Higgs Vacuum Stability & Physics Beyond the Standard Model

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AK & A. Spencer-Smith, Phys Lett B 722 (2013) 130 [arXiv:1301.2846]
AK & A. Spencer-Smith, JHEP 1308 (2013) 036 [arXiv:1305.7283]
AK & A. Spencer-Smith, arXiv:1404.4709

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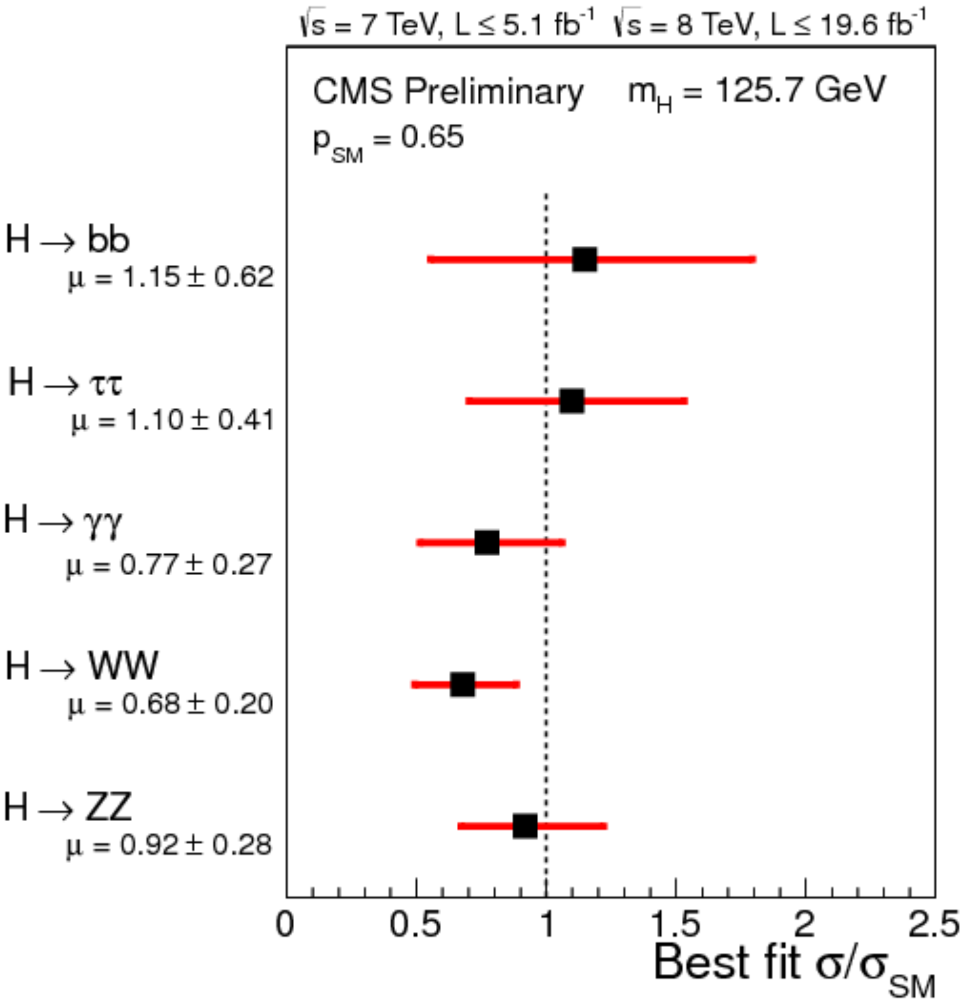
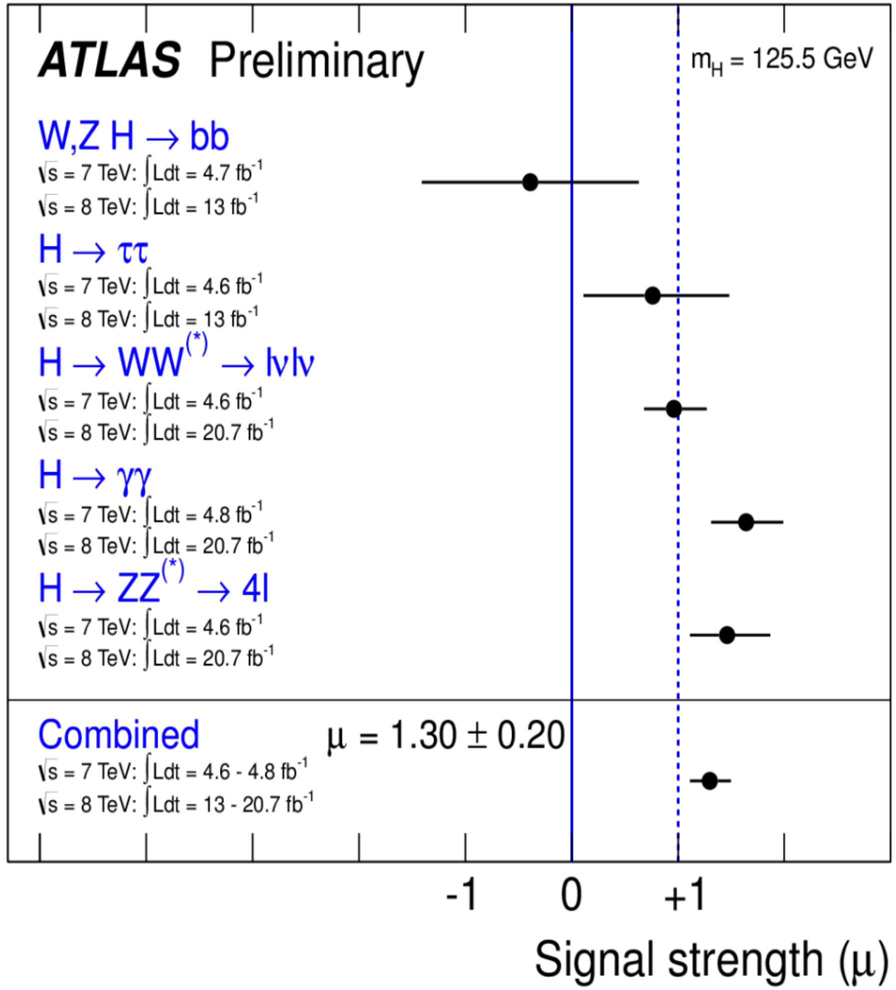
Combined 2012-13 data: It is a Higgs boson, maybe even the Higgs boson





The SM Higgs with M_H=125.9± 0.4 GeV

- Naturalness problem: somewhat heavy than typical prediction of the supersymmetric models and somewhat light than typical prediction of technicolour models.

- More notably, the Standard Model vacuum state $|0\rangle_{EW}$

$$_{EW}\langle 0|h|0\rangle_{EW} = v_{EW} \approx 246 \text{ GeV}$$

is a false (local) vacuum. The true vacuum state

$$\langle 0|h|0\rangle \sim M_P \approx 10^{18}\text{GeV} ,$$

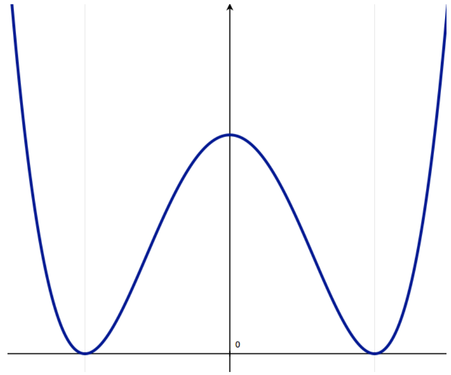
and it carries large negative energy density ~ - $(M_P)^4$.

- How long does the electroweak vacuum live?

EW vacuum lifetime: effective Higgs potential

- Electroweak Higgs doublet (in the unitary gauge): $H = \begin{pmatrix} 0 \\ h(x)/\sqrt{2} \end{pmatrix}$

$$V_{\rm H}^{(0)}(h) = \frac{\lambda}{8}\left(h^2 - v_{EW}\right)^2$$



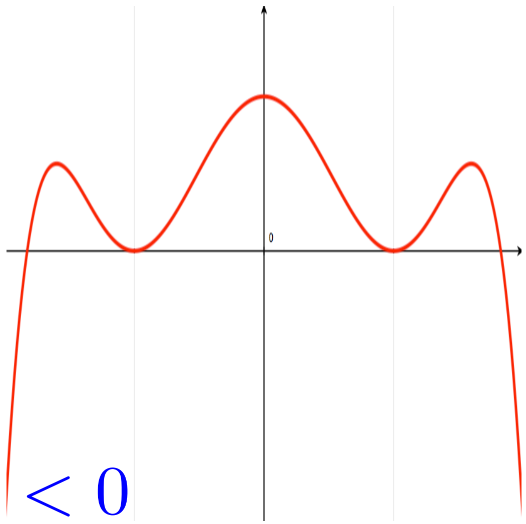
- Effective (quantum-corrected) potential

$$V_{\rm H}^{(1-\text{loop})}(h) = \frac{\lambda(h)}{8}\left(h^2 - v_{EW}\right)^2 \ ,$$

$$\lambda(h) = \lambda(\mu) + \beta_\lambda \ln(h/\mu)$$

$$(4\pi)^2\beta_\lambda = -6y_t^4 + 24\lambda^2 + \dots$$

$$y_t(m_t) \approx 1 \ , \ \lambda(m_h) \approx 0.13 \ \longrightarrow \beta_\lambda < 0$$



EW vacuum lifetime: RG extrapolation of SM parameters

- Previous calculations:

F. Bezrukov, M. Y. .Kalmykov, B. A. Kniehl and M. Shaposhnikov, JHEP 1210, 140 (2012) [arXiv:1205.2893 [hep-ph]];

G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, JHEP 1208, 098 (2012) [arXiv:1205.6497 [hep-ph]]; D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, JHEP 1312, 089 (2013) [arXiv:1307.3536].

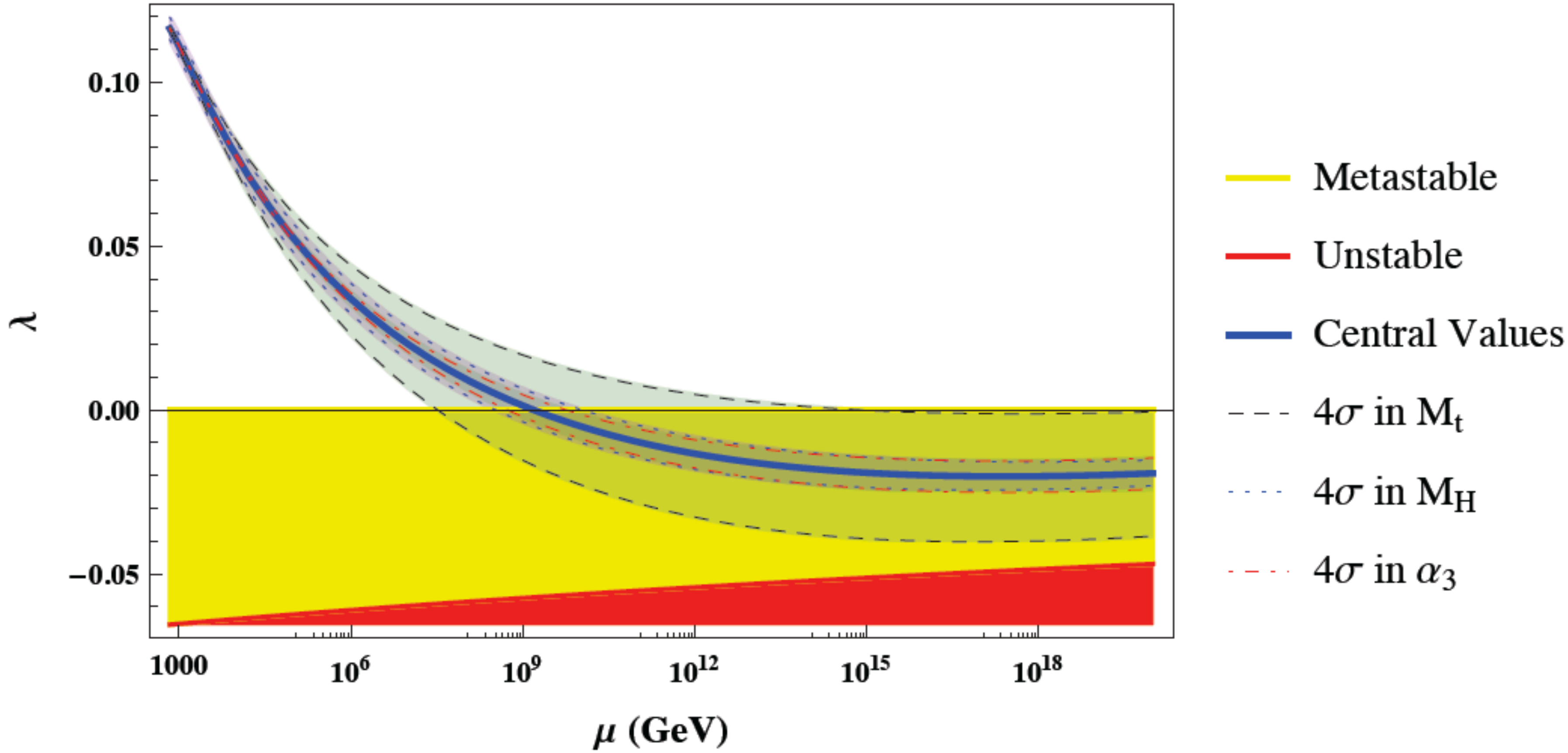
3-loop RGE's in mass-independent MSbar scheme; full 2-loop matching condition

- Our calculations:

AK & A. Spencer-Smith, arXiv:1404.4709; A. Spencer-Smith, arXiv:1405.1975

1-loop RGE in mass-dependent scheme, 2 – and 3 – loop RGE's in MSbar + corresponding matching condition

EW vacuum lifetime: RG running of λ in a mass-dependent scheme



Instability scale:

$[\lambda(\mu_i) = 0]$ $\quad \log_{10}\left(\frac{\mu_i}{\text{GeV}}\right) \approx 9.19 \pm 0.65_{M_t} \pm 0.19_{M_h} \pm 0.13_{\alpha_3} \pm 0.02_{\text{th}}$

EW vacuum lifetime: flat spacetime estimate

- Large field limit:

$$V_{\mathrm{H}} = \frac{\bar{\beta}_\lambda \ln(h/\mu_i)}{4} h^4 \ , \ \bar{\beta}_\lambda = \beta_\lambda|_{\mu=\mu_i}$$

$$h_* = \mu_i e^{-1/4}$$

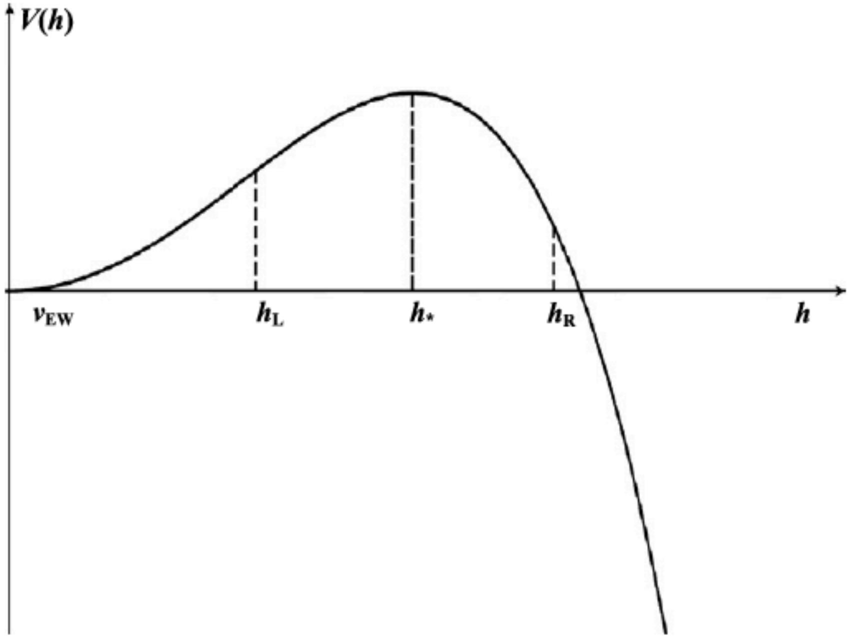


Fig. 1. The Higgs potential. For large values of the Higgs field h, the electroweak vacuum configuration is regarded as trivial, $v_{EW} \approx 0$.

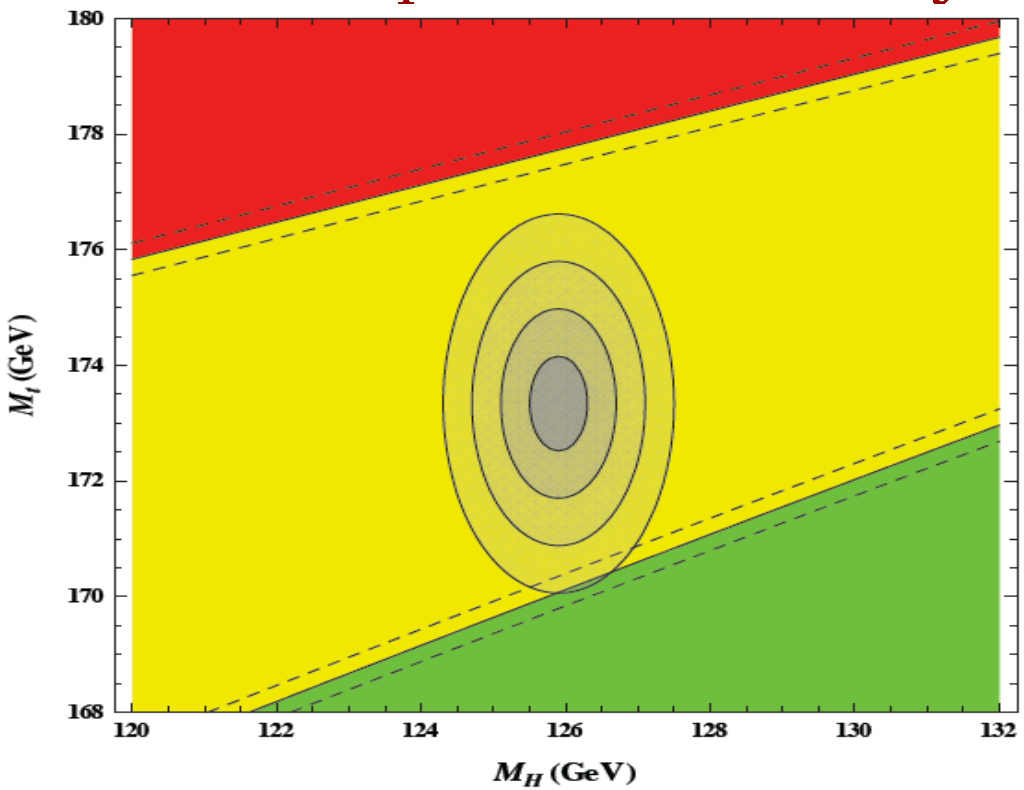
- Using Coleman's prescription, one can calculate that the decay of electroweak vacuum is dominated by small size Lee-Wick bounce solution,

$$R \sim 1/\mu_m \approx 10^{-17}/\mathrm{GeV}, \ \beta_\lambda|_{\mu=\mu_m} = 0$$

$$S_{\mathrm{LW}} = \frac{8\pi^2}{3|\lambda(\mu_m)|}, \ |\lambda(\mu_m)| \approx 0.01 - 0.02$$

$$P_{\mathrm{EW}} = e^{-p} \approx 1, \ p = (\mu_m/H_0)^4 \exp\left(-S_{\mathrm{LW}}\right) << 1$$

EW vacuum in flat spacetime: stability bound



Stability bound: $M_t < 170.16 \pm 0.22_{\alpha_3} \pm 0.13_{M_h} \pm 0.06_{\rm th}$ GeV

$$M_t^{\rm exp} = 173.34 \pm 0.76_{\alpha_3} \pm 0.3_{\rm QCD} \text{ GeV}$$

Absolute stability of the electroweak vacuum is excluded now at 99.98% CL [AK & A. Spencer-Smith, arXiv:1404.4709, A. Spencer-Smith, arXiv:1405.1975]

EW vacuum in an inflationary universe

- Electroweak vacuum decay may qualitatively differ in cosmological spacetimes:

  (i) Thermal activation of a decay process, $T_r < \mu_i$

  (ii) Production of large amplitude Higgs perturbations during inflation,

  $H_{\rm inf} < \mu_i$ [J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002]

  The bound that follows from the above consideration can be avoided, e.g., in curvaton models, or when $m_h^{\rm eff} > H_{\rm inf}$

- Actually, the dominant decay processes are due to instantons, (Hawking-Moss, or CdL-type) [AK & A. Spencer-Smith, Phys Lett B 722 (2013) 130 [arXiv: 1301.2846]]

$$V(h,\phi) = V_{\rm H}(h) + V_{\rm inf}(\phi) + V_{\rm H-inf}$$

$$V_{\rm inf}(\phi) = \mathcal{V}_{\rm inf} + V'_*(\phi - \phi_{\rm inf}) + 1/2V''_*(\phi - \phi_{\rm inf})^2 + ...$$

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'_*}{\mathcal{V}_{\rm inf}}\right)^2 << 1, \quad -1 << \eta = M_P^2\frac{V''_*}{\mathcal{V}_{\rm inf}} << 1$$

EW vacuum in inflationary universe

- Fixed background approximation: $\phi = \phi_{\rm inf},\ ds^2 = d\chi^2 + \rho^2(\chi)d\Omega_3,$

  $\rho(\chi) = H_{\rm inf}^{-1}\sin(H_{\rm inf}\chi),\ \chi = t^2 + r^2,\ \chi \in [0, \pi/H_{\rm inf}],\ H_{\rm inf}^2 = \mathcal{V}_{\rm inf}/3M_P^2$

- EoM for Higgs field:

$$\ddot{h} + 3H_{\rm inf}\cot(H_{\rm inf}\chi)\dot{h} = \frac{\partial V(\phi_{\rm inf}, h)}{\partial h}$$

$$\dot{h}(0) = \dot{h}(\pi/H_{\rm inf}) = 0$$

$$h_L(x_*) = h_R(x_*)$$

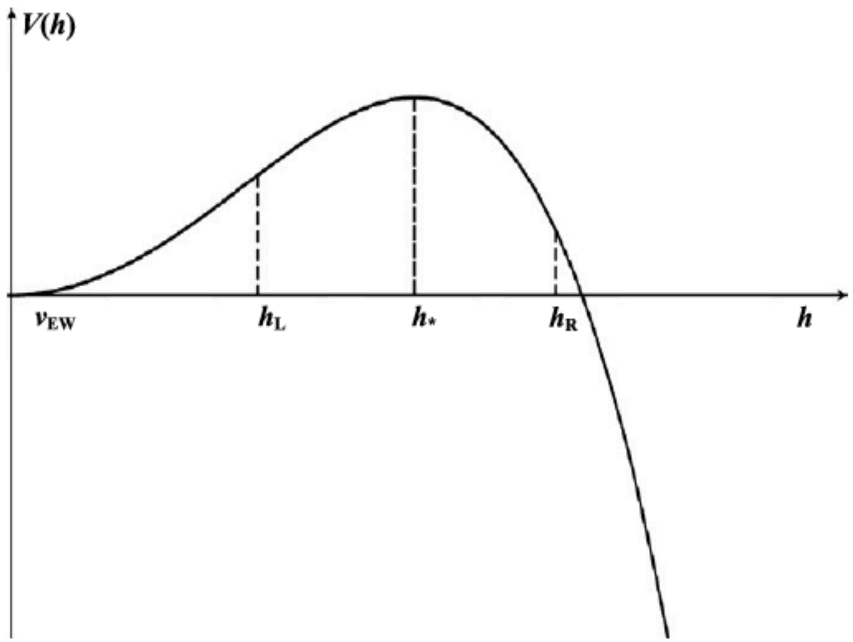


Fig. 1. The Higgs potential. For large values of the Higgs field h, the electroweak vacuum configuration is regarded as trivial, $v_{EW} \approx 0$.

EW vacuum in inflationary universe

- Hawking-Moss instanton: $\frac{\partial V}{\partial h} = 0 \ , h(x) = h_*,$

$$p \approx \exp\left\{-\frac{8\pi^2}{3}\frac{V_{\rm H}(h_*) + V_{\rm H-inf}(\phi_{\rm inf}, h_*)}{H_{\rm inf}^4}\right\}$$

- For $V_{\rm H-inf}(\phi_{\rm inf}, h_*) << V(h_*)$,

HM transition dominates

$$p = N_e^4 \exp\left(\frac{\pi^2 \bar{\beta}_\lambda}{2e}\frac{\mu_i^4}{H_{\rm inf}^4}\right)$$

EW vacuum in inflationary universe

HM transition generates a fast decay of the electroweak vacuum, unless

$$H_{\rm inf} < 10^9 (10^{12})~{\rm GeV}$$

$$m_h = 126~{\rm GeV}, \quad m_t = 174(172)~{\rm GeV}$$

- Together with $n_s < 1$, this implies that only small-field inflationary models are allowed with a negligible tensor/scalar:

$$r < 10^{-11} (10^{-5})$$

- This seems now is excluded by the BICEP2 results:

$$r = 0.2^{+0.07}_{-0.05}$$

EW vacuum in inflationary universe

- Consider, $V_{\mathrm{H-inf}} = \frac{\alpha}{2} h^2 \phi^2 \ (\alpha > 0), \ m_h^{\mathrm{eff}} = \alpha^{1/2}\phi_{\mathrm{inf}} > H_{\mathrm{inf}}$

[O. Lebedev & A. Westphal Phys.Lett. B719 (2013) 415]

[similar consideration applies $\frac{\xi}{2} R^2 h^2$]

$$h_* = (-\alpha/\lambda)^{1/2}\phi_{\mathrm{inf}} > \mu_i, \ (\lambda(h_*) < 0)$$

- Large-field chaotic inflation $\left[V_{\mathrm{inf}} = 1/2 m_\phi^2 \phi^2, \ m_\phi = 10^{-5} M_{\mathrm{P}}\right]$, with

$$10^{-6} > \alpha > 1.4\sqrt{|\lambda|}\left(H_{\mathrm{inf}}/\phi_{\mathrm{inf}}\right)^2 > 6 \cdot 10^{-12} \ .$$

- Naturalness constraint:

$$\alpha < 64\pi^2 \left(m_\phi/m_h\right)^2 \approx 2 \cdot 10^{-20}$$

Tuning is needed!

EW vacuum in inflationary universe

- In the limit $m_h^{\rm eff} >> H_{\rm inf}$

$$h'' + 3h'\chi = \frac{\partial V(\phi_{\rm inf}, h)}{\partial h}, \quad [x = m_h^{\rm eff}\chi]$$

$$h(x) = \begin{cases} 8h_{\rm R}\left(8 + \left(\frac{h_{\rm R}}{h_*}\right)^2 x^2\right)^{-1}, & 0 \leq x < x_* \\ \frac{x_* h_*}{x(J_1(ix_*)+iY_1(-ix_*))}\left(J_1(ix) + iY_1(-ix)\right), & x_* < x < \infty \end{cases},$$

$$x_* = \frac{2\sqrt{2}h_*}{h_{\rm R}}\left(\frac{h_{\rm R}}{h_*} - 1\right)^{1/2}.$$

$$B_{\rm CdL} = -\frac{2\pi^2}{\lambda} I < 0, \quad I = \int_0^\infty x^3 dx \left[h^2(x)\left(1 - \frac{h^2(x)}{2h_*^2}\right)\right] < 0, \quad \lambda(\mu > \mu_i) < 0.$$

$$p \propto \exp\{-B_{\rm CdL}\} >> 1$$ EW vacuum is unstable!

EW vacuum in inflationary universe

- Fast decay of EW ceases inflation globally (no eternal inflation)

$$e^{3H_{\rm inf}\tau}e^{-(\tau H_{\rm inf})^4 p}$$

$$\tau_{\rm stop} \approx (3/p)^{1/3} H_{\rm inf}^{-1} < 1.4 H_{\rm inf}^{-1}$$

- The above considerations applies to models with curvaton

$$\mathcal{A}_s' = \sqrt{\mathcal{A}_s^2 + \frac{g^2(X) H_{\rm inf}^2}{8\pi^2 M_{\rm P}^2}}, \quad r' = \frac{16\epsilon}{1+\epsilon g^2(X)}$$

In light of BICEP2 the electroweak vacuum within the Standard Model is unstable! New physics must enter at energies < 10^9 GeV.

Neutrino masses and vacuum stability

AK & A. Spencer-Smith, JHEP 1308 (2013) 036 [arXiv:1305.7283]

$$\beta_\lambda^{(1)} = 24\lambda^2 - 6y_t^4 + \frac{3}{4}g_2^4 + \frac{3}{8}\left(g_1^2 + g_2^2\right)^2 + \lambda\left(-9g_2^2 - 3g_1^2 + 12y_t^2\right)$$

① Extension of the electroweak gauge sector
② Extension of a scalar sector
③ Extension of the fermionic sector

- Working with MS-bar couplings:

① Modification of beta-functions above the particle mass threshold
② Finite threshold correction due to the matching of low and high energy theories

- Neutrino oscillations (= masses) provide the most compelling evidence for the physics beyond the Standard Model.

Neutrino masses and vacuum stability

AK & A. Spencer-Smith, JHEP 1308 (2013) 036 [arXiv:1305.7283]

- Type I see-saw models: additional massive sterile neutrinos – not capable to solve the vacuum stability problem
- Type III see-saw models: additional electroweak-triplet fermions – may solve the problem for very specific range of parameters

More promising candidates:

- Type II see-saw: additional electroweak-triplet scalar
- Left-right symmetric models: additional gauge bosons, scalars and fermions

Type II see-saw models and vacuum stability

Scalar potential:

$$V(\phi,\Delta) = -m_\phi^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 + m_\Delta^2 \mathrm{tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}(\mathrm{tr}(\Delta^\dagger\Delta))^2$$

$$+\frac{\lambda_2}{2}\left[(\mathrm{tr}(\Delta^\dagger\Delta))^2 - \mathrm{tr}(\Delta^\dagger\Delta)^2\right] + \lambda_4(\phi^\dagger\phi)\mathrm{tr}(\Delta^\dagger\Delta) + \lambda_5\phi^\dagger[\Delta^\dagger,\Delta]\phi + \left[\frac{\lambda_6}{\sqrt{2}}\phi^T i\sigma_2\Delta^\dagger\phi + \mathrm{h.c.}\right].$$

Neutrino masses:

$$\frac{1}{\sqrt{2}}(y_\Delta)_{fg} l_L^{Tf} C i\sigma_2 \Delta l_L^g + h.c.$$

Type II see-saw models and vacuum stability

Stability conditions:

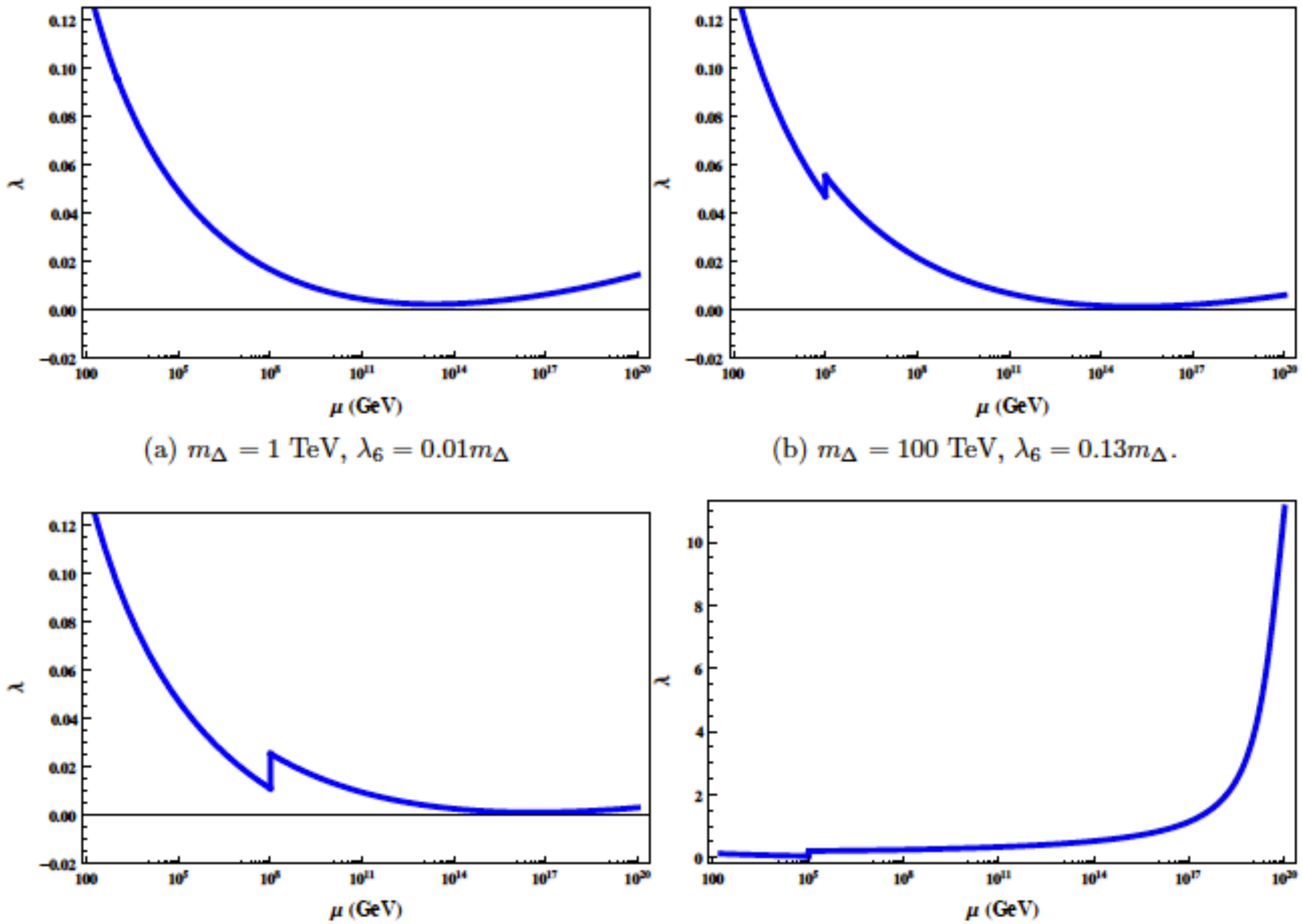
$$\lambda > 0,$$
$$\lambda_1 > 0,$$
$$\lambda_1 + \frac{\lambda_2}{2} > 0,$$
$$\lambda_4 \pm \lambda_5 + 2\sqrt{\lambda\lambda_1} > 0,$$
$$\lambda_4 \pm \lambda_5 + 2\sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} > 0.$$

Avoiding tachyonic instabilities:

$$\lambda_6 > 0,$$
$$-\lambda_5 v_\Delta < \lambda_6,$$
$$-2\lambda_5 v_\Delta - \frac{\lambda_2 v_\Delta^3}{v_{EW}^2} < \lambda_6,$$

Tree level matching condition: $$\lambda_h = \lambda - \frac{\lambda_6^2}{2m_\Delta^2}.$$

Type II see-saw models and vacuum stability



(a) $m_\Delta = 1$ TeV, $\lambda_6 = 0.01 m_\Delta$

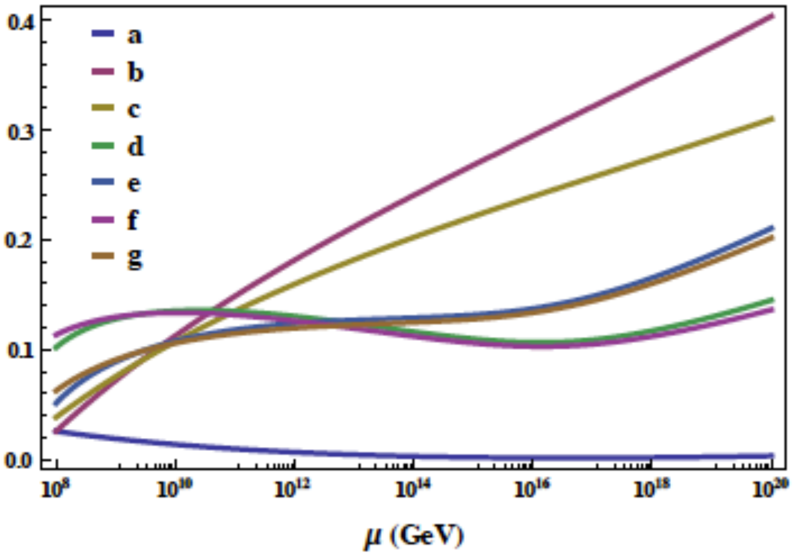
(b) $m_\Delta = 100$ TeV, $\lambda_6 = 0.13 m_\Delta$.

(c) $m_\Delta = 10^8$GeV, $\lambda_6 = 0.17 m_\Delta$.

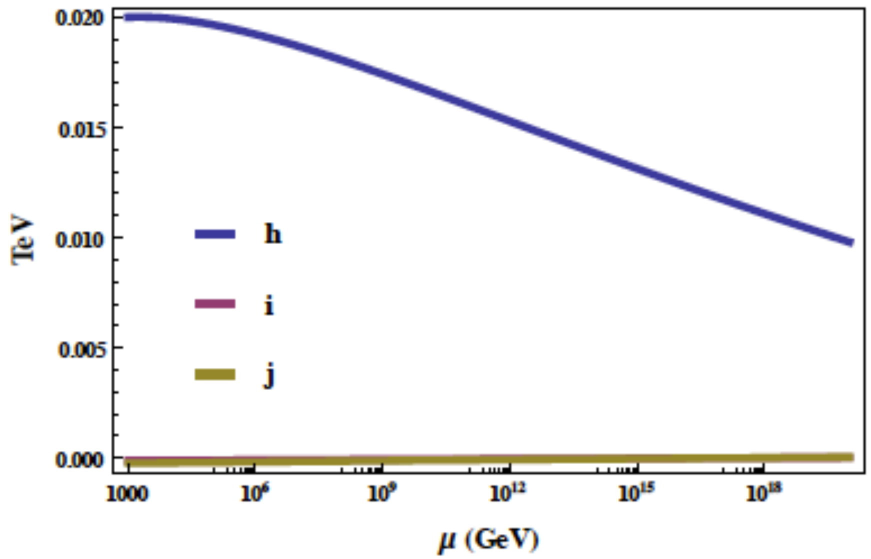
(d) $m_\Delta = 100$ TeV, $\lambda_6 = 0.57 m_\Delta$.

Figure 2: One loop running of the Higgs quartic coupling in the type-II seesaw model, with $m_h = 125$ GeV and $m_t = 173$ GeV.

Type II see-saw models and vacuum stability



(a) $m_\Delta = 10^8$GeV, $\lambda_6 = 0.17 m_\Delta$.



(b) $m_\Delta = 1$ TeV, $\lambda_6 = 0.01 m_\Delta$.

Figure 3: Conditions for stability and absence of tachyonic modes in the scalar potential of the type-II seesaw model, with $m_h = 125$ GeV and $m_t = 175$ GeV. Line labels correspond to the stability conditions (3.9)-(3.16) with $a \equiv \lambda$, $b \equiv \lambda_1$, $c \equiv \lambda_1 + \frac{\lambda_2}{2}$, $d/e \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda\lambda_1}$, $f/g \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)}$, $h \equiv \lambda_6$, $i \equiv -\lambda_5 v_\Delta$, $j \equiv -2\lambda_5 v_\Delta - \frac{\lambda_2 v_\Delta^3}{v_{EW}^2}$.

Alternating LR-symmetric model with universal see-saw for all fermion masses

$$SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{\rm B-L}$$

Scalar sector:

$$\phi_L \in (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1/2), \;\; \phi_R \in (\mathbf{1}, \mathbf{2}, \mathbf{1}, 1/2).$$

$$V(\phi_L, \phi_R) = -m^2 \left(\phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R\right) + \frac{\lambda}{2} \left(\phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R\right)^2 + \sigma \phi_L^\dagger \phi_L \phi_R^\dagger \phi_R.$$

New fermions:

$$\begin{aligned} N_L, N_R &\in (\mathbf{1}, \mathbf{1}, \mathbf{1}, 0), \\ E_L, E_R &\in (\mathbf{1}, \mathbf{1}, \mathbf{1}, -1), \\ U_{iL}, U_{iR} &\in (\mathbf{3}, \mathbf{1}, \mathbf{1}, 2/3), \\ D_{iL}, D_{iR} &\in (\mathbf{3}, \mathbf{1}, \mathbf{1}, -1/3), \end{aligned}$$

Alternating LR-symmetric model with universal see-saw for all fermion masses

Tree-level matching:

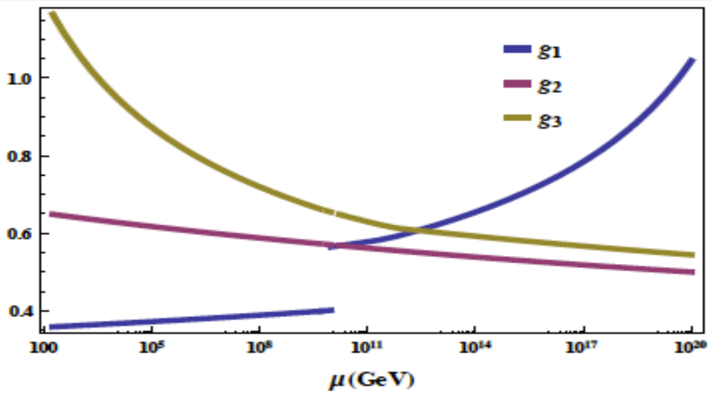
$$g_1 = \frac{g_R\, g_{B-L}}{\sqrt{g_R^2 + g_{B-L}^2}} = \frac{g_2\, g_{B-L}}{\sqrt{g_2^2 + g_{B-L}^2}},$$

$$y_{f_i} = y_{F_i}^2 \frac{v_R}{M_{F_i}}.$$

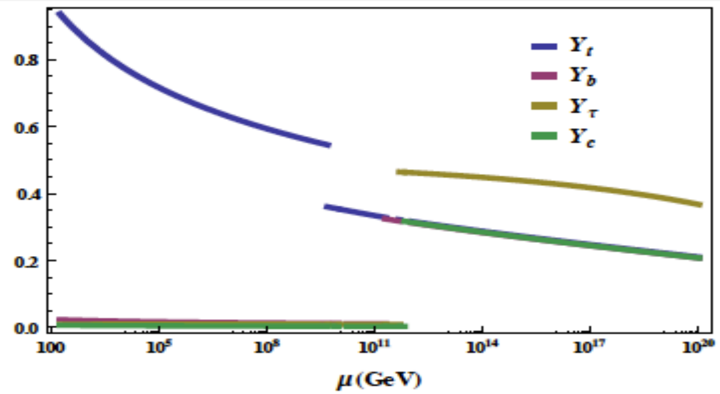
One-loop matching:

$$\frac{\lambda_{eff}}{8} = -\frac{\sigma}{4} - \frac{9\lambda^2}{256\pi^2}\left(1 - \ln\left[\frac{2m^2}{\mu^2}\right]\right) + \frac{3y^4}{16\pi^2}\left(\frac{1}{4} - \frac{5}{8}\ln\left[\frac{M_T^2 + \frac{m^2 y_T^2}{\lambda}}{\mu^2}\right]\right)$$

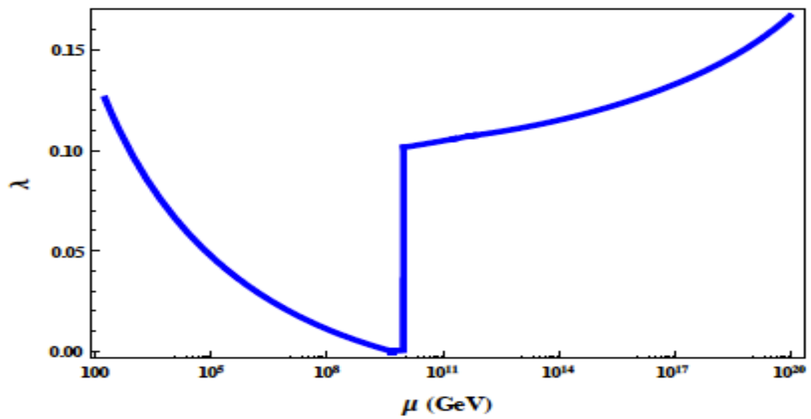
Alternating LR-symmetric model with universal see-saw for all fermion masses



(a) Running gauge couplings.



(b) Running Yukawa couplings.



(c) Running Higgs quartic coupling

Figure 4: Running couplings in the ALRSM with $M_T = 4.7 \times 10^9$ GeV and $m = 1 \times 10^{10}$ GeV.

Conclusions:

- The absolute stability of the electroweak vacuum is excluded at 99.98% CL
- In light of BICEP2 discovery of B-modes the electroweak vacuum within the Standard model is unstable. New physics, stabilizing the electroweak vacuum, must enter the game at scales < 10^9 GeV.
- The most promising models of neutrino masses with a stable electroweak vacuum are: type II see-saw and LR models. They may potentially be tested at LHC.