Exact result and phase structure of N=2* supersymmetric Yang-Mills theory

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A. Buchel, J. Russo, K.Z. 1301.1597
J. Russo, K.Z. 1302.6968, 1309.1004
X. Chen-Lin, J. Gordon, K.Z., 1408.6040
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K.Z., 1410.6114

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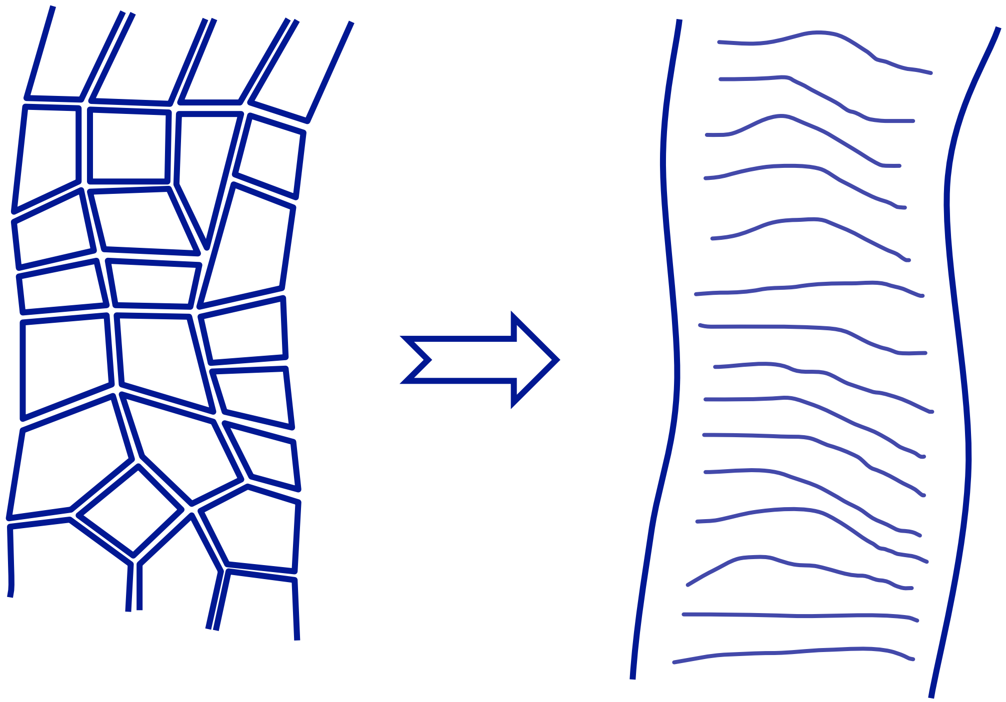
Planar diagrams: $N \to \infty, \qquad \lambda = g^2 N$ – fixed

't Hooft'74



AdS/CFT correspondence

$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$

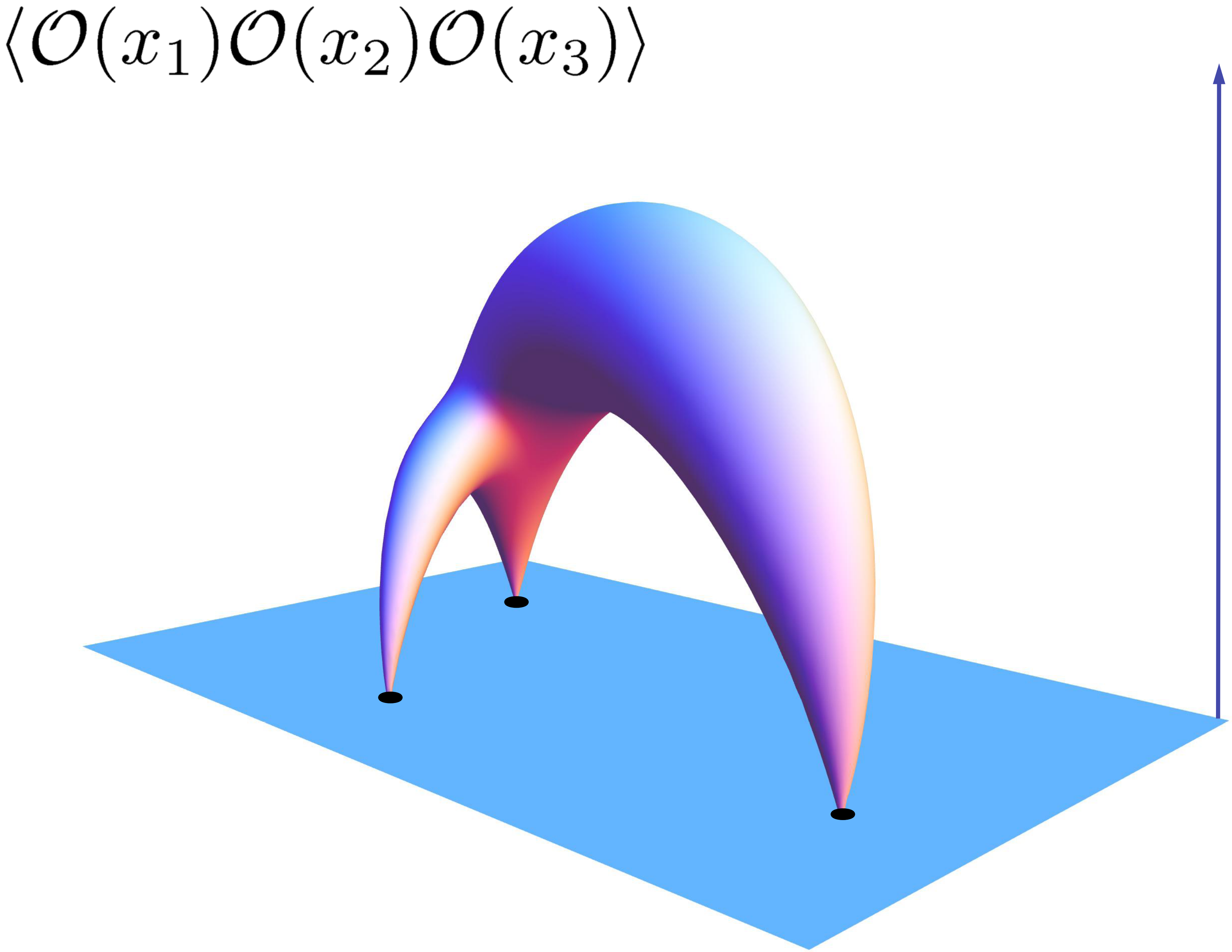
AdS/CFT correspondence

Yang-Mills theory with
N=4 supersymmetry

Exact equivalence ⇕

Maldacena'97
Gubser, Klebanov, Polyakov'98
Witten'98

String theory on
$AdS_5 x S^5$ background



$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle$

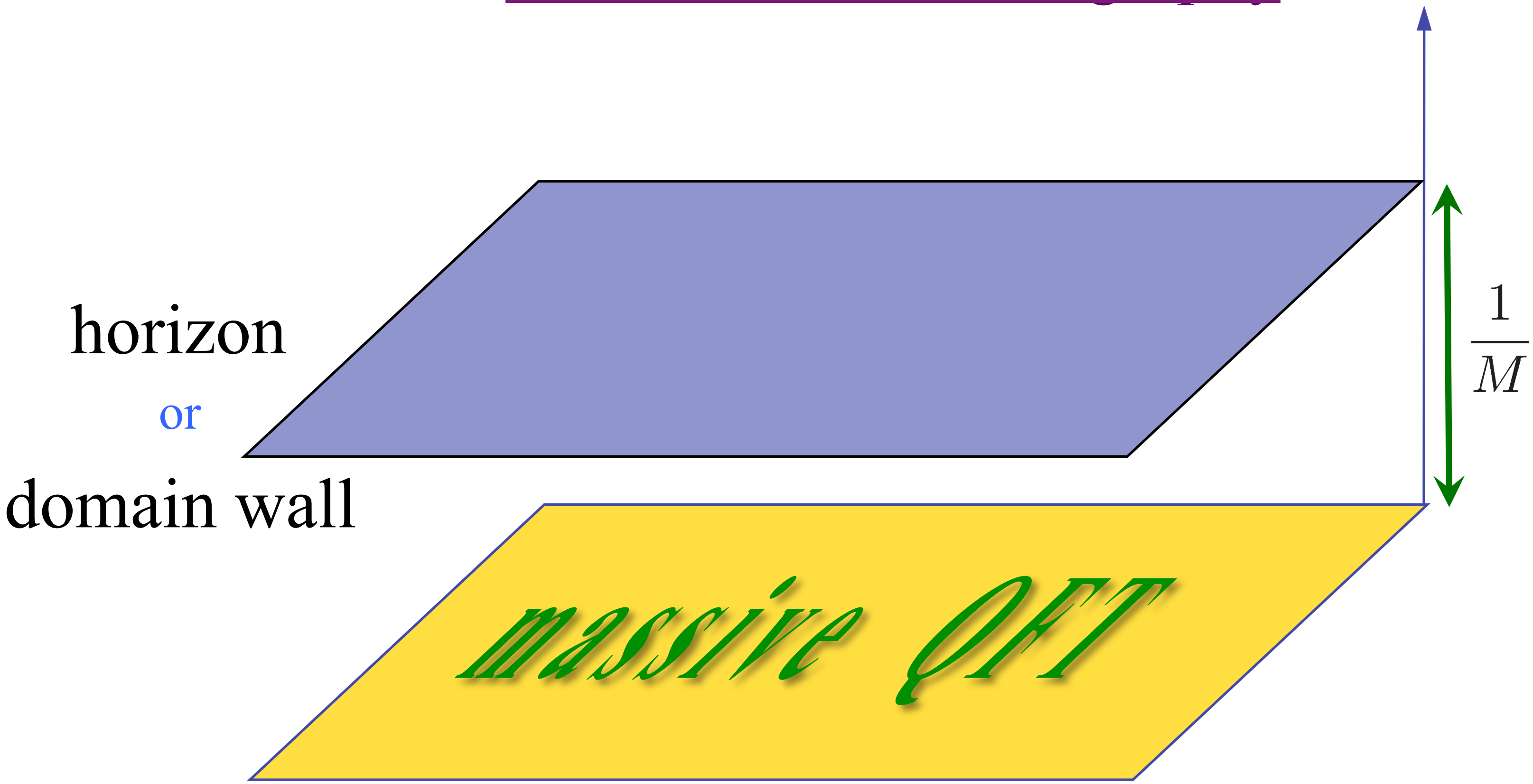
$\mathcal{N} = 4$ SYM	Strings on $AdS_5 \times S^5$
't Hooft coupling: $\lambda = g^2 N$	String tension: $T = \frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s = \frac{\lambda}{4\pi N}$
Large-N limit	Free strings
Strong coupling	Classical strings
Local operators Scaling dimension: Δ	String states Energy: E

String Theory (quantum gravity) $\equiv$ 4d CFT

$AdS_5 x S^5$ N=4 super-Yang-Mills

- Still a conjecture…
- Overwhelming number of <u>quantitative</u> tests
- Empirical "proof"

Non-conformal holography

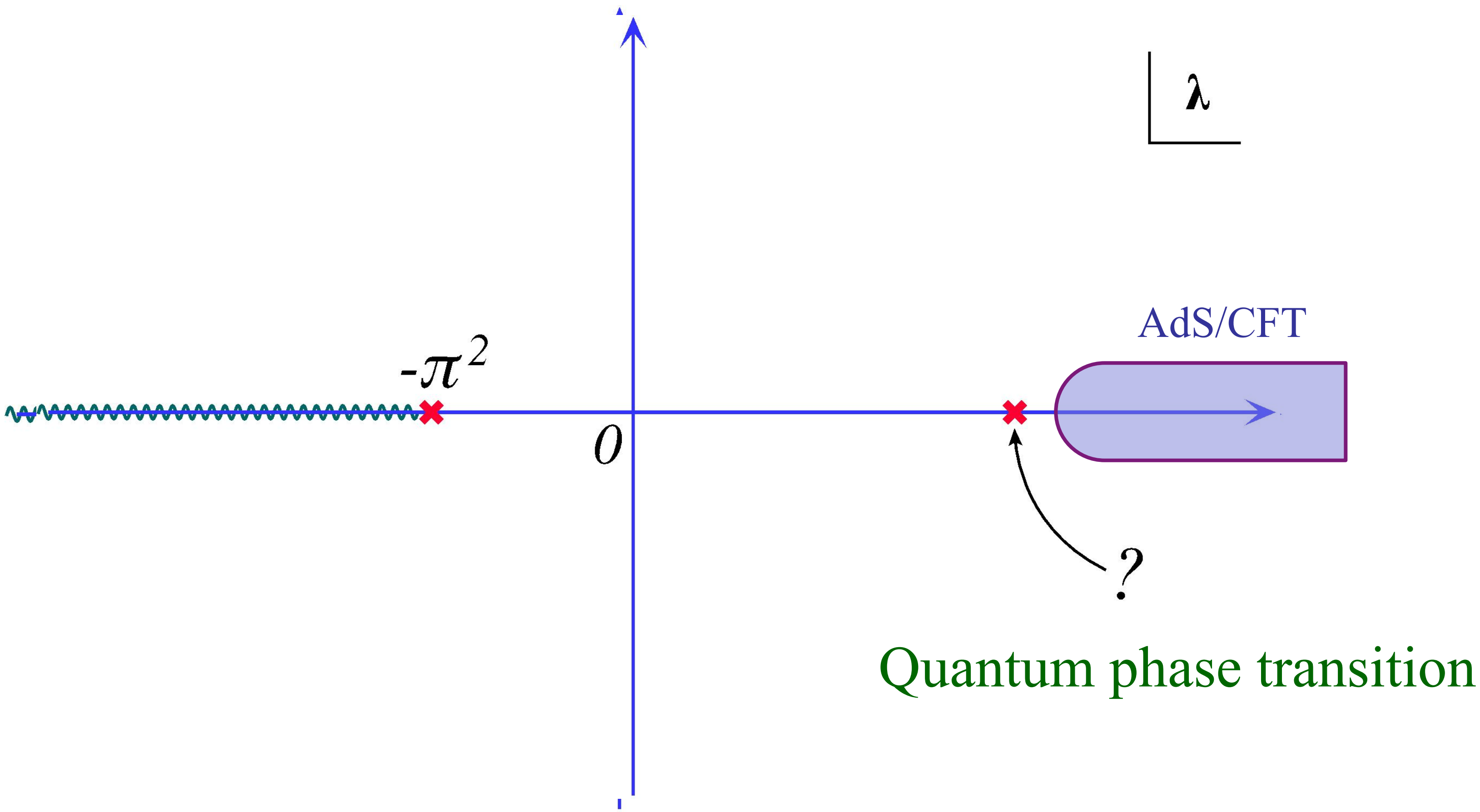


- Routinely used in many contexts
- Much less precise than AdS/CFT

Phase transitions

In this talk:
$N=\infty$
$\lambda=g^2N$

In N=4 super-Yang-Mills:



N=2* theory

$$\mathcal{L} = \frac{1}{g^2}\,\mathrm{tr}\left\{-\frac{1}{2}F_{\mu\nu}^2 + |D_\mu\Phi|^2 + D_\mu Z_i^\dagger D^\mu Z^i + \ldots\right.$$

← N=4 SYM

$$\left. -M^2 Z_i^\dagger Z^i - M\bar{\Psi}_i\Psi^i - M\,\mathrm{Im}\,\Phi\varepsilon^{ij}Z_iZ_j + \mathrm{c.c.}\right\}$$

hypermultiplets

Z^1

Ψ^1 Ψ^2 + conj.

Z^2

mass = $\pm M$

vector multiplet

A_μ

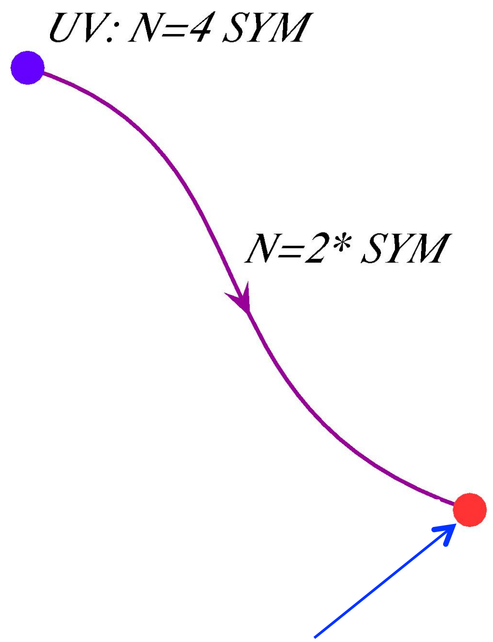
ψ λ

Φ

mass = 0

At $E \ll M$: integrate out hypermultiplet

- UV regularization of pure N=2 SYM



(weak coupling)

(strong coupling)

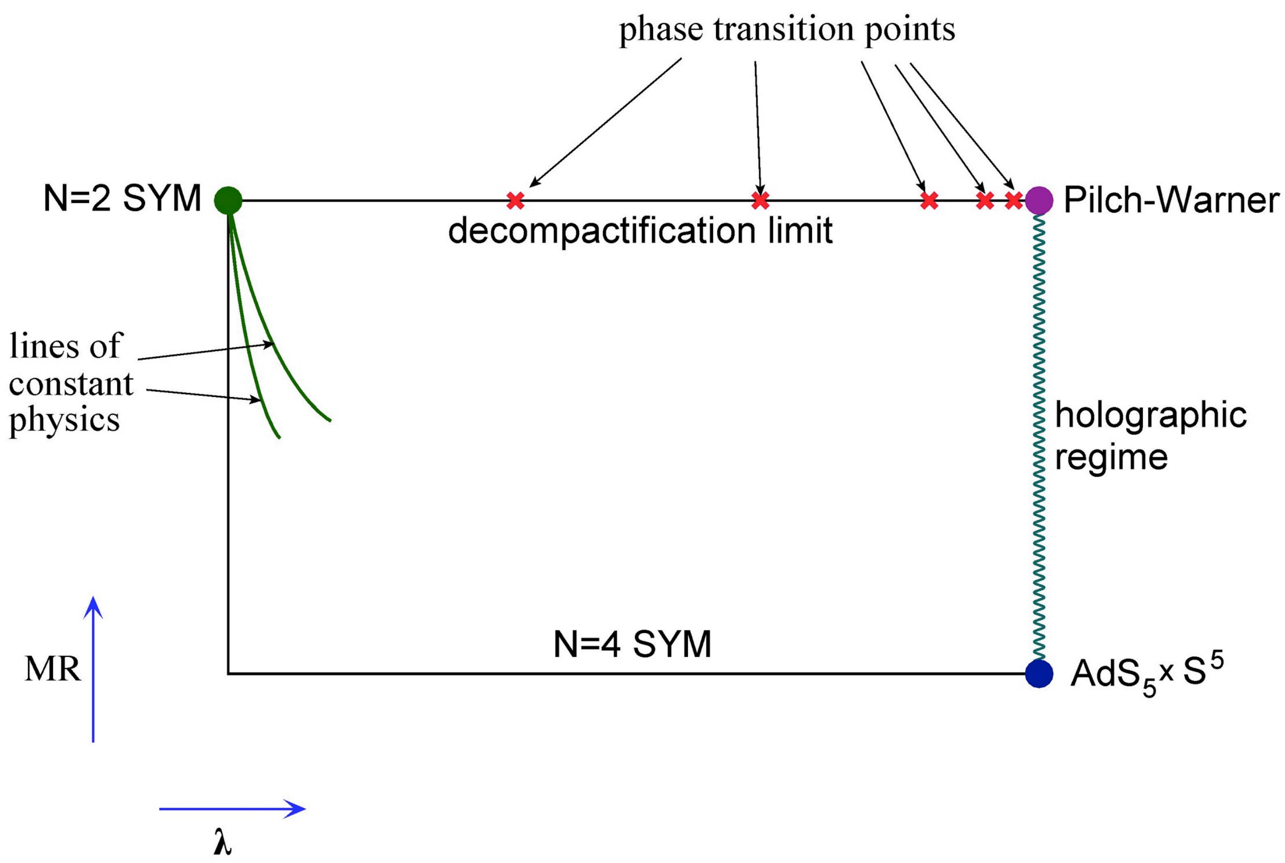
Dynamically generated scale: $\Lambda = M\, e^{-\frac{4\pi^2}{\lambda}}$ Hoyos'10

Interacting CFT_5

dimensional crossover due to Eguchi-Kawai mechanism

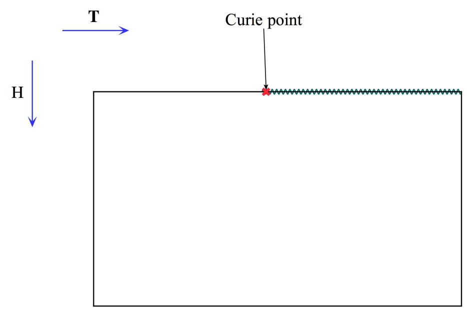
Young,Z.'14

Phase diagram



R: radius of S^4

Ising model:



Non-perturbative corrections

Dynamically generated scale: Λ

Kinematic scale: M

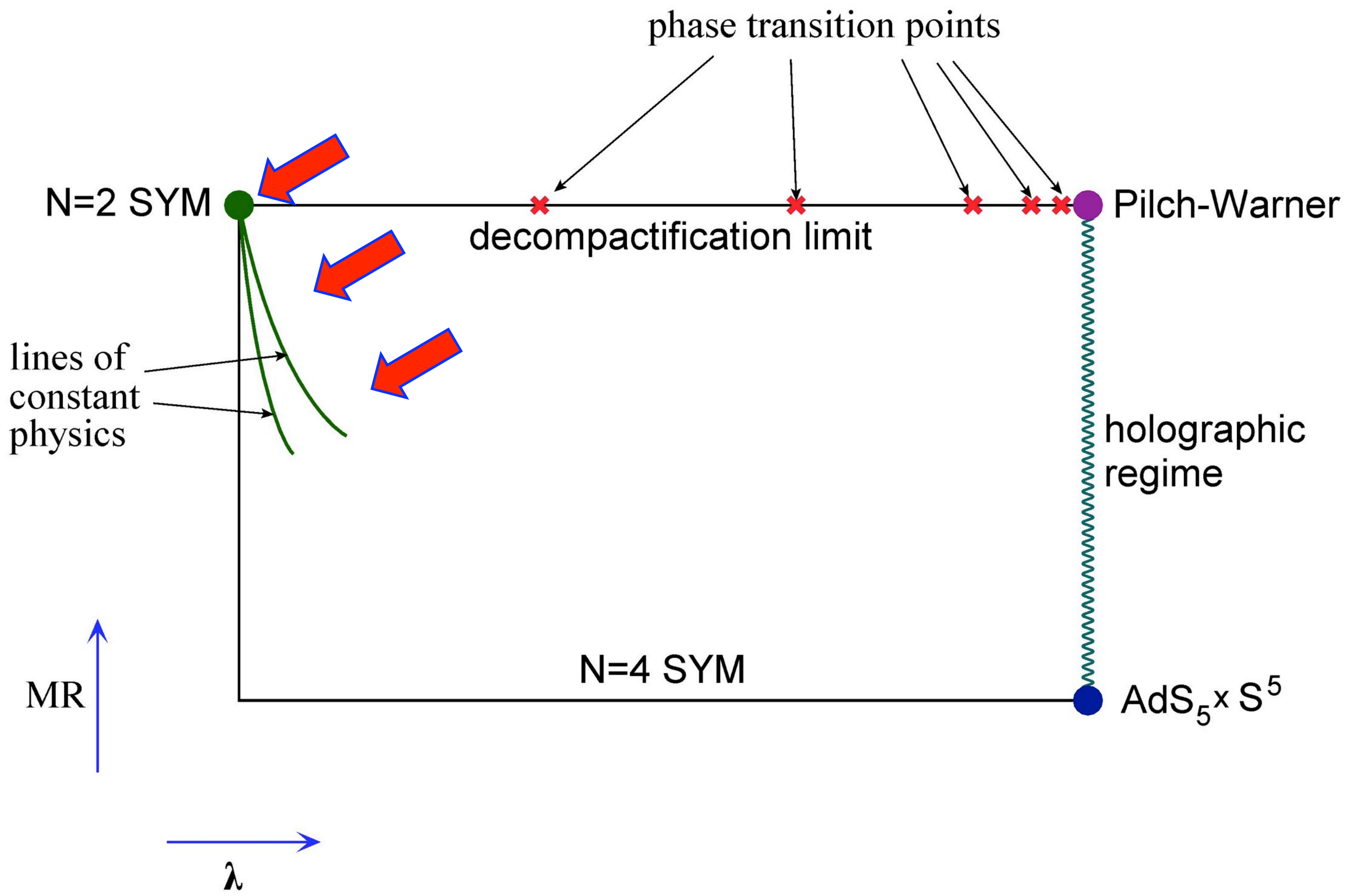
$$M \gg \Lambda$$

(perturbative regime)

$$\mathcal{A} = \text{perturbative} + \sum_{n=1}^{\infty} C_n \left(\frac{\Lambda}{M}\right)^{2n}$$

- not calculable in general
- usually parameterized by condensates – ITEP sum rules…
- goal: compute all C_n's in a solvable model

Effective field theory regime



$$\Lambda = M\,\mathrm{e}^{-\frac{4\pi^2}{\lambda}}$$

Expansion parameter of OPE: $\frac{\Lambda^2}{M^2} = \mathrm{e}^{-\frac{8\pi^2}{\lambda}}$

- weak-coupling expansion has no perturbative part

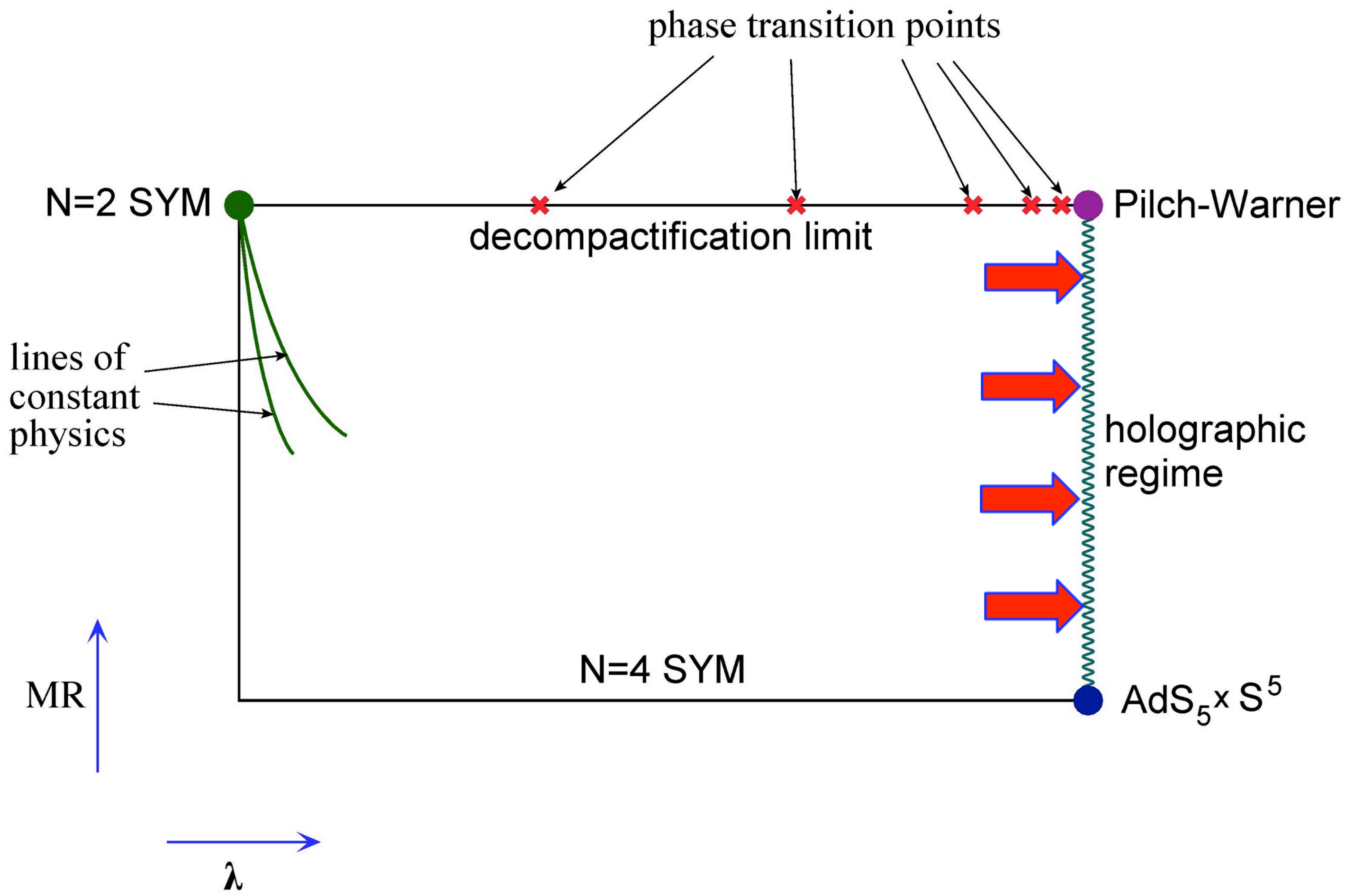
Example: exact free energy

$$f(\lambda) = 2\sum_{n=1}^{\infty} \ln\left(1 - (-1)^n\,\mathrm{e}^{-\frac{8\pi^2 n}{\lambda}}\right)$$

Russo, Z.'13

$$f(\lambda) = 2\,\mathrm{e}^{-\frac{8\pi^2}{\lambda}} - 3\,\mathrm{e}^{-\frac{16\pi^2}{\lambda}} + \frac{8}{3}\,\mathrm{e}^{-\frac{24\pi^2}{\lambda}} - \frac{7}{2}\,\mathrm{e}^{-\frac{32\pi^2}{\lambda}} + \frac{12}{5}\,\mathrm{e}^{-\frac{40\pi^2}{\lambda}} - 4\,\mathrm{e}^{-\frac{48\pi^2}{\lambda}} + \ldots$$

Strong coupling and holography



Perimeter law

Wilson loop:

$$W(C) = \left\langle \frac{1}{N} \,\mathrm{P} \exp \oint_C ds \; (i\dot{x}^\mu A_\mu + |\dot{x}|\Phi) \right\rangle$$

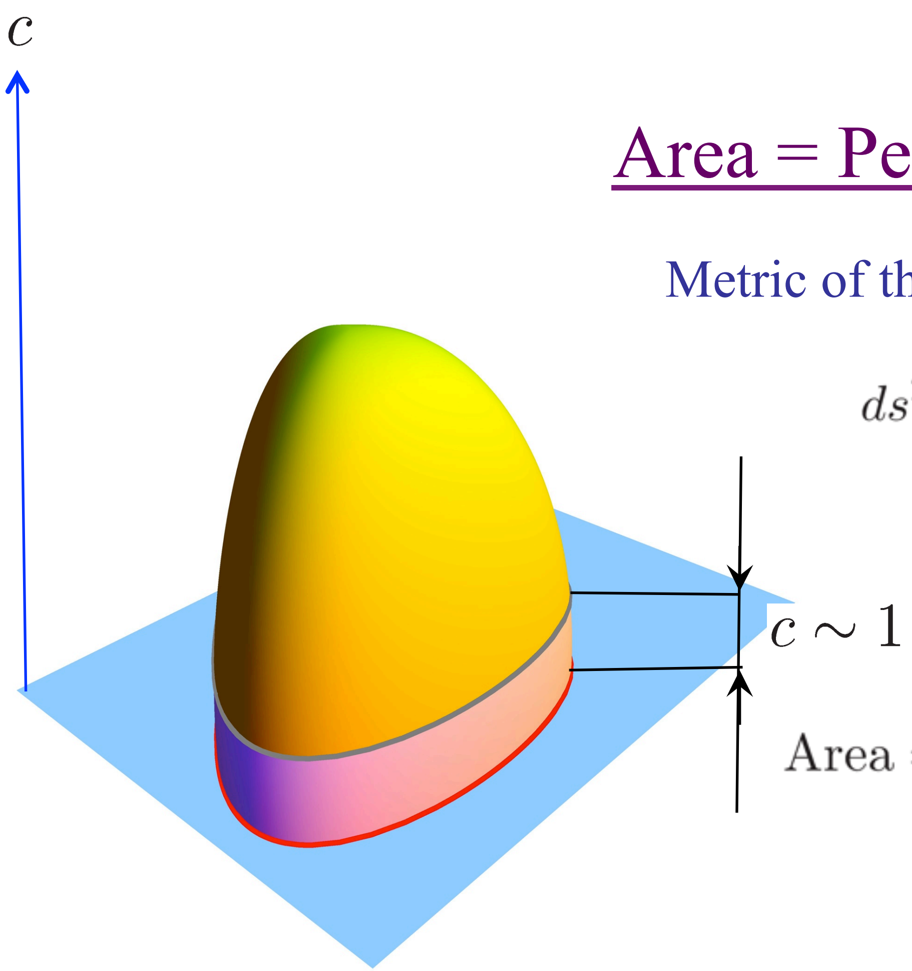
Exact result for asymptotically large loops:

$$\ln W(C) \simeq \frac{\sqrt{\lambda} M L}{2\pi} \qquad (\lambda \to \infty,\; ML \gg 1)$$

Buchel, Russo, Z.'13
Chen-Lin, Gordon, Z.'14

(perimeter law)

Area = Perimeter



Metric of the gravity dual (domain wall in AdS):

$$ds^2 = \frac{\rho^6}{c^2-1} M^2 dl^2 + \frac{1}{\rho^6 \left(c^2-1\right)^2} dc^2$$

Pilch, Warner'00

$$\text{Area} = ML \int_{1+\frac{\varepsilon^2 M^2}{2}}^{\infty} \frac{dc}{\left(c^2-1\right)^{\frac{3}{2}}} = \frac{L}{\varepsilon} - ML$$

renormalized away

$$\ln W(C) \simeq -\text{Tension} \times \text{Area}_{\text{ren}} = \frac{\sqrt{\lambda} ML}{2\pi}$$

$\frac{\sqrt{\lambda}}{2\pi}$ $\quad -ML$

agrees with field-theory calculation!

- free enegry also agrees

Bobev, Elvang, Freedman, Pufu'13

Conclusions

- holographic duality operates at strong coupling in QFT
- direct calculations in strongly coupled QFTs are (with a bit of luck) possible
- opening avenue for direct tests of holography
- N=2* is an interesting theory, with non-trivial phase structure
- what are the implications of the phase transitions AdS/CFT?