The Potential of Minimal Flavour Violation

Belén Gavela
Universidad Autónoma de Madrid (UAM) and IFT

with R. Alonso, L. Merlo, S. Rigolin

PACIFIC, Moorea September 8-13 2011
Beyond Standard Model because

1) Experimental evidence for new particle physics:

*** Neutrino masses
*** Dark matter
** Matter-antimatter asymmetry

2) Uneasiness with SM fine-tunings
We understand ordinary particles = excitations over the vacuum

We DO NOT understand the vacuum = state of lowest energy:
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- **The gravity vacuum**: cosmological cte. \( \Lambda \), \( \Lambda \sim 10^{-123} \, M_{\text{Planck}}^4 \)

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- **The gravity vacuum**: cosmological cte. $\Lambda$, $\Lambda \sim 10^{-123} \, M_{\text{Planck}}$

- *The QCD vacuum*: Strong CP problem, $\theta_{\text{QCD}} < 10^{-10}$

- *The electroweak vacuum*: Higgs-field, v.e.v. $\sim O(100) \, \text{GeV}$
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* The **electroweak** vacuum: Higgs-field, v.e.v.~$\sim O(100)$ GeV

The (Tevatron->) LHC allow us to explore it
The happiness in the air of the LHC era

... as we are almost “touching” the Higgs
We understand ordinary particles = excitations over the vacuum.

We DO NOT understand the vacuum = state of lowest energy:

• The gravity vacuum: cosmological cte. $\Lambda$, $\Lambda \sim 10^{-123} \, M_{\text{Planck}}$

* The QCD vacuum: Strong CP problem, $\theta_{\text{QCD}} < 10^{-10}$

* The electroweak vacuum: Higgs-field, v.e.v. $\sim O(100) \, \text{GeV}$

The Higgs excitation has the quantum numbers of the EW vacuum.
BSM because

1) Experimental evidence for new particle physics:
   *** Neutrino masses
   *** Dark matter
   ** Matter-antimatter asymmetry

2) Uneasiness with SM fine-tunings, i.e. electroweak:
   *** Hierarchy problem
   *** Flavour puzzle
BSM electroweak

* **HIERARCHY PROBLEM**
  Fine-tuning issue: if BSM physics, why Higgs so light
  
  Interesting mechanisms to solve it from SUSY;
  strong-int. Higgs, extra-dim….
  
  In practice, none without further fine-tunings

* **FLAVOUR PUZZLE**
* All quark flavour data are \(\sim\)consistent with SM

Kaon sector, B-factories, accelerators....

There are some \(\sim\)2-3 sigma anomalies around, though:

- -- sin 2\(\beta\) in CKM fit (Lunghi, Soni, Buras, Guadagnoli, UTfit, CKMfitter)
- -- anomalous like-sign dimuon charge asymmetry in B\(\_\)s decays (D0)
- -- B \(\rightarrow\) \(\tau\)\(\nu\) (UTfit)
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- \( \B \rightarrow \tau \nu \) (UTfit)

LHC
* Neutrino masses indicate BSM…. yet consistent with 3 standard families

-- in spite of some 2-3 sigma anomalies:

* Minos, 2 sigma, neutrinos differ from antineutrinos

* Hints of steriles: LSND and MiniBoone in antineutrinos, new deficit in Chooz nu_efluxes, Gallex deficit in antinu_e, cosmological-radiation, solar...
Disregarding some 2-3 $\sigma$ anomalies...

* All quark flavour data are $\sim$consistent with SM

* Neutrino masses indicate BSM.... yet consistent with 3 standard families

yet....we do NOT understand flavour
The Flavour Puzzle

Why 2 replicas of the first family?

when we only need one to account for the visible universe
The Flavour Puzzle

Why so different masses and mixing angles?
The Flavour Puzzle

Why has nature chosen the number and properties of families so as to allow for CP violation... and explain nothing? (i.e. not enough for matter-antimatter asymmetry)
Neutrino light on flavour?
The Higgs mechanism can accommodate masses in SM... but neutrinos (?)
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Neutrinos lighter because Majorana?
Lepton mixing in charged currents

Quarks

\[ V_{\text{CKM}} = \begin{pmatrix}
    c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\
    -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23} - s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13}
\end{pmatrix} \]
Lepton mixing in charged currents

Leptons

$$V_{PMNS} = \begin{pmatrix}
    c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\
    -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23} - s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}s_{13}s_{12}e^{-i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13}
\end{pmatrix} \begin{pmatrix}
e^{i\alpha} \\
e^{i\beta} \\
1
\end{pmatrix}$$
More wood for the Flavour Puzzle

<table>
<thead>
<tr>
<th></th>
<th>Leptons</th>
<th>Quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{PMNS}}$</td>
<td>$\begin{pmatrix} 0.8 &amp; 0.5 &amp; ?(&lt;10^\circ) \ -0.4 &amp; 0.5 &amp; -0.7 \ -0.4 &amp; 0.5 &amp; +0.7 \end{pmatrix}$</td>
<td>$\begin{pmatrix} \sim 1 &amp; \lambda &amp; \lambda^3 \ \lambda &amp; \sim 1 &amp; \lambda^2 \ \lambda^3 &amp; \lambda^2 &amp; \sim 1 \end{pmatrix}$</td>
</tr>
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Why so different?
### More wood for the Flavour Puzzle

#### Leptons

$$V_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & ?(<10^\circ) \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & +0.7 \end{pmatrix}$$

#### Quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix} \quad \lambda \sim 0.2$$

Maybe because of Majorana neutrinos?
Dirac o Majorana?

• The only thing we have really understood in particle physics is the gauge principle

• SU(3) x SU(2) x U(1) allow Majorana masses....

Lepton number was only an accidental symmetry of the SM

Anyway, it is for experiment to decide
* **HIERARCHY PROBLEM**

Fine-tuning issue: if BSM physics, why Higgs so light

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* **FLAVOUR PUZZLE**: no progress
  Understanding stalled since 30 years.
The Higgs mechanism can accommodate masses in SM... but neutrinos (?)
DARK FLAVOURS ?

(large angle MSW)

\[ v_1 \rightarrow v_2 \rightarrow v_3 \]

\[ d \rightarrow s \rightarrow b \]

\[ u \rightarrow c \rightarrow t \]
DARK FLAVOURS?
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  \[ \Lambda_{\text{electroweak}} \sim 1 \text{ TeV} \]

* **FLAVOUR PUZZLE**: no progress

  Understanding stalled since 30 years.

  Only new B physics data AND neutrino masses and mixings

  \[ \Lambda_f \sim 100's \text{ TeV} \]

  BSMs tend to worsen the flavour puzzle
The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop
i.e susy MSSM:

\[ \begin{array}{c}
\bar{f} \\
g, \tilde{g}, \tilde{\chi}^0 \\
r_i
\end{array} \]

\[ < 1 \text{ loop in SM} \quad \text{---=} \quad \text{Best (precision) window of new physics} \]

ii) **FCNC**

i.e susy MSSM:

\[ K^0 - \bar{K}^0 \text{ mixing} \]

\[ \mu \rightarrow e\gamma \]

competing with SM at one-loop
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< 1 loop in SM --->

ii) **FCNC**

i.e susy MSSM:

\[ K^0 - \bar{K}^0 \text{ mixing} \]

\[ \mu \rightarrow e \gamma \text{ conversion (MEG, } \mu 2e...\text{PRISM)} \]

competing with SM at one-loop
The FLAVOUR WALL for BSM

* The **QCD** vacuum: Strong CP problem, $\theta_{QCD} < 10^{-10}$

**BSM in general induce** $\theta_{QCD} > 10^{-10}$

i.e. **at one-loop**

(vs multiloop in SM)

* The **matter-antimatter asymmetry**: CP-violation from quarks in SM fails by $\sim 10$ orders of magnitude (+ too heavy Higgs)
How to advance in a model-independent way?

• In quark flavour puzzle

• In lepton flavour puzzle
How to go about it model-independent?...

**Effective field theory**

Mimic travel from Fermi’s beta decay to SM

\[ \mathcal{L} = \mathcal{L}_{U(1)_{\text{em}}} + \mathcal{O}_{\text{Fermi}} + \ldots + \frac{M^2}{\mathcal{O}} \]
From the Fermi theory to SM

\[ G_F \left( \bar{e}_L \gamma^\mu \nu^e_L \right) \left( \bar{u}_\mu d_L \right) \]

\( \text{U}(1)_{\text{em}} \) invariant
From the Fermi theory to SM

\[ G_F \left( \bar{e} \gamma_\mu \nu^e_L \right) \left( \bar{u} \gamma_\mu d_L \right) \]

U(1)\text{_{em}} invariant

\[ \frac{g^2}{M_W^2} \left( L_\alpha \gamma_\mu L_\alpha \right) \left( Q_{L\beta} \gamma_\mu Q_{\beta} \right) \]

SU(2) xU(1)\text{_{em}} gauge invariant
If new physics scale $M > v$

$$\mathcal{L} = \mathcal{L}_{SU(3) \times SU(2) \times U(1)} + \mathcal{O}^{d=5} + \mathcal{O}^{d=6} + \ldots \quad \text{with} \quad M, \quad M^2$$
\[ \mathcal{L} = \mathcal{L}_{SU(3) \times SU(2) \times U(1)} + O^{d=5} + O^{d=6} + \ldots. \]

\[ O^{d=6} : \text{conserve } B, \ L \ldots \text{ and lead to new flavour effects for quarks and leptons} \]

SU(2) xU(1)_{em} gauge invariant
From the Fermi theory to SM

\[ G_F (\bar{e}_L \gamma^\mu \nu^e_L)(\bar{u}_R \gamma^\mu d_L) \]

\[ \text{U(1)}_{\text{em}} \text{ invariant} \]

\[ \frac{g^2}{M_W^2} (L_\alpha \gamma_\mu L_\alpha) (Q_{L\beta} \gamma_\mu Q_{\beta}) \]

\[ \text{SU(2) xU(1)}_{\text{em}} \text{ gauge invariant} \]

\[ \beta \text{-decay} \]

From Majorana masses to Seesaw

\[ \lambda / M (L L H H) \]

\[ \Delta N \sum \]

Seesaw models

From the SM to the theory of flavour

\[ Q_\alpha \]

\[ Q_\beta \]

\[ Q_\gamma \]

\[ Q_\delta \]

\[ \text{SU(2) xU(1)}_{\text{em}} \text{ gauge invariant} \]

\[ ? \]

The Theory of Flavour
A humble ansatz:

- Minimal Flavour Violation

(Chivukula, Georgi)
(D’Ambrosio, Giudice, Isidori, Strumia)(Buras)
A humble ansatz:

- **Minimal Flavour Violation**

  ....taking laboratory data at face value

(Chivukula, Georgi)
(D’Ambrosio, Giudice, Isidori, Strumia)(Buras)
* All quark flavour data are consistent with SM

= consistent with CKM

= consistent with all flavour effects due to Yukawas
$Y_D = \begin{pmatrix}
y_d & 0 & 0 \\
0 & y_s & 0 \\
0 & 0 & y_b
\end{pmatrix}$

$Y_U = \mathcal{V}_{CKM}^\dagger \begin{pmatrix}
y_u & 0 & 0 \\
0 & y_c & 0 \\
0 & 0 & y_t
\end{pmatrix}$
Minimal Flavour violation (MFV)

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawas

MFV Hypothesis \( \equiv \) The Yukawas are the only sources \( (irreducible) \) of flavour violation in the SM and BSM

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• Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

\[
\text{MFV Hypothesis} \equiv \text{The Yukawas are the only sources (irreducible) of flavour violation. in the SM and BSM.}
\]


The global Flavour symmetry of the SM with massless fermions:

\[
G_f = SU(3)_Q L \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e
\]

\[
Q_L \rightarrow \Omega_L Q_L \quad D_R \rightarrow \Omega_d D_R \quad \ldots
\]

\[
D_R = (d_R, s_R, b_R) \sim (1, 1, 3)
\]
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- \(Q_L \to \Omega_L Q_L\)
- \(D_R \to \Omega_d D_R\) ... 
- \(D_R = (d_R, s_R, b_R) \sim (1, 1, 3)\)
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The global Flavour symmetry of the SM: Yukawas break it

\[ G_f = SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e \]

\[ Q_L \rightarrow \Omega_L Q_L \quad D_R \rightarrow \Omega_d D_R \quad \ldots \]

\[ D_R = (d_R, s_R, b_R) \sim (1, 1, 3) \]
$G_f = \text{SU}(3)_{Q_L} \times \text{SU}(3)_{U_R} \times \text{SU}(3)_{D_R}$

$Y_D$ ~ $(\overline{3}, 1, 3)$

$Y_U$ ~ $(\overline{3}, 3, 1)$
Minimal Flavour violation (MFV)

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

MFV Hypothesis $\equiv$ The Yukawas are the only sources (irreducible) of flavour violation. in the SM and BSM


The global Flavour symmetry of the SM: Yukawas break it unless

$$G_f = SU(3)_L \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$$

$$Q_L \rightarrow \Omega_L Q_L \quad D_R \rightarrow \Omega_d D_R \quad \ldots \quad Y_D \rightarrow \Omega_L Y_u \Omega_d^+ \ldots$$

$$D_R = (d_R, s_R, b_R) \sim (1, 1, 3)$$

$$Q_L Y_D D_R H \quad Y_D \sim (3, 1, \bar{3})$$
Minimal Flavour violation (MFV)

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

MFV Hypothesis ≡ The Yukawas are the only sources (irreducible) of flavour violation in the SM and BSM


It is very predictive for quarks:

\[ L = L_{SM} + c^{d=6} O^{d=6} + \ldots \]

\[ C^{d=6} \sim \frac{Y_{\alpha\beta}}{2} Y_{\gamma\delta} \]

\[ O^{d=6} \sim Q_{\alpha} Q_{\beta} \bar{Q}_{\gamma} Q_{\delta} \]
From the Fermi theory to SM

\[ \beta\text{-decay} \]

\[ V_F \ (\bar{e}_L \gamma^\mu \nu^e_L)(\bar{u}_L \gamma^\mu d_L) \]

U(1)_{em} invariant

\[ \frac{g^2}{M_W^2} (L_\alpha \gamma^\mu L_\alpha) (Q_{L_\beta} \gamma^\mu Q_{\beta}) \]

SU(2) xU(1)_{em} gauge invariant

From the SM to the theory of flavour

The Theory of Flavour
MFV IS NOT A MODEL OF FLAVOUR

IT REMAINS AT THIS LEVEL

From the SM to the theory of flavour

SU(2) x U(1)_{em} gauge invariant

The Theory of Flavour
MFV IS NOT A MODEL OF FLAVOUR
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\[ C^{d=6} \sim \frac{Y^+_{\alpha\beta} Y_{\gamma\delta}}{\Lambda_{\text{flavour}}^2} \]

From the SM to the theory of flavour

SU(2) x U(1)\text{em} gauge invariant

The Theory of Flavour
A rationale for the MFV ansatz?

• Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

• Inspired in "condensate" flavour physics a la Froggat-Nielsen (Yukawas $\sim \langle \Psi \bar{\Psi} \rangle^n / \Lambda_f^n$, rather than in susy-like options

• It makes you think on the relation between scales: electroweak vs. flavour vs lepton number scales
* MFV can reconcile $\Lambda_f$ and $\Lambda_{\text{electroweak}}$:

$$\Lambda_f \sim \Lambda_{\text{electroweak}} \sim \text{TeV}$$

... and induce observable flavour changing effects
**WHY MFV?**

**FOR QUARKS**

- Hierarchy Problem points to $\Lambda \sim \text{TeV}$

| $\mathcal{O}_{d=6}^i$ | $\Lambda_f$ | $C_{d=6}$
|-----------------------|------------|----------|
| $(s_L \gamma^\mu d_L)^2$ | $9.8 \times 10^2$ | $1.6 \times 10^4$
| $(s_R d_L)(s_L d_R)$ | $1.8 \times 10^4$ | $3.2 \times 10^5$
| $(\bar{c}_L \gamma^\mu u_L)^2$ | $1.2 \times 10^3$ | $2.9 \times 10^3$
| $(\bar{c}_R u_L)(\bar{c}_L u_R)$ | $6.2 \times 10^3$ | $1.5 \times 10^4$
| $(\bar{b}_L \gamma^\mu d_L)^2$ | $5.1 \times 10^2$ | $9.3 \times 10^2$
| $(\bar{b}_R d_L)(\bar{b}_L d_R)$ | $1.9 \times 10^3$ | $3.6 \times 10^3$
| $(\bar{b}_L \gamma^\mu s_L)^2$ | $1.1 \times 10^2$ | $1.1 \times 10^2$
| $(\bar{b}_R s_L)(\bar{b}_L s_R)$ | $3.7 \times 10^2$ | $3.7 \times 10^2$

**WITHOUT MFV:** $\Lambda_f > \sim 10^2 \text{ TeV}$

**WITH MFV:** $\Lambda_f \sim \text{TeV}$

$c_{d=6} \equiv c_{d=6}(Y_u, Y_d)$

G. Isidori, Y. Nir, G. Perez, 1002.09
1. NA62 main targets are the rare $K$ decays ($Br \ll 10^{-11}$), e.g. $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$Smith$

$MFV$ region

$SM$
MFV suggests that $Y_U$ & $Y_D$ have a dynamical origin at high energies .......

$$Y \sim \langle \Phi \rangle \text{ or } \langle \Phi \chi \rangle \text{ or } \langle (\cdot)^n \rangle ...$$

Spontaneous breaking of flavour symmetry dangerous

--> i.e. gauge it  
(Grinstein, Redi, Villadoro, 2010)  
(Feldman, 2010)  
(Guardagnoli, Mohapatra, Sung, 2010)
MFV suggests that $Y_U$ & $Y_D$ have a dynamical origin at high energies .......

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(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
MFV suggests that $Y_U$ & $Y_D$ have a dynamical origin at high energies .......

$$Y \sim \langle \Phi \rangle \quad \text{or} \quad \langle \Phi \chi \rangle \quad \text{or} \quad \langle \quad \rangle^n \ldots$$

That scalar field or aggregate of fields may have a potential

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
The Dynamics Behind MFV

MFV suggests that $Y_U$ & $Y_D$ have a dynamical origin at high energies .......

$$Y \sim <\Phi> \text{ or } <\Phi \chi> \text{ or } <(\ldots)^n> ...$$

*What is the potential of Minimal Flavour Violation?

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
The Dynamics Behind MFV

MFV suggests that $Y_U$ & $Y_D$ have a dynamical origin at high energies ......

$$Y \sim \langle \Phi \rangle \text{ or } \langle \Phi \chi \rangle \text{ or } \langle (\ )^n \rangle \ldots$$

*What is the potential of Minimal Flavour Violation?*

*Can its minimum correspond \textbf{naturally} to the observed masses and mixings?*

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
We constructed the scalar potential for both 2 and 3 families, for scalar fields:

1) $Y \rightarrow$ one single scalar $\Sigma \sim (3, 1, 3)$

2) $Y \rightarrow$ two scalars $\chi \chi^+ \sim (3, 1, 3)$

3) $Y \rightarrow$ two fermions $\Psi \bar{\Psi} \sim (3, 1, 3)$
We constructed the scalar potential for both 2 and 3 families, for scalar fields:

1) $Y \rightarrow$ one single scalar $\Sigma \sim (3, 1, \overline{3})$
   
   $d=5$ operator

2) $Y \rightarrow$ two scalars $\chi \chi^+ \sim (3, 1, \overline{3})$
   
   $d=6$ operator

3) $Y \rightarrow$ two fermions $\overline{\Psi}\Psi \sim (3, 1, 3)$
   
   $d=7$ operator
1) **Y --\(\rightarrow\)** one single field \(\Sigma\)

\[
Y \sim \frac{< \Sigma >}{\Lambda_f}
\]

*What is the general potential \(V(\Sigma, H)\) invariant under \(SU(3) \times SU(2) \times U(1)\) and \(G_f\)?*
Construction of the Potential

* two families: 5 invariants at renormalizable level:
  (Feldman, Jung, Mannel)

$$\text{Tr} \left( \Sigma_u \Sigma_u^+ \right) \text{ det} \left( \Sigma_u \right)$$

$$\text{Tr} \left( \Sigma_d \Sigma_d^+ \right) \text{ det} \left( \Sigma_d \right)$$

$$\text{Tr} \left( \Sigma_u \Sigma_u^+ \Sigma_d \Sigma_d^+ \right)$$

* non-renormalizable terms are simply functions of those!
We constructed the most general potential:

\[ V (\Sigma_u, \Sigma_d) = \sum_i \left[ -\mu_i^2 \text{Tr}(\Sigma_i \Sigma_i^+) - \tilde{\mu}_i^2 \det(\Sigma_i) \right] \]

\[ + \sum_{i,j} \left[ \lambda_{ij} \text{Tr}(\Sigma_i \Sigma_i^+) \text{Tr}(\Sigma_j \Sigma_j^+) + \tilde{\lambda}_{ij} \det(\Sigma_i) \det(\Sigma_j) \right] + \ldots \]

It only relies on \( G_f \) symmetry

and analyzed its minima

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
The invariants can be written in terms of masses and mixing

* two families:

\[
<\Sigma_d> = \Lambda_f \cdot \text{diag}(y_d) ; \quad<\Sigma_u> = \Lambda_f \cdot V_{\text{Cabibbo}} \cdot \text{diag}(y_u)\
\]

\[
Y_D = \begin{pmatrix} y_d & 0 \\ 0 & y_s \end{pmatrix}, \quad Y_U = \mathcal{V}_C \begin{pmatrix} y_u & 0 \\ 0 & y_c \end{pmatrix} \quad V_{\text{Cabibbo}} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}
\]

\[
<\text{Tr} (\Sigma_u \Sigma_u^+) > = \Lambda_f^2 (y_u^2 + y_c^2) ; \quad <\text{det} (\Sigma_u) > = \Lambda_f^2 y_u y_c
\]

\[
<\text{Tr} (\Sigma_u \Sigma_u^+ \Sigma_d \Sigma_d^+) > = \Lambda_f^4 \left[ (y_c^2 - y_u^2) (y_s^2 - y_d^2) \cos 2\theta + ... \right]/2
\]

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
The minimum of the Potential is given by:
\[ \frac{\partial V}{\partial y_i} = 0 \quad \frac{\partial V}{\partial \theta_i} = 0 \]

Take the angle for example:
\[ \frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2) \,(y_s^2 - y_d^2) \sin 2\theta_c = 0 \]

Non-degenerate masses \[\Rightarrow \sin 2\theta_c = 0\] No mixing!

Notice also that \[\frac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J}\] (Jarlskog determinant)

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
Non-degenerate masses

\[ \frac{\partial V}{\partial y_i} = 0 \quad \frac{\partial V}{\partial \theta_i} = 0 \]

Take the angle for example:

\[ \frac{\partial V}{\partial \theta_c} \propto (y_c^2 - y_u^2) (y_s^2 - y_d^2) \sin 2\theta_c = 0 \]

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

\( \sin 2\theta_c = 0 \)  No mixing!

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)
Non-degenerate masses

\[ \sin 2\theta_c = 0 \quad \text{No mixing!} \]

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms...

* Without fine-tuning, for two families the spectrum is degenerate

* To accommodate realistic mixing one must introduce wild fine tunnings of \( O(10^{-10}) \) and nonrenormalizable terms of dimension 8

(Alonso, Gavela, Merlo, Rigolin)
Y --> one single field $\Sigma$

**three families**

* at renormalizable level: 7 invariants instead of the 5 for two families

\[
\begin{align*}
\text{Tr} \left( \Sigma_u \Sigma_u^\dagger \right) & \overset{\text{vev}}{=} \Lambda_f^2 \left( y_t^2 + y_c^2 + y_u^2 \right), \\
\text{Tr} \left( \Sigma_d \Sigma_d^\dagger \right) & \overset{\text{vev}}{=} \Lambda_f^2 \left( y_b^2 + y_s^2 + y_d^2 \right), \\
\text{Tr} \left( \Sigma_u \Sigma_u^\dagger \Sigma_u \Sigma_u^\dagger \right) & \overset{\text{vev}}{=} \Lambda_f^4 \left( y_t^4 + y_c^4 + y_u^4 \right), \\
\text{Tr} \left( \Sigma_d \Sigma_d^\dagger \Sigma_d \Sigma_d^\dagger \right) & \overset{\text{vev}}{=} \Lambda_f^4 \left( y_b^4 + y_s^4 + y_d^4 \right), \\
\text{Tr} \left( \Sigma_u \Sigma_u^\dagger \Sigma_d \Sigma_d^\dagger \right) & \overset{\text{vev}}{=} \Lambda_f^4 \left( P_0 + P_{\text{int}} \right),
\end{align*}
\]

**Interesting angular dependence:**
\[
P_0 \equiv - \sum_{i<j} \left( y_{u_i}^2 - y_{u_j}^2 \right) \left( y_{d_i}^2 - y_{d_j}^2 \right) \sin^2 \theta_{ij},
\]
\[
P_{\text{int}} \equiv \sum_{i<j,k} \left( y_{d_i}^2 - y_{d_k}^2 \right) \left( y_{u_j}^2 - y_{u_k}^2 \right) \sin^2 \theta_{ik} \sin^2 \theta_{jk} +
\]
\[
- \left( y_d^2 - y_s^2 \right) \left( y_c^2 - y_t^2 \right) \sin^2 \theta_{12} \sin^2 \theta_{13} \sin^2 \theta_{23} +
\]
\[
+ \frac{1}{2} \left( y_d^2 - y_s^2 \right) \left( y_c^2 - y_t^2 \right) \cos \delta \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13},
\]

Sad conclusions as for 2 families:

needs non-renormalizable + super fine-tuning
Spectrum for flavons $\Sigma$ in the bifundamental:

* 3 generations: for the largest fraction of the parameter space, the stable solution is a degenerate spectrum

\[
\begin{pmatrix}
  y_u & y_c & y_t \\
  y_u & y_c & y_t \\
\end{pmatrix}
\sim
\begin{pmatrix}
  y & y & y \\
  y & y & y \\
\end{pmatrix}
\]

instead of the observed hierarchical spectrum, i.e.

\[
\begin{pmatrix}
  y_u & y_c & y_t \\
  y_u & y_c & y_t \\
\end{pmatrix}
\sim
\begin{pmatrix}
  0 & 0 & y \\
  0 & 0 & y \\
\end{pmatrix}
\]

(at leading order)
Spectrum: the hierarchical solution is unstable in most of the parameter space.

**Stability:**

\[
\frac{\tilde{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}
\]

\[
V^{(4)} = \sum_{i=u,d} (-\mu^2_i A_i + \tilde{\mu}_i B_i + \lambda_i A^2_i + \lambda'_i A_{ii}) + g_{ud}A_u A_d + \lambda_{ud}A_{ud}.
\]

ie, the u-part:

\[
V^{(4)} = -\mu^2_u A_u + \tilde{\mu}_u B_u + \lambda_u A^2_u + \lambda'_u A_{uu}
\]
Spectrum: the hierarchical solution is unstable in most of the parameter space.

\[
V^{(4)} = \sum_{i=u,d} \left( -\mu_i^2 A_i + \bar{\mu}_i B_i + \lambda_i A_i^2 + \lambda_i' A_{ii} \right) + g_{ud} A_u A_d + \lambda_{ud} A_{ud}.
\]

\[\frac{\bar{\mu}^2}{\mu^2} < \frac{2\lambda'^2}{\lambda}\]

ie, the u-part:

\[V^{(4)} = -\mu_u^2 A_u + \bar{\mu}_u B_u + \lambda_u A_u^2 + \lambda_u' A_{uu}\]

Nardi emphasized this solution (and extended the analysis to include also U(1) factors)
The real, unavoidable, problem is again mixing:

* Just one source:

$$\text{Tr} \left( \sum_u \sum_u^+ \sum_d \sum_d^+ \right) = \Lambda^4_f (P_0 + P_{\text{int}})$$

$P_0$ and $P_{\text{int}}$ encode the angular dependence,

$$P_0 \equiv - \sum_{i<j} \left( y_{u_i}^2 - y_{u_j}^2 \right) \left( y_{d_i}^2 - y_{d_j}^2 \right) \sin^2 \theta_{ij},$$

$$P_{\text{int}} \equiv \sum_{i<j,k} \left( y_{d_i}^2 - y_{d_j}^2 \right) \left( y_{u_j}^2 - y_{u_k}^2 \right) \sin^2 \theta_{ik} \sin^2 \theta_{jk} +$$

$$- \left( y_d^2 - y_s^2 \right) \left( y_c^2 - y_t^2 \right) \sin^2 \theta_{12} \sin^2 \theta_{13} \sin^2 \theta_{23} +$$

$$+ \frac{1}{2} \left( y_d^2 - y_s^2 \right) \left( y_c^2 - y_t^2 \right) \cos \delta \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13}.$$
Summary

--> **Dynamical** MFV scalars in the bifundamental of $G_f$ do not provide realistic masses and mixings (at least in the minimal realization)
2) $Y \rightarrow$ quadratic in fields $\chi$

\[
Y \sim \frac{<\chi \chi^+>}{\Lambda_f^2}
\]

i.e. $Y_D \sim \frac{\chi^L_d (\chi^R_d)^+}{\Lambda_f^2} \sim (3, 1, 1) (1, 1, 3) \sim (3, 1, 3)$
2) \[ Y \rightarrow \text{quadratic in fields } \chi \]

\[ Y \sim \frac{\langle \chi \chi^+ \rangle}{\Lambda_f^2} \]

Automatic strong mass hierarchy and one mixing angle! already at the renormalizable level

Holds for 2 and 3 families!
Y $\rightarrow$ quadratic in fields $\chi$

It is very simple:

- a square matrix built out of 2 vectors

$$
\begin{pmatrix}
    d \\
    e \\
    f \\
    \vdots
\end{pmatrix}
\begin{pmatrix}
    a \\
    b \\
    c \\
    \ldots
\end{pmatrix}
$$

has only one non-vanishing eigenvalue

strong mass hierarchy at leading order:
- only 1 heavy “up” quark
- only 1 heavy “down” quark

only $|\chi|$’s relevant for scale
Y \rightarrow \text{quadratic in fields } \chi

The invariants are:

\[ \chi^L_u \chi^L_u, \quad \chi^R_u \chi^R_u, \quad \chi^L_d \chi^L_d, \quad \chi^R_d \chi^R_d, \quad \chi^L_u \chi^L_d = |\chi^L_u| |\chi^L_d| \cos \theta_c. \]

\[ \theta_c \] is the angle between up and down L vectors.
Y \rightarrow \text{quadratic in fields } \chi

Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:

\[ \chi_u^L \chi_u^L, \quad \chi_u^R \chi_u^R, \quad \chi_d^L \chi_d^L, \]
\[ \chi_d^R \chi_d^R, \quad \chi_u^L \chi_d^L = |\chi_u^L||\chi_d^L| \cos \theta_c. \]

We can fit the angle and the masses in the Potential; as an example:

\[ V' = \lambda_u \left( \chi_u^L \chi_u^L - \frac{\mu_u^2}{2\lambda_u} \right)^2 + \lambda_d \left( \chi_d^L \chi_d^L - \frac{\mu_d^2}{2\lambda_d} \right)^2 \]
\[ + \lambda_{ud} \left( \chi_u^L \chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}} \right)^2 + \cdots \]

Whose minimum sets (2 generations):

\[ y_c^2 = \frac{\mu_u^2}{2\lambda_u \Lambda_f^2}, \quad y_s^2 = \frac{\mu_d^2}{2\lambda_d \Lambda_f^2}, \quad \cos \theta = \frac{\mu_{ud} \sqrt{\lambda_u \lambda_d}}{\mu_u \mu_d \lambda_{ud}} \]
Towards a realistic 3 family spectrum

e.g. replicas of $\chi_L$, $\chi^R_u$, $\chi^R_d$

???
Towards a realistic 3 family spectrum

e.g. replicas of $\chi^L, \chi^R_u, \chi^R_d$

Suggests sequential breaking:

\[
\begin{align*}
SU(3)^3 & \quad \xrightarrow{mt, mb} \quad SU(2)^3 \quad \xrightarrow{mc, ms, \theta_C} \quad \cdots \\
Y_u & = \frac{\langle \chi^L \rangle \langle \chi^R_u \rangle}{\Lambda_f^2} + \frac{\langle \chi'^L_u \rangle \langle \chi'^R_u \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin \theta y_c & 0 \\ 0 & \cos \theta y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \\
Y_d & = \frac{\langle \chi^L \rangle \langle \chi^R_d \rangle}{\Lambda_f^2} + \frac{\langle \chi'^L_d \rangle \langle \chi'^R_d \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}.
\end{align*}
\]
Towards a realistic 3 family spectrum

\[ Y \rightarrow \text{quadratic in fields } \chi \]

e.g. replicas of \( \chi^L, \chi^R_u, \chi^R_d \)

Suggests sequential breaking:

\[
\begin{array}{ccc}
\text{SU(3)}^3 & \rightarrow & \text{SU(2)}^3 \\
\text{mt, mb} & \rightarrow & \text{mc, ms, } \theta_C
\end{array}
\]

\[
Y_d = \frac{\langle \chi^L \rangle \langle \chi^R_d \rangle}{\Lambda_f^2} + \frac{\langle \chi'^L_d \rangle \langle \chi'^R_d \rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}.
\]

Maybe some connection to: Berezhiani+Nesti; Ferretti et al., Calibbi et al. ??
Towards a realistic 3 family spectrum

Combining fundamentals and bi-fundamentals

i.e. combining d=5 and d =6 Yukawa operators

\[ \Sigma_u \sim (3, \overline{3}, 1), \quad \Sigma_d \sim (3, 1, \overline{3}), \quad \Sigma_R \sim (1, 3, \overline{3}), \]

\[ \chi_u^L \in (3, 1, 1), \quad \chi_u^R \in (1, 3, 1), \quad \chi_d^L \in (3, 1, 1), \quad \chi_d^R \in (1, 1, 3). \]

The Yukawa Lagrangian up to the second order in \(1/\Lambda_f\) is given by:

\[ \mathcal{L}_Y = \overline{Q}_L \left[ \frac{\Sigma_d}{\Lambda_f} + \frac{\chi_d^L \chi_d^R}{\Lambda_f^2} \right] D_R H + \overline{Q}_L \left[ \frac{\Sigma_u}{\Lambda_f} + \frac{\chi_u^L \chi_u^R}{\Lambda_f^2} \right] U_R \tilde{H} + \text{h.c.}, \]
* From bifundamentals: \[<\Sigma_u> = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & y_t
\end{pmatrix}\]

\[<\Sigma_d> = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & y_b
\end{pmatrix}\]

* From fundamentals \(\chi\): \(y_c\), \(y_s\) and \(\theta_C\)
At leading (renormalizable) order:

\[
Y_u \equiv \frac{\langle \Sigma_u \rangle + \langle \chi_u^L \rangle \langle \chi_u^R \rangle}{\Lambda_f} = \begin{pmatrix}
0 & \sin \theta_c y_c & 0 \\
0 & \cos \theta_c y_c & 0 \\
0 & 0 & y_t
\end{pmatrix},
\]

\[
Y_d \equiv \frac{\langle \Sigma_d \rangle + \langle \chi_d^L \rangle \langle \chi_d^R \rangle}{\Lambda_f} = \begin{pmatrix}
0 & 0 & 0 \\
0 & y_s & 0 \\
0 & 0 & y_b
\end{pmatrix}.
\]

without unnatural fine-tunings

The masses of the first family and the other angles from non-renormalizable terms or other corrections or replicas?
Are these constructions non-minimal MFV? NMFV

* When the Yukawa is a combination, the interpretation of the minima of the potential is not straightforward

* Fundamentals $\chi$ lead to different hierarchy of FCNC operators than bifundamentals $\Sigma$:

$$\overline{D}_R \Sigma^\dagger_d \Sigma_u \Sigma^\dagger_u Q_L \sim \text{[mass]}^6 \quad \leftrightarrow \quad \overline{D}_R \chi^R_d \chi^L_u Q_L \sim \text{[mass]}^5$$

- possible different phenomenology than for minimal MFV
What is the scalar potential of MFV including Majorana υs?

- Work ongoing right now

- It should allow to answer the question - within MFV - of whether leptonic mixing differs from quark mixing because of the different nature of mass
Conclusions

We constructed the general Scalar Potential for MFV and explored its minima

* The flavor symmetry imposes strong restrictions: just a few invariants allowed at the renormalizable and non-renormalizable level. Quark masses and mixings difficult to accommodate

* Flavons in the bifundamental alone (Y ~ <Σ>/Λf) do NOT lead naturally to realistic mixing

* Flavons in the fundamental are tantalizing (Y ~ <χ^2>/Λ^2), inducing naturally:
  - strong mass hierarchy
  - non-trivial mixing !!

-- We are exploring the leptonic MFV scalar potential
Back-up slides
In fact, MFV **assumes** more, e.g. top dominance:

\[
\left[ Y^u(Y^u)^\dagger \right]_{i\neq j}^n \approx y_t^{2n} V_{ti}^* V_{tj}
\]

\[ A(d^i \rightarrow d^j)_{\text{MFV}} = (V_{ti}^* V_{tj}) \ A_{\text{SM}}^{(\Delta F=1)} \left[ 1 + a_1 \frac{16\pi^2 M_W^2}{\Lambda^2} \right] \]

\[ \text{d-d} \sim \text{s-d} \sim \text{b-s} \text{ transitions of } \sim \text{equal strength} \]

while it may not be so...

**for instance for SM+ 2 Higgses (automatic Z}_3) \text{ light quarks may dominate}**

(Branco, Grimus, Lavoura)
Minimal Flavour violation (MFV)

• Unitarity of CKM first row:
  \[
  |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006
  \]

• *Restrict to flavour blind ops. -> 4 operators

• Correction is only multiplicative to $\beta$ and $\mu$ decay rate

The direct experimental limit puts strong constraints on all 4 operators, at the level of the colliders constraints or better.

\[
\Delta_{CKM} = -(0.1 \pm 0.6) \cdot 10^{-3}
\]

\[
\Lambda_{eff} > 11 \text{TeV (90\% CL)}
\]
\( \Sigma \) are bifundamentals of \( G_f \):

\[
\overline{Q}_L \frac{\Sigma_u}{\Lambda} U_R H \quad \Sigma_u \sim (3, \bar{3}, 1)
\]

\[
\overline{Q}_L \frac{\Sigma_d}{\Lambda} D_R H \quad \Sigma_d \sim (3, 1, \bar{3})
\]

\( Y \rightarrow \) one single field \( \Sigma \)

\[ V (\Sigma_u \Sigma_u H) \]
The fundamentals of $G_f$ are vectors, similar to quarks and leptons.

$$\mathcal{L}_Y = \overline{Q}_L \frac{\chi^L_d \chi^R_d}{\Lambda^2_f} D_R H + \overline{Q}_L \frac{\chi^L_u \chi^R_u}{\Lambda^2_f} U_R \hat{H} + \text{h.c.},$$

i.e. $Y_D \sim \frac{\chi^L_d (\chi^R_d)^+}{\Lambda^2_f} \sim (3, 1, 1) (1, 1, \overline{3}) \sim (3, 1, \overline{3})$

$\chi^L_u, \chi^L_d \sim (3, 1, 1); \quad \chi^R_u \sim (1, 3, 1); \quad \chi^R_d \sim (1, 1, 3)$
It holds also for 3 families: one heavy “up”, one heavy “down”, one angle

\[ Y_D = \frac{\langle \chi_d^L \chi_d^{R\dagger} \rangle}{\Lambda_f^2} \quad Y_U = \frac{\langle \chi_u^L \chi_u^{R\dagger} \rangle}{\Lambda_f^2} \]

The Yukawas are composed of two 'vectors'. Such a structure has only one eigenvalue, one mass. This fact becomes evident when rotating the v.e.v.s of the fields to the form:

\[
V_L^\dagger Y_D V_{DR} = \frac{|\chi_d^L| |\chi_d^R|}{\Lambda_f^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
V_L^\dagger Y_U V_{UR} = \frac{|\chi_u^L| |\chi_u^R|}{\Lambda_f^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\]

This means a hierarchy among the masses and an angle only by construction! already at renormalizable level
Can its minimum correspond naturally to the observed masses and mixings?

i.e. with all dimensionless $\lambda$’s $\sim 1$

and dimensionful $\mu$’s $\lesssim \Lambda_f$
ν masses beyond the SM

The Weinberg operator

Dimension 5 operator:

$\frac{\lambda}{M} (L L H H) \rightarrow \frac{\lambda \nu^2}{M} (\nu \nu)$

It’s unique → very special role of ν masses:
lowest-order effect of higher energy physics
ν masses beyond the SM

The Weinberg operator

Dimension 5 operator:

\[ \frac{\lambda}{M} (L L H H) \rightarrow \frac{\lambda}{M} \nu^2 (\nu \nu) \]

It’s unique → very special role of ν masses: lowest-order effect of higher energy physics

This mass term violates lepton number (B-L) → Majorana neutrinos
ν masses beyond the SM

The Weinberg operator

Dimension 5 operator:

\[ \frac{\lambda}{M} (L \ L \ H \ H) \rightarrow \frac{\lambda}{M} \nu^2 \nu (\nu \nu) \]

It’s unique → very special role of ν masses:
lowest-order effect of higher energy physics

This mass term violates lepton number (B-L) → Majorana neutrinos

\[ O^{d=5} \] is common to all models of Majorana ņs
From the Fermi theory to SM

\[ G_F \left( \bar{e}_L \gamma_\mu \nu_{L}^e \right) \left( \bar{u}_\mu d_L \right) \]

\( \beta \)-decay

U(1)\(_{\text{em}} \) invariant

\[ \frac{g^2}{M_W^2} \left( L_\alpha \gamma_\mu L_\alpha \right) \left( Q_{L\beta} \gamma_\mu Q_{L\beta} \right) \]

SU(2) xU(1)\(_{\text{em}} \) gauge invariant

From Majorana masses to Seesaw

\[ \lambda / M \left( L L H H \right) \]

\( \nu \)-mass

SU(2) xU(1)\(_{\text{em}} \) inv.
From the Fermi theory to SM

\[ G_F (\bar{e}_L \gamma^\mu \nu^e_L)(\bar{u}_\mu d_L) \]

\( \text{U}(1)_{\text{em}} \) invariant

\[ \frac{g^2}{M_W^2} (L_\alpha \gamma_\mu L_\alpha) (Q_{L\beta} \gamma_\mu Q_{\beta}) \]

\( \text{SU}(2) \times \text{U}(1)_{\text{em}} \) gauge invariant

From Majorana masses to Seesaw

\[ \lambda / M (L L H H) \]

\( \frac{\lambda_{\alpha\beta}}{M} \)

\( \text{SU}(2) \times \text{U}(1)_{\text{em}} \) inv.

2 x 2 = 1 + 3

\( \beta \)-decay

\( \nu \)-mass
From the Fermi theory to SM

\[ G_F \left( \bar{e}_L \gamma_\mu \nu^e_L \right) \left( \bar{u}_\mu d_L \right) \]

\[ U(1)_{em} \text{ invariant} \]

\[ \frac{g^2}{M_W^2} \left( L_\alpha \gamma_\mu L_\alpha \right) \left( Q_{L\beta} \gamma_\mu Q_{\beta} \right) \]

\[ SU(2) \times U(1)_{em} \text{ gauge invariant} \]

From Majorana masses to Seesaw

\[ \lambda / M \left( L L H H \right) \]

\[ SU(2) \times U(1)_{em} \text{ inv.} \]

\[ \nu \text{-mass} \]

\[ \beta \text{-decay} \]

Seesaw models