Transition Radiation by Active Neutrinos

J. C. D'Olivo & J. A. Loza

Instituto de Ciencias Nucleares
Universidad Nacional Autónoma de México

Neutrino EM Properties

- A great effort is currently being done in order to determine the precise values of the masses and mixing angles of the neutrinos.
- Neutrino oscillations give information on the differences of neutrino squared masses and other techniques can provide valuable complementary information.
- Beside their own relevance, neutrino electromagnetic (EM) properties, may have important consequences in a variety of physical, astrophysical, and cosmological contexts. (C. Giunti & A. Studenikin, 2009)

"... the most likely model for the "neutron" seems to be ... that at rest it is a magnetic dipole with a certain moment μ." (W. Pauli, 1930)

- Up to now there is no evidence confirming a nonzero value for any intrinsic EM properties of neutrinos.
- Because of the chiral symmetry obeyed by massless neutrinos, in the Standard Model $\mu_\nu = 0$.
- In the minimally extended SM, with singlet right-handed neutrinos and $m_\nu \neq 0$, μ_ν is non-vanishing, but extremely small:

$$\mu_\nu \approx 3 \times 10^{-19} \left(\frac{m_\nu}{1\text{eV}} \right) \mu_B,$$

$\mu_B = e/2m_e$ Bohr magneton

- Experimental studies are stimulated by the hope to observe any deviation from the predictions of the SM, which would have a profound implication for the search of new physics.

Experimental limits on μ_ν

- Laboratory limits, low energy $\nu_e - e$ scattering
  Reactor neutrinos

$$\mu_\nu < 3.2 \times 10^{-11} \mu_B \quad (GEMMA, 2009)$$

  Solar neutrinos

$$\mu_\nu < 5.4 \times 10^{-11} \mu_B \quad (Borexino, 2008)$$

- Astrophysical constraints
  Nonstandard energy-loss in globular clusters (G. Raffelt, 1990)

$$\mu_\nu \lesssim 3 \times 10^{-12} \mu_B$$

  Energy in SN collapse not entirely carried away by ν_R
  (Barbieri & Mohapatra, 1988, Ayala, Torres & JCD, 1999)

Neutrino Properties in Matter

The properties of neutrinos that propagate through a medium can be substantially different compared to their properties in the vacuum.

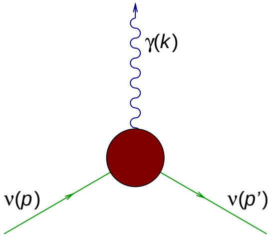
- The energy-momentum relation of active neutrinos are affected by its coherent interactions with the particles in a background. From the neutrino self-energy

$$V_\ell = \left(1 - Re\, n_\ell\right), \qquad \ell = e, \mu, \tau$$

V_ℓ: Potential energy n_ℓ: Refraction index

- In normal matter (e, p, n) $n_e \neq n_{\mu,\tau} \Rightarrow$ non trivial phase Pattern of neutrino oscillations significantly modified. Flavor transformation can be amplified in a resonant-like fashion (MSW effect).

- Trough their weak interactions with the charged leptons and nucleons, neutrinos acquire an effective coupling to the electromagnetic field.



- In the presence of an external magnetic field **B** there are anisotropic contributions to the neutrino refraction index

$$V_\ell = b_\ell + c_\ell e\,\hat{\boldsymbol{k}} \cdot \boldsymbol{B},$$

e: electron charge. (Nieves, Pall, & JCD, 1996)

- Flavor transformations affected in a way that preserves quirality, contrary to what happens for neutrino oscillations caused by transition magnetic moments.
- The resonant flavor transformations ($\nu_\ell \to \nu_s$) of neutrinos diffusing in different directions respect to the magnetic field occur at different depths within a neutron protostar. This can induce a momentum flux asymmetry and is the basis of a neutrino-driven pulsar kick. (KS mechanism).

Neutrino EM Process

Several process can happens in a medium even if neutrinos do not have any non vanishing intrinsic EM-property.

- Plasmon Decay (Adams, Ruderman, & Woo, 1963)

$$\gamma \to \nu + \bar{\nu}$$

Effective mechanism of neutrino energy losses in red giants, presupernovae and in the cores of white dwarfs.

- Neutrino decay

$$\nu \to \nu' + \gamma$$

Not suppressed by GIM mechanism.

- Cherenkov Radiation

$$\nu \to \nu + \gamma$$

Kinematically allowed whenever $n > 1$ and $\upsilon > 1/n$, where n is the refraction index of the medium and υ the neutrino velocity.

Transition Radiation

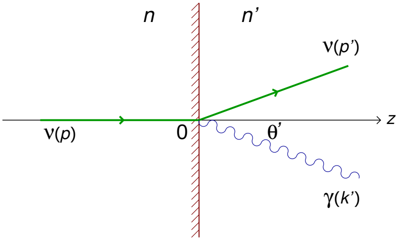
Radiation emitted when a charged particle goes across the boundary between two media with different indices of refraction. More generally, when it moves trough a nonuniform medium. (Ginzburg & Frank, 1945)

- The phenomenon takes place even if the conditions for Cherenkov radiation are not satisfied.
- Also produced by neutral particle having a non vanishing dipole moment.
- TR by an intrinsic (magnetic, electric, or toroidal) dipole moment of a neutrino has been considered by several authors.
- Possible new technique to measure μ_ν (M. Sakuda, 1994).

TR by the Neutrino EM Vertex

When Neutrinos cross the interface between two media will emit TR because of their effective EM interaction in matter (Loza & JCD, 2010).

Neutrino beam crossing the boundary between a medium and the vacuum ($n' = 1$)



Neutrinos move along the z axis, perpendicularly to a plane interface.

Transition Probability

$$d\mathcal{W} = \frac{V d^3\wp'}{(2\pi)^3} \frac{V d^3\kappa'}{(2\pi)^3} |S_{fi}|^2,$$

where ($S_{fi} = 0$ for $z > 0$)

$$|S_{fi}|^2 = \frac{\pi^3}{\upsilon V^2} \frac{|\mathcal{M}|^2}{\mathcal{E}\mathcal{E}'\omega}\, \delta(\wp_x - \wp'_x - \kappa_x)\, \delta(\wp_y - \wp'_y - \kappa_y) \times \delta(\mathcal{E} - \mathcal{E}' - \omega) \left| \int_{-\ell/2}^{0} \mathrm{d}z \exp[i(\wp - \wp'_z - \kappa_z) z] \right|^2 .$$

$V = \ell^3$ volume of the transition region
$\upsilon = \ell/\tau$ neutrino velocity (τ: time interval of the process)

4-momentum of the initial and final neutrino:

$$p^{\mu} = (\mathcal{E}, \boldsymbol{\wp}), \quad \boldsymbol{\wp} = \wp\,\hat{z}, \ \ \wp = |\boldsymbol{\wp}|$$

$$p'^{\mu} = (\mathcal{E}', \boldsymbol{\wp}')$$

4-momentum of the emitted photon:

$$k^{\mu} = (\omega, \boldsymbol{\kappa}) \qquad \text{medium}$$

$$k'^{\mu} = (\omega', \boldsymbol{\kappa}') \qquad \text{vacuum}$$

$$\omega' = \omega, \qquad \boldsymbol{\kappa}' \neq \boldsymbol{\kappa}$$

$\nu\nu\gamma$ amplitude

$$\mathcal{M} = -i\sqrt{\mathcal{Z}}\,\bar{\mathcal{U}}(p')\Gamma^{\mu}\mathcal{U}(p)\epsilon_{\mu}(k,\lambda)\,,$$

- $\sqrt{\mathcal{Z}} \cong 1$ renormalization factor
- $\mathcal{U}(p)$ Dirac spinor with momentum p $\qquad (p'^2 = p^2 = m_\nu^2)$
- $\epsilon_\mu(k,\lambda) = (0, \boldsymbol{\epsilon}(k,\lambda)) \quad (\lambda = 1,2) \quad$ Polarization vectors

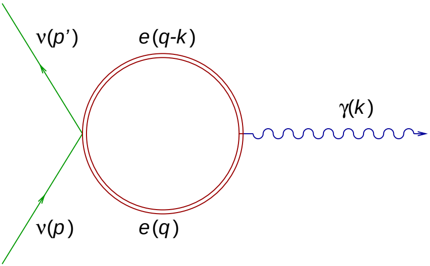
  $\epsilon(k,\lambda) \cdot \epsilon(k,\lambda') = \delta_{\lambda\lambda'}$

  $\epsilon(k,\lambda) \cdot k = \epsilon(k,\lambda) \cdot u = 0$

  $u^\mu = (1, \mathbf{0})$: 4-velocity of the medium
- Γ^μ Neutrino EM vertex in matter

The background contribution of electrons and nucleons to Γ^μ has been determined by a one-loop calculation using the methods of the Thermal Field Theory (Oraevskiĭ, Plakhov, Semikoz, & Smorodinskiĭ, 1988; Nieves, Pal & JCD, 1989).

To leading order in the Fermi constant G_F, for $T \ll M_W$



$$\Gamma^\mu = eG_F\sqrt{2}\gamma^\nu \int \frac{d^4p}{(2\pi)^4} \, \mathrm{Tr}\left[\gamma^\nu S_F(p)\gamma^\rho(a + b\gamma_5)S_F(p-k)\right]$$

Electron propagator:

$$S_F = (\not{q} + m)\left[\frac{1}{q^2 - m_e^2} + 2\pi i \delta(p^2 - m_e^2)\eta(E)\right],$$

$$\eta(E) = \frac{\Theta(E)}{e^{(E-\mu)/T} + 1} + \frac{\Theta(-E)}{e^{(-E+\mu)/T} + 1}$$

$\Theta(E)$: step function $\qquad E = p \cdot u$
μ: chemical potential

$$\Gamma_\mu = -\sqrt{2}\,\frac{G_F}{e}(a\,\Pi_{\mu\nu} + b\,\Pi^A_{\mu\nu})\gamma^\nu L\,,$$

$a = 2\sin^2\theta_W \pm \frac{1}{2}\,, \quad b = \mp\frac{1}{2}$ (upper sign ν_e and lower sign $\nu_{\mu,\tau}$)

$L = \frac{1}{2}(1 - \gamma_5)$

$$\begin{aligned} \Pi_{\mu\nu}(k) &= \pi_T R_{\mu\nu} + \pi_L Q_{\mu\nu} && \text{EM polarization tensor} \\ \Pi^A_{\mu\nu}(k) &= \pi_A P_{\mu\nu} && \text{axial polarization tensor} \end{aligned}$$

$$\pi_{T,L,A} = \pi_{T,L,A}(\omega, \kappa) \qquad \left(\kappa = |\boldsymbol{\kappa}| = \sqrt{k^2 - \omega^2}\,, \quad \omega = k \cdot u\right)$$

$$\begin{aligned} R_{\mu\nu} &= -\sum_{\lambda=1,2} \epsilon_\mu^{(\lambda)} \epsilon_\nu^{(\lambda)} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} - Q_{\mu\nu} \\ Q_{\mu\nu} &= \epsilon_\mu^{(3)} \epsilon_\nu^{(3)} = -\frac{k^2}{\kappa^2} \left(u_\mu - \frac{\omega}{k^2} k_\mu\right) \left(u_\nu - \frac{\omega}{k^2} k_\nu\right) \\ P_{\mu\nu} &= i\, \epsilon_{\mu\nu\alpha\beta} k^\alpha u^\beta / \kappa \end{aligned}$$

$$R_{\mu\nu} Q^{\mu\nu} = R_{\mu\nu} P^{\mu\nu} = Q_{\mu\nu} P^{\mu\nu} = 0$$

$$R_{\mu\nu} R^{\mu\nu} = 2, \quad Q_{\mu\nu} Q^{\mu\nu} = 1, \quad P_{\mu\nu} P^{\mu\nu} = -2$$

$$k^\mu \Pi_{\mu\nu} = k^\mu \Pi^A_{\mu\nu} = 0 \quad \Rightarrow \quad k^\mu \Gamma_\mu = 0 \qquad \text{(gauge invariance)}$$

Performing the integrations on $d^3\wp'$

Energy radiated into the vacuum

$$\frac{d^2\mathcal{S}}{d\omega d\theta'} = \omega \frac{d^2\mathcal{W}}{d\omega d\theta'} = \frac{1}{32\pi^2 \upsilon} \frac{\omega^2 |\overline{\mathcal{M}}|^2 \sin\theta'}{\mathcal{E}\wp'_z(\kappa_z + \wp'_z - \wp)^2},$$

$$\wp'_z = \sqrt{\wp^2 - 2\mathcal{E}\omega + \omega^2 \cos^2\theta'}, \qquad \kappa_z = \kappa\cos\theta,$$

θ, θ': angles of the photon with the z axis, within and outside the medium. Snell's law ($\kappa_{x,y} = \kappa'_{x,y}$):

$$\sin\theta' = n\sin\theta \quad (n' = 1),$$

with $0 \le \theta \le \pi/2$.

Photon dispersion relation in the medium $\omega(\kappa)$

$$\omega^2 - \kappa^2 = \pi_T(\omega, \kappa)$$

Braaten & Segel (1993):

$$\pi_T(\omega,\kappa) = \frac{4\alpha}{\pi}\,\frac{\omega^2-\kappa^2}{\kappa^2}\int_0^\infty dp\,\eta(E)\frac{p^2}{E}\left(\frac{\omega^2}{\omega^2-\kappa^2} - \frac{\omega E}{2\kappa p}\ln\frac{\omega E+\kappa p}{\omega E-\kappa p}\right)$$

$$= \omega_p^2\left[1+\frac{1}{2}G(v_e^2\kappa^2/\omega^2)\right], \qquad \alpha = e^2/4\pi$$

$0 \le v_e \le 1$: “typical” velocity of the electrons in the plasma

$$\omega_p^2 = \frac{4\alpha}{\pi}\int_0^\infty dp\frac{p^2}{E}\left(1-\frac{p^2}{3E^2}\right)\eta(E) \qquad \text{plasma frequency}$$

$$G(x) = \frac{3}{x}\left[1 - \frac{2x}{3} - \frac{(1-x)}{2\sqrt{x}}\log\frac{1+\sqrt{x}}{1-\sqrt{x}}\right], \quad 0 \le x \le 1.$$

$$0 \le G(x) \le 1, \quad G'(x) > 0$$

Good approximation:

- Solve for $G(x) = \frac{2}{5}x + ...$

$$\frac{\kappa^2}{\omega^2} = \frac{\omega^2 - \omega_p^2}{\omega^2 + \frac{\upsilon_e^2\,\omega_p^2}{5}}$$

- Make κ^2/ω^2 equals to this value in the argument of G

$$\kappa^2 \cong \omega^2 - \frac{\omega_p^2}{2} G\left(\upsilon_e^2 \frac{\omega^2 - \omega_p^2}{\omega^2 + \frac{\upsilon_e^2\omega_p^2}{5}}\right)$$

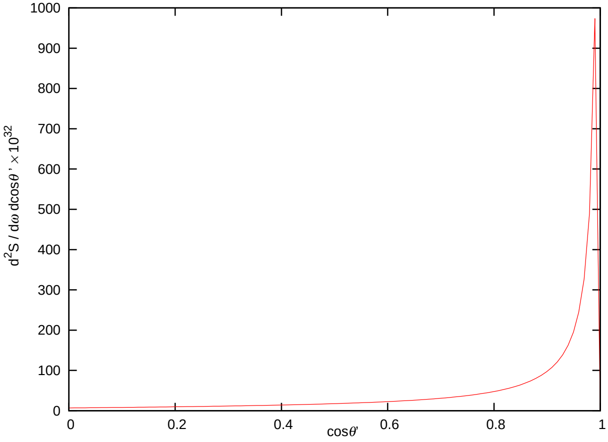
Limiting cases

- **Classical** $(m_e - \mu \gg T)$
  Non relativistic and non degenerated gas
  $v_e = \sqrt{5T/m_e} \qquad \omega_p^2 = \frac{4\pi\alpha n_e}{m_e}\left(1 - \frac{5T}{2m_e}\right)$
- **Relativistic** $(T \gg m_e)$
  $v_e = 1 \qquad \omega_p^2 = \frac{4\alpha}{3\pi}\left(\mu^2 + \frac{\pi^2 T^2}{3}\right)$
- **Degenerated** $(T \to 0)$
  $v_e = v_F = p_F/E_F \qquad \omega_p^2 = \frac{4\pi\alpha n_e}{\mathcal{E}_F} = \frac{4\alpha}{3\pi} p_F^2 v_F$

$$\pi_A(\omega,\kappa) \ll \pi_T(\omega,\kappa)$$

Discarding π_A and taking into account that $Q^{\mu\rho}\epsilon_\mu(k,\lambda) = 0$ for $\lambda = 1, 2$

$$|\overline{\mathcal{M}}|^2 \cong \frac{G_F^2}{\pi\alpha} a^2 \pi_T^2 \left(\mathcal{E}\mathcal{E}' - \wp\,\wp_z' \cos^2\theta + \kappa\wp\cos\theta\sin^2\theta\right)$$



Angular distribution of TR energy as a function of $\cos\theta'$

$\omega = 0.4$ MeV and $\omega_p = 5$ keV

The upper value for the angle in vacuum depends on ω and it is simpler to integrate over the angle in the medium ($0 \leqslant \theta \leqslant \pi/2$).

Using $d\theta'/d\theta = n\cos\theta/\sqrt{1 - n^2\sin\theta}$, for a relativistic neutrino ($\mathcal{E} \cong \wp$) we find:

Energy spectrum

$$\frac{d\mathcal{S}}{d\omega} = \frac{G_F^2\, a^2}{32\pi^3 \alpha\, \upsilon}\, \kappa^2 |\pi_T|^2 \mathcal{F}(\omega)$$

$$\mathcal{F}(\omega) = \int_0^1 \frac{\mathrm{d}\zeta\, \zeta}{\sqrt{1 - n^2(1-\zeta^2)}} \times \frac{\wp - \omega - \wp_z'\zeta^2 + \kappa\zeta(1-\zeta^2)}{\wp_z'(\wp_z' + \kappa\zeta - \wp)^2},$$

with $\zeta = \cos\theta$ and $\kappa' = \omega$ in vacuum.

Analytic approximation for $\mathcal{F}(\omega)$: $\sqrt{1 - n^2(1-\zeta^2)} \to 1$

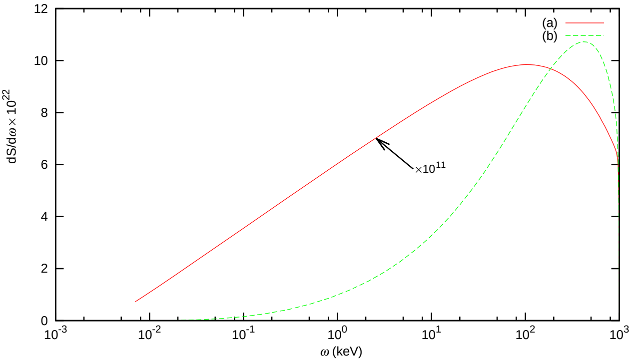
$$\frac{d\mathcal{S}}{d\omega} = \frac{G_F^2\, a^2}{64\pi^3\alpha\, \upsilon}\,(\omega^2-\kappa^2)^2 \left[\left(1-\frac{\omega}{\wp}\right)\mathcal{I}(s) - \frac{\wp^2}{2\kappa^2}\mathcal{L}(s) + \frac{\wp^2}{4\kappa^2}\left(\frac{2\kappa^2}{\wp^2}-1+\varrho\right)\mathcal{J}(s)\right]_{s_0}^{s_1}$$

where $\varrho = [(\wp-\omega)^2-\kappa^2]/\wp^2$, $s_1 = (\wp-\omega+\kappa)/\wp$, and $s_0 = |\varrho|^{1/2}$.

$$\begin{aligned}
\mathcal{I}(s) &= \frac{1-\varrho}{1-s} + \frac{\varrho}{s} + 2\varrho\ln(1-s) - 2\varrho\ln s\,, \\
\mathcal{J}(s) &= \frac{(1-\varrho)^2}{1-s} - \frac{\varrho^2}{2s^2}(4s+1) \\
&\quad + (3\varrho+1)(1-\varrho)\ln(1-s) + (3\varrho-2)\varrho\ln s\,, \\
\mathcal{L}(s) &= \frac{s}{4}(s+4) + \frac{\varrho^2}{4s^2}(4s+1) \\
&\quad + (1-\varrho)^2\ln(1-s) + (1-\varrho)\varrho\ln s\,.
\end{aligned}$$

Energy Spectrum

TR emitted by a 1 MeV incident ν_e vs. the photon energy



(a) Classical electron gas with $\omega_p = 20$ eV (polypropylene) at room temperature ($\upsilon_e = \sqrt{5T/m_e} \cong 0$).
(b) Degenerate gas with $\omega_p = 5$ KeV and $\upsilon_e = \upsilon_F = 0.3$.

- For a degenerated plasma the TR energy is much higher and the spectrum is more sharp than in the classical limit.
- Total radiated energy (classical gas):

$$\mathcal{S} \cong 0.8 \times 10^{-34}\text{eV} \quad \approx 100 \times \mathcal{S}_T$$

$\mathcal{S}_T$: TR energy for an intrinsic toroidal dipole moment.

- (Sakuda & Kurihara, 1995)

$$\mathcal{S}_M = 4.5 \times 10^{-13} \left(\frac{\mu_\nu}{\mu_B}\right)^2 \mathcal{E}, \quad \omega_p \gg m_\nu$$

Taking $\mathcal{S}_M = 0.8 \times 10^{-34}$ eV and $\mathcal{E} = 1$ MeV

$$\begin{aligned} \mu_\nu &\simeq 1.4 \times 10^{-14} \mu_B \\ &\simeq 2 \times 10^6 (\mu_\nu)_{SM} \qquad (m_\nu = 0.1\text{eV}) \end{aligned}$$

Conclusions

- The new phenomena under consideration is a prediction based solely on the physics of the SM. It would happen even if the electron neutrino were massless.
- The $\mathcal{S}_M > \mathcal{S}$ for μ_ν closer to the present experimental limits. But such limits might be quite poor, as astrophysical arguments seem to indicate.
- $\mu_\nu \sim 10^{-14}\mu_B$ corresponding to $\mathcal{S}_M = \mathcal{S}$ is of the same order as the model-independent upper bound on μ_ν^D generated by physics above the scale of electroweak symmetry breaking (N. F. Bell, et al., 2005).
- The emitted radiation increases enormously if, instead of a classical gas, we consider a degenerated electron plasma similar to those existing in stellar objects.
- Further studies to fully understand the implications in such astrophysical environments and other physical situations.