PACIFIC 2015
UCLA Symposium on Particle Astrophysics and Cosmology Including Fundamental Interactions
September 12 - 19, 2015, Moorea, French Polynesia

Exact anomalous dimensions of 4-dimensional N=4 super-Yang-Mills theory in 'tHooft limit

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Outline

•Yang-Mills gauge theories are the mathematical basis of our description of Nature, but they are very difficult to deal with, especially beyond the weak coupling regime. Lattice Monte-Carlo simulations don't always lead to precise and transparent results.

•Can we hope on analytic understanding of Yang-Mills theories in strong coupling regime?

•A remarkable example of integrable (=solvable) 4D quantum gauge field theory is a remote relative of QCD - the superconformal N=4 Yang-Mills theory in the planar ('t Hooft) approximation $N_c=\infty$.

•In a certain sense, planar N=4 SYM is completely solvable, i.e. any reasonable physical quantity (not only in BPS sector!) can be computed at any force of coupling.

•Exact solution is due to AdS/CFT correspondence to string theory on $AdS_5\times S^5$ background and to *quantum integrability* of string sigma-model.

•I will describe the origins and the form of exact equations for anomalous dimensions, called Quantum Spectral Curve (QSC, and review some results of computations using QSC.

Planar Graphs as String Worldsheets

't Hooft

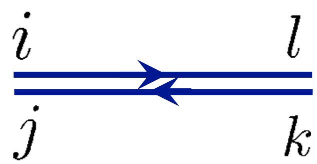
- Yang-Mills theory

$$\mathcal{S}_{YM} = \frac{1}{g_{YM}^2} \int d^4x \, \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} \,, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$A_\mu^{ij} \,, \qquad i,j = 1, \ldots N_c$$

- Propagators

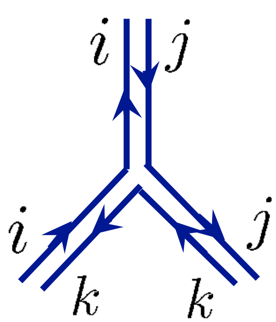
$$\langle A^{ij}(y) A^{kl}(x) \rangle_0 = \delta^{il} \delta^{jk} \, D(x-y)$$

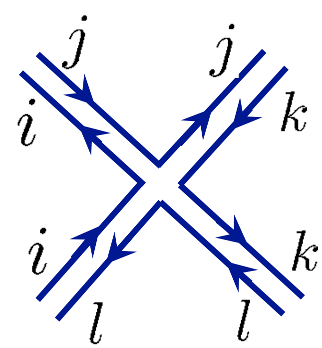


$$i,j,k,l = 1 \ldots N_c$$

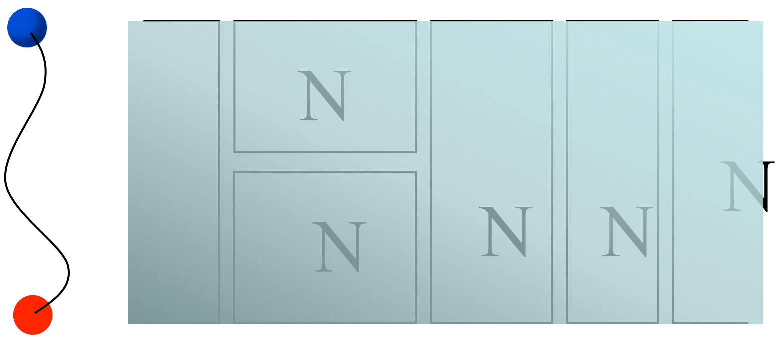
Index is conserved along each line

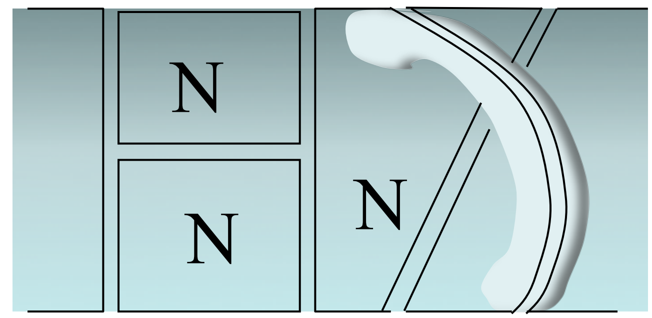
- Vertices





Planar graphs and large-N expansion





$$(g_{YM}^2 N)^5$$

$$(g_{YM}^2 N)^5 \times \frac{1}{N^2}$$

Double expansion:

Perturbative, in ‘t Hooft coupling: $\lambda = g_{YM}^2 N$

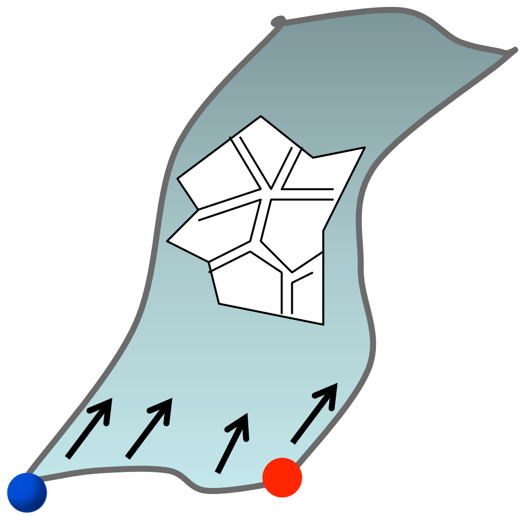
Topological, in string coupling: $g_s = \frac{1}{N^2}$

- Successfully applied for matrix approach in 2d quantum gravity and non-critical strings – early example of “AdS/CFT”.

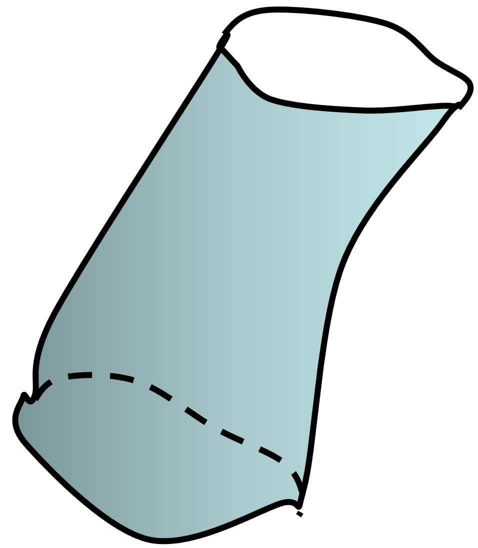
David
V.K.

String picture in QCD

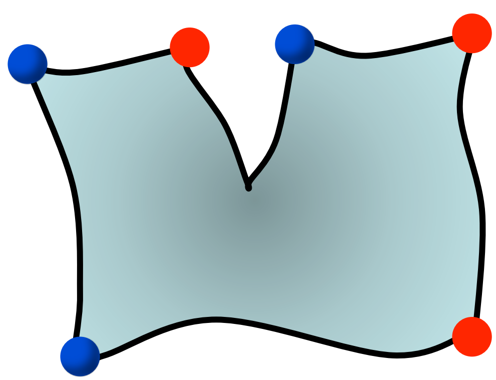
- Meson: quark and antiquark connected by an open string

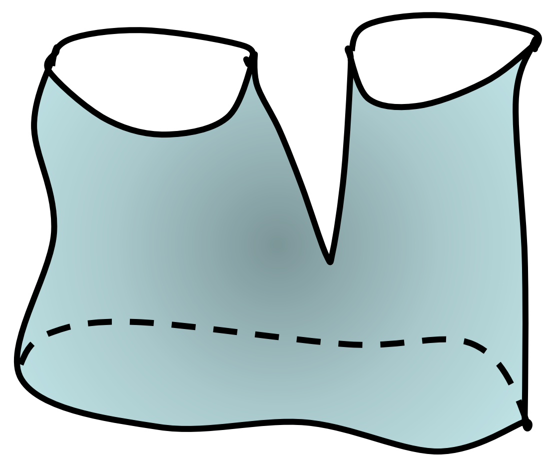


- Glueball: closed string



- Amplitudes





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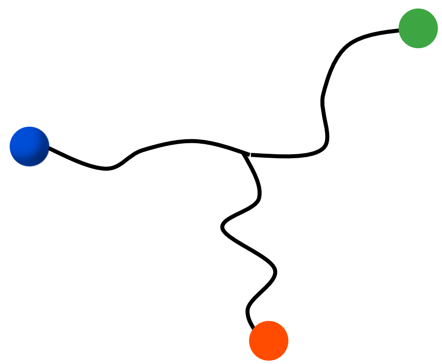
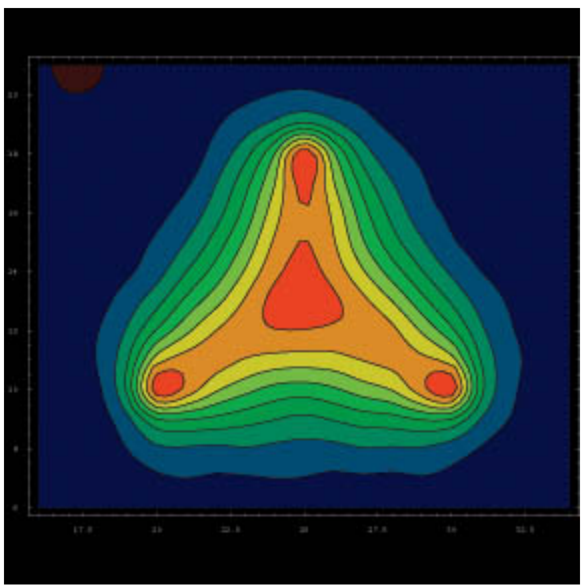
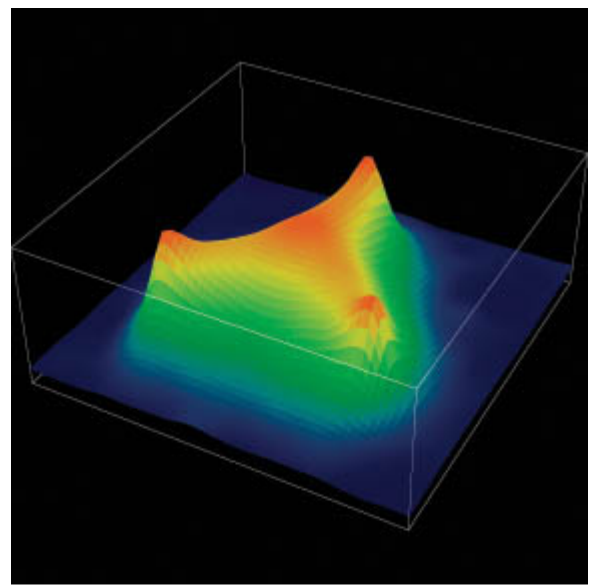
Barion flux tube formation (lattice QCD)…







Ichie, Bornyakov, Streuer , Schierholz

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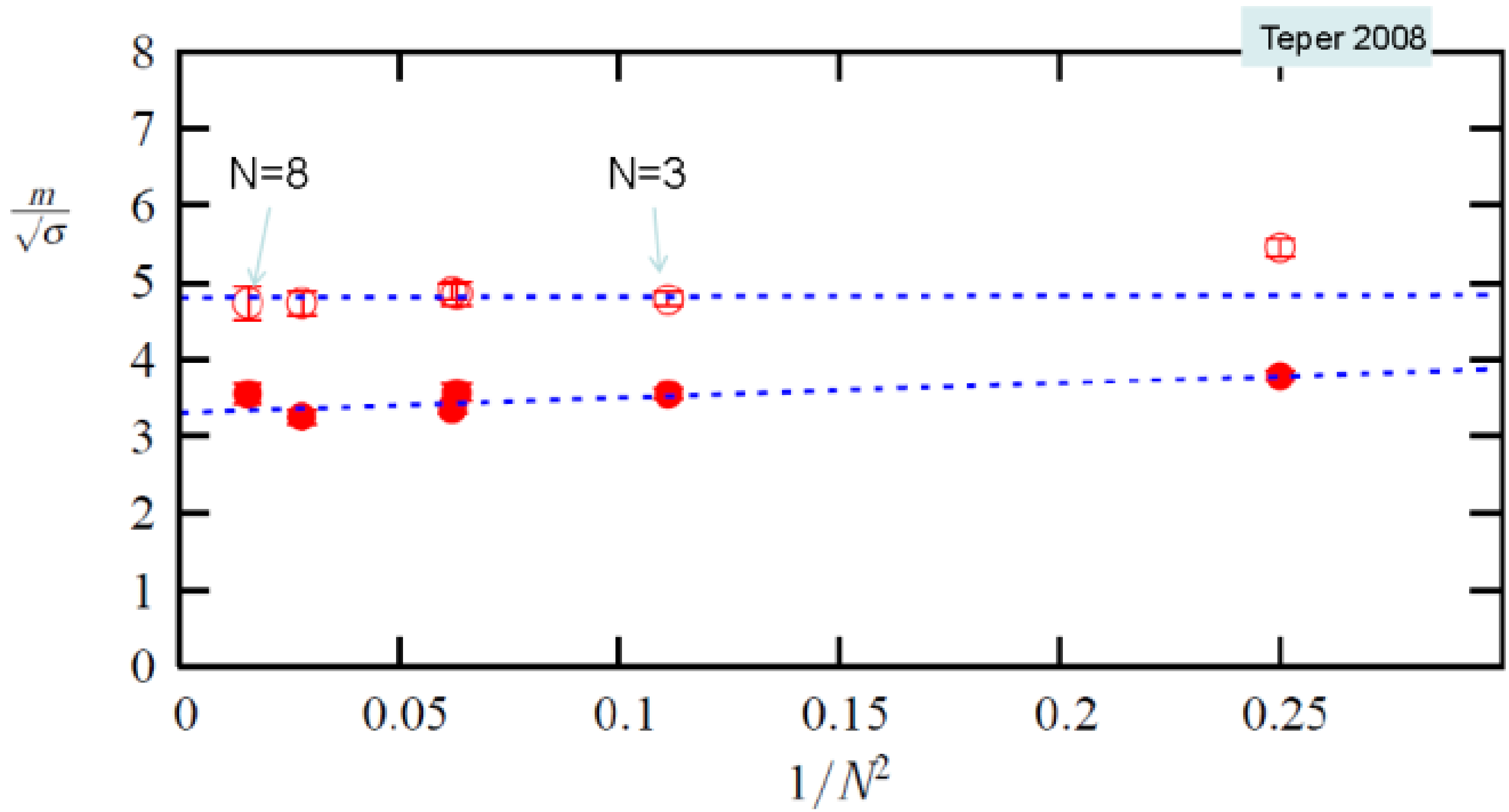
Glueball masses as functions of number of colors N_c from lattice QCD simulations



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N=4 Super-Yang-Mills theory

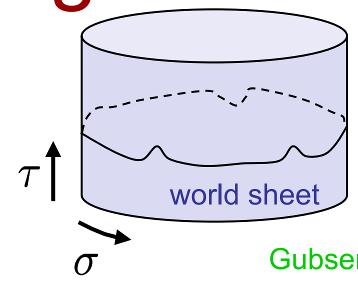
AdS/CFT correspondence
and integrability

N=4 SYM dual to superstring on $AdS_5 \times S^5$

$$\mathcal{S}_{SYM} = \frac{1}{g^2}\int d^4x\, \mathrm{Tr}\left(F^2 + (\mathcal{D}\Phi)^2 + \bar\Psi\mathcal{D}\Psi + \bar\Psi\Phi\Psi + [\Phi,\Phi]^2\right)$$

super-conformal theory:
β-function=0, no massive particles

CFT/AdS duality
weak / strong



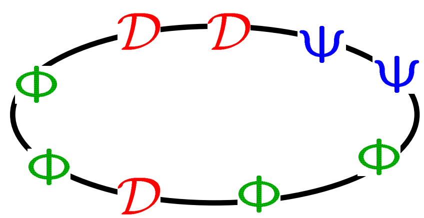
Maldacena
Gubser,Klebanov,Polyakov
Witten

$$\mathcal{S}_{sigma} = g\int d\tau \int_0^L d\sigma\, \left[\left(\partial\vec{X}\right)^2 + \left(\partial\vec{Y}\right)^2 + \text{fermions}\right]$$

Metsaev, Tseytlin

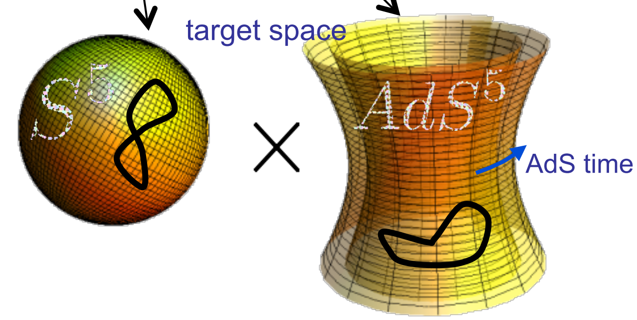
Local operators:

$$\mathcal{O}(x) = \mathrm{Tr}\,[\mathcal{D}\mathcal{D}\Psi\Psi\Phi\Phi\mathcal{D}\Psi\ldots](x)$$



$\longleftrightarrow$

operators / states



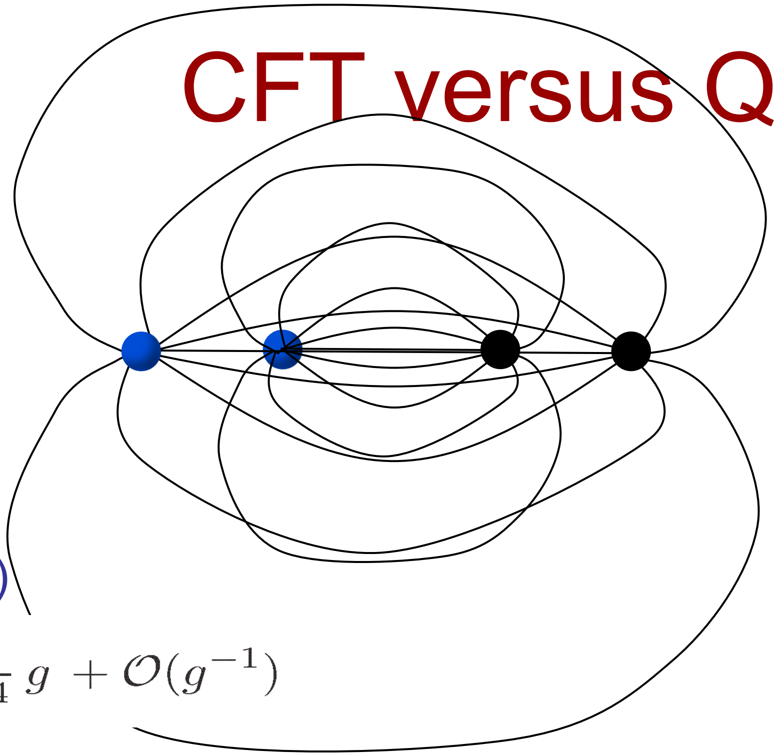
Anomalous dimension $\Delta_{\mathcal{O}}(g)$ = Energy of the dual string state

- Super-conformal symmetry PSU(2,2|4) isometry of string target space
- Gamma-deformed N=4 SYM is also conformal&integrable (N=4 $\rightarrow$ N=0)

$$[A,B] \to e^{i\gamma_{AB}}\, AB - e^{-i\gamma_{AB}}\, BA$$

Leigh, Strassler
Frolov
Lunin,Maldacena
Beisert, Roiban

CFT versus QCD



- CFT

$$V_\lambda(r) = -\frac{e^2(g)}{4\pi r}$$

- N=4 SYM (planar limit, strong coupling)

$$e^2(g) = -\frac{2^{3/2}\pi}{\Gamma(1/4)^4}\, g + \mathcal{O}(g^{-1})$$

Maldacena

- QCD: stretched string at large separation

$$V(r) = V(r_0) + \sigma\, r + \frac{c(r)}{r},$$

$$\text{where} \quad c(r) = -\frac{D-2}{24} + O(1/r)$$

- 4D Correlators:

scaling dimensions

structure constants

non-trivial functions of 't Hooft coupling *g*

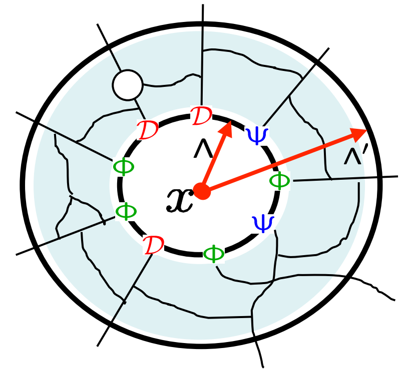
$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \frac{\delta_{ij}}{|x|^{2\Delta_j(g)}}$$

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle = \frac{C_{ijk}(g)}{|x_{12}|^{\Delta_i+\Delta_j-\Delta_k}|x_{23}|^{\Delta_j+\Delta_k-\Delta_i}|x_{31}|^{\Delta_i+\Delta_k-\Delta_j}}$$

They describe the whole conformal theory via operator product expansion

Dilatation operator in SYM perturbation theory

- Dilatation operator $\hat{D}$ from point-splitting and renormalization



$$\mathcal{O}_j^{\Lambda'}(x) = \left[\left(\frac{\Lambda'}{\Lambda} \right)^{\hat{D}} \right]_{jk} \mathcal{O}_k^{\Lambda}(x)$$

- Conformal dimensions are eigenvalues of dilatation operator $\hat{D}$

$$\hat{D}_{jk} \mathcal{O}_k(x) = \Delta_j \mathcal{O}_j$$

- Can be computed from perturbation theory in $g^2 = N g_{YM}^2$

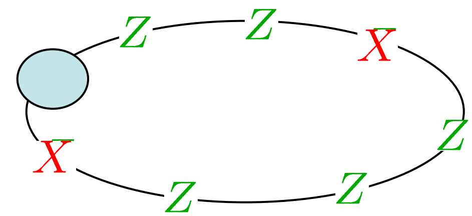
$$\hat{D} = \hat{D}^{(0)} + g^2 \hat{D}^{(2)} + g^4 \hat{D}^{(4)} + \dots$$

$$\Delta = \Delta^{(0)} + g^2 \Delta^{(2)} + g^4 \Delta^{(4)} + \dots$$

Exact spectrum at one loop (su(2)-sector)

- Dilatation operator = Heisenberg Hamiltonian, integrable by Bethe ansatz!

Tr$Z^L(x)$ - vacuum:



$$\hat{D} = L + g^2 \sum_{l=1}^{L} \left(1 - \sigma_l \cdot \sigma_{l+1}\right)$$

Minahan, Zarembo

$$+g^4 \sum_{l=1}^{L} \left(\left(1 - \sigma_l \cdot \sigma_{l+2}\right) - 4\left(1 - \sigma_l \cdot \sigma_{l+1}\right)\right) + O(g^6)$$

Beisert, Kristijansen,Staudacher

also integrable!

- Twisted periodic boundary condition with twist parameter γ
  It corresponds to a particular case of gamma-deformed, N=1 SYM.
- One loop N=4 SYM anomalous dimensions given by condition of analyticity (polynomiality) of two Baxter functions

$$Q_1(u) = e^{iu\gamma} \prod_{j=1}^{J} (u - u_j), \qquad Q_2(u) = e^{-iu\gamma} \prod_{j=1}^{L-J} (u - \tilde{u}_j)$$

related by so called QQ-relation:

$$Q_1(u+i/2)Q_2(u-i/2) - Q_1(u-i/2)Q_2(u+i/2) = 2\sin(\gamma)\, u^L$$

- One-loop anomalous dimensions in terms of solutions of QQ relations:

$$\Delta = L + i\partial_u \log \frac{Q(u-i/2)}{Q(u+i/2)}\Big|_{u=0}$$

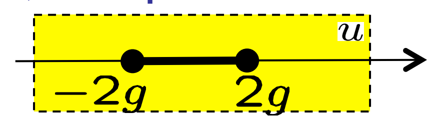
Bethe'31

Gromov, V.K., Vieira
Gromov, V.K., Leurent, Volin
V.K., Leurent, Volin (to appear)

Quantum Spectral Curve for exact N=4 SYM dimensions

- QSC generalizes the integrability and analyticity principals from quantum spin chains to the AdS5xS5 string sigma-model.
- We brought spectral AdS/CFT problem from infinite AdS/CFT Y-system to a finite number of non-linear integral QSC equations.
- Basic functions with "simple" analytic properties (one cut, or i-periodic cuts)

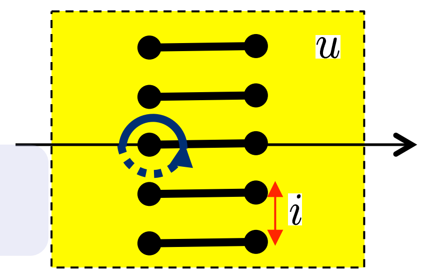
$$\mathbf{P}_a(u), \qquad a = 1,2,3,4;$$



$$\mu_{ab}(u) = -\mu_{ba}(u), \qquad a,b = 1,2,3,4 \qquad \mathrm{Pf}(\mu) = 1$$

$$\tilde{\mu}_{ab}(u) = \mu_{ab}(u+i)$$

- analytic continuation of through the cut on



- Large u asymptotics defined by angular momenta on the sphere S^5 $\quad \mathbf{J} = \begin{pmatrix} J_1 \\ J_2 \\ J_3 \end{pmatrix}$

$$\mathbf{P}_a(u) \sim u^{\frac{1}{2}(\aleph \cdot \mathbf{J})_a} \qquad , \qquad u \to \infty \qquad \aleph = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

- If we include twists of gamma-deformation: $\gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix}$
- No other singularities for all functions!

Gromov, V.K., Leurent, Volin 2013

Quantum Spectral Curve Equations (Pμ -system)

Reduction for SL(2) operators $\mathrm{Tr}(\nabla^S Z^L)$

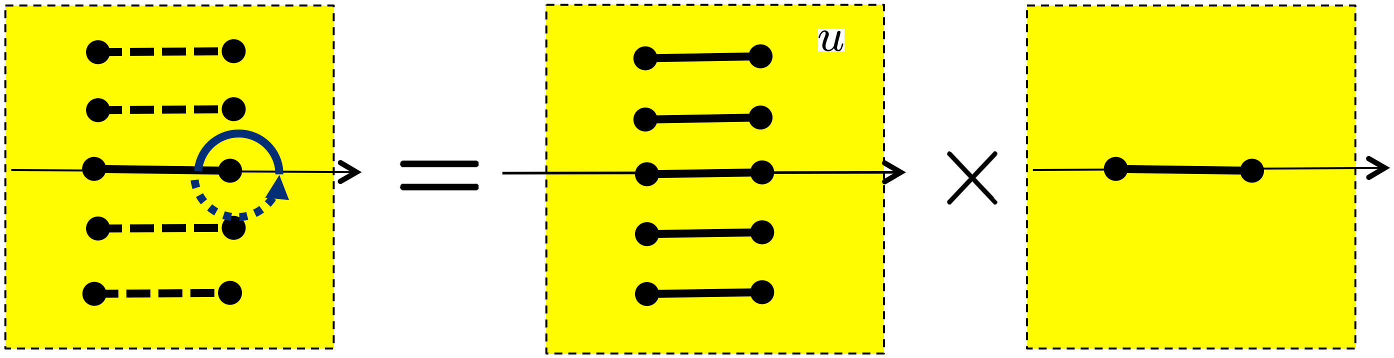
$$\mathbf{P}^a = -\chi^{ab}\mathbf{P}_b$$

$$\chi = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- Monodromy around the branch point:

$$\tilde{\mathbf{P}}_a = \mu_{ab}\mathbf{P}^b$$

$$\tilde{\mathbf{P}}_a \quad = \quad \mu_{ab} \quad \times \quad \mathbf{P}^b$$



- Pμ -system contains also an equation for monodromy of μ:

$$\tilde{\mu}_{ab} - \mu_{ab} = \mathbf{P}_a\tilde{\mathbf{P}}_b - \mathbf{P}_b\tilde{\mathbf{P}}_a$$

- Anomalous dimension can then be found from other asymptotics, e.g.

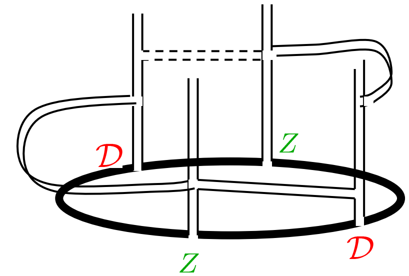
$$\mu_{12} \simeq u^{\Delta - J_1}$$

Some Results for SYM Spectrum from Exact Equations

Perturbative Konishi dimension of N=4 SYM from QSC

131,000 graphs!

$$\begin{aligned}
\Delta \;=\;& 4+12g^2-48g^4+336g^6+g^8\left(-2496+576\,\zeta_3-1440\,\zeta_5\right)\\
&+g^{10}\left(15168+6912\,\zeta_3-5184\,\zeta_3^2-8640\,\zeta_5+30240\,\zeta_7\right)\\
&+g^{12}\left(-7680-262656\,\zeta_3-20736\,\zeta_3^2+112320\,\zeta_5+155520\,\zeta_3\zeta_5+75600\,\zeta_7-489888\,\zeta_9\right)\\
&+g^{14}\big(-2135040+5230080\,\zeta_3-421632\,\zeta_3^2+124416\,\zeta_3^3-229248\,\zeta_5+411264\,\zeta_3\zeta_5\\
&\quad-993600\,\zeta_5^2-1254960\,\zeta_7-1935360\,\zeta_3\zeta_7-835488\,\zeta_9+7318080\,\zeta_{11}\big)\\
&+g^{16}\Big(54408192-83496960\,\zeta_3+7934976\,\zeta_3^2+1990656\,\zeta_3^3-19678464\,\zeta_5-4354560\,\zeta_3\zeta_5\\
&\quad-3255552\,\zeta_3^2\zeta_5+2384640\,\zeta_5^2+21868704\,\zeta_7-6229440\,\zeta_3\zeta_7+22256640\,\zeta_5\zeta_7\\
&\quad+9327744\,\zeta_9+23224320\,\zeta_3\zeta_9+\frac{65929248}{5}\zeta_{11}-106007616\,\zeta_{13}-\frac{684288}{5}Z_{11}^{(2)}\Big)\\
&+g^{18}\Big(-1014549504+1140922368\,\zeta_3-51259392\,\zeta_3^2-20155392\,\zeta_3^3+575354880\,\zeta_5\\
&\quad-14294016\,\zeta_3\zeta_5-26044416\,\zeta_3^2\zeta_5+55296000\,\zeta_5^2+15759360\,\zeta_3\zeta_5^2-223122816\,\zeta_7\\
&\quad+34020864\,\zeta_3\zeta_7+22063104\,\zeta_3^2\zeta_7-92539584\,\zeta_5\zeta_7-113690304\,\zeta_7^2-247093632\,\zeta_9\\
&\quad+119470464\,\zeta_3\zeta_9-245099520\,\zeta_5\zeta_9-\frac{186204096}{5}\zeta_{11}-278505216\,\zeta_3\zeta_{11}-25386566\\
&\quad+1517836320\,\zeta_{15}+\frac{15676416}{5}Z_{11}^{(2)}-1306368\,Z_{13}^{(2)}+1306368\,Z_{13}^{(3)}\Big)\\
&+g^{20}\Big(16445313024-13069615104\,\zeta_3-1509027840\,\zeta_3^2+578949120\,\zeta_3^3\\
&\quad-14929920\,\zeta_3^4-11247547392\,\zeta_5+1213581312\,\zeta_3\zeta_5+1234206720\,\zeta_3^2\zeta_5\\
&\quad-70170624\,\zeta_3^3\zeta_5-1390279680\,\zeta_5^2-654842880\,\zeta_3\zeta_5^2+\frac{6966252288}{175}\zeta_5^3\\
&\quad+377212032\,\zeta_7-1610841600\,\zeta_3\zeta_7+154680192\,\zeta_3^2\zeta_7+222341760\,\zeta_5\zeta_7\\
&\quad+133788672\,\zeta_3\zeta_5\zeta_7+868662144\,\zeta_7^2+4915257984\,\zeta_9-332646912\,\zeta_3\zeta_9\\
&\quad-91072512\,\zeta_3^2\zeta_9+1099699200\,\zeta_5\zeta_9+2275620480\,\zeta_7\zeta_9+\frac{9793211904}{5}\zeta_{11}\\
&\quad-2334572928\,\zeta_3\zeta_{11}+2713772160\,\zeta_5\zeta_{11}-\frac{787483944}{175}\zeta_{13}+3372969600\,\zeta_3\zeta_{13}\\
&\quad-\frac{4308536566944}{875}\zeta_{15}-21661960320\,\zeta_{17}+\frac{752219136}{5}Z_{11}^{(2)}-\frac{5070791808}{175}Z_{13}^{(2)}\\
&\quad-\frac{7159104}{7}Z_{13}^{(3)}+\frac{2716063488}{175}Z_{15}^{(2)}-\frac{17895168}{25}Z_{15}^{(3)}+11943936\,\zeta_3 Z_{11}^{(2)}\Big)+\mathcal{O}(g^{22}),
\end{aligned}$$



Marbeu, Volin
(10-loops from quantum spectral curve)

Always expressed through rationals times Riemann multi-zeta numbers

Confirmed up to 5 loops by direct graph calculus

Fiamberti,Santambrogio,Sieg,Zanon
Velizhanin
Eden,Heslop,Korchemsky,Smirnov,Sokatchev

- Integrability is far more efficient than summing Feynman diagrams!

Strong coupling and numerics from exact QSC equations

- 1/g-expansion for dimension of Konishi operator from our exact equations

Gubser, Polyakov, Klebanov

Gromov, V.K., Vieira

Gromov, Levkovich-Maslyuk, Valatka

- Numerics of extremely high precision from QSC (easily 20 orders!)

Gromov, Levkovich-Maslyuk, Sizov

Confirms earlier results of
Gromov, V.K., Vieira
Frolov

0.1	4.115 506 377 945	0.2	4.418 859 880 802
0.3	4.826 948 662 284	0.4	5.271 565 182 595
0.5	5.712 723 424 787	0.6	6.133 862 814 488
0.7	6.531 606 077 852	0.8	6.907 504 206 024
0.9	7.264 169 587 439	1.	7.604 070 717 047
1.1	7.929 294 264 157	1.2	8.241 563 441 148
1.3	8.542 302 872 295	1.4	8.832 699 939 316
1.5	9.113 754 048 916	1.6	9.386 314 656 368
1.7	9.651 110 426 530	1.8	9.908 771 708 559
1.9	10.159 848 013 162	2.	10.404 821 743 441

- Exact AdS/CFT QSC equations pass all known tests!

BFKL Dimension from Quantum Spectral Curve

- Balitsky-Fadin-Kuraev-Lipatov limit for twist-2 operator:

- LO and NLO known from the direct Feynman graph resummation.
- We realized the analytic continuation in spin S and BFKL limit for QSC equations

Alfimov, Gromov, V.K.

Costa, Concalves, Penedones

LO :

Jaroszevicz, Lipatov, Kotikov, Lipatov

Reproduced from QSC by Alfimov, Gromov, V.K.

NLO :

Kotikov, Lipatov

NNLO :

Gromov, Levkovich-Maslyuk, Sizov

Multi-zeta functions:

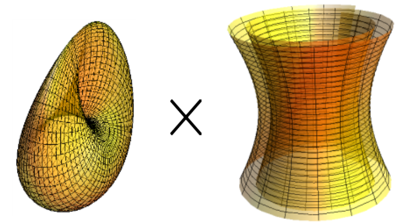
$$
\begin{aligned}
\frac{1}{256}F_3 = \\
&-\frac{5S_{-5}}{8}-\frac{S_{-4,1}}{2}+\frac{S_1S_{-3,1}}{2}+\frac{S_{-3,2}}{2}-\frac{5S_2S_{-2,1}}{4} \\
&+\frac{S_{-4}S_1}{4}+\frac{S_{-3}S_2}{8}+\frac{3S_{3,-2}}{4}-\frac{3S_{-3,1,1}}{2}-S_1S_{-2,1,1} \\
&+S_{2,-2,1}+3S_{-2,1,1,1}-\frac{3S_{-2}S_3}{4}-\frac{S_5}{8}+\frac{S_{-2}S_1S_2}{4} \\
&+\pi^2\left[\frac{S_{-2,1}}{8}-\frac{7S_{-3}}{48}-\frac{S_{-2}S_1}{12}+\frac{S_1S_2}{48}\right] \\
&+\zeta_3\left[-\frac{7S_{-1,1}}{4}+\frac{7S_{-2}}{8}+\frac{7S_{-1}S_1}{4}-\frac{S_2}{16}\right] \\
&+\left[2\mathrm{Li}_4-\frac{\pi^2\log^2 2}{12}+\frac{\log^4 2}{12}\right](S_{-1}-S_1)-\pi^4\left[\frac{2S_{-1}}{45}-\frac{S_1}{96}\right] \\
&+\frac{\log^5 2}{60}-\frac{\pi^2\log^3 2}{36}-\frac{2\pi^4\log 2}{45}-\frac{\pi^2\zeta_3}{24}+\frac{49\zeta_5}{32}-2\mathrm{Li}_5
\end{aligned}
$$

Found from iterative solution of QSC

- Might help to find the NNLO leading pomeron trajectory for QCD

Conclusions and prospects

- We have a few examples of exactly solvable planar gauge theories at D>2, such as 4D N=4 SYM and 3D ABJM model, dual to CP^3 x AdS_4

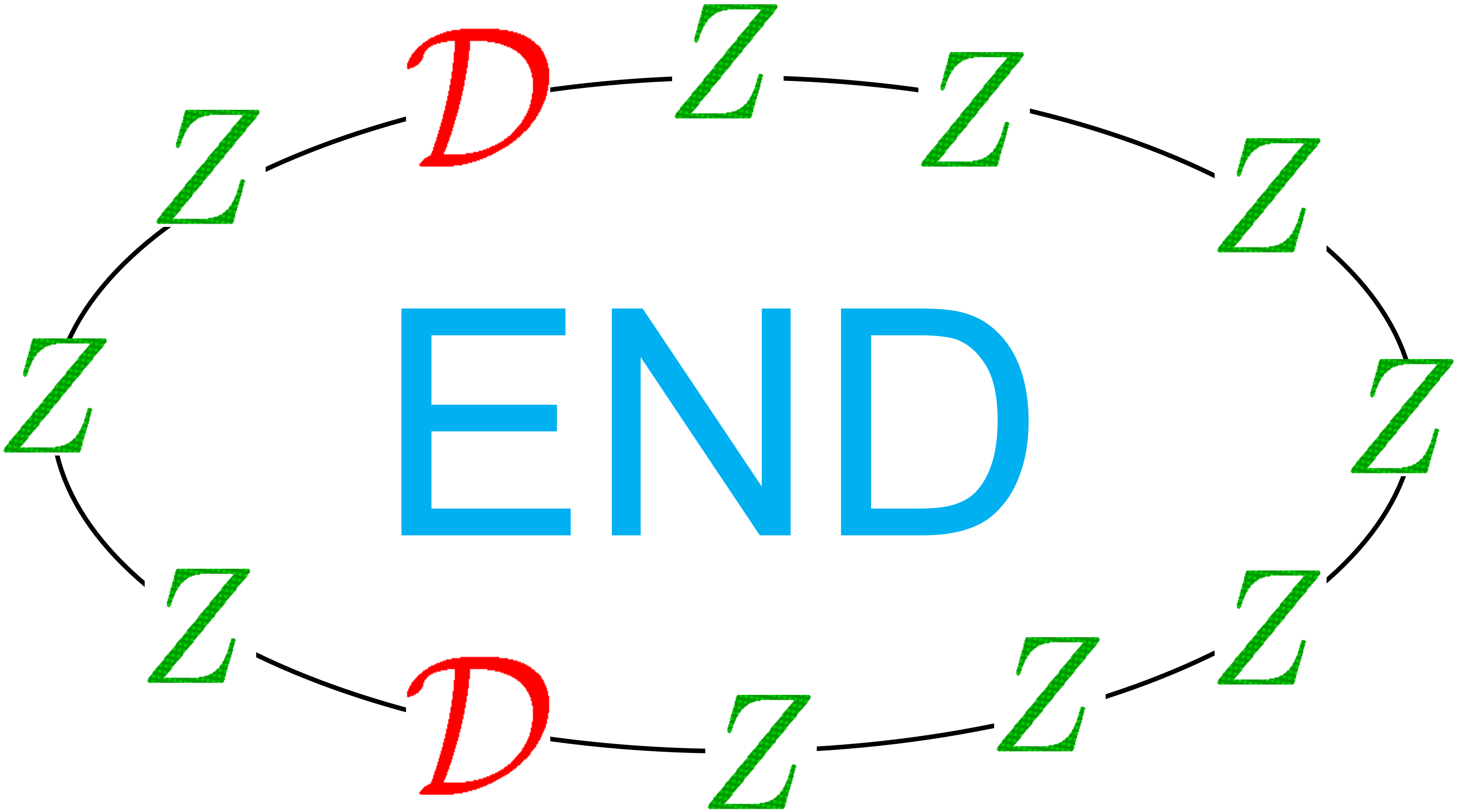


- The problem of computing the anomalous dimensions of local operators is reduced to a finite set of non-linear Riemann-Hilbert type equations -- Quantum Spectral Curve – a unique tool for analytic and numerical study of anomalous dimensions at any coupling.

- Many other physical quantities might be exactly computable: n-point correlation f-ns, Wilson loops, gluon scattering amplitudes, 1/N-corrections

- Can we use it as a zero order approximation to realistic gauge theories, such as QCD?

- What is the origin of this integrability on the gauge side?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