

Finite-size corrections to Fermi's golden rule: Implications to neutrino physics

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References:

- (1)K.Ishikawa and Y.Tobita,P.T.E.P;073 (2013) B02,
- (2)K.I and Y.Tobita , Annals of Physics ,344 (2014), 118-178
- (3)K.I,T.Nozaki,M.Sentoku,and Y.Tobita, arXiv:1405.0582[hep-ph]
- (4)K.I, T.Tajima, and Y.Tobita, arXiv:1409.4339[hep-ph]
- (5)K.I and Shimomura,PTP(2005), and others

Contents

1. *Introduction : "Fermi's golden rule and correction"*
 2. *Field theory and boundary conditions*
 3. $\pi \rightarrow \nu + l, \mu \rightarrow \nu + e + \bar{\nu}$
 4. $\nu + \gamma(E, B) \rightarrow \nu + \gamma$
 5. $\mu \rightarrow e + \gamma, A \rightarrow B + \gamma, \text{ others}$
 6. *Summary*
-

The probability of the event is computed in Quantum mechanics

The event is specified by the initial state and final states

The states are expressed by the boundary conditions

$$i\hbar \frac{\partial}{\partial t} \Psi(t, \vec{x}) = (H_0 + H_{int}) \Psi(t, \vec{x})$$

1-1. Transition rate at T is computed with the amplitude,

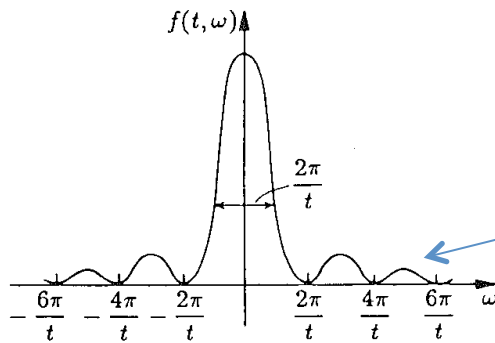
$$\int_0^T dt e^{i\omega t} = \frac{1}{i\omega} (e^{i\omega T} - 1) = e^{i\omega T/2} \frac{2 \sin \omega T/2}{\omega}$$

$$P = g^2 \int d\omega \left(\frac{2 \sin \omega T/2}{\omega} \right)^2 |f_{\alpha,\beta}|^2$$

$$= g^2 T \int dx \left(\frac{2 \sin x}{x} \right)^2 \left[g(0) + \frac{x}{T} g'(0) + \dots \right]$$

$$= g^2 2\pi T g(0) \left[1 + \frac{1}{T} \times \infty + \dots \right],$$

$$x = \omega T/2, g = |f_{\alpha,\beta}|^2 \quad \omega = (E_i - E_f)/\hbar$$



11.8: 関数 $f(t, \omega)$ は $t \rightarrow \infty$ において $2\pi t \delta(\omega)$ となる。すなわち $\omega = 0$ のピークは鋭くなる。

1. Amplitude at Large T is determined by the states of

$$\omega \approx 0 \quad P = T\Gamma_0$$

Golden rule (Dirac(1927), Fermi(1949))

2. (1) 1/T correction ?

(2) If $g(0)=0$, ?

$$P = T \left(\Gamma_0 + \frac{P^{(d)}}{T} \right)$$

Tails give 1/T corrections

Kinetic-energy non-conservation

$$E_k = E_{total} - E_{int} \neq E_{total}, \text{ for } E_{int} \neq 0$$

1-2. Transition probability P (of time interval T)

$$P = T\Gamma_0 + P^{(d)} \quad : T - \text{linear} + \text{constant}$$

Γ_0 : *Fermi's golden rule*

$P^{(d)}$: *correction* (our work)

1-1. $P^{(d)}$ is very different from Γ_0
 Γ_0 : *particle – like behavior* (ν flavor oscillation)

$P^{(d)}$: *wave – like behavior* (ν diffraction)

1-2. $P^{(d)}$ has been ignored in most places,
but affects the transitions of non-stationary waves. .

1-3. $P^{(d)}$ is important in ν , γ reactions

2 .Quantum Field Theory

$$L = \bar{\psi}\gamma_{\mu}(p^{\mu} + eA^{\mu})\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} (QED), \text{ or } L(QFD) \\ + Higgs$$

Symmetry: Poincare Invariance

Conserved quantities: Energy- momentum, angular momentum, and others

Boundary conditions

1. Vacuum and excitations follow Poincare invariance

$$|0\rangle, |p_i\rangle, |p_{i_1}, p_{i_2}\rangle, \dots$$

2. In matter, matter effects, detector effects(many –body effects) would violate some invariance and be different.

3. In scattering or decays, boundary conditions are important

$S[T]$ is constructed with LSZ(1955) formula

$$\lim_{t \rightarrow \pm T/2} \langle \alpha | \phi^f(t) | \beta \rangle = \langle \alpha | \phi_{in}^f | \beta \rangle$$

$$\phi^f(t) = i \int d^3x f^*(\vec{x}, t) \partial_0 \phi(\vec{x}, t)$$

$$S[T] = \Omega_-^\dagger(T) \Omega_+(T)$$

$$\Omega_\pm(T) = \lim_{t \rightarrow \mp T/2} U(t)^\dagger U_0(t)$$

$$S[T] S^\dagger[T] = 1$$

$$\langle \alpha | S[T] | \beta \rangle = \langle \alpha | S^n[T] | \beta \rangle + \langle \alpha | S^{(d)}[T, f] | \beta \rangle$$

$$E_\alpha = E_\beta, \quad E_\alpha \neq E_\beta$$

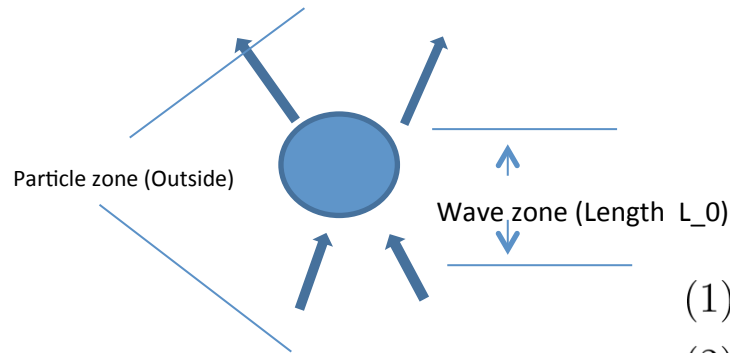
$$T\Gamma_0, \quad P^{(d)}$$

wave packet



- (1) Boundary conditions at $T = T/2, -T/2$, depend on $f(x)$.
- (2) Amplitude and probability depend on $f(x)$.
- (3) $f(x; \vec{p}, \vec{X}, T) = N \int d\vec{k} e^{-\sigma/2(\vec{k}-\vec{p})^2 + i(E(\vec{k})(t-T) - \vec{k}(\vec{x}-\vec{X}))}$, $\sigma = \text{nuclear size}$
- (4) Position \vec{x}, T and momentum \vec{p} for a complete set.

2-1 Asymptotic states are particles



ν Pion or charged leptons

$$S[\infty], L = c \times \infty$$

$$P_{pion}(T) = T\Gamma_0$$

$$(1) [S[\infty], H_0] = 0$$

(2) *Kinetic energy is conserved. (kinetic energy = total energy)*

(3) *Fermi's golden rule, S - matrix, $e^{-\epsilon|t|} H_{int}$*

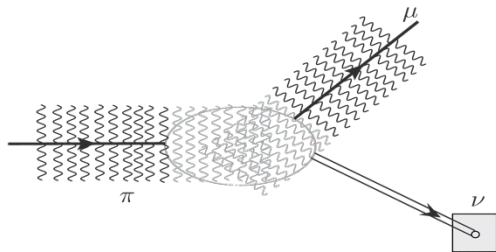
2-2. Asymptotic states are waves

Amplitude of the events that ν is detected at (\vec{X}_ν, T_ν)

Neutrino phenomena

$$S[T], L = cT$$

$$P_\nu(T) = T\Gamma_0 + P^{(d)}$$



$$(1) [S[T], H_0] \neq 0$$

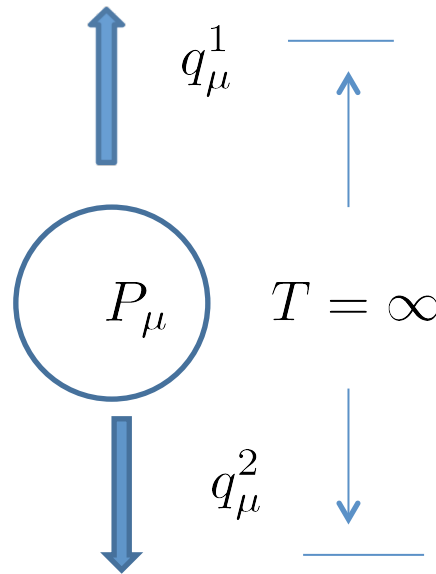
(2) *kinetic energy non - conservation,*

(3) *Fermi's golden rule has a correction*

Wave's characteristic features appear in space - time dependence and are probed with wave packets

Waves overlap in large area, and have finite interaction energy

Particle-like behavior; no-overlap

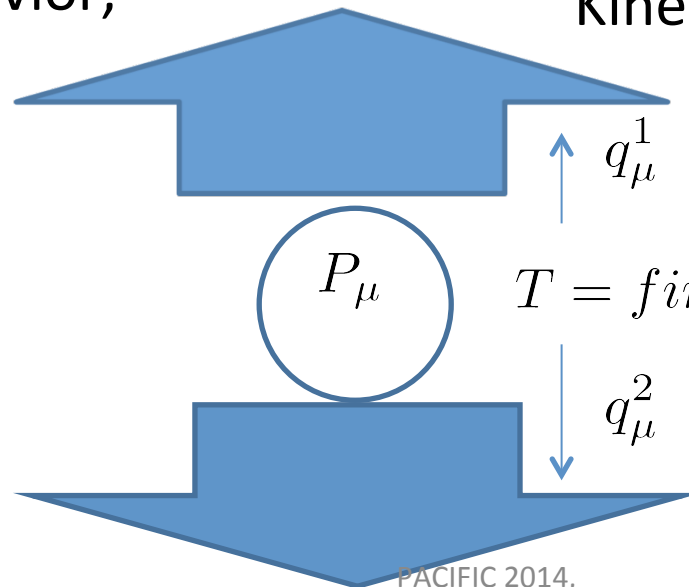


Kinetic-energy conserved

$$P_\mu = q_\mu^1 + q_\mu^2$$

$$P = \Gamma_0 T$$

Wave-like behavior; overlap



Kinetic-energy non-conservation

$$P_\mu \neq q_\mu^1 + q_\mu^2$$

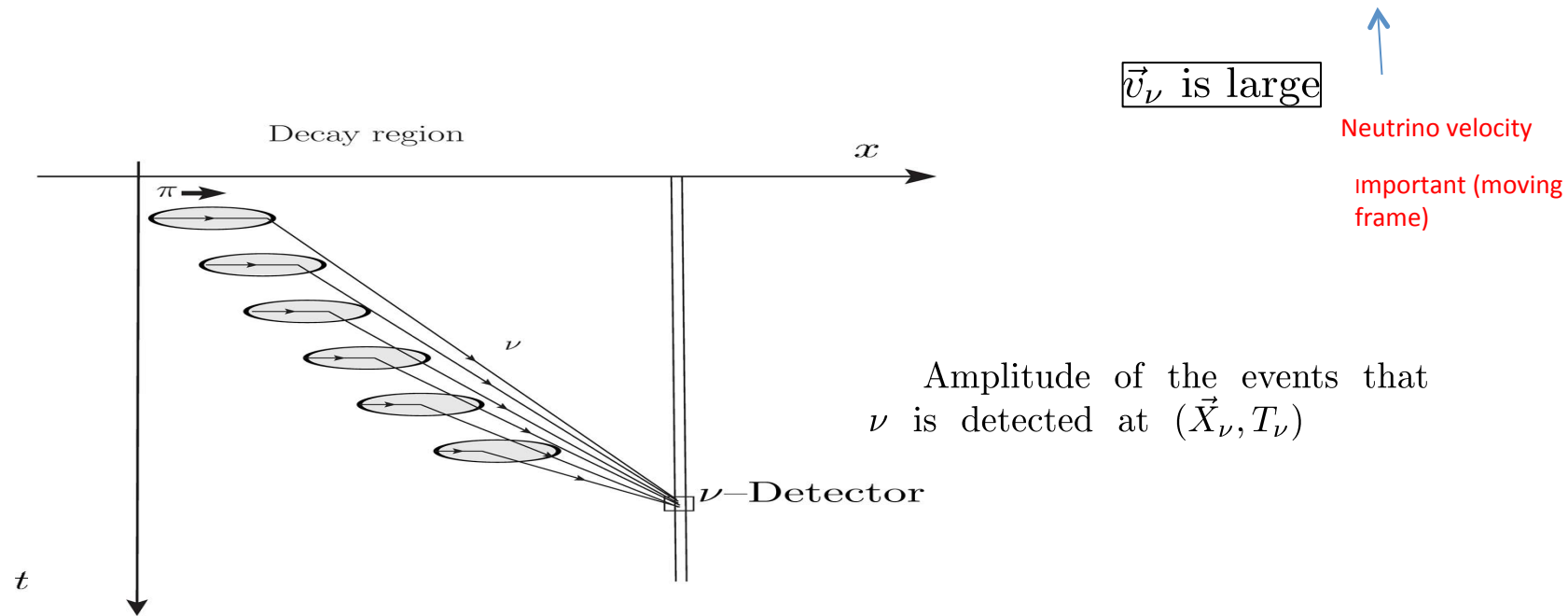
$$P = P^{(d)} + T\Gamma_0$$

3. Pion decay amplitude : neutrino is detected at $L=cT$ $S[T]$

- * By using wave packets, probability amplitude of the event is given [LSZ]. ν
- * $P^{\{d\}}$ is caused by the kinetic-energy non-conserving term .

$$\bar{M} = \int d^4x \langle l, \nu | H_w(x) | \pi \rangle$$

$$= \int d^4x N_1 \langle 0 | J_{V-A}^\mu | \pi \rangle \bar{u}(\vec{p}_l) \gamma_\mu (1 - \gamma_5) \nu(p_\nu) e^{ip_l \cdot x + ip_\nu \cdot (x - X_\nu)} e^{-\frac{1}{2\sigma_\nu} (\vec{x} - \vec{X}_\nu - \vec{v}_\nu(t - T_\nu))^2}$$



Rigorous calculation : light-cone singularity+short- range

$$\int \frac{d^3 p_l}{(2\pi)^3} \sum_{spin} |T|^2 = \frac{N_3}{E_\nu} \int d^4 x_1 d^4 x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{X}_\nu - \vec{v}_\nu (t_i - T_\nu))^2}$$

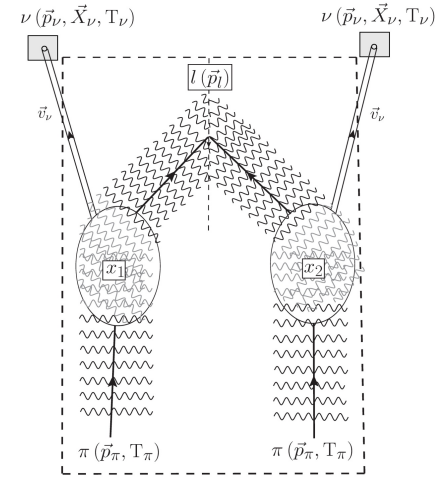
$$\times \Delta_{\pi,l}(x_1 - x_2) e^{i p_\nu \cdot (x_1 - x_2)}$$

$$\Delta_{\pi,l}(x_1 - x_2) = \frac{1}{(2\pi)^3} \int \frac{d^3 p_l}{E(p_l)} [(2p_\pi \cdot p_\nu)(p_\pi \cdot p_l) - m_\pi^2 (p_l \cdot p_\nu)]$$

$$\times e^{-i(p_\pi - p_l) \cdot (x_1 - x_2)}$$

$$= \delta((x_1 - x_2)^2) + short - range$$

Light-cone singularity



$$e^{i p_\nu \cdot (x_1 - x_2)} \delta((x_1 - x_2)^2) = e^{i(E_\nu - p_\nu)(x_1 - x_2)^0} = e^{i \frac{m^2}{2E} (x_1^0 - x_2^0)} \rightarrow P(d)$$

$$e^{i p_\nu \cdot (x_1 - x_2)} \times (short - range) \rightarrow T\Gamma_0$$

\mathcal{V} v_ν large and $\approx constant$ small $\frac{m^2}{2E}$

$$P_\nu(T) = T\Gamma_0 + P^{(d)}$$

$$P^{(d)} = CT\tilde{g}\left(\frac{m_\nu^2 T}{2E_\nu}\right) \approx C \frac{2E_\nu}{m_\nu^2} (macro.T), \tilde{g}(0) = \pi$$

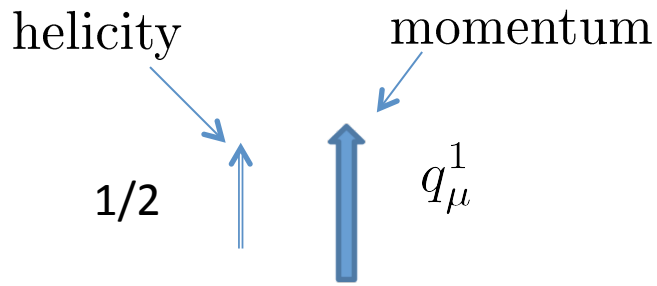
energy scale: $\frac{(m_\nu c^2)^2}{E_\nu}$

charged leptons or pion large $\frac{m^2}{2E}$

v_l small and varies

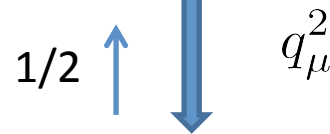
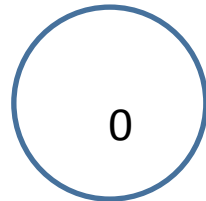
$$P = T\Gamma_0$$

Light fermion



Forbidden for $m_l=0$
 Ruderman-Finkelstein-, Steinberger,
 Sasaki-et al,-('49) .

$$\Gamma_0 = 0$$



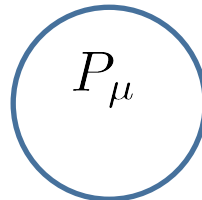
$$H_w = (V - A)_\mu (V - A)^\mu$$

Waves of light fermion



Allowed (*KI and YT*)

$$P^{(d)} \neq 0$$



$$P_\mu \neq q_\mu^1 + q_\mu^2$$

$$0^- \rightarrow l + \nu_l \quad P = T\Gamma_0(l, \nu_l) + P^{(d)}(l, \nu_l)$$

(I) Helicity suppression due to V-A interaction

$$(1) \Gamma_0(l, \nu_l) = m_l^2 C_0 \approx 0 \quad (l = e)$$

$$(2) \Gamma_0(e, \nu_e) / \Gamma_0(\mu, \nu_\mu) = 10^{-4}$$

(II) Unique properties of $P^{(d)}(l, \nu_l)|_{\text{observed particle}}$

(1) depends on observed particle; $P^{(d)}(e, \nu_e)|_{\nu_e} \neq P^{(d)}(e, \nu_e)|_e$

(2) hold universality. $P^{(d)}(e, \nu_e)|_{\nu_e} = P^{(d)}(\mu, \nu_\mu)|_{\nu_\mu}$

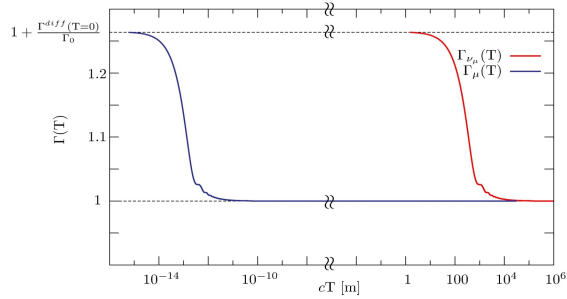
(3) $P^{(d)}$ depends on absolute neutrino mass, m_ν .

$$P^{(d)} = \frac{8}{15} g^2 m_\pi^4 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^4 \left(1 + \frac{3m_l^2}{m_\pi^2}\right) \frac{m_\pi^2 \sigma_l p_\pi}{m_l^2}$$

l : lepton or ν Due to $\frac{1}{m_l^2}$, large symmetry breaking

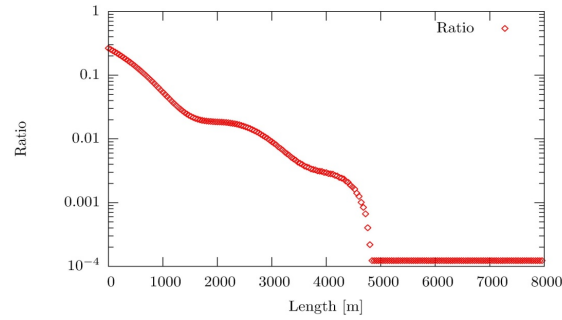
Include τ_π

Rate(ν , T) vs Rate(μ , T)



$L = cT$

Rate(ν_e)/Rate(ν_μ)



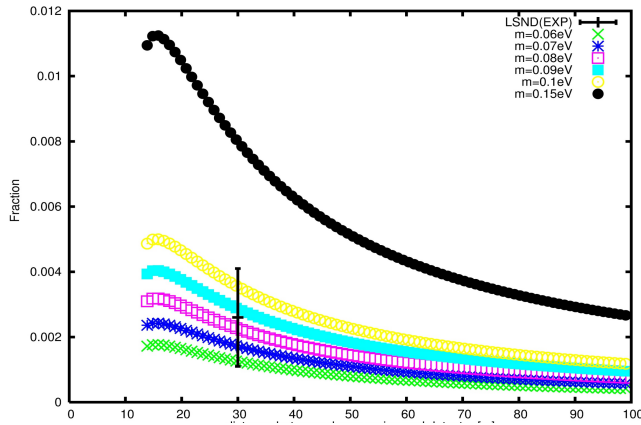
$L = cT$

Exp.

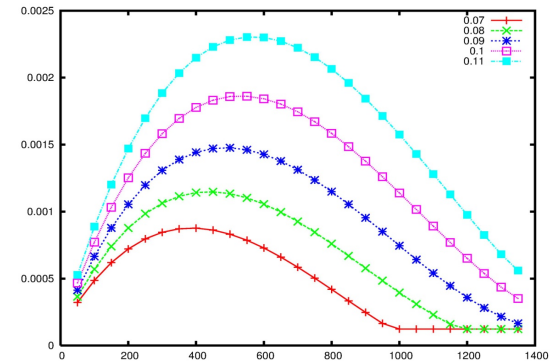
ν_e (LSND) vs m_ν P_{ν_e}/P_{ν_μ}

Absolute neutrino mass (LSND)

ν_e appearance at near detector (T2K)



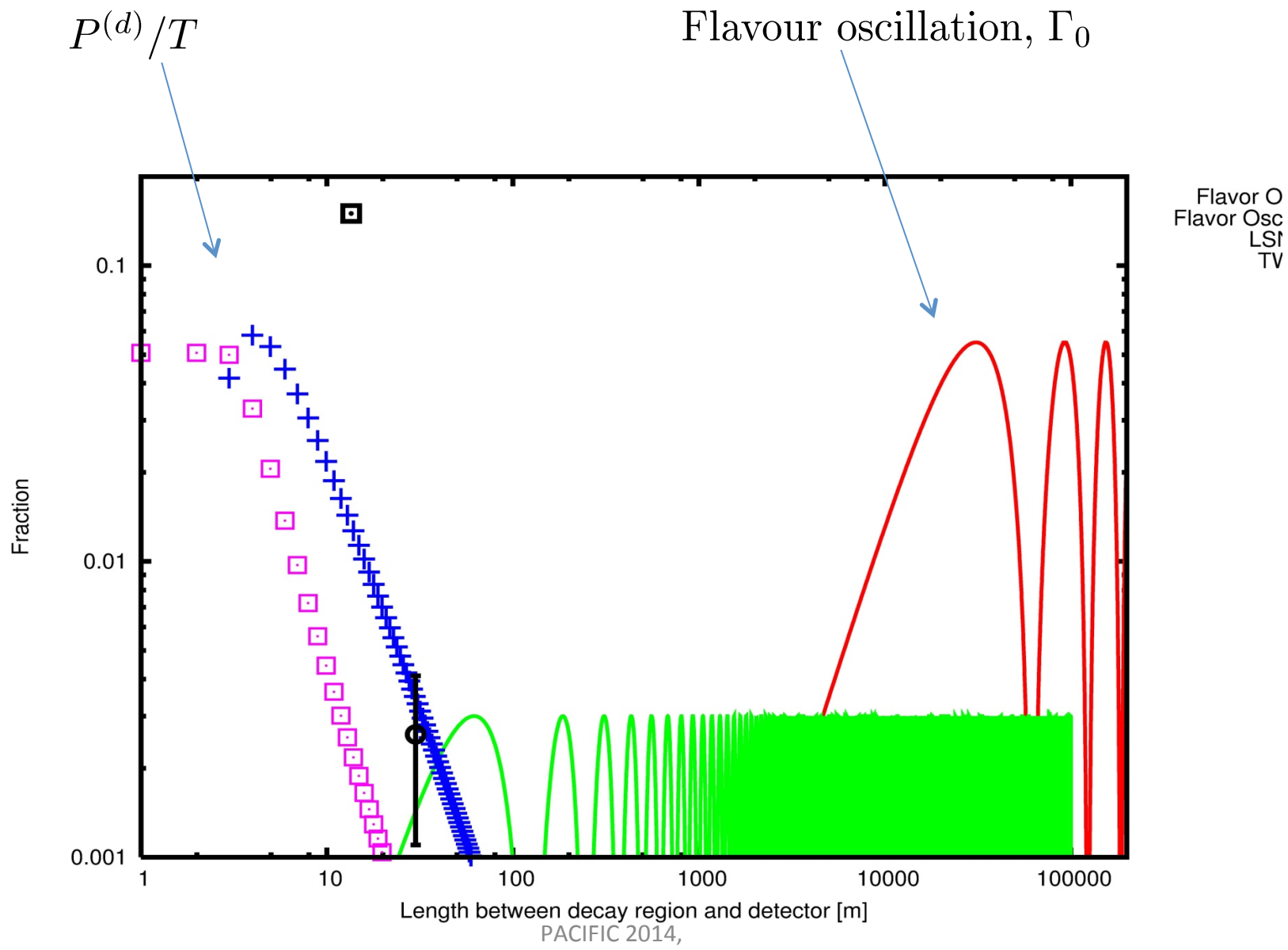
$L = cT$



$$\frac{\hbar}{\tau_\pi} \approx \frac{m_\nu^2}{E_\nu}, \text{ sensitive to } m_\nu$$

$m_{\nu_l} = 0.085 \pm 0.022 eV/c^2, m_{\nu_h} = 0.098 \pm 0.022, eV/c^2$; normal hierarchies
 $m_{\nu_l} = 0.070 \pm 0.026 eV/c^2, m_{\nu_h} = 0.083 \pm 0.026 eV/c^2$; inverted hierarchies

$L_{\text{decay-detector}} = 170[m]$,
 • axis = 2.5 degree,
 Detector 3[m]x3[m]



3 – 2. *Muon decay (Ishikawa – Nozaki – Sentoku – Tobita)*

$$\mu \rightarrow \bar{\nu}_e + e + \nu_\mu$$

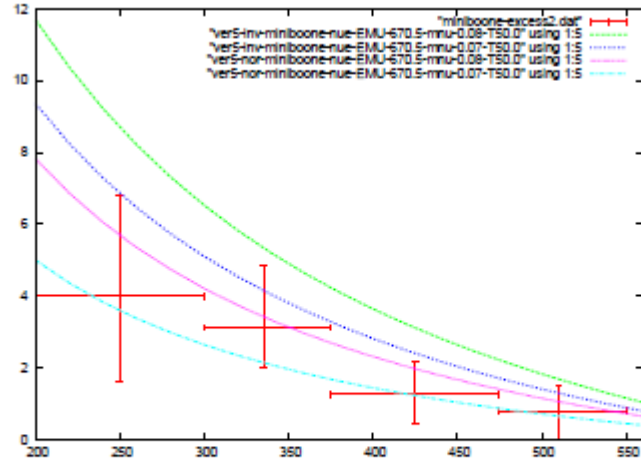
Lehman's spectrum representation $\rho(m^2)$ for (e and ν_μ)

$$P^{(d)} = \frac{2G_F^2}{(2\pi)^3 E_\mu} \int \frac{d\vec{p}_{\nu_e}}{(2\pi)^3 E_{\nu_e}} 2\pi p_\mu \cdot p_{\nu_e} \int dm^2 \rho(m^2) [\sigma_{\nu_e} C \tilde{g}(\omega_{\nu_e}, T; \tau_\mu) + \text{"Normal"}]$$

LSND, KARMEN, MiniBooNE, decay in flight : Γ_0

LSND, MiniBoone, decay at rest. $\Gamma_0 + P^{(d)}$

MiniBooNE



Comparison with Exp.

FIG. 5. $P_{\nu_e}^{(d)}/P_{\nu_e}^{(0)}(\mu)$ is compared with MiniBooNE data including statistic and systematic errors [16]. For numerical calculation, $m_{\nu_h} = 0.07$ eV (blue: inverted, light blue: normal), $m_{\nu_h} = 0.08$ eV (green: inverted, magenta: normal), $E_\mu = 670$ MeV, and $E_\pi = 1.15$ GeV are used. [5]

DAR: LSND ,KAEMEN

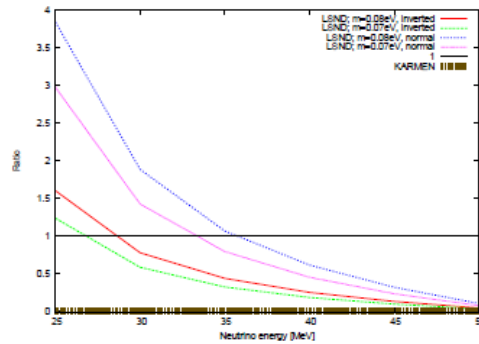


FIG. 8. The ratios $\frac{dP^{(d)}}{dE}$ are shown. For LSND, $T_\mu = 0$, $cT = 0.8\text{m}$ ($T \sim 2.5\text{ns}$), $L = 29.8\text{m}$, $\sigma_{\nu_e}^{\text{LSND}}$, $\Delta m_{LSND}^2 = 1.2\text{eV}^2$ and $\sin^2 2\theta_{LSND} = 0.003$ are used. Red curve shows inverted hierarchy of $m_{\nu_h} = 0.08$ eV, green curve shows inverted of $m_{\nu_h} = 0.07$ eV, blue curve shows normal hierarchy of $m_{\nu_h} = 0.08$ eV, and magenta curve shows normal of $m_{\nu_h} = 0.07$ eV. For KARMEN, $T_\mu = 0.3\mu\text{s}$, $\Delta t = 5\mu\text{s}$, $cT = 0.3\text{m}$ ($T \sim 1.0\text{ns}$), $m_{\nu_h} = 0.08$ eV, $L = 17.7\text{m}$, angle between proton beam and detector $\theta = 100^\circ$, $\sigma_{\nu_e}^{\text{KARMEN}}$, $\Delta m_{LSND}^2 = 1.2\text{eV}^2$ and $\sin^2 2\theta_{LSND} = 0.003$ are used. The time resolution T_R is set to 20 ns in both cases. A geometry for μ^+ DAR is shown in Fig.7 and a relation between T and θ is given in Appendix. [5]

ν_e energy spectrum

red : normal, green : $P^{(d)}$

(top) $cT = 1$ meter :

(bottom) $cT = 10$ meter

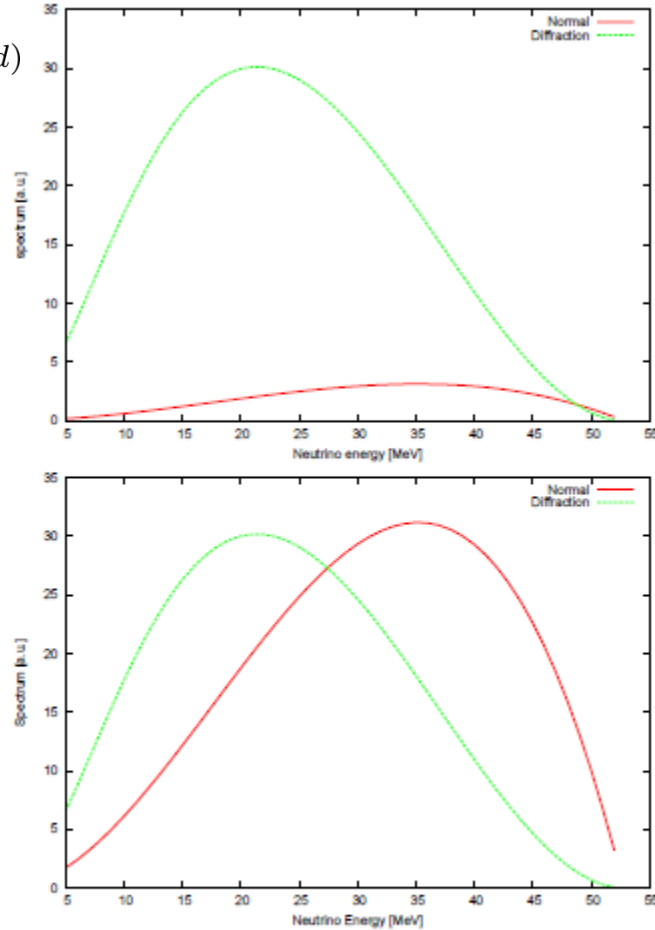


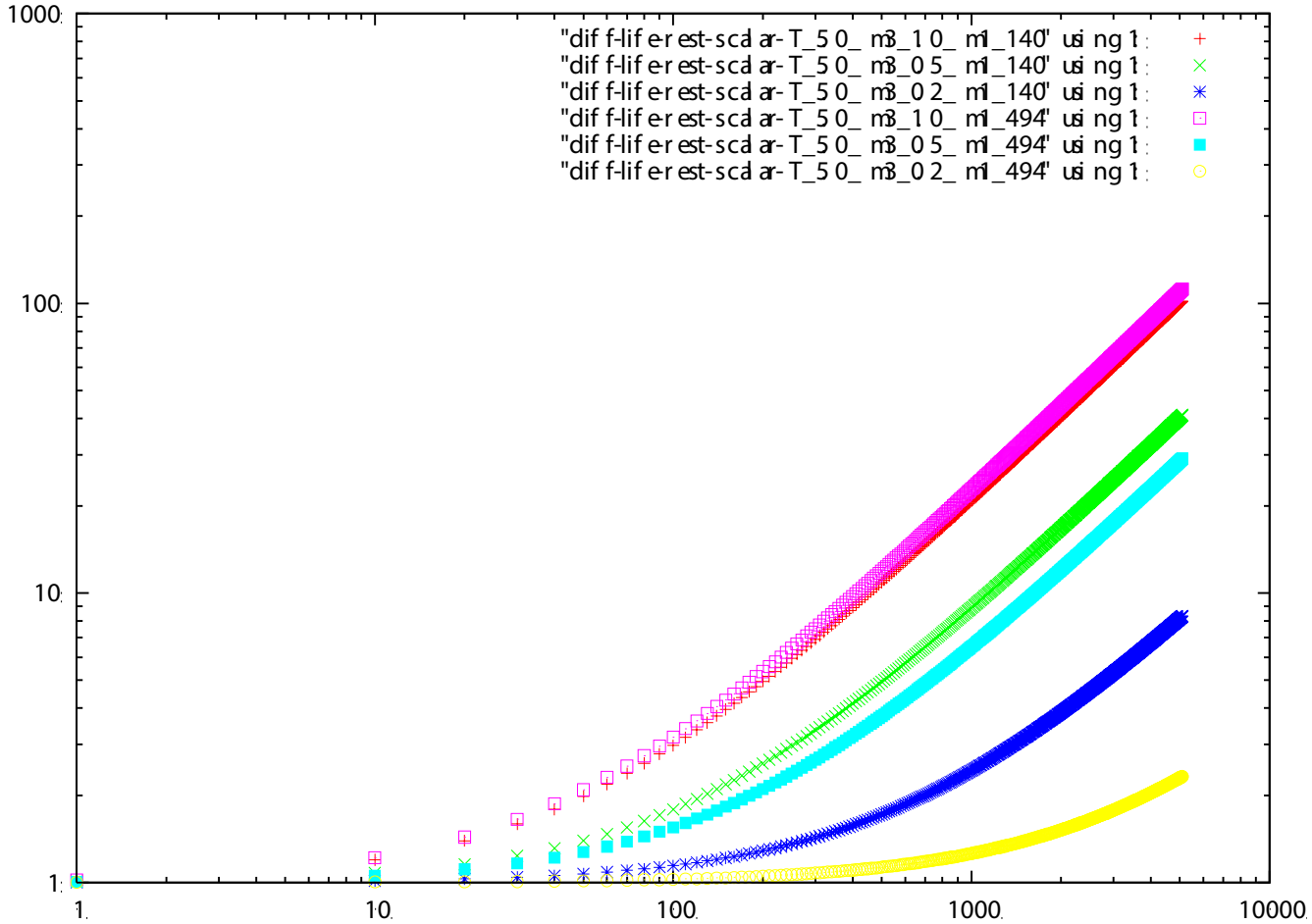
FIG. 3. (Color online) ν_e spectrum in μ DAR. The red curve shows normal component given in Eq. (86) and green curve shows diffraction component given in Eq. (85). $m_{\nu_e} = 0.08$ eV, and $2\sigma_{\nu_e} = 12^{\frac{2}{3}}/m_{\pi}^2$ (^{12}C carbon target) are used for numerical computation. Here, detector is located at $L = 0$. The top figure is for $cT = cT_D = 1$ and bottom one is for $cT = cT_D = 10$ m.

Weak decays: enhancement factor vs size of wave packet σm_π^2

Pi,K decays

$$T = 1.7 \times 10^{-8} \text{ s}$$

$$\frac{\Gamma_0 + P^{(d)}/T}{\Gamma_0}$$



size of wave packet σm_π^2

$P^{(d)}$ causes reactions extended to macroscopic areas:

Differences

Kinetic energy conservation

Lorentz invariance

Particle characteristics

helicity suppression

micro vs macro

$\lambda(\text{typical length})$

$$L_{int} = \frac{d}{dt}G$$

contributes to natural phenomena

Γ_0 $P^{(d)}$

Yes *No*

Yes *No*

particle *wave*

Yes *No*

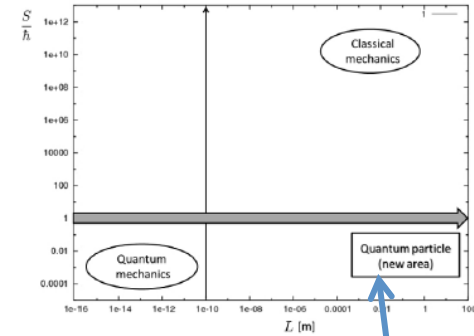
micro *macro*

$$\frac{\hbar}{p} \quad \frac{\hbar}{mc} \frac{E}{mc^2}$$

$$= 0 \quad \neq 0$$

Yes + *Yes*

1-1 quantum vs classical
length L vs action S



1-2 electron mass and Bohr radius

$$m_e = 0.5 MeV/c^2$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = r_e \alpha^{-2} = 0.5 \times 10^{-10} M(\text{Bohr radius})$$

**A new area of
quantum**

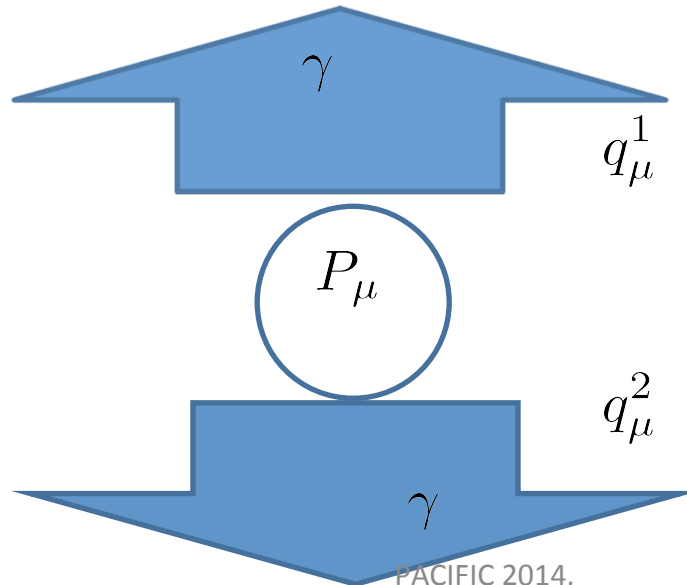
4.1⁺ meson(*c* \bar{c} , *e*⁻ *e*⁺, ν $\bar{\nu}$) \rightarrow $\gamma\gamma$ (*Ishikawa – Tajima – Tobita*)



$\Gamma_0 = 0$ (Landau-Yang)

$H_{int} = arbitrary$

Wave zone



$P^{(d)} \neq 0$

$P_\mu \neq q_\mu^1 + q_\mu^2$

Landau-Yang's theorem

2photon state $|\gamma, \gamma\rangle$

$$(j_{z1}, j_{z2}) = | + 1, +1\rangle, | - 1, -1\rangle, | + 1, -1 \rangle \pm | - 1, +1\rangle$$

$$(J, J_z) = (2, +2), (2, -2), (2, 0), \pm (0, 0)$$

$$\begin{array}{ll} \text{Momentum 1;} & (0, 0, p) \\ \text{Momentum 2;} & (0, 0, -p) \end{array} \quad \Gamma_0 = 0 \quad (\text{Landau - Yang})$$

$$\begin{array}{ll} \text{Momentum 1;} & (\delta p_1, \delta p_2, p + \delta p_3) \\ \text{Momentum 2;} & (\delta p'_1, \delta p'_2, -p + \delta p'_3) \end{array} \quad P^{(d)} \neq 0$$

m_{eff} : photon's effective mass

1. In vacuum : $m_{eff} = 0$ (gauge invariance)

2. In matter :

$$(2 - 1 : \text{low } \omega) \quad n = 1 + \epsilon$$

$$(2 - 2 : \text{high } \omega) \quad n = 1 - \frac{\omega_p^2}{\omega^2}$$

$$E(p)^2 = \frac{p^2}{n^2} = p^2 + 2\omega_p^2$$

$$\omega_p = \sqrt{\frac{n_e e^2}{m_e^2}}$$

triangle diagram

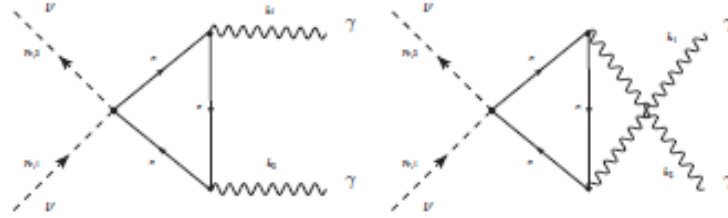


Figure 1: Tri-angle diagram of the electron loop which gives contributions to $1^+(\bar{l}) \rightarrow \gamma\gamma$, $\nu + \gamma \rightarrow \nu + \gamma$ and $\nu + \bar{\nu} \rightarrow \gamma + \gamma$.

$$\Gamma_{5,\alpha} = \langle 0 | J_{5,\alpha}(0) | k_1, k_2 \rangle = -i \frac{e^2}{4\pi} 2f_1 \epsilon^{\mu_1} \epsilon^{\mu_2}$$

$$\times [(k_{1,\mu_2} \epsilon_{\mu_1} \nu_1 \nu_2 \alpha - k_{2,\mu_1} \epsilon_{\mu_2} \nu_1 \nu_2 \alpha) k_1^{\nu_1} k_2^{\nu_2} + (k_1 k_2) \epsilon_{\mu_1 \mu_2 \nu \alpha} (k_1 - k_2)^\nu]$$

$$f_1 = \frac{1}{4k_1 k_2}; k_1 k_2 \gg m_e^2$$

$$= \frac{1}{2m_e^2}; k_1 k_2 \ll m_e^2$$

Effective action :axial vector decay $\phi^\mu \rightarrow \gamma + \gamma$

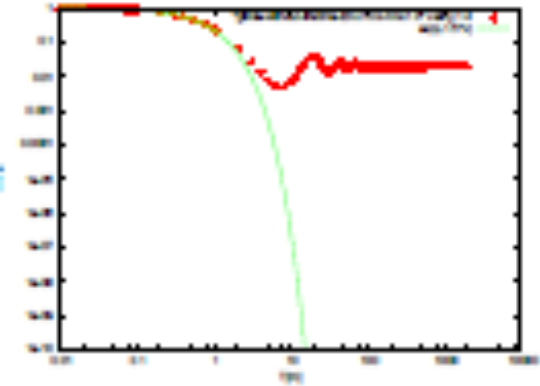
$$S_{int} = \int dx \partial_\mu (g \phi^\mu \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta})$$

$$= \int_{surface} dS_\mu g \phi^\mu(x) \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

The surface term gives no bulk effect, $\Gamma = 0$, but $P^{(d)} \neq 0$.

$$P = P^{(d)}$$

$$P^{(d)} = \frac{(\pi - 2)^2}{1536} \frac{(g\alpha)^2}{4\sqrt{\pi\sigma_\gamma} m_\gamma^2} \frac{E_c + 2p_c}{E_c(E_c + p_c)}$$



decreases with T as $e^{-T \beta(\omega_\gamma T)}$

Neutrino light scattering

$$(1) \nu + \gamma \rightarrow \nu + \gamma (\Gamma = 0; \text{Gell - Mann}(1964))$$

Effective action

$$S_{\nu\gamma} = \frac{\alpha G_F}{2\pi\sqrt{2}} \int d^4x \frac{\partial}{\partial x_\mu} (J_\mu^A \tilde{F}_{\alpha\beta} F^{\alpha\beta})$$

$$J_{mu}^A = \bar{\nu}(x)(1 - \gamma_5)\gamma_\mu\nu(x)$$

$$P^{(d)} = \frac{G_F^2 \alpha^2}{96\pi^2} \frac{1}{\sqrt{\pi\sigma_\gamma}} \frac{p^2}{m_\gamma^2}; \text{CM high energy}$$

$$(2) \nu + (B, E) \rightarrow \nu + \gamma$$

$$P^{(d)} = \frac{G_F^2 \alpha}{\pi^2} \left(\frac{eE(cB)}{m_e c^2} \right)^2 \frac{C(m_\nu) p_\nu^3}{\sqrt{\pi\sigma_\gamma} m_\gamma^2 m_e^2}; \text{high energy}$$

$$P^{(d)} \approx O(1); eE = 100 \text{MeV/meter}, p_\nu = 1000 \text{TeV}$$

Magnitudes of Probabilities

$$m_\gamma c^2 = 10^{-9} \text{eV}^2$$

In magnetic or electric field

$$P_B = 6.4 \times 10^{-27} \left(\frac{B}{B_0} \right)^2 \left(\frac{p_\nu}{p_\nu^{(0)}} \right)^3; B_0 = 1\text{T}, p_\nu^{(0)} = 10\text{MeV}$$

$$P_E = 2.8 \times 10^{-20} \left(\frac{E}{E_0} \right)^2 \left(\frac{p_\nu}{p_\nu^{(0)}} \right)^3; E_0 = 10^3 \text{GV}/m, p_\nu^{(0)} = 10\text{MeV}$$

l_ν : *mean free path of ν*

Sun

$$l_\nu = 5 \times 10^{15} \left(\frac{p_\nu^0}{p_\nu} \right)^2 \text{m}; m_\gamma c = 1\text{eV}, n_A = 10^{29} / (\text{cm})^3; p_\nu^0 = 10\text{MeV}$$

SuperNovae

$$l_\nu = 6 \times 10^4 \text{meters}; \frac{p_\gamma}{m_\gamma c} = 10^7, n_A = 10^{25} / (\text{cm})^3;$$

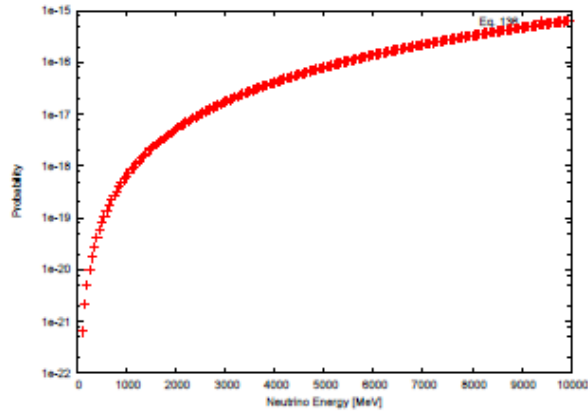


Figure 2: E_ν dependence of the probability Eq. (136) is shown. $B = 10$ Tesla is used for the calculation.

$$P_{\nu+B}(E_\nu); B = 10\text{T}$$

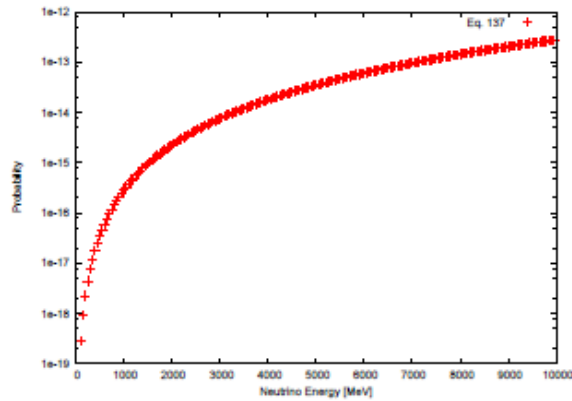


Figure 3: E_ν dependence of the probability Eq. (137) is shown. $E = 100$ GVolt/m is used for the calculation.

$$P_{\nu+E}(E_\nu); E = 100\text{GV/m}$$

Implications of $\nu + \gamma$

1. *high energy neutrino + "laser"*

$$N_{event} > 10^{-15} \times 10^{20} = 10^5$$

2. *neutrino + magnetic field*

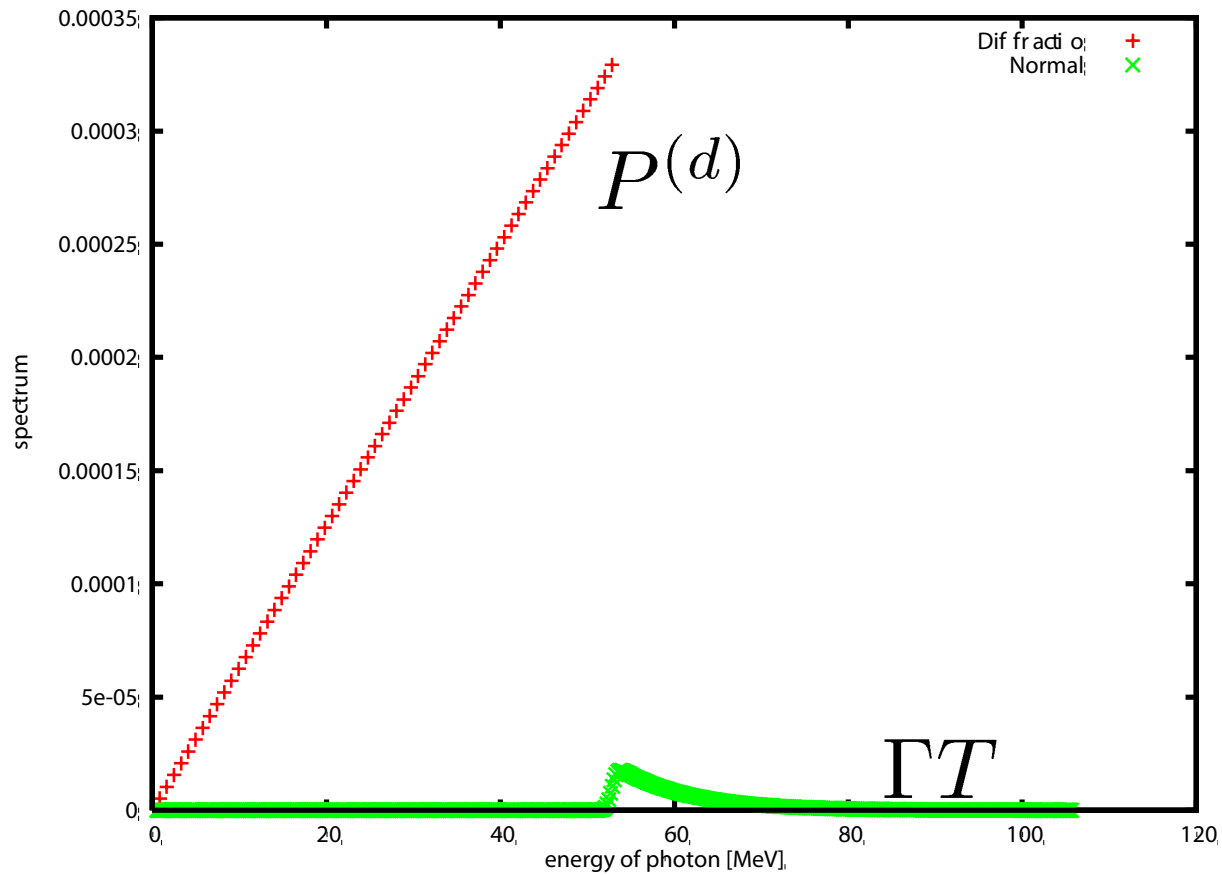
3 *neutrino + electric field (Lightning, --)*

4 *neutrino + photons (Sun, SuperNovae,)*

5 *Relic neutrino detection*

5. *Di – pole transition, nucleus transition, QED processes*

$$\mu \rightarrow e + \gamma, \frac{P^{(d)}(E)}{T\Gamma_0}, \text{ vs } E_\gamma$$



$$T = 1.7 \times 10^{-8} \text{ sec}, m_\gamma c^2 = 1 \text{ eV}, \sigma_\gamma m_\pi^2 = 100$$

E_γ

Examples of $P^{(d)}$

- (1). *Positron annihilation*
- (2). *Nuclear reactions*
- (3). *Chemical reactions*
- (4). *Others*

(1) e^+ *annihilation*

carbon hydride(chemical reaction)

Experiment: Enhancement for e^+ annihilation ; Z_{eff}/Z_c

Ethane C_2H_6 37

Propane C_3H_8 135

Hexane C_6H_{14} 2400

Octane C_8H_{18} 8800

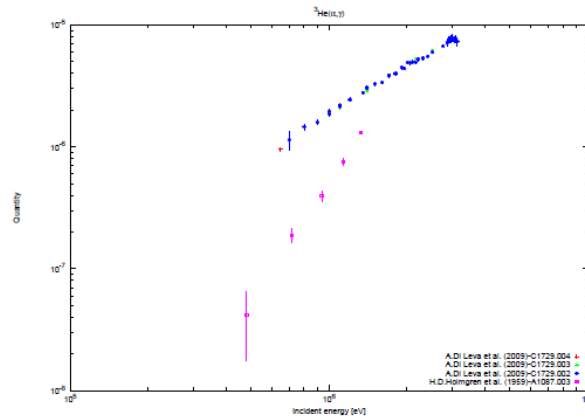
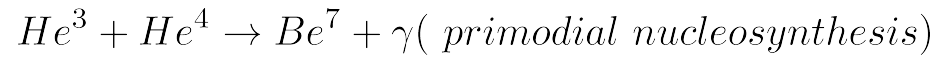
Dodecane $C_{12}H_{26}$ 18000

Ref.G.F.Gribakin,et.al,Rev.Mod.Physics,82,2557(2010)

Anomaly in e^+ annihilation

$$P^{(d)} / \Gamma_0 T$$

(2) Nuclear reactions



(HU Nuclear data – center)



(3) Absorption Coefficients of Ozone

Table

Ozone absorption coefficients (cm^{-1})

Wave length Inn Hearn Demore Criggs

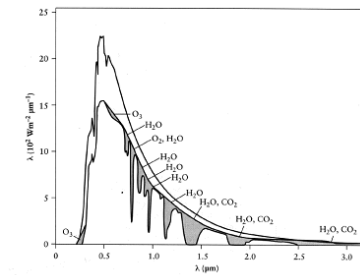
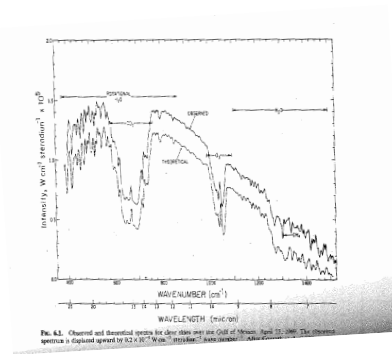
2536.5 133.1 133.9 135.0 131.8

2893.6 17.1 17.2 17.3 17.3

2967.3 6.72 6.97 6.85 7.00

3341.5 0.067 0.0498 — 0.065

year 1953 1961 1964 1968



Is $P^{(d)}$ necessary?

6. Summary – 1

Process	Γ_0	$P^{(d)}$:useful for
1. $\pi \rightarrow e\nu_e$	≈ 0	$\frac{E\sigma}{m_\nu^2}$	"determine m_ν "
2. $\nu + \gamma(B, E) \rightarrow \nu + \gamma$	0	$\alpha^2 G_F^2 p^2$	(<i>super novae</i> , --)
3. <i>Others</i>			$e^- + e^+ \rightarrow \gamma\gamma; \gamma + e \rightarrow \gamma + e$
			($He^3 + He^4 \rightarrow Be^7 + \gamma, \dots$), $\mu \rightarrow e\bar{\nu}_e\nu_\mu$, - - - - -
	$Q\bar{Q}$		transitions($B^- \rightarrow J/\Psi + K^-$)

Summary – 2

1. $P^{(d)}$ important in small m , high E , small T , large σ
2. $A' \rightarrow A + \gamma$; "negligible" for atoms
3. $\Gamma_0 = 0, P = P^{(d)}$
4. $P^{(d)}$ gives macroscopic quantum effects