FIRE5: a C++ implementation of Feynman Integral REduction

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Fire

The goal of this talk is to present the new version FIRE.

- FIRE - Feynman Integral REduction

FIRE can be downloaded from `http://git.sander.su/fire`

Feynman integrals

- Feynman integrals over loop momenta:

$$\mathcal{F}(a_1,\ldots,a_n) \;=\; \int\cdots\int \frac{\mathrm{d}^d k_1 \ldots \mathrm{d}^d k_h}{E_1^{a_1}\ldots E_n^{a_n}}\,.$$

- Currently one needs to evaluate millions of Feynman integrals with diffeent indices a_i coresponding to a particular diagram, so evaluating each of them analytically turns into an unreal task.

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- FIRE
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- FIRE5 — Mathematica **and c++**

n	1	2	3	4
Mathematica	640	1453	2061	3958
C++, 1 thread, disk mode	134	168	255	552
C++, 4 threads, disk mode	76	100	163	323
C++, 1 thread, RAM mode	107	136	210	473
C++, 4 threads, RAM mode	49	74	108	237

Table : Timings for the on-shell massless double box example, n is the total power of irreducible numerators

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- Either download a binary package from `http://git.sander.su/fire/downloads` and hope that it works for your system
- Or download the source with git and build it yourself (this also gives you access to dev versions of FIRE)

Building from sources requires KyotoCabinet and Snappy libraries, but they are shipped with FIRE.

Installation from sources:

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- Switch to dev version: git checkout dev

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- bin/FLink — auxilirary binary used to run fermat from Mathematica

0. Loading FIRE in Mathematica

```
SetDirectory[<path to the folder with FIRE>];
Get["FIRE5.m"];
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SetDirectory[<path to the folder with FIRE>];
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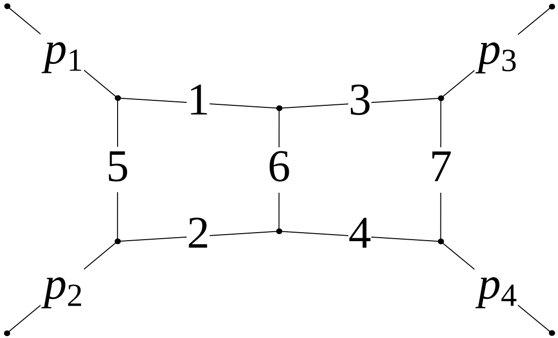
or

```
FIREPath = <path to the folder with FIRE>;
Get[FIREPath<>"FIRE5.m"];
```


p_1
p_2
p_3
p_4
1
2
3
4
5
6
7

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- `PrepareIBP[]; Prepare[AutoDetectRestrictions -> True]; SaveStart["doublebox"];`

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  or a better way for multiple integrals:
- ```
  EvaluateAndSave[{{1, {1, 1, 1, 1, 1, 1, 1,
  -1, -1}},
  {1, {1, 1, 1, 1, 1, 1, 1, 0,
  -2}}},"doublebox.tables"]
  ```
  ```

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  Later load a new kernel, load the start file and...
- `LoadTables["doublebox.tables"];`

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- The same is true for the c++ FIRE, however it can also work in parallel with sectors of the same level (the number of positive indices).

2. Finding equivalents between master integrals

During the main reduction phase FIRE cannot find equivalents between master integrals in different sectors.

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0}}, {1, {0, 1, 1, 0, 0, 1, 0, 0, 0}}, {1, {0, 1,
1, 0, 1, 1, 1, 0, 0}}, {1, {1, 0, 0, 1, 0, 1, 0,
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1, 0, 0}}, {1, {1, 1, 1, 1, 0, 0, 0, 0, 0}}, {1,
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1, 1, -1, 0}}}
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- Internal = {k1, k2};
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- `FindRules[MasterIntegrals[]]`
  Or save rules to a file with
- `WriteRules[MasterIntegrals[], FIREPath <> "examples/doublebox"];`

2. Finding equivalents between master integrals

- The file contains 4 lines:

```
G[1, {0, 0, 1, 1, 1, 1, 1, 0, 0}] ->
{{1, G[1, {1, 1, 0, 0, 1, 1, 1, 0, 0}]}};
G[1, {1, 0, 0, 1, 1, 1, 1, 0, 0}] ->
{{1, G[1, {0, 1, 1, 0, 1, 1, 1, 0, 0}]}};
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{{1, G[1, {0, 1, 1, 0, 0, 1, 0, 0, 0}]}};
```

- It can be loaded with

```
LoadRules[FIREPath <> "examples/doublebox", 1];
```

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```
#threads 4
#variables d, s, t
#start
#folder examples/
#problem 1 doublebox.start
#integrals doublebox.m
#output doublebox.tables
```

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- `FIREPath = <path to the folder with FIRE>;`
- `SetDirectory[FIREPath <> "extra/LiteRed/Setup/"];`
- `Get["LiteRed.m"];`
- `Get[FIREPath <> "FIRE5.m"];`
- `Internal = {l, r};`
- `External = {p, q};`
- `Propagators = (-Power[##, 2]) & /@ {l - r, l, r, p - l, q - r, p - l + r, q - r + l};`
- `Replacements = {p^2 -> 0, q^2 -> 0; p q -> -1/2};`

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- `CreateNewBasis[v2, Directory -> FIREPath <> "temp/v2.dir"];`
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- `SolvejSector /@ UniqueSectors[v2];`
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- `DiskSave[v2];`

The solution stage is not guaranteed to work, but at least symmetries normally help.

4. Usage of LiteRed

LiteRed files can be converted so that they can be used by the c++ FIRE.

- FIREPath = <path to the folder with FIRE>;
- Get[FIREPath <> "FIRE5.m"];
- LoadStart[FIREPath <> "examples/v2"];
- TransformRules[FIREPath <> "temp/v2.dir", FIREPath <> "examples/v2.lbases", 2];
- SaveSBases[FIREPath <> "examples/v2"];

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- `Internal, External, Propagators, problem.tables` → `problem.rules` *(detection of equivalent masters)*

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- `Internal`, `External`, `Propagators`, `problem.tables` → `problem.rules` *(detection of equivalent masters)*
- `Internal`, `External`, `Propagators` → folder with `LiteRed` rules and symmetries *(loading LiteRed)*

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- `Internal`, `External`, `Propagators` → folder with `LiteRed` rules and symmetries *(loading LiteRed)*
- `problem.start`, folder with `LiteRed` rules and symmetries → `problem.sbases` and `problem.lbases` *(transforming LiteRed rules)*

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- Non-global symmetry rules produced by LiteRed can improve performance;
- Equivalent master integrals slow the reduction a lot. One can first make a test run only detecting masters, then produce rules for equivalents, then do the final run.

Optimization hints. Hardware

- FIRE has two different modes — disk mode and RAM mode (depending on the way coefficients and other things are stored while FIRE works)

Optimization hints. Hardware

1. Disk mode. All databases for all sectors are stored on disk. It is essential to use a fast local hard disk in this case. It is also important to have caching set up in your operating system like an intermidiate buffer before disk access. While FIRE works with a number of sectors in parallel, those databases are open (plus one global database).

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2. RAM mode. The open databases are in-memory databases. However after work is over in a sector, it is dumped to disk. This mode does not need caching and requires more RAM than in the disk mode, but it does not rely that much on the disk speed.

Optimization hints. Hardware

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- Choose an appropriate mode for you. In both cases one should have enough RAM to prevent swapping and enough disk space to avoid crashes with the "can't write" message.
- The more threads are in use, the more RAM FIRE needs. Sometimes one has to use less threads (the `#threads` setting) than the number of processor cores globally or only at the substitution stage (the `#sthreads` setting). In this case the number of fermat processes can be increased with the `#fthreads` setting.

- There is a nice program FIRE that you can use for integral reduction
- :)


FIRE