Evolution of Scalar Fields in the Early Universe

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The Motivation

- The recent discovery of the Higgs boson with mass

\[ M_h = 125.7 \pm 0.4 \text{ GeV} \]

[Particle Data Group 2014]

[Dario Buttazzo et al. JHEP 1312 (2013) 089]
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- During inflation, the scalar field with a shallow potential can obtain a large vacuum expectation value (VEV).
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  - \( \Rightarrow \) possibility for Leptogenesis

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Outline

1. Quantum Fluctuations in the Inflationary Universe
2. Classical Motion of Scalar Fields
3. Possible New Physics
4. Issue with Isocurvature Perturbations
Quantum Fluctuations in the Inflationary Universe
Quantum fluctuations in the inflationary universe

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[Figure from A. Linde - arXiv: 0503203]
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  1. long correlation length $l$
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=> behave like (quasi) classical field.

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Quantum fluctuations in the inflationary universe

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In a pure de Sitter spacetime, a scalar field with mass $m$ can obtain a large VEV

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- In the inflationary universe, the exponential expansion period exists for a finite time \( t \)

\[
\langle \phi^2 \rangle \approx \frac{H^2}{2(2\pi)^3} \int_{H-e^{-Ht}}^H \frac{d^3k}{k} = \frac{H^3}{4\pi^2}t \approx \frac{H^2}{4\pi^2}N
\]

for \( m^2 = 0 \) or \( m^2 \ll H^2 \) with \( t \lesssim 3H/m^2 \). \( N \approx Ht \) is the number of e-folds. [A. Linde, Phys. Lett. B116, 335 (1982)]
One can also understand the fluctuation as both the **scalar field** $\phi(x)$ and the **metric** $g_{\mu\nu}(x)$ experience quantum jumps.

\[ V(\phi) = A e^{S_{\text{Eu}}(\phi_i) - S_{\text{Eu}}(\phi_f)}, \]

where $S_{\text{Eu}}(\phi)$ is the Euclidean action and $A$ is some $O(m^4)$ prefactor.
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**The Hawking-Moss instanton**

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\Gamma(\phi_i \rightarrow \phi_f) = A e^{S_E(\phi_i) - S_E(\phi_f)}, \quad \text{where} \quad S_E(\phi) = -\frac{3m_4^4}{8V(\phi)}
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The entire process can then be viewed as the fields are underdoing **Brownian motion** and can be described by **diffusion equation**.
**Stochastic approach & Hawking-Moss tunneling**

- $P_c(\phi, t)$: the probability distribution of finding $\phi$ at time $t$
- Diffusion equation

\[
\frac{\partial P_c}{\partial t} = -\frac{\partial j_c}{\partial \phi}
\quad \text{where} \quad -j_c = \frac{\partial}{\partial \phi} \left( \frac{H^3 P_c}{8\pi^2} \right) + \frac{P_c}{3H} \frac{dV}{d\phi}
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[A. A. Starobinsky (1982); A. Vilenkin (1982)]

Evolution of Scalar Fields in the Early Universe (slide 8)
**Stochastic approach & Hawking-Moss tunneling**

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- In equilibrium \( \frac{\partial P_c}{\partial t} = 0, j_c = 0 \). One obtain the distribution

\[
P_c(\phi) = e^{S_E(\phi_{\text{min}}) - S_E(\phi)} \\
\approx \exp \left[ \frac{-3m_{pl}^4}{8} \Delta V(\phi) \frac{V(\phi_{\text{min}})^2}{V(\phi) - V(\phi_{\text{min}})} \right]
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for \( \Delta V = V(\phi) - V(\phi_{\text{min}}) \ll V(\phi_{\text{min}}) \).
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- The variance of the fluctuation is

\[
\langle \phi^2 \rangle = \frac{\int \phi^2 P_c(\phi)d\phi}{\int P_c(\phi)d\phi}
\]
Quantum fluctuation of the Higgs field

- Example: the Higgs field $\phi$ on the inflationary background (inflaton $I$).

$$V(\phi, I) = V_H(\phi) + V_I(I) + ... \approx \frac{1}{4} \lambda_{\text{eff}} \phi^4 + \Lambda_I^4 + ...$$
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- The quantum transition of the Higgs field from 0 to $\phi$ is not suppressed if

\[ \frac{1}{4} \lambda_{\text{eff}} \phi^4 < \frac{8}{3} \left( \frac{\Lambda_I^2}{m_{\text{pl}}} \right)^4 \sim H_I^4 \Rightarrow |\phi| < 0.62 \lambda_{\text{eff}}^{-1/4} H_I \]
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Even though $\langle\phi\rangle = 0$ due to the even potential, the variance of the fluctuation of $\phi$ is not zero.

$$\phi_0 = \sqrt{\langle\phi^2\rangle} \approx 0.36\lambda_{\text{eff}}^{-1/4}H_I$$
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- Generally, during inflation, we expect the scalar field to obtain a large VEV $\phi_0$ such that

$$ V_H(\phi_0) \sim H_I^4 $$
Classical Motion of Scalar Fields
Slow rolling during inflation

- Scalar field in an expanding universe

\[
\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi\dot{\phi} + \frac{\partial V}{\partial \phi} = 0
\]
Slow rolling during inflation

■ Scalar field in an expanding universe

\[ \ddot{\phi} + 3H\dot{\phi} + \Gamma_{\phi\phi} + \frac{\partial V}{\partial \phi} = 0 \]

■ During inflation, the scalar field can be in slow-roll.

\[ \dot{\phi} \ll \frac{\partial V}{\partial \phi} \quad \text{and} \quad \dot{\phi}^2 \ll V \]
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- The slow-roll conditions are
  \[ 9H^2 \gg \frac{\partial^2 V (\phi, I)}{\partial \phi^2} = m_{\text{eff}}^2 (\phi) \quad \text{and} \quad \sqrt{48\pi} \frac{V(\phi, I)}{m_{\text{pl}}} \gg \left| \frac{\partial V (\phi, I)}{\partial \phi} \right| . \]
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- The first condition can be understood as the time scale for rolling down
  \[ \tau \sim m_{\text{eff}}^{-1} = \left( \sqrt{\frac{\partial^2 V}{\partial \phi^2}} \right)^{-1} \gg H^{-1}. \]
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- As long as \( m_{\text{eff}} (\phi) \ll H \), there is insufficient time for the scalar field to roll down.
For $\frac{1}{4} \lambda \phi^4$ or the Higgs potential, the slow-roll conditions are

$$|\phi| \ll 3 \lambda_{\text{eff}}^{-1/2} H_I \quad \text{and} \quad |\phi| \ll \left(\frac{27}{4 \pi}\right)^{1/6} \lambda_{\text{eff}}^{-1/3} (m_{\text{pl}} H_I^2)^{1/3}.$$
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- The conditions for all the quantum fluctuations to be unable to roll are:

$$\lambda_{\text{eff}} \ll 4800 \quad \text{and} \quad \lambda_{\text{eff}} \ll 3 \times 10^5 \left(\frac{m_{\text{pl}}}{\Lambda_I}\right)^2,$$

which are easily satisfied when $\Lambda_I < m_{\text{pl}}$. 

⇒ a large Higgs VEV is developed.
The slow rolling of the Higgs field

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$\Rightarrow$ a large Higgs VEV is developed.
**Brief summary**

**Quantum fluctuation**

Brings the field to a VEV $\phi_0$ such that

$$V_\phi (\phi_0) \sim H^4$$

**Slow rolling**

The field won’t roll down if

$$m_{\text{eff}}^2 \ll H^2$$
As inflation ends, the inflaton enters the coherent oscillations regime, $H < m_{\text{eff}}(\phi_0)$. The Higgs field is no longer in slow-roll.
Relaxation of the Higgs field after inflation

- As inflation ends, the inflaton enters the coherent oscillations regime, $H < m_{\text{eff}}(\phi_0)$. The Higgs field is no longer in slow-roll.
- The Higgs then rolls down and oscillates around $\phi = 0$ with decreasing amplitude within $\tau_{\text{roll}} \sim H^{-1}$.

$\Lambda_I = 10^{16}$ GeV  
$\Gamma_I = 10^3$ GeV  
$T_{\text{max}} = 6.4 \times 10^{12}$ GeV  
$\lambda_{\text{eff}} = 0.003$  
$\phi_0 = 3.7 \times 10^{13}$ GeV  
$H_I = 2.4 \times 10^{13}$ GeV
Relaxation of the Higgs field after inflation

- During the oscillation of the Higgs field, the Higgs condensate can decay into several product particles:
  - Non-perturbative decay: W and Z bosons.

\[ \Lambda_I = 10^{15} \text{ GeV and } \Gamma_I = 10^9 \text{ GeV for IC-1} \]

- Perturbative decay (thermalization): top quark.
- Those decay channels do affect the oscillation of the Higgs field but they become important only after several oscillations.
Possible New Physics

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    2. Out of thermal equilibrium


Similar idea for axion


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$\equiv$ Roll down of the Higgs field
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Issue with Isocurvature Perturbations
One issue for applying to Leptogenesis

\[ \phi_0 = \sqrt{\langle \phi^2 \rangle} \]

is the average over several Hubble volumes. Each Hubble volume has different initial \( \phi_0 \) value. When inflation ends, each patch of the observable universe began with different value of \( \phi \).

If \[ L \propto \partial_0 \left| \phi^2 \right| \]

\( \Rightarrow \)

Different asymmetry in each Hubble volume \( \Rightarrow \) Large isocurvature perturbations, which are constrained by current CMB observation.

[Figure from Lauren Pearce]
Isocurvature perturbations

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Solutions to the isocurvature perturbation issue

Solutions:

1. IC-1: Second Minimum at Large VEVs
   \[ \phi \gg v_{EW} \]  
   Example: 
   \[ L_{\text{lift}} = \phi^{10} \Lambda^{6} \]

2. IC-2: Inflaton-Higgs coupling
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   \[ L_{\Phi I} = - \frac{1}{2} I^{2} n_{M}^{2} - 2 \phi^{2} \]
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  1. At large VEVs, Higgs potential is sensitive to higher-dimensional operators.
  
  \[ L_{\text{lift}} = \phi^{10} \Lambda^6 \]

  2. There seems to be a Planckian minimum below our electroweak (EW) vacuum. Our EW vacuum is not stable.

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4. Higgs field rolls down from the second minimum.
IC-1: Second minimum at large VEV

$\Lambda_I = 10^{15} \text{ GeV}$
$\Gamma_I = 10^9 \text{ GeV}$
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IC-2: Inflaton-Higgs coupling

- Introduce coupling between the Higgs and inflaton field. E.g.

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- If \( m_{\text{eff},\phi}(\langle I \rangle) \gg H \) in the early stage of inflation, the slow roll condition is not satisfied.
IC-2: Inflaton-Higgs coupling

1 In the early stage of inflation, \( \langle I \rangle \) is large. Higgs potential is steep. Slow-roll condition is not satisfied. The Higgs VEV stay at \( \phi = 0 \).

\[
\langle \phi^2 \rangle \sim 0 \\
\text{Quantum jumps} \\
\text{Rolls down classically} \\
\text{Early stage of inflation} \\
V(\phi) \\
H^4
\]
1. In the early stage of inflation, $\langle I \rangle$ is large. Higgs potential is steep. Slow-roll condition is not satisfied. The Higgs VEV stay at $\phi = 0$.

2. At the last $N_{\text{last}}$ e-folds of inflation, $\langle I \rangle \downarrow, m_{\text{eff},\phi}(\langle I \rangle) < H_I$, Higgs VEV starts to develop.
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4. The Higgs VEV then rolls down from $\phi_0$. 
IC-2: Inflaton-Higgs coupling

- For $N_{\text{last}} = 5 - 8$, the isocurvature perturbation only develops on the small angular scales which are not yet constrained.

\[
< \text{IC2} > \\
\Lambda_I = 10^{17} \text{ GeV} \\
\Gamma_I = 10^8 \text{ GeV} \\
N_{\text{last}} = 8 \\
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Summary

- During inflation, the Higgs field can obtain a large VEV through quantum fluctuation, but the field cannot roll down due to inflationary background.
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Summary

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Thank you for your listening!
The Universe appears to be almost homogeneous and isotropic today
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The Universe appears to be almost homogeneous and isotropic today ⇒ Inflation

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Inflation from a real scalar field: Inflaton \( I(x) \)

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\mathcal{L}_I = \frac{1}{2} g^{\mu\nu} \partial_\mu I \partial_\nu I - V_I(I)
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Inflation from a real scalar field: Inflaton $I(x)$

$$\mathcal{L}_I = \frac{1}{2} g^\mu\nu \partial_\mu I \partial_\nu I - V_I (I)$$

The equation of motion is

$$\ddot{I} + 3H \dot{I} + \Gamma_I \dot{I} + \frac{dV_I (I)}{dI} = 0, \quad \text{with} \quad H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3m^2_{pl}} (\rho_I + \rho_{other})$$

where we assume a uniform field configuration and a FRW spacetime

$$ds^2 = dt^2 - a(t)^2 (dr^2 + r^2 d\Omega^2).$$
1 **Slow-roll** (inflation) regime: $\ddot{I} \ll \frac{dV}{dI}$ and $\dot{I}^2 \ll V$.

- **Slow-Roll**
  - $V(I)$
  - $\Lambda_I$
  - $I$
  - **Coherent Oscillations**

- **Radiation-dominated regime**
  - $a(t) \propto (t - t_i)^{1/2}$
  - At $t = 1/\Gamma_I$, most of the inflatons decay into $\rho_R$, and the reheating is complete.

- **End of Inflation**
  - $t = 1/\Gamma_I$
  - $T_R$ max

- **Evolution of Scalar Fields in the Early Universe (slide 28)**
1 **Slow-roll** (inflation) regime: \( \ddot{I} \ll \frac{dV}{dI} \) and \( \dot{I}^2 \ll V \).

- \( \Gamma_I \) is not active.

\[ 3H\dot{I} \approx -\frac{dV}{dI}, \quad \text{and} \quad H^2 \approx \frac{8\pi}{3m_{pl}^2} V_I(I) \]

\[ a(t) \propto e^{Ht} \]

**Coherent Oscillations** regime: \( a(t) \propto (t-t_i)^{2/3} \)

Inflaton acts like a non-relativistic particle. The Universe is matter-dominated.

Inflaton then decays into relativistic particles \( \rho_R \).

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\[ \rho_I(t) = \Lambda I a(t)^3 e^{-\Gamma_I t} \]

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Evolution of Scalar Fields in the Early Universe (slide 28) PACIFIC 2015
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   - Inflaton then decays into relativistic particles $\rho_R$.
   - $\dot{\rho}_I + 3H\rho_I + \Gamma_I \rho_I = 0 \Rightarrow \rho_I(t) = \frac{\Lambda^4_I}{a(t)^3} e^{-\Gamma_I t}$
The Brief History of the Early Universe

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\dot{\rho}_I + 3H\rho_I + \Gamma_I\rho_I = 0 \quad \Rightarrow \quad \rho_I(t) = \frac{\Lambda_I^4}{a(t)^3} e^{-\Gamma_I t}
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3 **Radiation-dominated** regime: \( a(t) \propto (t - t_i)^{1/2} \)
   - At \( t = 1/\Gamma_I \), most of the inflatons decay into \( \rho_R \), and the reheating is complete.
If $|V(\phi_f) - V(\phi_i)| \ll V(\phi_i)$, we have

$$S_E(\phi_i) - S_E(\phi_f) = -\frac{3m_{pl}^4}{8} \left[ \frac{1}{V(\phi_i)} - \frac{1}{V(\phi_f)} \right] \approx -\frac{3m_{pl}^4}{8} \frac{V(\phi_f) - V(\phi_i)}{V(\phi_i)^2}$$
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The transition rate is then

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\frac{\Gamma}{V} \propto \exp \left( -\frac{3m_{pl}^4}{8} \frac{V(\phi_f) - V(\phi_i)}{V(\phi_i)^2} \right)
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The Hawking-Moss Tunneling

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- Thus, the transition is not suppressed as long as

\[
V(\phi_f) - V(\phi_i) < \frac{8}{3m_{pl}^4} V(\phi_i)^2
\]
As inflation ends, the inflatons enter the coherent oscillations regime, the Higgs field is no longer in slow-roll. In this case, we have to consider the full equation of motion

$$\ddot{\phi} + 3H \dot{\phi} + \Gamma_\phi \dot{\phi} = - \frac{\partial V_H (\phi)}{\partial \phi}.$$
Reheating

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\[ \ddot{\phi} + 3H \dot{\phi} + \Gamma \dot{\phi} = -\frac{\partial V_H (\phi)}{\partial \phi}. \]

- The Hubble parameter and the temperature of the plasma are determined by

\[ \dot{\rho}_r + 4H \rho_r = \Gamma_I \rho_I, \]

\[ H^2 = \frac{8\pi G}{3} (\rho_I + \rho_r), \]

\[ \rho_r = \frac{\pi^2}{30} g^* T^4. \]
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- While the decay of Higgs may produce some non-zero lepton number by itself, most of the plasma are generated by the decay of inflaton.
Perturbative decay (thermalization) to top quark

- Thermalization rate is comparable to the Hubble parameter only after the maximum reheating has been reached.

$H(t)$ vs $\Gamma_H(t)$ through top quark for IC-1, with the parameters $\Lambda_I = 10^{15}$ GeV and $\Gamma_I = 10^9$ GeV. The vertical lines: the first time the Higgs VEV crosses zero, and the time of maximum reheating, from left to right.