p\bar{p}$ collisions

* Experimentally measured as $A_{FB} = \dfrac{\sigma(\Delta y_+^{t\bar{t}}) - \sigma(\Delta y_-^{t\bar{t}})}{\sigma(\Delta y_+^{t\bar{t}}) + \sigma(\Delta y_-^{t\bar{t}})}$ $\quad \Delta y_\pm^{t\bar{t}} = \theta(\pm\Delta y^{t\bar{t}})$

* NLO: first non-vanishing order. NNLO: first correction

$$\text{Unexpanded form: } A_{FB} = \frac{\alpha_s^3 N_3 + \alpha_s^4 N_4 + \mathcal{O}(\alpha_s^5)}{\alpha_s^2 D_2 + \alpha_s^3 D_3 + \alpha_s^4 D_4 + \mathcal{O}(\alpha_s^5)}$$



Summary And Outlook

Summary

- We computed the NNLO corrections to $q\bar{q} \to t\bar{t} + X$ with antenna subtraction including leading-color and most fermionic contributions
  - All necessary massive antennae computed and integrated
  - Subtraction terms derived and tested
    - Verified convergence of $\mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NNLO}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{T}}_{\mathrm{NNLO}}$ to $\mathrm{d}\hat{\sigma}^{\mathrm{RR}}_{\mathrm{NNLO}}$ and $\mathrm{d}\hat{\sigma}^{\mathrm{RV}}_{\mathrm{NNLO}}$
    - Demonstrated analytic cancellation of all IR singularities
- We constructed a fully differential parton-level event generator
  - All differential distributions can be efficiently obtained in a single run

Outlook

- Complete remaining fermionic contributions: N_h and identical-quark
- Complete remaining partonic channels: gg, qg, qq'