Probing Flavor Ratios and Flavor Transitions Mechanisms of Astrophysical Neutrinos by Neutrino Telescopes
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K.-C. Lai, G.-L. Lin and T. C. Liu, 
F.-S. Lee, G.-L. Lin, T. C. Liu and Y. Yang, in progress
Outline

• Review on possible types of astrophysical neutrino sources
• What can we learn by detecting these neutrinos?
  (1) the original neutrino flavor ratio at astrophysical source-assuming three flavor oscillations
  (2) the neutrino flavor transition mechanism during its propagation from source to Earth—with a clear knowledge on the source flavor ratio
• Answering the above questions by flavor discriminations in neutrino telescopes
Common astrophysical neutrino sources

From $\pi$ decays

$\nu_e$ (1,0,0)

$\nu_\mu$ (0,1,0)

$\nu_\tau$ (0,0,1)

$\Phi_0 = \left( \phi_0(\nu_e), \phi_0(\nu_\mu), \phi_0(\nu_\tau) \right)$

$\phi_0(\nu_e) + \phi_0(\nu_\mu) + \phi_0(\nu_\tau) = 1$
Pion source (1/3,2/3,0)

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \mu^+ \rightarrow \bar{\nu}_\mu + e^+ + \nu_e \]

Energies of various neutrinos are comparable, i.e., muon decays before losing its energy by interactions.

Cosmogenic (GZK) neutrinos produced by

\[ p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \rightarrow n + \pi^+ \]

and the subsequent pion decay fit into this category.
Muon damped source (0,1,0)

\[ \pi^+ \rightarrow \mu^+ + \nu_{\mu} \]

\[ \mu^+ \rightarrow \bar{\nu}_{\mu} + e^+ + \nu_e \]

Muon loses significant amount of energy before it decays:

(1) muon interacts with matter

J. P. Rachen and P. Meszaros, 1998

(2) Muon interacts with background photon field


Neutrino flux from muon decays is negligible

See more detailed studies in


Transition from pion source to muon-damped source
⇒due to particle density and background field strength at the source
Systematically studying sources on Hillas plot

\[ \phi(E_p) \propto E_p^{-2} \]

Sources with significant $\nu_\tau$ fractions

Neutrinos from WIMP annihilations

$$\chi\chi \rightarrow \tau^+\tau^- , b\bar{b}$$

Tau lepton and $b$ can decay into $\nu_\tau$
Reconstructing the neutrino flavor ratio at the source

\[
\begin{pmatrix}
\phi(v_e) \\
\phi(v_\mu) \\
\phi(v_\tau)
\end{pmatrix} = 
\begin{pmatrix}
P_{ee} & P_{e\mu} & P_{e\tau} \\
P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\
P_{\tau e} & P_{\tau\mu} & P_{\tau\tau}
\end{pmatrix}
\begin{pmatrix}
\phi_0(v_e) \\
\phi_0(v_\mu) \\
\phi_0(v_\tau)
\end{pmatrix}
\]

Measured flux \(\Phi\)

Source flux \(\Phi_0\)

\[
P_{\alpha\beta} \equiv P(v_\beta \rightarrow v_\alpha) = \sum_{i=1}^{3} |U_{\beta i}|^2 |U_{\alpha i}|^2, \text{ where } v_\alpha = U_{\alpha i}^* v_i
\]

Flavor Eigenstate

Mass Eigenstate

\(U_{\alpha i}\) contains 3 mixing angles--\(\theta_{12}, \theta_{23},\) and \(\theta_{13}\)

one CP phase \(\delta\)
Reconstructing the neutrino flavor ratio at the source--continued

• How well can we distinguish astrophysical sources with different neutrino flavor ratio, assuming three flavor neutrino oscillations?

• This depends on our understanding of neutrino mixing parameters and flavor discrimination capabilities in neutrino telescopes.

\[
\begin{align*}
\sin^2 \theta_{12} &= 0.304^{+0.022,0.066}_{-0.016,0.054}, & \sin^2 \theta_{23} &= 0.5^{+0.07,0.17}_{-0.06,0.14}, & \sin^2 \theta_{13} &= 0.01^{+0.009}_{-0.006}; \\
\sin^2 \theta_{13} &\leq 0.35 \quad 3\sigma \quad \text{Normal hierarchy}
\end{align*}
\]

Flavor discrimination capability

At water Cherenkov detectors such as ANTARES, IceCube and KM3NeT, track to shower event ratio can be used to extract the flux ratio

\[ R = \frac{\phi(\nu_\mu)}{\phi(\nu_e) + \phi(\nu_\tau)} \]

In appropriate energy window, one can further identify tau shower so that one can measure

\[ S = \frac{\phi(\nu_e)}{\phi(\nu_\tau)} \]

Flavor discrimination--continued

In newly proposed Askaryan Radio Array (ARA) with $E_{\nu} > 10^{17}$ eV, $\nu_e$ may be separated from other flavors by LPM effect. One can determine

$$R' = \frac{\phi(\nu_e)}{\phi(\nu_\mu) + \phi(\nu_\tau)}$$


At this energy, it is difficult to measure

$$S' = \frac{\phi(\nu_\mu)}{\phi(\nu_\tau)}$$

ARA sensitivity on GZK neutrinos

<table>
<thead>
<tr>
<th>Model &amp; references</th>
<th>$N_\nu$:</th>
<th>ANITA-II, (2008 flight)</th>
<th>ARA, 3 years</th>
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<tbody>
<tr>
<td><strong>Baseline cosmogenic models:</strong></td>
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<tr>
<td>Protheroe &amp; Johnson 1996 [27]</td>
<td>0.6</td>
<td>59</td>
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<td>Engel, Seckel, Stanev 2001 [28]</td>
<td>0.33</td>
<td>47</td>
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<tr>
<td>Kotera, Allard, &amp; Olinto 2010 [29]</td>
<td>0.5</td>
<td>59</td>
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<td><strong>Strong source evolution models:</strong></td>
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<tr>
<td>Engel, Seckel, Stanev 2001 [28]</td>
<td>1.0</td>
<td>148</td>
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<tr>
<td>Kalashev et al. 2002 [30]</td>
<td>5.8</td>
<td>146</td>
<td></td>
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<tr>
<td>Barger, Huber, &amp; Marfatia 2006 [32]</td>
<td>3.5</td>
<td>154</td>
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<td>Yuksel &amp; Kistler 2007 [33]</td>
<td>1.7</td>
<td>221</td>
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<td><strong>Mixed-Iron-Composition:</strong></td>
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<tr>
<td>Ave et al. 2005 [34]</td>
<td>0.01</td>
<td>6.6</td>
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<tr>
<td>Stanev 2008 [35]</td>
<td>0.0002</td>
<td>1.5</td>
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<tr>
<td>Kotera, Allard, &amp; Olinto 2010 [29] upper</td>
<td>0.08</td>
<td>11.3</td>
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<tr>
<td>Kotera, Allard, &amp; Olinto 2010 [29] lower</td>
<td>0.005</td>
<td>4.1</td>
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<tr>
<td><strong>Models constrained by Fermi cascade bound:</strong></td>
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<td>Ahlers et al. 2010 [36]</td>
<td>0.09</td>
<td>20.7</td>
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<td><strong>Waxman-Bahcall (WB) fluxes:</strong></td>
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<tr>
<td>WB 1999, evolved sources [37]</td>
<td>1.5</td>
<td>76</td>
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<tr>
<td>WB 1999, standard [37]</td>
<td>0.5</td>
<td>27</td>
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Water Cherenkov detectors (astrophysical sources)

Only $R$ is measured

Both $R$ and $S$ are measured

15% accuracy

($\sim$100 events)
Radio wave detectors (GZK neutrinos)

Only $R'$ is measured

Both $R'$ and $S'$ are measured

15% accuracy

($\sim 100$ events)
Summary for part I

• We have reviewed various sources of astrophysical neutrinos with different neutrino flavor ratios.

• The possibility of reconstructing the above flavor ratios through flavor discrimination in neutrino telescopes is discussed.
Probing neutrino flavor transition mechanisms—model independent parameterization

The flavor transition mechanisms of astrophysical neutrinos might be probed.

Earlier discussions on this issue:
G. Barenboim and C. Quigg, Phys. Rev. D 2003,
J. Beacom et al. Phys. Rev. Lett. 2003 ...

Work out $P$ model by model and calculate the resultant $\Phi$ which is to be tested by neutrino telescope.

However, we perform a transformation

$$Q = A^{-1}PA.$$ 

Classification of flavor transition models can be done easily on $Q$. Fit $Q$ to the measurement

K.-C. Lai, G.-L. Lin and T. C. Liu,
\[\Phi_0 = \frac{1}{3} (1,1,1)^T + a(0,-1,1)^T + b(2,-1,-1)^T, \quad -1/6 \leq b \leq 1/3\]

\[\left(\phi_0(\nu_e), \phi_0(\nu_\mu), \phi_0(\nu_\tau)\right)^T\]

\[\phi_0(\nu_e) + \phi_0(\nu_\mu) + \phi_0(\nu_\tau) = 1\]

\[\Phi = P\Phi_0\]

\[\Phi = \kappa(1,1,1)^T + \rho(0,-1,1)^T + \lambda(2,-1,-1)^T\]

\[\kappa = 1/3\] corresponds to conservation of neutrino flux
A simple transformation

\[
\begin{pmatrix}
\kappa \\
\rho \\
\lambda
\end{pmatrix} = 
\begin{pmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{pmatrix} \begin{pmatrix}
1/3 \\
a \\
b
\end{pmatrix}, \text{ where}

P_{\alpha \beta} \equiv P(\nu_\beta \rightarrow \nu_\alpha)

\[
\begin{pmatrix}
Q_{11} & Q_{12} & Q_{13} \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{pmatrix} = A^{-1} \begin{pmatrix}
P_{ee} & P_{e\mu} & P_{e\tau} \\
P_{\mu e} & P_{\mu\mu} & P_{\mu\tau} \\
P_{\tau e} & P_{\tau\mu} & P_{\tau\tau}
\end{pmatrix} A
\]

\[
A = \begin{pmatrix}
1 & 0 & 2 \\
1 & -1 & -1 \\
1 & 1 & -1
\end{pmatrix}
\]

\[
\rho = \frac{1}{2}(\phi(\nu_\tau) - \phi(\nu_\mu)), \quad \lambda = \frac{1}{3}\left(\phi(\nu_e) - \frac{\phi(\nu_\mu) + \phi(\nu_\tau)}{2}\right)
\]

\[
\phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = 3\kappa
\]

\[
\sum_{\alpha} P_{\alpha\beta} = 1 \Rightarrow Q_{11} = 1, Q_{12} = Q_{13} = 0
\]

This then implies \( \kappa = 1/3 \)

Gives the meaning of \( Q \) matrix
Classify flavor transition models

Flux conservation

\[
Q = \begin{pmatrix}
1 & 0 & 0 \\
Q_{21} & Q_{22} & Q_{23} \\
Q_{31} & Q_{32} & Q_{33}
\end{pmatrix}
\]

\[\theta_{23} = 45^\circ, \theta_{12} = 0^\circ \text{ limit}\]

Flux conservation+\(\nu_\mu --\nu_\tau\) symmetry

\[
Q = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
Q_{31} & 0 & Q_{33}
\end{pmatrix}
\]

Values for \(Q_{31}\) and \(Q_{33}\) determine the model

Fit \(Q_{31}\) and \(Q_{33}\) to the data

Recent T2K result makes it more complicated.
Q matrix for standard oscillation

\[ Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \]

\[ \text{i.e., } Q_{31} = 0, \ Q_{33} = 1/3 \]

Evaluated in tribimaximal limit:
\[ \sin^2 \theta_{23} = 1/2, \]
\[ \sin^2 \theta_{12} = 1/3, \]
\[ \sin^2 \theta_{13} = 0 \]

To the first order in
\[ \epsilon \equiv 2 \cos 2\theta_{23} / 9 + \sqrt{2} \sin \theta_{13} \cos \delta / 9 \]
\[ -3\epsilon = -0.24/3 \text{ Normal hierarchy} \]
\[ = -0.27/3 \text{ Inverted hierarchy} \]

Take T2K best fit: \( \sin^2 2 \theta_{13} = 0.11 \) (N), 0.14 (I) at \( \delta = 0 \)

Also assume \( \theta_{23} = \pi/4 \)
Q matrix for neutrino decays—just for illustration

Normal hierarchy

\[
Q = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
1/2 & 0 & 0
\end{pmatrix}
\]
i.e., \(Q_{31} = 0.5, \ Q_{33} = 0\)

Inverted hierarchy

\[
Q = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
-1/2 & 0 & 0
\end{pmatrix}
\]
i.e., \(Q_{31} = -0.5, \ Q_{33} = 0\)
Observing pion source and muon damped source to determine $Q_{31}$ and $Q_{33}$

**Pion source**

\[
\begin{pmatrix}
1/3 \\
\rho \\
\lambda
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
Q_{31} & 0 & Q_{33}
\end{pmatrix} \begin{pmatrix}
1/3 \\
-1/3 \\
0
\end{pmatrix} \Rightarrow \lambda = Q_{31}/3
\]

**Muon-damped source**

\[
\lambda \equiv \frac{1}{3} \left( \phi(v_e) - \frac{\phi(v_\mu) + \phi(v_\tau)}{2} \right)
\]

\[
\begin{pmatrix}
1/3 \\
\rho \\
\lambda
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
Q_{31} & 0 & Q_{33}
\end{pmatrix} \begin{pmatrix}
1/3 \\
-1/2 \\
-1/6
\end{pmatrix} \Rightarrow \lambda = Q_{31}/3 - Q_{33}/6
\]
Compare oscillation with neutrino decays (H, M→L)

\[
\Delta R_\pi/R_\pi = \Delta R_\mu/R_\mu = 10\%
\]

\[
R = \frac{\phi(v_\mu)}{\phi(v_e) + \phi(v_\tau)}
\]

Water Cherenkov detector ~50 events
Change the input model
Change the input model--continued
GZK neutrino dominates at $E_{\nu} > 10^{17}$ eV

We have a pure pion source

$$
\begin{pmatrix}
1/3 \\
\rho \\
\lambda \\
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
Q_{31} & 0 & Q_{33} \\
\end{pmatrix} \begin{pmatrix}
1/3 \\
\end{pmatrix}, \text{ so}
$$

$$
\lambda \equiv \frac{1}{3} \left( \phi(v_e) - \frac{\phi(v_\mu) + \phi(v_\tau)}{2} \right) = Q_{31}/3
$$

Pion source can only probe $Q_{31}$
Compare oscillation with neutrino decays (H, M→L)

\[ \Delta R' / R' = 10\% \]

\[ R' = \frac{\phi(v_e)}{\phi(v_\mu) + \phi(v_\tau)} \]

\[ \Delta R' / R' = 20\% \]

\[ R'_{osc} = 0.5 \]

Radio wave detection

~50 events
Change the input model

\[ R_{\text{NH}} = 2 \]

\[ \Delta R' / R' = 10\% \]

\[ \Delta R' / R' = 20\% \]
Change the input model

\[ R'_{IH} = 0 \]

Assume \( \Delta R'_{IH} = 0.1 \)
Summary (I)

- We have proposed a model-independent parameterization, the Q matrix, for flavor transition mechanisms (standard oscillations and beyond) of astrophysical neutrinos.
- Each row of matrix Q carries a definite physical meaning. $Q_{1i}$ for normalization, $Q_{2i}$ for $\mu$-$\tau$ symmetry breaking and $Q_{3i}$ governing the flux difference

\[
\phi(\nu_\tau) - \frac{\phi(\nu_\mu) + \phi(\nu_\tau)}{2}
\]

- In the $\mu$-$\tau$ symmetry limit with flux conservation, only $Q_{31}$ and $Q_{33}$ are non-vanishing. They are useful for classifying neutrino flavor transition models.
Summary (II)

- Kilometer size neutrino telescopes such as IceCube and KM3NeT are suitable for detecting neutrinos with energies up to few tens of PeV. They are capable of distinguishing track and shower signals. The parameters $Q_{31}$ and $Q_{33}$ can both be probed by simultaneously observing pion source and muon-damped source. However, an astrophysical neutrino source is generally a mixture of the two, and the degree of the mixture depends on the neutrino energy.
Summary (III)

- For $E_\nu > 10^{17}$ eV, one expects the dominance of GZK neutrino flux which is a pion source.
- The Askaryan Radio Array (ARA) experiment is optimized for observing GZK neutrino flux. The discrimination of $\nu_e$ from $\nu_\mu$ and $\nu_\tau$ can help to probe the parameter $Q_{31}$.
- A 20% accurate measurement on flavor ratio is generally sufficient to distinguish neutrino decay models ($H, M \rightarrow L$) from standard neutrino oscillations at the $3\sigma$ level.