Higgs Vacuum Stability & Physics Beyond the Standard Model

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AK & A. Spencer-Smith, arXiv:1404.4709

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Combined 2012-13 data: It is a Higgs boson, maybe even the Higgs boson.

$\sqrt{s} = 7$ TeV, $L \leq 5.1$ fb$^{-1}$  $\sqrt{s} = 8$ TeV, $L \leq 19.6$ fb$^{-1}$

CMS Preliminary  $m_H = 125.7$ GeV  $\rho_{SM} = 0.65$

- $H \rightarrow bb$  $\mu = 1.15 \pm 0.62$
- $H \rightarrow \tau\tau$  $\mu = 1.10 \pm 0.41$
- $H \rightarrow \gamma\gamma$  $\mu = 0.77 \pm 0.27$
- $H \rightarrow WW$  $\mu = 0.68 \pm 0.20$
- $H \rightarrow ZZ$  $\mu = 0.62 \pm 0.28$

ATLAS Preliminary  $m_H = 125.5$ GeV

- $W,Z H \rightarrow bb$
  - $\sqrt{s} = 7$ TeV: $L_{dL} = 4.7$ fb$^{-1}$
  - $\sqrt{s} = 8$ TeV: $L_{dL} = 13$ fb$^{-1}$
- $H \rightarrow \tau\tau$
  - $\sqrt{s} = 7$ TeV: $L_{dL} = 4.6$ fb$^{-1}$
  - $\sqrt{s} = 8$ TeV: $L_{dL} = 20.7$ fb$^{-1}$
- $H \rightarrow WW^{(*)} \rightarrow llv$
  - $\sqrt{s} = 7$ TeV: $L_{dL} = 4.6$ fb$^{-1}$
  - $\sqrt{s} = 8$ TeV: $L_{dL} = 20.7$ fb$^{-1}$
- $H \rightarrow \gamma\gamma$  $\mu = 1.30 \pm 0.20$
  - $\sqrt{s} = 7$ TeV: $L_{dL} = 4.0 - 5.0$ fb$^{-1}$
  - $\sqrt{s} = 8$ TeV: $L_{dL} = 13 - 20.7$ fb$^{-1}$
- $H \rightarrow ZZ^{(*)} \rightarrow 4l$
The SM Higgs with $M_H=125.9\pm0.4$ GeV

- **Naturalness problem:** somewhat heavy than typical prediction of the supersymmetric models and somewhat light than typical prediction of technicolour models.

- More notably, the Standard Model vacuum state $|0\rangle_{EW}$

$$EW \langle 0|h|0 \rangle_{EW} = v_{EW} \approx 246 \text{ GeV}$$

is a false (local) vacuum. The true vacuum state

$$\langle 0|h|0 \rangle \sim M_P \approx 10^{18} \text{ GeV} ,$$

and it carries large negative energy density $\sim - (M_P)^4$.

- How long does the electroweak vacuum live?
EW vacuum lifetime: effective Higgs potential

- Electroweak Higgs doublet (in the unitary gauge): 
  \[ H = \begin{pmatrix} 0 \\ h(x)/\sqrt{2} \end{pmatrix} \]

- Effective (quantum-corrected) potential

  \[ V_{H}^{(0)}(h) = \frac{\lambda}{8} (h^2 - v_{EW})^2 \]

  \[ V_{H}^{(1-loop)}(h) = \frac{\lambda(h)}{8} (h^2 - v_{EW})^2, \]

  \[ \lambda(h) = \lambda(\mu) + \beta_\lambda \ln(h/\mu) \]

  \[ (4\pi)^2 \beta_\lambda = -6y_t^4 + 24\lambda^2 + \ldots \]

  \[ y_t(m_t) \approx 1, \quad \lambda(m_h) \approx 0.13 \quad \rightarrow \beta_\lambda < 0 \]
EW vacuum lifetime: RG extrapolation of SM parameters

- **Previous calculations:**

3-loop RGE’s in mass-independent MSbar scheme; full 2-loop matching condition

- **Our calculations:**
  1-loop RGE in mass-dependent scheme, 2 – and 3 – loop RGE’s in MSbar + corresponding matching condition
EW vacuum lifetime: RG running of $\lambda$ in a mass-dependent scheme

Instability scale:

$$[\lambda(\mu_i) = 0] \quad \log_{10} \left( \frac{\mu_i}{\text{GeV}} \right) \approx 9.19 \pm 0.65_{M_t} \pm 0.19_{M_h} \pm 0.13_{\alpha_3} \pm 0.02_{\text{th}}$$
EW vacuum lifetime: flat spacetime estimate

- Large field limit:

\[ V_H = \frac{\overline{\beta}_\lambda \ln(h/\mu_i)}{4} h^4, \ \overline{\beta}_\lambda = \beta_\lambda|_{\mu=\mu_i} \]

\[ h_* = \mu_i e^{-1/4} \]

- Using Coleman’s prescription, one can calculate that the decay of electroweak vacuum is dominated by small size Lee-Wick bounce solution,

\[ R \sim 1/\mu_m \approx 10^{-17}/\text{GeV}, \ \beta_\lambda|_{\mu=\mu_m} = 0 \]

\[ S_{\text{LW}} = \frac{8\pi^2}{3|\lambda(\mu_m)|}, \ |\lambda(\mu_m)| \approx 0.01 - 0.02 \]

\[ P_{\text{EW}} = e^{-p} \approx 1, \ p = (\mu_m/H_0)^4 \exp (-S_{\text{LW}}) << 1 \]
EW vacuum in flat spacetime: stability bound

Stability bound: \[ M_t < 170.16 \pm 0.22 \alpha_3 \pm 0.13 M_h \pm 0.06_{\text{th}} \text{ GeV} \]

\[ M_t^{\text{exp}} = 173.34 \pm 0.76 \alpha_3 \pm 0.3_{\text{QCD}} \text{ GeV} \]

EW vacuum in an inflationary universe

- Electroweak vacuum decay may qualitatively differ in cosmological spacetimes:
  
  (i) Thermal activation of a decay process, \( T_r < \mu_i \)
  
  (ii) Production of large amplitude Higgs perturbations during inflation, \( H_{\text{inf}} < \mu_i \) [J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002]

  The bound that follows from the above consideration can be avoided, e.g., in curvaton models, or when \( m_h > H_{\text{inf}} \)

- Actually, the dominant decay processes are due to instantons, (Hawking-Moss, or CdL-type) [AK & A. Spencer-Smith, Phys Lett B 722 (2013) 130 [arXiv: 1301.2846]]

\[
V(h, \phi) = V_H(h) + V_{\text{inf}}(\phi) + V_{H-\text{inf}}
\]

\[
V_{\text{inf}}(\phi) = V_{\text{inf}} + V'_*(\phi - \phi_{\text{inf}}) + 1/2V''_*(\phi - \phi_{\text{inf}})^2 + ... \\

\epsilon = \frac{M^2_P}{2} \left( \frac{V'_*}{V_{\text{inf}}} \right)^2 \ll 1, \quad -1 \ll \eta = M^2_P \frac{V''_*}{V_{\text{inf}}} \ll 1
\]


EW vacuum in inflationary universe

- Fixed background approximation: \( \phi = \phi_{\text{inf}}, \; ds^2 = d\chi^2 + \rho^2(\chi)d\Omega_3, \)

\[
\rho(\chi) = H_{\text{inf}}^{-1} \sin(H_{\text{inf}}\chi), \; \chi = t^2 + r^2, \; \chi \in [0, \pi/H_{\text{inf}}], \; H_{\text{inf}}^2 = V_{\text{inf}}/3M_P^2
\]

- EoM for Higgs field:

\[
\ddot{h} + 3H_{\text{inf}} \cot(H_{\text{inf}}\chi)\dot{h} = \frac{\partial V(\phi_{\text{inf}}, h)}{\partial h}
\]

\[
\dot{h}(0) = \dot{h}(\pi/H_{\text{inf}}) = 0
\]

\[
h_L(x_*) = h_R(x_*)
\]

Fig. 1. The Higgs potential. For large values of the Higgs field \( h \), the electroweak vacuum configuration is regarded as trivial, \( v_{\text{EW}} \approx 0 \).
EW vacuum in inflationary universe

- Hawking-Moss instanton: \( \frac{\partial V}{\partial h} = 0 \), \( h(x) = h_* \),

\[
p \approx \exp \left\{ -\frac{8\pi^2}{3} \frac{V_H(h_*) + V_{H-\text{inf}}(\phi_{\text{inf}}, h_*)}{H_{\text{inf}}^4} \right\}
\]

- For \( V_{H-\text{inf}}(\phi_{\text{inf}}, h_*) << V(h_*) \),

HM transition dominates

\[
p = N_e^4 \exp \left( \frac{\pi^2 \bar{\beta} \lambda}{2e} \frac{\mu_i^4}{H_{\text{inf}}^4} \right)
\]
EW vacuum in inflationary universe

HM transition generates a fast decay of the electroweak vacuum, unless

\[ H_{\text{inf}} < 10^9 (10^{12}) \text{ GeV} \]

\[ m_h = 126 \text{ GeV}, \quad m_t = 174(172) \text{ GeV} \]

- Together with \( n_s < 1 \), this implies that only small-field inflationary models are allowed with a negligible tensor/scalar:

\[ r < 10^{-11} (10^{-5}) \]

- This seems now is excluded by the BICEP2 results:

\[ r = 0.2^{+0.07}_{-0.05} \]
EW vacuum in inflationary universe

- Consider, \( V_{H-\inf} = \frac{\alpha}{2} h^2 \phi^2 \quad (\alpha > 0) \), \( m_{h}^{\text{eff}} = \alpha^{1/2} \phi_{\inf} > H_{\inf} \)


[similar consideration applies \( \frac{\xi}{2} R^2 h^2 \)]

\[ h_{\ast} = (-\alpha/\lambda)^{1/2} \phi_{\inf} > \mu_{i}, \quad (\lambda(h_{\ast}) < 0) \]

- Large-field chaotic inflation \( [V_{\inf} = 1/2 m_{\phi}^2 \phi^2, \quad m_{\phi} = 10^{-5} M_{P}] \)

with

\[ 10^{-6} > \alpha > 1.4 \sqrt{|\lambda|} (H_{\inf}/\phi_{\inf})^2 > 6 \cdot 10^{-12}. \]

- Naturalness constraint:

\[ \alpha < 64\pi^2 (m_{\phi}/m_{h})^2 \approx 2 \cdot 10^{-20} \]

Tuning is needed!
EW vacuum in inflationary universe

- In the limit \( m_h^{\text{eff}} \gg H_{\text{inf}} \)

\[
h'' + 3h' \chi = \frac{\partial V(\phi_{\text{inf}}, h)}{\partial h}, \quad [x = m_h^{\text{eff}} \chi] \]

\[
h(x) = \begin{cases} 
8h_R \left( 8 + \left( \frac{h_R}{h_*} \right)^2 x^2 \right)^{-1}, & 0 \leq x < x_* \\

\frac{x_* h_*}{x(J_1(ix_*) + iY_1(-ix_*))} \left( J_1(ix) + iY_1(-ix) \right), & x_* < x < \infty
\end{cases}
\]

\[
x_* = \frac{2\sqrt{2}h_*}{h_R} \left( \frac{h_R}{h_*} - 1 \right)^{1/2}.
\]

\[
B_{\text{CdL}} = -\frac{2\pi^2}{\lambda} I < 0, \quad I = \int_0^\infty x^3 dx \left[ h^2(x) \left( 1 - \frac{h^2(x)}{2h_*^2} \right) \right] < 0, \quad \lambda(\mu > \mu_i) < 0.
\]

\[
p \propto \exp\{-B_{\text{CdL}}\} \gg 1 \quad \text{EW vacuum is unstable!}
\]
EW vacuum in inflationary universe

- Fast decay of EW ceases inflation globally (no eternal inflation)

\[ e^{3H_{\text{inf}} \tau} e^{-(\tau H_{\text{inf}})^4 p} \]

\[ \tau_{\text{stop}} \approx (3/p)^{1/3} H^{-1}_{\text{inf}} < 1.4 H^{-1}_{\text{inf}} \]

- The above considerations applies to models with curvaton

\[ A'_{s} = \sqrt{A_{s}^2 + \frac{g^2(X)H_{\text{inf}}^2}{8\pi^2 M_{P}^2}}, \quad r' = \frac{16\epsilon}{1 + \epsilon g^2(X)} \]

In light of BICEP2 the electroweak vacuum within the Standard Model is unstable! New physics must enter at energies < 10^9 GeV.

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Neutrino masses and vacuum stability

\[ \beta^{(1)}_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{4}g_2^4 + \frac{3}{8}(g_1^2 + g_2^2)^2 + \lambda (-9g_2^2 - 3g_1^2 + 12y_t^2) \]

1. Extension of the electroweak gauge sector
2. Extension of a scalar sector
3. Extension of the fermionic sector

- Working with MS-bar couplings:
  1. Modification of beta-functions above the particle mass threshold
  2. Finite threshold correction due to the matching of low and high energy theories

- Neutrino oscillations (= masses) provide the most compelling evidence for the physics beyond the Standard Model.
Neutrino masses and vacuum stability

- Type I see-saw models: additional massive sterile neutrinos – not capable to solve the vacuum stability problem

- Type III see-saw models: additional electroweak-triplet fermions – may solve the problem for very specific range of parameters

More promising candidates:

- Type II see-saw: additional electroweak-triplet scalar

- Left-right symmetric models: additional gauge bosons, scalars and fermions
Type II see-saw models and vacuum stability

Scalar potential:
\[ V(\phi, \Delta) = -m^2_\phi \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + m_\Delta \text{tr}(\Delta^\dagger \Delta) + \frac{\lambda_1}{2} (\text{tr}(\Delta^\dagger \Delta))^2 \]
\[ + \frac{\lambda_2}{2} [(\text{tr}(\Delta^\dagger \Delta))^2 - \text{tr}(\Delta^\dagger \Delta)^2] + \lambda_4 (\phi^\dagger \phi) \text{tr}(\Delta^\dagger \Delta) + \lambda_5 \phi^\dagger [\Delta^\dagger, \Delta] \phi + \left[ \frac{\lambda_6}{\sqrt{2}} \phi^T i \sigma_2 \Delta^\dagger \phi + h.c. \right]. \]

Neutrino masses:
\[ \frac{1}{\sqrt{2}} (y_\Delta)_{fg} l^T_L l^g_L C i \sigma_2 \Delta l^g_L + h.c. \]
Type II see-saw models and vacuum stability

Stability conditions:

\[
\begin{align*}
\lambda &> 0, \\
\lambda_1 &> 0, \\
\lambda_1 + \frac{\lambda_2}{2} &> 0, \\
\lambda_4 \pm \lambda_5 + 2\sqrt{\lambda_1} &> 0, \\
\lambda_4 \pm \lambda_5 + 2\sqrt{\lambda_1} &> 0.
\end{align*}
\]

Avoiding tachyonic instabilities:

\[
\begin{align*}
\lambda_6 &> 0, \\
-\lambda_5 v_\Delta &< \lambda_6, \\
-2\lambda_5 v_\Delta - \frac{\lambda_2 v_\Delta^3}{v_{EW}^2} &< \lambda_6,
\end{align*}
\]

Tree level matching condition:

\[
\lambda_h = \lambda - \frac{\lambda_6^2}{2m_\Delta^2}.
\]
Type II see-saw models and vacuum stability

Figure 2: One loop running of the Higgs quartic coupling in the type-II seesaw model, with $m_h = 125$ GeV and $m_t = 173$ GeV.
Type II see-saw models and vacuum stability

Figure 3: Conditions for stability and absence of tachyonic modes in the scalar potential of the type-II seesaw model, with $m_h = 125$ GeV and $m_t = 175$ GeV. Line labels correspond to the stability conditions \((3.9)-(3.16)\) with:

- $a \equiv \lambda$
- $b \equiv \lambda_1$
- $c \equiv \lambda_1 + \frac{\lambda_2}{2}$
- $d/e \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda \lambda_1}$
- $f/g \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda \left(\lambda_1 + \frac{\lambda_2}{2}\right)}$
- $h \equiv \lambda_6$
- $i \equiv -\lambda_5 v_\Delta$
- $j \equiv -2\lambda_5 v_\Delta - \frac{\lambda_2 v_\Delta^3}{v_{EW}^2}$. 

\[(a) \quad \mu (\text{GeV}) \quad (b) \quad \mu (\text{TeV})\]
Alternating LR-symmetric model with universal seesaw for all fermion masses

\[ SU(2)_L \times SU(2)_R \times U(1)_{B-L} \]

Scalar sector:

\[ \phi_L \in (1, 1, 2, 1/2), \quad \phi_R \in (1, 2, 1, 1/2). \]

\[ V(\phi_L, \phi_R) = -m^2 \left( \phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R \right) + \frac{\lambda}{2} \left( \phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R \right)^2 + \sigma \phi_L^\dagger \phi_L \phi_R^\dagger \phi_R. \]

New fermions:

\[ N_L, N_R \in (1, 1, 1, 0), \]
\[ E_L, E_R \in (1, 1, 1, -1), \]
\[ U_{iL}, U_{iR} \in (3, 1, 1, 2/3), \]
\[ D_{iL}, D_{iR} \in (3, 1, 1, -1/3), \]
Alternating LR-symmetric model with universal seesaw for all fermion masses

Tree-level matching:

\[
g_1 = \frac{g_R \, g_{B-L}}{\sqrt{g_R^2 + g_{B-L}^2}} = \frac{g_2 \, g_{B-L}}{\sqrt{g_2^2 + g_{B-L}^2}},
\]

\[
y_{f_i} = y_{F_i}^2 \frac{v_R}{M_{F_i}}.
\]

One-loop matching:

\[
\frac{\lambda_{eff}}{8} = -\frac{\sigma}{4} - \frac{9\lambda^2}{256\pi^2} \left(1 - \ln \left[\frac{2m^2}{\mu^2}\right]\right) + \frac{3y^4}{16\pi^2} \left(\frac{1}{4} - \frac{5}{8} \ln \left[\frac{M_T^2 + \frac{m^2 y_T^2}{\lambda}}{\mu^2}\right]\right)
\]
Alternating LR-symmetric model with universal seesaw for all fermion masses

Figure 4: Running couplings in the ALRSM with $M_T = 4.7 \times 10^9$ GeV and $m = 1 \times 10^{10}$ GeV.
Conclusions:

- The absolute stability of the electroweak vacuum is excluded at 99.98% CL.

- In light of BICEP2 discovery of B-modes the electroweak vacuum within the Standard model is unstable. New physics, stabilizing the electroweak vacuum, must enter the game at scales $< 10^9$ GeV.

- The most promising models of neutrino masses with a stable electroweak vacuum are: type II see-saw and LR models. They may potentially be tested at LHC.