Will Planck Observe Gravity Waves?

Qaisar Shafi

Bartol Research Institute
Department of Physics and Astronomy
University of Delaware

in collaboration with G. Dvali, R. K. Schaefer, G. Lazarides, N. Okada, K.Pallis, M. Rehman, N. Senoguz, J. Wickman.

Moorea, September, 2014

Inflationary Cosmology

[Guth, Linde, Albrecht & Steinhardt, Starobinsky, Mukhanov, Hawking, . . .]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r, $dn_s/d\ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Physics Beyond the SM?

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2, \ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k}, \ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

  where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.
- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta, \ r \simeq 16\epsilon.$$

- The tensor to scalar ratio r can be related to the energy scale of inflation via

$$V(\phi_0)^{1/4} = 3.3 \times 10^{16}\, r^{1/4} \text{ GeV}.$$

- The amplitude of the curvature perturbation is given by

$$\Delta^2_{\mathcal{R}} = \frac{1}{24\pi^2} \left(\frac{V/m_p^4}{\epsilon} \right)_{\phi=\phi_0} = 2.43 \times 10^{-9} \text{ (WMAP7 normalization)}.$$

- The spectrum of the tensor perturbation is given by

$$\Delta^2_h = \frac{2}{3\,\pi^2} \left(\frac{V}{m_P^4} \right)_{\phi=\phi_0}.$$

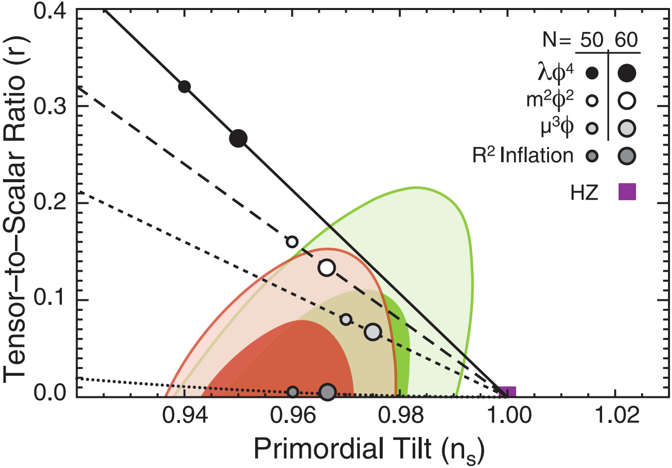
- The number of e-folds after the comoving scale $l_0 = 2\,\pi/k_0$ has crossed the horizon is given by

$$N_0 = \frac{1}{m_p^2} \int_{\phi_e}^{\phi_0} \left(\frac{V}{V'} \right) d\phi.$$

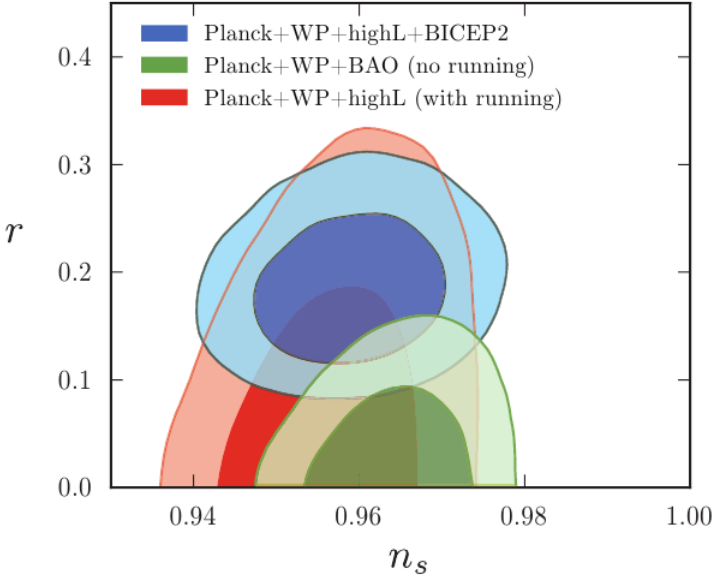
Inflation ends when $\max[\epsilon(\phi_e), |\eta(\phi_e)|] = 1$.

BICEP 2 Result

- BICEP 2 a few months ago surprised many people with their results that $r \sim 0.2$ (0.16).
- Some tension with the Planck upper bound $r < 0.11$.
- Somewhat earlier WMAP 9 stated that $r < 0.13$.



WMAP nine year data

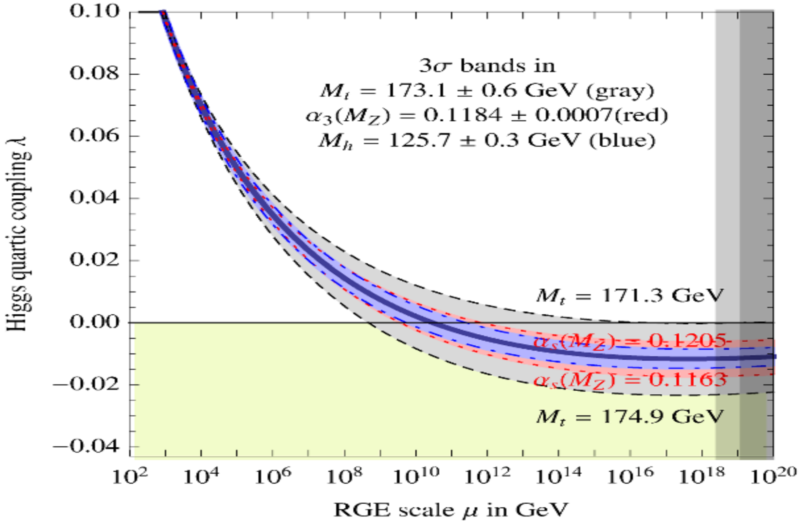


Baumann, Amsterdam 2014

SM Higgs Inflation?

Update of RGE analysis (@ 3-loop level)

Buttazzo et al., JHEP 12 (2013) 089

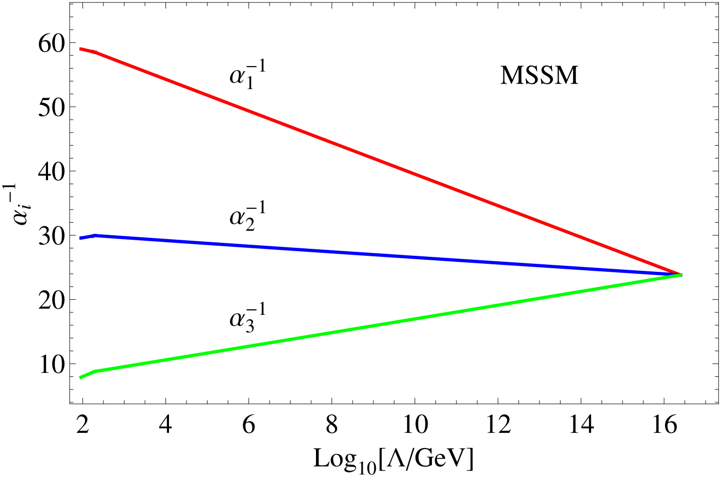


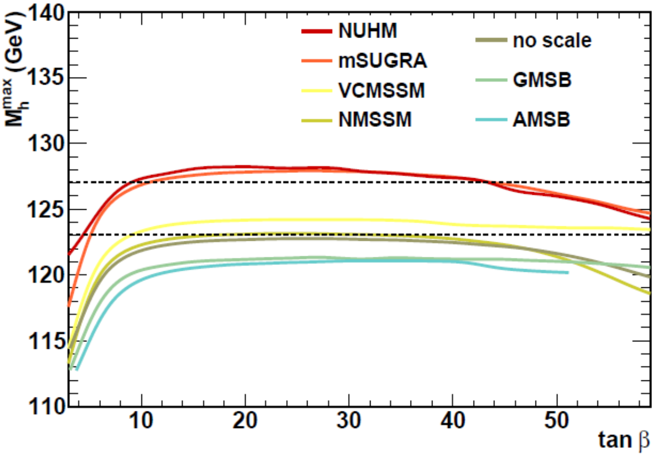
Supersymmetry

- Resolution of the gauge hierarchy problem
- Predicts plethora of new particles which LHC should find
- Unification of the SM gauge couplings at $M_{GUT} \sim 2 \times 10^{16}$ GeV
- Cold dark matter candidate (LSP)
- Radiative electroweak breaking
- String theory requires supersymmetry (SUSY)

Alas, SUSY not yet seen at LHC

Why Supersymmetry?





A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, Phys. Lett. B **708**, 162 (2012)

SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]
[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\, (\Phi\, \overline{\Phi} - M^2)$$

$S =$ gauge singlet superfield, $(\Phi\,,\overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\,,\overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\,\overline{\Phi} \to \Phi\,\overline{\Phi}, \;\; S \to e^{i\alpha}\, S, \;\; W \to e^{i\alpha}\, W$$

$\Rightarrow$ W is a unique renormalizable superpotential

- Some examples of gauge groups:

$G = U(1)_{B-L}$, (Supersymmetric superconductor)

$G = SU(5) \times U(1), \quad (\Phi = 10), \quad$ (Flipped $SU(5)$)

$G = 3_c \times 2_L \times 2_R \times 1_{B-L}, \ (\Phi = (1, 1, 2, +1))$

$G = 4_c \times 2_L \times 2_R, \ (\Phi = (\overline{4}, 1, 2)),$

$G = SO(10), \ (\Phi = 16)$

- At renormalizable level the SM displays an 'accidental' global $U(1)_{B-L}$ symmetry.

- Next let us 'gauge' this symmetry, so that $U(1)_{B-L}$ is now promoted to a local symmetry. In order to cancel the gauge anomalies, one may introduce 3 SM singlet (right-handed) neutrinos.

  This has several advantages:

- See-saw mechanism is automatic and neutrino oscillations can be understood.

- RH neutrinos acquire masses only after $U(1)_{B-L}$ is spontaneously broken; Neutrino oscillations require that RH neutrino masses are $\lesssim 10^{14}\mathrm{GeV}$.

- RH neutrinos can trigger leptogenesis after inflation, which subsequently gives rise to the observed baryon asymmetry;

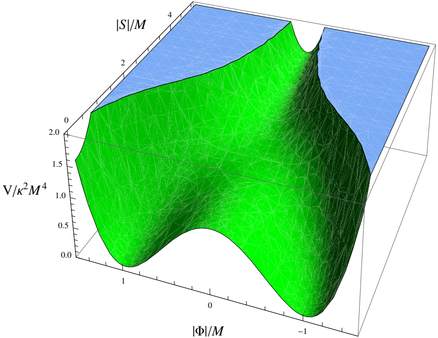
- Last but not least, the presence of local $U(1)_{B-L}$ symmetry enables one to explain the origin of Z_2 'matter' parity of MSSM. (It is contained in $U(1)_{B-L} \times U(1)_Y$, if $B-L$ is broken by a scalar vev, with the scalar carrying two units of $B-L$ charge.)

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle\overline{\Phi}\rangle| = |\langle\Phi\rangle| = M, \ \langle S\rangle = 0$$



Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_\pm^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\rm 1loop} = \frac{1}{64\pi^2} {\rm Str}[\mathcal{M}^4(S)(\ln \frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4+1\right) \ln \frac{\left(x^4-1\right)}{x^4} + 2x^2 \ln \frac{x^2+1}{x^2-1} + 2 \ln \frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

Full Story

Also include supergravity corrections + soft SUSY breaking terms

- The minimal Kähler potential can be expanded as

$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2$$

- The SUGRA scalar potential is given by

$$V_F = e^{K/m_p^2} \left(K_{ij}^{-1} D_{z_i} W D_{z_j^*} W^* - 3 m_p^{-2} |W|^2 \right)$$

where we have defined

$$D_{z_i} W \equiv \frac{\partial W}{\partial z_i} + m_p^{-2} \frac{\partial K}{\partial z_i} W; \; K_{ij} \equiv \frac{\partial^2 K}{\partial z_i \partial z_j^*}$$

and $z_i \in \{\Phi, \overline{\Phi}, S, ...\}$

[Senoguz, Shafi '04; Jeannerot, Postma '05]

- Take into account sugra corrections, radiative corrections and soft SUSY breaking terms:

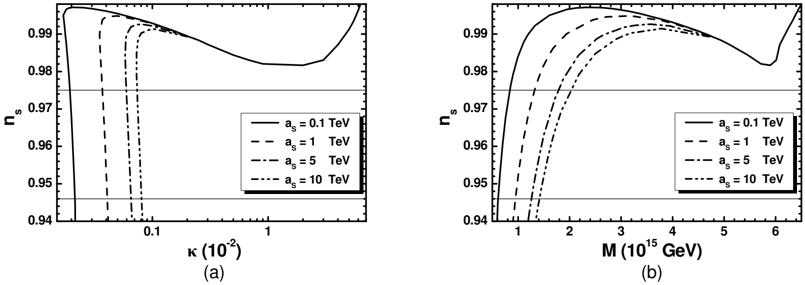
$$V \simeq \kappa^2 M^4 \left(1 + \left(\frac{M}{m_p}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a_s \left(\frac{m_{3/2} x}{\kappa M}\right) + \left(\frac{m_{3/2} x}{\kappa M}\right)^2\right)$$

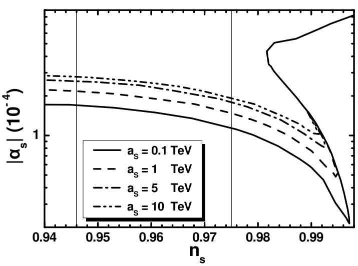
where $a_s = 2\,|2 - A| \cos[\arg S + \arg(2 - A)]$, $x = |S|/M$ and $S \ll m_P$.

Note: No 'η problem' with minimal (canonical) Kähler potential !

Results

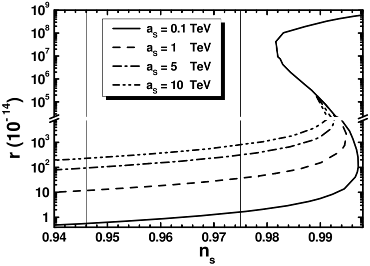
[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]





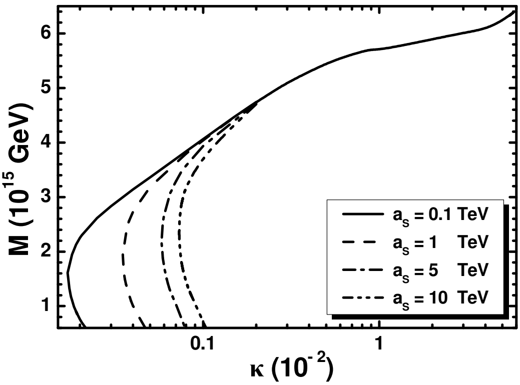
|α_s| (10^-4)
1
0.94
0.95
0.96
0.97
0.98
0.99
n_s
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV

(a)

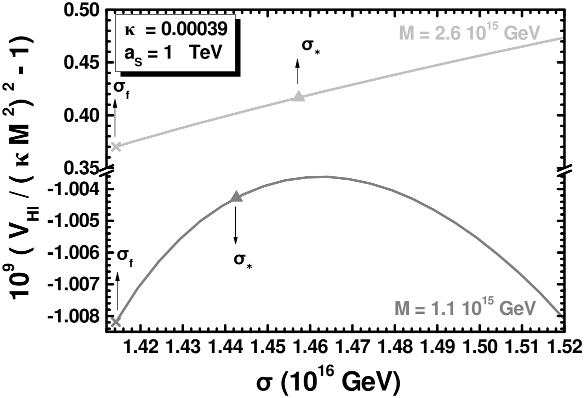


r (10^-14)
10^9
10^8
10^7
10^6
10^5
10^3
10^2
10
1
0.94
0.95
0.96
0.97
0.98
0.99
n_s
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV

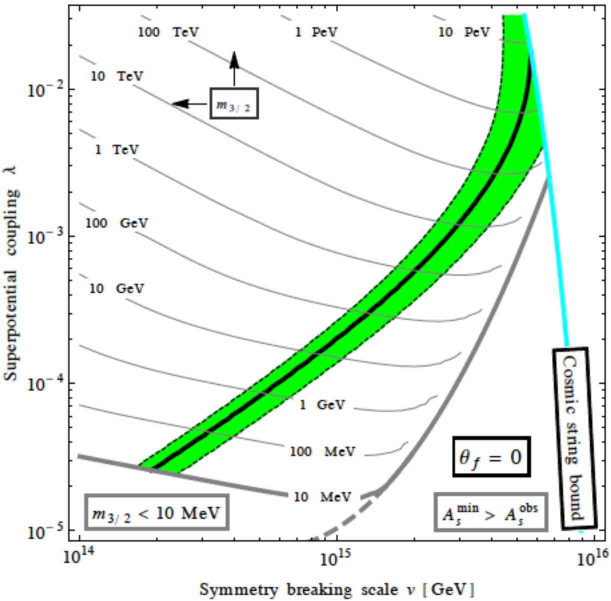
(b)



M (10^15 GeV)
6
5
4
3
2
1
0.1
1
κ (10^-2)
a_s = 0.1 TeV
a_s = 1 TeV
a_s = 5 TeV
a_s = 10 TeV



κ = 0.00039
a_S = 1 TeV
M = 2.6 10^15 GeV
M = 1.1 10^15 GeV
σ_*
σ_f
σ_*
σ_f
10^9 (V_HI / (κ M^2)^2 - 1)
σ (10^16 GeV)
0.50
0.45
0.40
0.35
-1.004
-1.005
-1.006
-1.007
-1.008
1.42 1.43 1.44 1.45 1.46 1.47 1.48 1.49 1.50 1.51 1.52



100 TeV
1 PeV
10 PeV
10 TeV
$m_{3/2}$
1 TeV
100 GeV
10 GeV
1 GeV
100 MeV
10 MeV
$m_{3/2} < 10$ MeV
$\theta_f = 0$
$A_s^{min} > A_s^{obs}$
Cosmic string bound
Superpotential coupling λ
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{14}
10^{15}
10^{16}
Symmetry breaking scale v [GeV]

Non-Minimal SUSY Hybrid Inflation and Tensor Modes

- Minimal SUSY hybrid inflation model yields tiny r values $\lesssim 10^{-10}$
- A more general analysis with a non-minimal Kähler potential can lead to larger r-values;
- The Kähler potential can be expanded as:

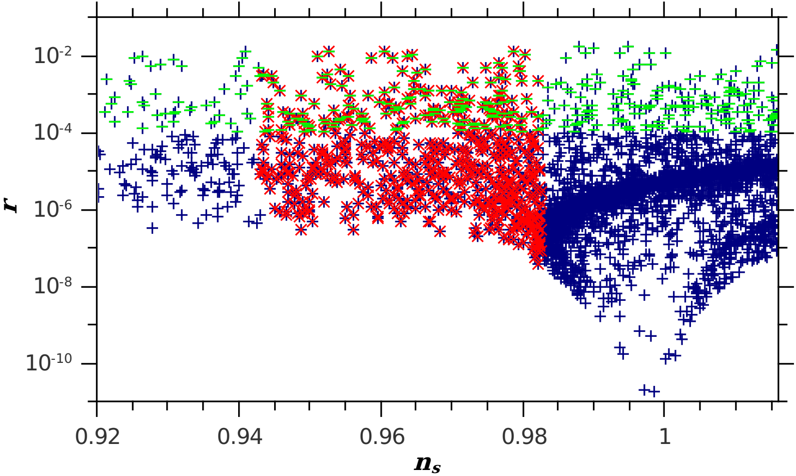
$$K = |S|^2 + |\Phi|^2 + |\overline{\Phi}|^2 + \frac{\kappa_S}{4}\frac{|S|^4}{m_P^2} + \frac{\kappa_\Phi}{4}\frac{|\Phi|^4}{m_P^2} + \frac{\kappa_{\overline{\Phi}}}{4}\frac{|\overline{\Phi}|^4}{m_P^2} + \kappa_{S\Phi}\frac{|S|^2|\Phi|^2}{m_P^2} + \kappa_{S\overline{\Phi}}\frac{|S|^2|\overline{\Phi}|^2}{m_P^2} + \kappa_{\Phi\overline{\Phi}}\frac{|\Phi|^2|\overline{\Phi}|^2}{m_P^2} + \frac{\kappa_{SS}}{6}\frac{|S|^6}{m_P^4} + \cdots ,$$

The scalar potential becomes

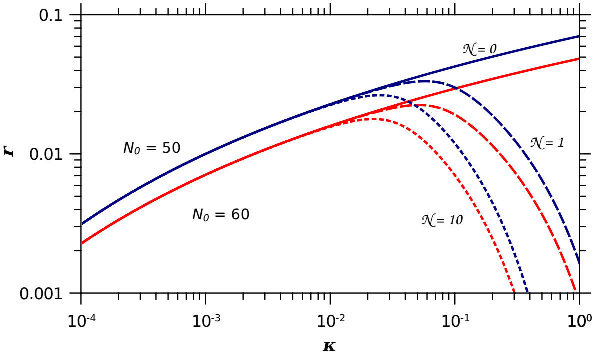
$$V \simeq \kappa^2 M^4 \left(1 - \kappa_S \left(\frac{M}{m_P}\right)^2 x^2 + \gamma_S \left(\frac{M}{m_P}\right)^4 \frac{x^4}{2} + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x) + a \left(\frac{m_{3/2}\, x}{\kappa\, M}\right) + \left(\frac{M_S\, x}{\kappa\, M}\right)^2\right)$$

with (leading order) non-minimal Kähler, SUGRA, radiative, and soft SUSY-breaking corrections, and where

$$\gamma_S \equiv 1 - \frac{7}{2}\kappa_S + 2\kappa_S^2 - 3\kappa_{SS}$$



10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
r
0.92
0.94
0.96
0.98
1
n_s



While radiative corrections are subdominant at large r, they play a crucial role in limiting the size of r. This limiting behavior comes in *indirectly* via the number of e-foldings N_0.

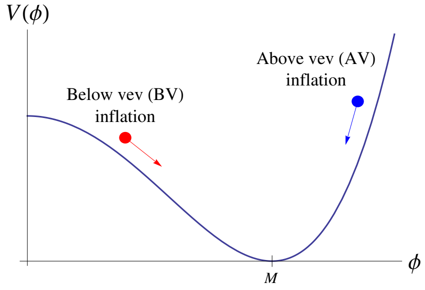
Tree Level Gauge Singlet Higgs Inflation

[Kallosh and Linde, 07; Rehman, Shafi and Wickman, 08]

- Consider the following Higgs Potential:

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{M}\right)^2\right]^2 \quad \longleftarrow \text{(tree level)}$$

Here ϕ is a gauge singlet field.

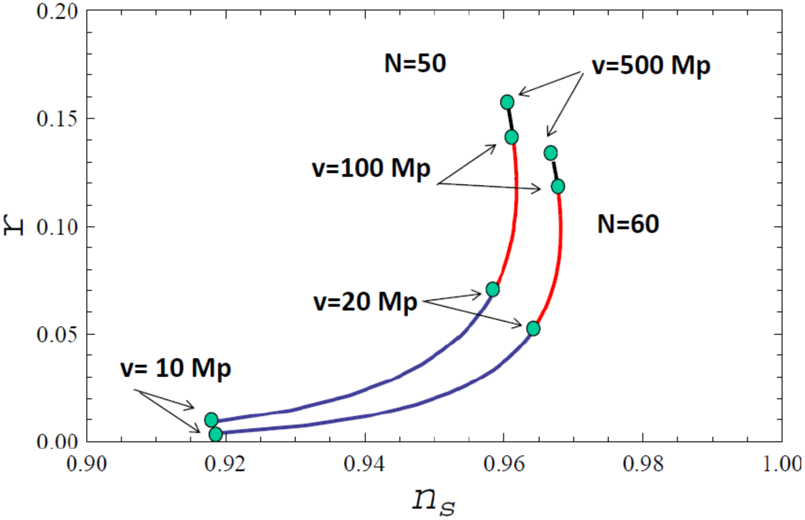


- WMAP/Planck data favors BV inflation ($r \lesssim 0.1$).
- BUT now BICEP2 may have found $r \approx 0.2$.

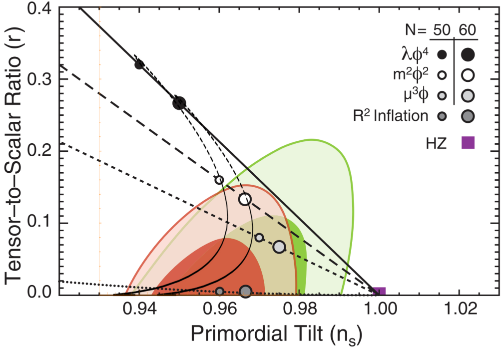
Inflation of the B-L scalar field:

$$V = \frac{1}{4}\lambda(\phi^2 - v^2)^2 \text{ , where } \phi/\sqrt{2} = \mathcal{R}[\phi]$$

We consider inflation with the initial inflation VEV: $\phi < v$

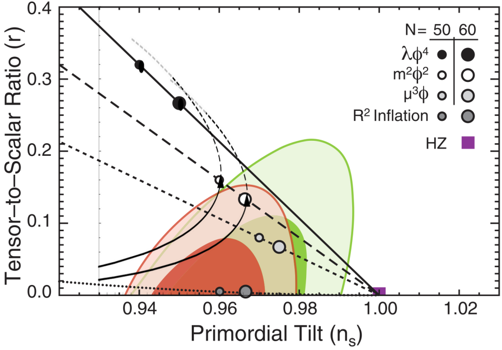


Coleman–Weinberg Potential:

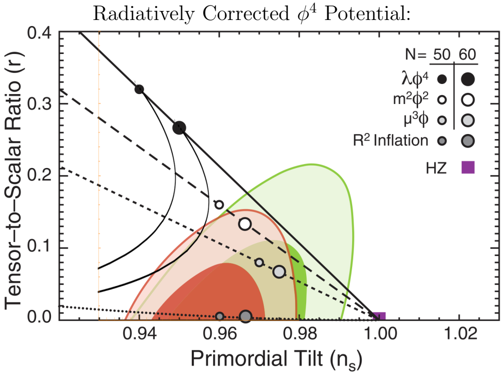


n_s vs. r for Coleman–Weinberg potential. The dashed portions are for $\phi > v$. N is taken as 50 (left curves) and 60 (right curves).

Radiatively Corrected ϕ^2 Potential:



n_s vs. r for radiatively corrected ϕ^2 potential. The dashed portions are for $\kappa < 0$. The one loop radiative correction is larger than the tree level potential in the portions displayed in gray. N is taken as 50 (left curves) and 60 (right curves).



n_s vs. r for radiatively corrected ϕ^4 potential. The dashed portions are for $\kappa < 0$. The one loop radiative correction is larger than the tree level potential in the portions displayed in gray. N is taken as 50 (left curves) and 60 (right curves).

Quartic Inflation with non-minimal coupling to gravity

- We consider a quartic inflaton potential with a non-minimal gravitational coupling.

- The basic action of non-minimal ϕ^4 inflation is given in the Jordan frame

$$S_J^{\rm tree} = \int d^4x\sqrt{-g}\left[-\left(\frac{1+\xi\phi^2}{2}\right)\mathcal{R} + \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4!}\phi^4\right]$$

- The inflation potential in the Einstein frame is

$$V_E(\sigma_E(\phi)) = \frac{\frac{1}{4!}\lambda(t)\phi^4}{(1+\xi\,\phi^2)^2}.$$

Quartic Inflation with non-minimal coupling to gravity

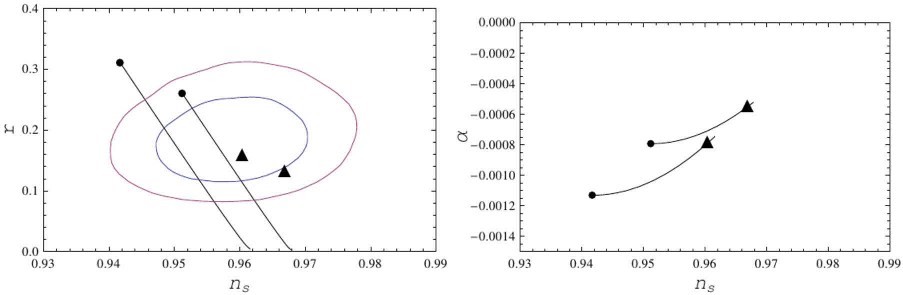


Figure 6. ϕ^4 potential with non-minimal gravitational coupling: n_s vs. r (left panel) and n_s vs. α (right panel) for various ξ values, along with n_s vs. r the contours (at the confidence levels of 68% and 95%) given by the BICEP2 collaboration (Planck+WP+highL+BICEP2). The black points and triangles are predictions in the textbook quartic and quadratic potential models, respectively. N is taken as 50 (left curves) and 60 (right curves).

Summary

- If r lies close to 0.15, with n_s around 0.96, then chaotic inflation with ϕ^2 potential is an especially simple scenario. However, transplanckian field values remain a concern.

- If $r \sim 0.1 - 0.05$, then inflation models based on the Higgs / Coleman-Weinberg potentials can provide simple / realistic frameworks for inflation.

- If $r \leq 0.01$, then supersymmetric hybrid inflation models are especially interesting. These work with inflaton field values below M_{Planck}, and supergravity corrections are under control. The simplest versions employ TeV scale SUSY, and hopefully LHC 14 will find it.