FIRE5: a C++ implementation of Feynman Integral REduction

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FIRE5: a C++ implementation of Feynman Integral REduction
The goal of this talk is to present the new version FIRE.

- **FIRE** - Feynman Integral REduction

FIRE can be downloaded from [http://git.sander.su/fire](http://git.sander.su/fire)
Feynman integrals

- Feynman integrals over loop momenta:

\[ F(a_1, \ldots, a_n) = \int \cdots \int \frac{d^d k_1 \ldots d^d k_h}{E_1^{a_1} \ldots E_n^{a_n}}. \]

- Currently one needs to evaluate millions of Feynman integrals with different indices \( a_i \) corresponding to a particular diagram, so evaluating each of them analytically turns into an unreal task.
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Multiple programs for Feynman integral reduction

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- FIRE
- Reduze
- LiteRed
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Versions of FIRE

- FIRE1, FIRE2 — mythical versions that existed only in private, based mostly on Gröbner bases
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- FIRE5 — Mathematica and C++
<table>
<thead>
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<th>2</th>
<th>3</th>
<th>4</th>
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<td>1453</td>
<td>2061</td>
<td>3958</td>
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<tr>
<td>C++, 1 thread, disk mode</td>
<td>134</td>
<td>168</td>
<td>255</td>
<td>552</td>
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<tr>
<td>C++, 4 threads, disk mode</td>
<td>76</td>
<td>100</td>
<td>163</td>
<td>323</td>
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<tr>
<td>C++, 1 thread, RAM mode</td>
<td>107</td>
<td>136</td>
<td>210</td>
<td>473</td>
</tr>
<tr>
<td>C++, 4 threads, RAM mode</td>
<td>49</td>
<td>74</td>
<td>108</td>
<td>237</td>
</tr>
</tbody>
</table>

**Table**: Timings for the on-shell massless double box example, $n$ is the total power of irreducible numerators
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- Either download a binary package from http://git.sander.su/fire/downloads and hope that it works for your system
- Or download the source with git and build it yourself (this also gives you access to dev versions of FIRE)

Building from sources requires KyotoCabinet and Snappy libraries, but they are shipped with FIRE.
Installation from sources:

- git clone https://bitbucket.org/feynmanIntegrals/fire.git
  && cd fire/FIRE5
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- `make dep` (to build the dependencies)
- `make`
- To download the latest version: `git pull`
- Switch to dev version: `git checkout dev`
Structure of FIRE

- The Mathematica part FIRE5.m — used as the main frontend to provide input and to analyze output
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- bin/FLink — auxiliary binary used to run fermat from Mathematica
0. Loading FIRE in Mathematica

SetDirectory[<path to the folder with FIRE>];
Get["FIRE5.m"];

or
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SetDirectory[<path to the folder with FIRE>];
Get["FIRE5.m"];

or

FIREPath = <path to the folder with FIRE>;
Get[FIREPath<>"FIRE5.m"];
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Usage of FIRE
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- \texttt{External} = \{p1, p2, p3\};
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- Internal = \{k1, k2\};
- External = \{p1, p2, p3\};
- Propagators = \{-k1^2, -(k1 + p1 + p2)^2, -k2^2, - (k2 + p1 + p2)^2, -(k1 + p1)^2, -(k1 - k2)^2, -(k2 - p3)^2, -(k2 + p1)^2, -(k1 - p3)^2\};
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- PrepareIBP[]; Prepare[AutoDetectRestrictions \rightarrow True]; SaveStart["doublebox"];
1. Reduction in Mathematica

- `LoadStart["doublebox",1]; Burn[];`
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- `LoadStart["doublebox",1]; Burn[];
  You can load multiple start files a time!
- `F[1, {1, 1, 1, 1, 1, 1, 1, -1, -1}]`
1. Reduction in Mathematica

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- F[1, {1, 1, 1, 1, 1, 1, 1, -1, -1}] or a better way for multiple integrals:

- EvaluateAndSave[{{1, {1, 1, 1, 1, 1, 1, 1, -1, -1}}},
  {1, {1, 1, 1, 1, 1, 1, 1, 0, -2}}],"doublebox.tables"]
1. Reduction in Mathematica

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  {1, {1, 1, 1, 1, 1, 1, 1, 0, -2}}},"doublebox.tables"]
  Later load a new kernel, load the start file and...
- LoadTables["doublebox.tables"];
FIRE automatically starts the reduction from the top-level sectors, then lower and lower (already with the knowledge of what integrals are required at this level). Then it performs the backward-substitutions;
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The same is true for the C++ FIRE, however it can also work in parallel with sectors of the same level (the number of positive indices).
2. Finding equivalents between master integrals

During the main reduction phase FIRE cannot find equivalents between master integrals in different sectors. MasterIntegrals[]
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MasterIntegrals[]

{{1, {0, 0, 0, 0, 1, 1, 1, 0, 0}}, {1, {0, 0, 1, 1, 1, 0, 0, 0}}, {1, {0, 1, 1, 0, 0, 1, 0, 0}}, {1, {1, 0, 0, 1, 0, 1, 0, 0}}, {1, {1, 0, 0, 1, 1, 1, 0, 0}}, {1, {1, 1, 0, 0, 1, 1, 0, 0}}, {1, {1, 1, 0, 0, 1, 1, 0, 0}}, {1, {1, 1, 1, 1, 1, 1, 0, 0}}, {1, {1, 1, 1, 1, 1, 1, 0, 0}}, {1, {1, 1, 1, 1, 1, 1, 0, 0}}}}
2. Finding equivalents between master integrals

- **Internal** = \{k_1, k_2\};

- **External** = \{p_1, p_2, p_3\};

- **Propagators** = \{-k_1^2, -(k_1 + p_1 + p_2)^2, -k_2^2, -(k_2 + p_1 + p_2)^2, -(k_1 + p_1)^2, -(k_1 - k_2)^2, -(k_2 - p_3)^2, -(k_2 + p_1)^2, -(k_1 - p_3)^2\};

- **Replacements** = \{p_1^2 \rightarrow 0, p_2^2 \rightarrow 0, p_3^2 \rightarrow 0, p_1 \ p_2 \rightarrow s/2, p_1 \ p_3 \rightarrow t/2, p_2 \ p_3 \rightarrow -1/2 (s + t)\};
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- FindRules[MasterIntegrals[]]  
  Or save rules to a file with  
- WriteRules[MasterIntegrals[],  
  FIREPath <> "examples/doublebox"];
2. Finding equivalents between master integrals

The file contains 4 lines:

\[
\begin{align*}
G[1, \{0, 0, 1, 1, 1, 1, 0, 0\}] & \rightarrow \\
\{\{1, G[1, \{1, 1, 0, 0, 1, 1, 1, 0, 0\}]\}\}; \\
G[1, \{1, 0, 0, 1, 1, 1, 1, 0, 0\}] & \rightarrow \\
\{\{1, G[1, \{0, 1, 1, 0, 1, 1, 1, 0, 0\}]\}\}; \\
G[1, \{1, 1, 0, 0, 0, 1, 1, 0, 0\}] & \rightarrow \\
\{\{1, G[1, \{0, 0, 1, 1, 1, 1, 0, 0, 0\}]\}\}; \\
G[1, \{1, 0, 0, 1, 0, 0, 1, 0, 0\}] & \rightarrow \\
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  \{\{1, \text{G}[1, \{0, 1, 1, 0, 0, 1, 0, 0\}]\}\};
  \]

- It can be loaded with

  \text{LoadRules[FIREPath <> "examples/doublebox", 1];}
3. The c++ reduction

To run the reduction one has to create a configuration file and run the c++ FIRE with bin/FIRE5 -c ConfigFileName
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#threads 4
#variables d, s, t
#start
#folder examples/
#problem 1 doublebox.start
#integrals doublebox.m
#output doublebox.tables
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One can produce LiteRed rules and use them with FIRE. This is demonstrated on one of the examples coming with LiteRed — a vertex 2-loop diagram.
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- `FIREPath = <path to the folder with FIRE>``;
- `SetDirectory[FIREPath <> "extra/LiteRed/Setup/"];``
- `Get["LiteRed.m"]``;
- `Get[FIREPath <> "FIRE5.m"]``;
- `Internal = {l, r};``
- `External = {p, q};``
- `Propagators = (-Power[##, 2]) & /@ {l - r, l, r, p - l, q - r, p - l + r, q - r + l};``
- `Replacements = {p^2 -> 0, q^2 -> 0; p q -> -1/2};`
4. Usage of LiteRed

- `CreateNewBasis[v2, Directory -> FIREPath <> "temp/v2.dir"];`
- `GenerateIBP[v2];`
- `AnalyzeSectors[v2, {0, _}];`
  (basic operations including zero sector detection)
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- `DiskSave[v2];`

The solution stage is not guaranteed to work, but at least symmetries normally help.
4. Usage of LiteRed

LiteRed files can be converted so that they can be used by the C++ FIRE.

- FIREPath = <path to the folder with FIRE>;
- Get[FIREPath <> "FIRE5.m"];
- LoadStart[FIREPath <> "examples/v2"];
- TransformRules[FIREPath <> "temp/v2.dir", FIREPath <> "examples/v2.lbases", 2];
- SaveSBases[FIREPath <> "examples/v2"];
FIRE workflow
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FIRE workflow

- Internal, External, Propagators \(\rightarrow\) problem.start (initial input)
- problem.config, problem.start or problem.sbases, problem.m (list of integrals), problem.rules (if exists), problem.tbases (if exists) \(\rightarrow\) problem.tables (reduction)
FIRE workflow

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- Internal, External, Propagators, problem.tables → problem.rules (*detection of equivalent masters*)
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- Internal, External, Propagators, problem.tables → problem.rules *(detection of equivalent masters)*
- Internal, External, Propagators → folder with LiteRed rules and symmetries *(loading LiteRed)*
FIRE workflow

- Internal, External, Propagators → problem.start (initial input)
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- Internal, External, Propagators, problem.tables → problem.rules (detection of equivalent masters)
- Internal, External, Propagators → folder with LiteRed rules and symmetries (loading LiteRed)
- problem.start, folder with LiteRed rules and symmetries → problem.sbases and problem.lbases (transforming LiteRed rules)
Optimization hints. Organization

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- Non-global symmetry rules produced by LiteRed can improve performance;
- Equivalent master integrals slow the reduction a lot. One can first make a test run only detecting masters, then produce rules for equivalents, then do the final run.
Optimization hints. Hardware

- FIRE has two different modes — disk mode and RAM mode (depending on the way coefficients and other things are stored while FIRE works)
Optimization hints. Hardware

1 Disk mode. All databases for all sectors are stored on disk. It is essential to use a fast local hard disk in this case. It is also important to have caching set up in your operating system like an intermediate buffer before disk access. While FIRE works with a number of sectors in parallel, those databases are open (plus one global database).
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2. RAM mode. The open databases are in-memory databases. However after work is over in a sector, it is dumped to disk. This mode does not need caching and requires more RAM than in the disk mode, but it does not rely that much on the disk speed.
Choose an appropriate mode for you. In both cases one should have enough RAM to prevent swapping and enough disk space to avoid crashes with the “can’t write” message.
Optimization hints. Hardware

- Choose an appropriate mode for you. In both cases one should have enough RAM to prevent swapping and enough disk space to avoid crashes with the “can’t write” message.
- The more threads are in use, the more RAM FIRE needs. Sometimes one has to use less threads (the \#threads setting) than the number of processor cores globally or only at the substitution stage (the \#stthreads setting). In this case the number of fermat processes can be increased with the \#fthreads setting.
Conclusion
There is a nice program FIRE that you can use for integral reduction

:)
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