Classical scale invariance and physics beyond the standard model

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CoEPP
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1. Some advertising

CoEPP: Started March 2011.
AUD 33m funding for 7 years.
U Melb led group inc. U Adelaide, Monash U and U Sydney.
Expt. (ATLAS) & theory collab.
~20 postdoc and 4 faculty positions.

International partners: U Penn (Trodden), Cambridge (Parker), Geneva (Clark), Freiburg (Jacobs), INFN Milano (Meroni), Duke (Kruse).

Collaborators welcome!

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2. Motivation and review

Setting all bare masses in the SM to zero increases the symmetry at the classical level:

$$B(x) \rightarrow \lambda B(\lambda x) \qquad F(x) \rightarrow \lambda^{3/2} F(\lambda x)$$

Scale invariance. **(B is boson, F is fermion.)**

Bare masses are: Higgs and RH Majorana neutrino.

Adding gravity: Planck mass also.

μ_ϕ^2 term in Higgs potential $\Rightarrow$

gauge hierarchy problem.

Scale invariance removes this term.

But scale invariance is anomalous: masses are generated at the quantum level via dimensional transmutation.

Is the gauge hierarchy problem really solved?

Automatically if you use dimensional regularisation.

W.A. Bardeen FERMILAB-CONF-95-391-T
K.A. Meissner and H. Nicolai, PLB648, 312 (2007); PLB660, 260 (2008)
R. Foot, A. Kobakhidze and RV, PLB655, 156 (2007)
See also: M. Shaposhnikov and D. Zenhausern, PLB671, 162 (2009)
M. Shaposhnikov and F. Tkachov, arXiv:0905.4857

Define the quantum theory to violate scale invariance in the "least possible way".

Momentum cut-off or Pauli-Villars regularisation breaks scale invariance in a hard way (impose WI on counterterms).

In DR, however:

$$\int \frac{d^4k}{(2\pi)^4} \to (\tilde{\mu})^{2\epsilon} \int \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}} \qquad (\tilde{\mu})^{2\epsilon} = 1 + \epsilon \ln \tilde{\mu}^2 + \ldots \quad \text{as } \epsilon \to 0$$

$\tilde{\mu}$ **explicitly breaks scale invariance, but it always occurs under a logarithm.**

With classical scale invariance and DR, there simply are no mass parameters available to even radiatively generate a μ_ϕ^2 term, let alone produce a quadratic divergent one. For example:

$$\int \frac{d^d k_E}{(2\pi)^d} \frac{1}{(k_e^2 + \Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(n - \frac{d}{2}\right)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}}$$

Standard DR formula. For d=4-2ε and n=1, you get

$$\int \frac{d^{4-2\epsilon} k_E}{(2\pi)^{4-2\epsilon}} \frac{1}{k_E^2 + \Delta} = \frac{1}{(4\pi)^{2-\epsilon}} \Gamma(-1+\epsilon) \Delta^{1-\epsilon}$$

Scale invariance ⇒ Δ=0 ⇒ quad. div. integral is zero.

The scale anomaly manifests via logarithms:

- **running parameters**
- **Coleman-Weinberg potential**

The scale anomaly can generate symmetry breaking scales:

Strong coupling examples:

- **QCD with massless quarks ⇒ Λ_{QCD} (DχSB)**
- **Technicolour ⇒** $(\Lambda_{\rm EW})^3 \sim \langle \overline{T} T \rangle$

Weak coupling example:

- ***Coleman-Weinberg breaking***

S. Coleman and E. Weinberg, PRD7, 1888 (1973)
E. Gildener and S. Weinberg, PRD13, 3333 (1976)

Gildener & S. Weinberg explained how to analyse CW symmetry breaking for weakly-coupled massless scalar field theories:

- **1-loop CW potential dominates along flat direction of tree-level potential**
- **Quartic couplings are running parameters $\lambda_i = \lambda_i(\mu)$**
- **Get flat direction by suitable relation amongst λ_i at certain scale $\mu=\Lambda$.**

The relation replaces one λ_i with quantally-generated scale Λ: dimensional transmutation (not fine-tuning!!).

Λ is free parameter; all masses related to it.

OK, so what exactly do we want?

A fully-realistic theory with classical scale invariance!

Fully-realistic means:

- **Generates acceptable EW Higgs mass**
- **Has nonzero neutrino masses**
- **Has dark matter**
- **Has baryogenesis**
- **Solves strong CP problem**
- **Has acceptable inflationary cosmology**
- **Explains origin of Planck scale**
- **Accommodates dark energy**

I'll discuss how to do some of the above, especially how to fine-tune an appropriate cosmological constant.

Hierarchy of scales is required:

$$m_{\rm DE} \sim 10^{-3}\ {\rm eV}$$

$$m_\nu \sim 0.1\ {\rm eV}$$

$$m_{\rm EW} \sim 100\ {\rm GeV}$$

$$m_{\rm leptogen} \sim 10^9\ {\rm GeV}$$

$$m_{\rm PQ} \sim 10^{10}\ {\rm GeV}$$

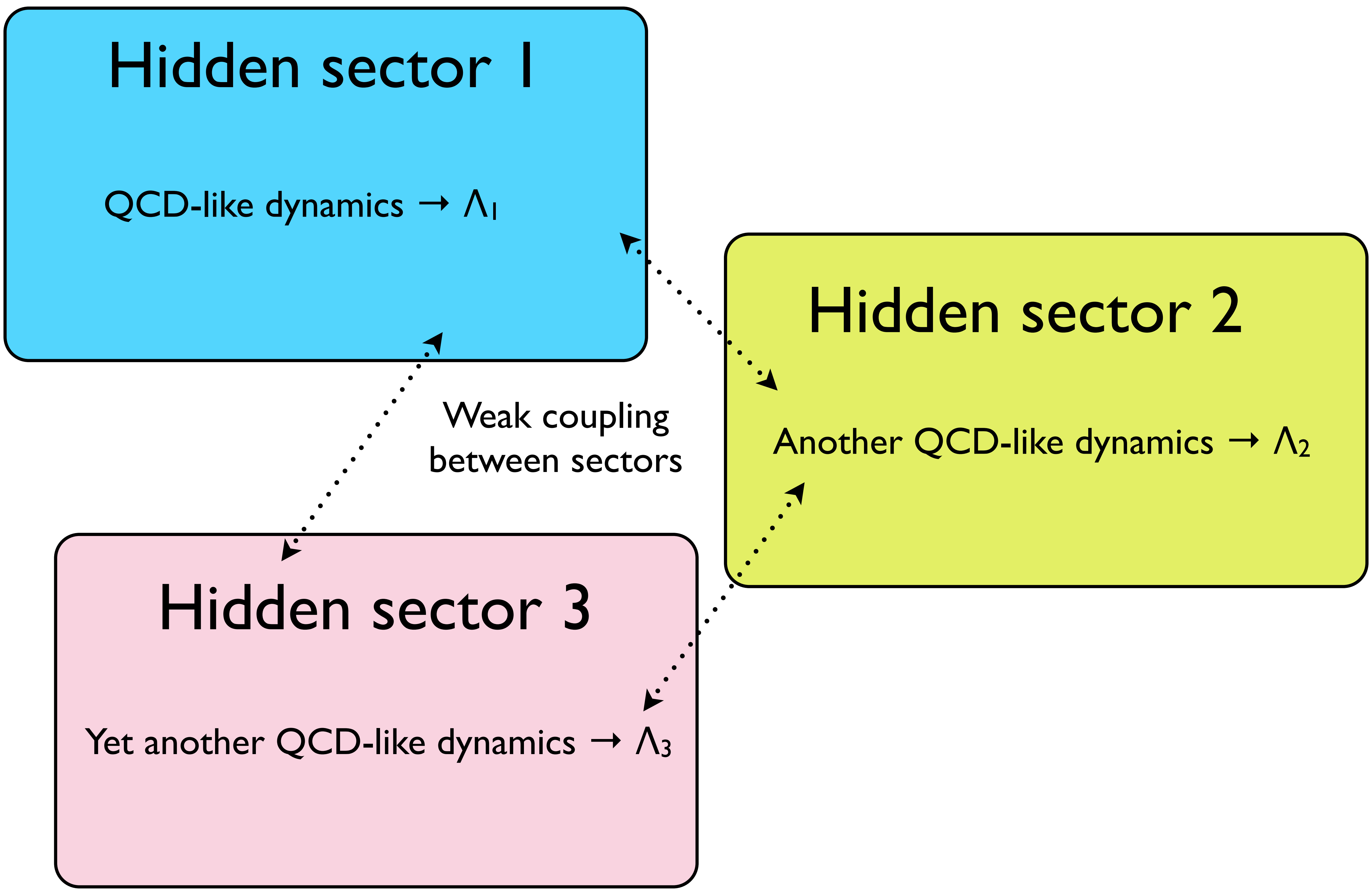
$$m_{\rm DM} \sim 10\ {\rm keV} - 10^{16}\ {\rm GeV}$$

$$m_P \sim 10^{19}\ {\rm GeV}$$

How to get rich set of scales from scale-invariance?

Obvious role for hidden sectors.

Strong coupling example:



Weak coupling scheme:

Φ = EW Higgs doublet **S_1, S_2, ... gauge singlets**

In limit where S-sector decouples from SM-sector:

$$V(\phi, S_1, S_2, \ldots) = V(\phi) + V(S_1, S_2, \ldots)$$

V(Φ) is the SM Coleman-Weinberg potential. Because of large top mass, it fails to radiatively induce a nonzero VEV for Φ.

But V(S1, S2, ...) can imply nonzero VEVs for S fields:

$$\langle \phi \rangle = 0 \lll \langle S \rangle \neq 0$$

Now switch on small coupling between sectors:

$$\sum_i \lambda_x^i \phi^\dagger \phi S_i^2$$

Negative λ_x induce negative squared-mass for Φ, hence nonzero VEV for Φ.

But as $\lambda_x \to 0$, we must get $\langle \Phi \rangle \to 0$, so

$$\frac{\langle \phi \rangle}{\langle S \rangle} \ll 1$$

is a technically-natural hierarchy.

3. Fine tuning CC to be tiny

Classical potential: $V_0(S_i) = \lambda_{ijkl} S_i S_j S_k S_l$

Hyperspherical rep: modulus r, angles θ_i

$$V_0(r, \theta_i) = r^4 f(\lambda_{ijkl}, \theta_i)$$

Classical CC = $V_{0,\mathrm{min}} = 0$ **(classical scale invariance)**

Effective potential when classical scale inv. holds:

$$V = A[g(\mu), m(\mu), \theta(\mu), \mu] r(\mu)^4$$
$$+ B[g(\mu), m(\mu), \theta(\mu), \mu] r(\mu)^4 \ln\left(\frac{r(\mu)^2}{\mu^2}\right)$$
$$+ C[g(\mu), m(\mu), \theta(\mu), \mu] r(\mu)^4 \left[\ln\left(\frac{r(\mu)^2}{\mu^2}\right)\right]^2 + \ldots$$

B. Kastening PLB283, 287 (1992)
M. Bando et al., PLB301, 83 (1993)

Extremum condition $\frac{\partial V}{\partial r} = 0$ **with** $\langle r \rangle \neq 0 \Rightarrow$

$$2A(\mu = \langle r \rangle) + B(\mu = \langle r \rangle) = 0$$

Dimensional transmutation: generation of scale $\langle r \rangle$

Fine-tuning the CC to zero: $V_{\rm min} = 0 \Rightarrow A(\langle r \rangle) = 0$

So A = B = 0 at scale <r>. A ≈ $A^{(0)}$ ≈ 0 is approx. Gildener-Weinberg condition.

The PGB thus gets mass at 2-loops at best:

$$m^2_{\rm PGB} = 8C(\langle r \rangle)\langle r \rangle^2$$

This must be positive for the CC fine-tuning to work.

From the RG Eqn expressing μ-independence of V:

$$C(\langle r \rangle) = \frac{1}{4}\mu \, \frac{dB}{d\mu}\Big|_{\mu=\langle r \rangle}$$ **with**

$$B^{(1-{\rm loop})}(\langle r \rangle) = \frac{1}{64\pi^2\langle r \rangle^2} \, \left[3{\rm Tr}m_V^4 + {\rm Tr}m_S^4 - 4{\rm Tr}m_F^4\right]\Big|_{\mu=\langle r \rangle}$$

Thus we can only accept theories which allow:

$$C^{(2-\text{loop})}(\langle r\rangle) = \frac{1}{64\pi^2\langle r\rangle^2}\left[3\text{Tr}m_V^4\gamma_V + \text{Tr}m_S^4\gamma_S - 4\text{Tr}m_F^4\gamma_F\right]\Big|_{\mu=\langle r\rangle} > 0$$

Υ's are anomalous dimensions

4. Explicit Model

R. Hempfling, PLB379, 153 (1996)
R. Foot, A. Kobakhidze and RV, PLB655, 156 (2007); PRD82, 035005 (2010); **arXiv: 1012.4848**
R. Foot, A. Kobakhidze, K. McDonald and RV, PRD76, 075014 (2007); PRD77, 035006 (2008)
K.A. Meissner and H. Nicolai, PLB648, 312 (2007); Eur. Phys. J C57, 493 (2008); PRD80, 086005 (2009)
W.F. Chang, J.N. Ng and J.M.S. Wu, PRD75, 115016 (2007)
J.R. Espinosa and M. Quiros, Phys.Rev.D76:076004 (2007)
S. Iso, N. Okada and Y. Orikasa, PLB676, 81 (2009)
M. Holthausen, M. Lindner and M.A. Schmidt, arXiv:0911.0710
L. Alexander-Nunneley and A. Pilaftsis, arXiv:1006.5916

Ingredients: Φ, S_1 and S_2 with

$$\langle \phi \rangle \ll \langle S_2 \rangle \ll \langle S_1 \rangle$$

EW **see-saw** **Planck**

Scale-invariant tree-level potential for singlets:

$$V_0(S_1, S_2) = \frac{\lambda_1}{4} S_1^4 + \frac{\lambda_2}{4} S_2^4 + \frac{\lambda_3}{2} S_1^2 S_2^2$$

($S_1 \to -S_1$ imposed)

Interested in $\lambda_3 < 0$ s.t. $\lambda_1 \lambda_2 \geq \lambda_3^2$ (boundedness). Let

$$S_1 = r \cos \omega, \;\; S_2 = r \sin \omega$$

$$V_0 = \frac{1}{4} S_1^4 \left(\lambda_1 + 2\lambda_3 \tan^2 \omega + \lambda_2 \tan^4 \omega \right)$$

For given S_1, minimum is at $\tan^2 \omega = \frac{|\lambda_3|}{\lambda_2}$ **with:**

$$V_{\rm min} = \frac{\lambda_1 \lambda_2 - \lambda_3^2}{4\lambda_2} \; S_1^4$$

Approx. flat direction when

$$\lambda_3(\langle r \rangle) \simeq -\sqrt{\lambda_1(\langle r \rangle)\lambda_2(\langle r \rangle)}$$

Dimensional transmutation

The flat direction is along:

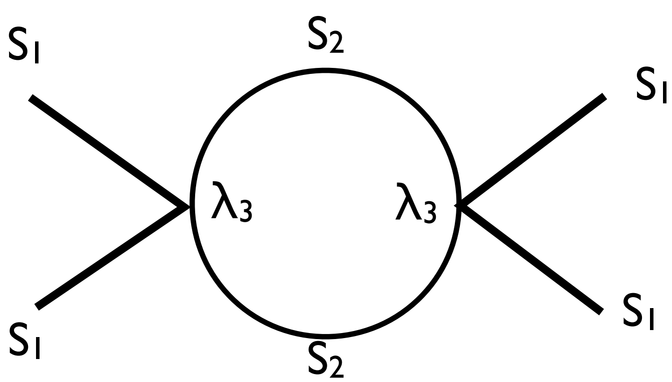
$$S_1 = r\sqrt{\frac{1}{1+\epsilon}} \equiv v, \qquad S_2 = \sqrt{\epsilon} v$$

$$\epsilon \equiv \tan^2 \omega = \sqrt{\frac{\lambda_1(\Lambda)}{\lambda_2(\Lambda)}} \qquad \Lambda = \langle r \rangle$$

Choose $\lambda_{1,2}(\Lambda)$ so that $\epsilon \sim (M_{\text{see-saw}}/M_P)^2 \ll 1$ **i.e.**

$$\lambda_1(\Lambda) = -\epsilon\lambda_3(\Lambda) = \epsilon^2\lambda_2(\Lambda)$$

$$\lambda_1(\Lambda) \ll |\lambda_3(\Lambda)| \ll \lambda_2(\Lambda)$$



$$\delta\lambda_1 \sim \frac{\lambda_3^2}{16\pi^2} = \frac{\epsilon^2\lambda_2^2}{16\pi^2} = \frac{\lambda_1\lambda_2}{16\pi^2}$$

hierarchy stable for perturbative λ_2

Generate Planck scale through $\mathcal{L} \supset \sqrt{-g}\; S_1^2\; R$

Generate see-saw scale through $\mathcal{L} \supset \lambda_M \overline{\nu}_R (\nu_R)^c S_2 + H.c.$

Scalar masses: $m_S^2 = 2(\lambda_1 - \lambda_3) v^2$ where $S \equiv \sin\omega S_1' - \cos\omega S_2'$

(primes denote shifted fields)

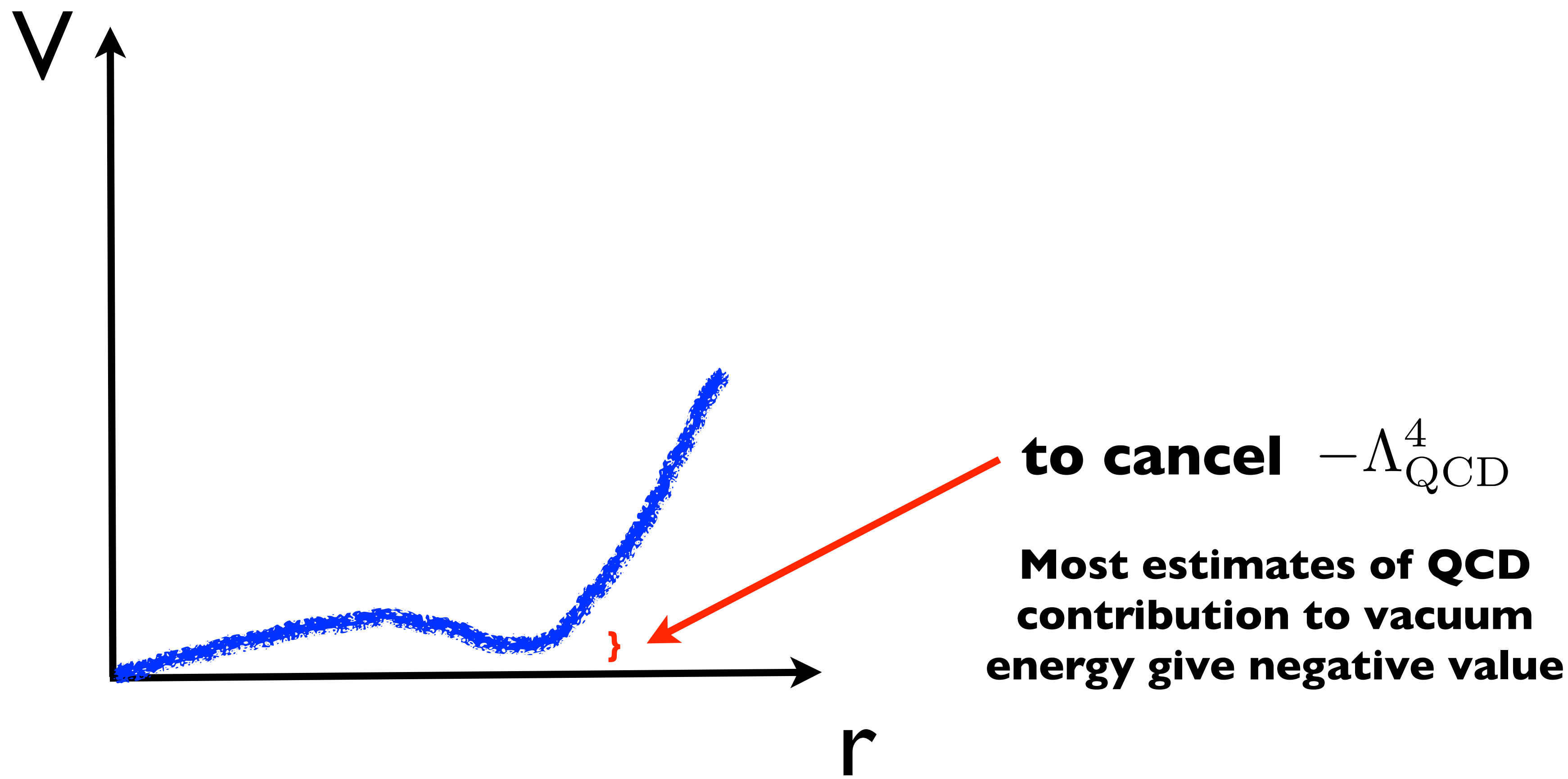
$s \equiv \cos\omega S_1' + \sin\omega S_2'$ **is PGB for scale invariance.**

Fine-tuning the CC to zero requires: $m_S^4 \simeq 2 \sum_{i=1}^{3} M_{\nu_{iR}}^4$

Compute $C^{2-\mathrm{loop}} = \frac{3\lambda_1 \lambda_2^2}{128\pi^4} \left[2 - y + (1-y)\frac{y}{\sqrt{6}} \right]$

$y \equiv \frac{6 M_{\nu_R}^4}{m_S^4} \simeq 1$ **(degen. ν_R)** **so** $C^{(2-\mathrm{loop})} > 0$

This model is OK



False vacuum

3. Conclusions

- Realistic models with classical scale invariance are feasible
- Hierarchies of scales can be generated in technically-natural way (modulo quantum gravity uncertainties)
- CC fine-tuning is a non-trivial constraint on such models