Vacuum Energy decay

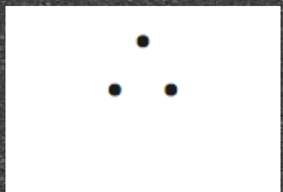
Is the vacuum energy stable?

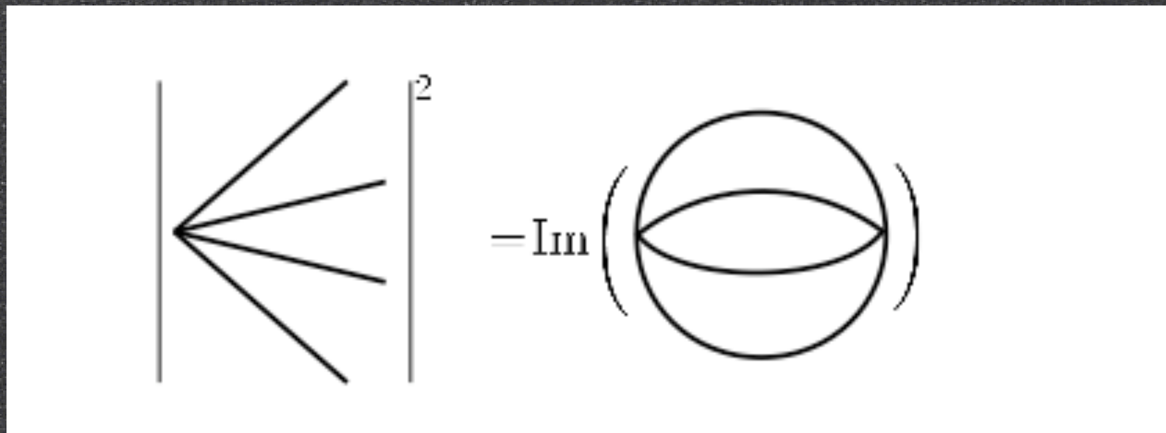
Abbott and Deser: Classical stability

(Fluctuations within the horizon)

Naively, quantum interactions destabilize the vacuum (spin not importat at this level)

Momentum is not conserved at vertices





No universal definition of vacuum state; no energetic argument

Width = V.T only true in some frames (Fermi)

S-matrix not well defined; nor are particles to be decayed into

Back to basics:

Naive analytic continuation

From de Sitter...

$$ds^2 = \frac{l^2 dz^2 - \sum_i (dx^i)^2}{z^2}$$

to euclidean (anti) de Sitter

$$ds^2 = \frac{l^2 dz^2 + \sum_i (dx^i)^2}{z^2}$$

No imaginary part in a one-loop BF computation (EA & RV)

Survival amplitude =Self-overlap

$$\mathcal{A}(t_f, t_i) \equiv \langle \psi(t_f) | \psi(t_i) \rangle$$

Cauchy-Schwarz:

$$|\mathcal{A}(t_f, t_i)| \leq 1$$

$$\Gamma(T) \equiv -\frac{2}{T}\,\mathrm{Log}\,\left|\mathcal{A}\left(t_f, t_i\right)\right|$$

Were it independent of T=tf-ti, it would have been a true width; otherwise it is an useful observable

Introducing sources

$$M[J] \equiv \langle \text{out}|\text{in}\rangle|_J = \int d\mu[\phi_f(\vec{x})]\, d\mu[\phi_i(\vec{x})] \;\; \Psi_{t_f}[\phi_f]^* \langle \phi_f(\vec{x}) t_f | \phi_i(\vec{x}) t_i \rangle \big|_J \;\; \Psi_{t_i}[\phi_i]$$

$$\Psi_{t_f}[\phi_f] \equiv \langle \phi_f(\vec{x})\; t_f|\text{out}\rangle$$
$$\Psi_{t_i}[\phi_i] \equiv \langle \phi_i(\vec{x})\; t_i|\text{in}\rangle$$

Schrodinger functional (Feynman kernel)

$$K[J][\phi_f\; t_f, \phi_i\; t_i] \equiv \langle \phi_f(\vec{x}) t_f | \phi_i(\vec{x}) t_i \rangle|_J$$

Vacuum wavefunctional

$$\Psi_0[\phi] = N\, e^{-\frac{1}{2}\int \frac{d^{n-1}k}{(2\pi)^{n-1}}\omega_k\, \phi_{-k}\phi_k}$$

Scrodinger's functional

$$K[J][\phi' t', \phi t] \equiv \int_{\phi(.,t)=\phi}^{\phi(.,t')=\phi'} \mathcal{D}\phi \, e^{i \int_{\Sigma_t}^{\Sigma_{t'}} d^n x \, L[\phi]}$$

$$\phi_k(t) = \phi_k^c(t) + \chi_k(t)$$

$$S_0 = \int \frac{d^{n-1}k}{(2\pi)^{n-1}} \int_{t_i}^{t_f} dt \left(\left(\frac{1}{2}(\dot{\phi}_k^c)^2 - \frac{\omega_k^2}{2}(\phi_k^c)^2 - j_k(t)\phi_k^c(t) \right) + \left(\frac{1}{2}\dot{\chi_k}^2 - \frac{\omega_k^2}{2}\chi_k^2 \right) + \right.$$

$$\left. \left(\dot{\phi}_k^c \dot{\chi}_k - \omega_k^2 \phi_k^c \chi_k - j_k(t)\chi_k(t) \right) \right) \qquad (2$$

$$\int_{t_i}^{t_f} dt \left(\frac{d}{dt} \left(\dot{\phi}_k^c \chi_k \right) - \chi_k \ddot{\phi}_k^c - \omega_k^2 \phi_k^c \chi_k - j_k(t)\chi_k(t) \right) =$$

$$\dot{\phi}_k^c(t_f)\chi_k(t_f) - \dot{\phi}_k^c(t_i)\chi_k(t_i) - \int_{t_i}^{t_f} dt\ \chi_k \left(\ddot{\phi}_k^c + \omega_k^2 \phi_k^c + j_k(t) \right)$$

$$\ddot{\phi}_k^c + \omega_k^2 \phi_k^c + j_k(t) = 0$$

There is also a contribution from the wavefunctional

$$-\int \frac{d^{n-1}k}{(2\pi)^{n-1}}\frac{1}{2}\omega_k\left((\phi_k^f)^2+(\phi_k^i)^2\right) = -\int \frac{d^{n-1}k}{(2\pi)^{n-1}}\frac{1}{2}\omega_k\left(((\phi^c)_k^f)^2+((\phi_k^c)^i)^2\right)+$$
$$\frac{1}{2}\omega_k\left((\chi_k^f)^2+(\chi_k^i)^2\right)+\omega_k\left((\phi_k^c)^f\chi_k^f+\chi_k^i(\phi_k^c)^i\right) \quad (2.6)$$

All boundary terms:

$$i\dot{\phi}_k^c(t_f)\chi_{-k}(t_f) - i\dot{\phi}_k^c(t_i)\chi_{-k}(t_i) - \omega_k\left(\phi_k^c(t_f)\chi_{-k}(t_f)+\phi_k^c(t_i)\chi_{-k}(t_i)\right)$$

B.C. for the propagator

$$i\dot{\phi}_k^c(t_f) - \omega_k\phi_k^c(t_f) = 0$$
$$i\dot{\phi}_k^c(t_i) + \omega_k\phi_k^c(t_i) = 0$$

The classical field reads

$$\phi^c(x) \equiv \int d^n x' \Delta_T(x,x') J(x')$$

After some arrangement the action can be written as

$$\frac{1}{2}\omega_k(\phi_k^c(t_f))^2 + \frac{1}{2}\omega_k(\phi_k^c(t_i))^2 - \frac{i}{2}\int_{t_i}^{t_f} dt dt' J_k(t) \Delta_T^k(t,t') J_k(t')$$

The interacting Schrodinger's functional reads

$$K_0[J] = e^{-\frac{i}{2}\int_{t_i}^{t_f} dtdt' \ J(t)\Delta_T(t,t')J(t')} \ K_0[0]$$

Free Schrodinger Functional

The free case

$$K_0[0] \equiv \int \mathcal{D}\chi \; e^{i\int d^{n-1}k \int_{t_i}^{t_f} dt \left(\frac{1}{2}|\dot{\chi_k}|^2 - \frac{\omega_k^2}{2}|\chi_k|^2\right)}$$

$$\chi_k(t_i) = \chi_k^i \qquad \chi_k(t_f) = \chi_k^f$$

$$K_0[0] = e^{i\int \frac{d^{n-1}k}{(2\pi)^{n-1}} \; \frac{\omega_k}{2\sin \;\omega_k T}\left[\left(|\chi_k^i|^2 + |\chi_k^f|^2\right)\cos \;\omega_k T - 2\mathrm{Re}\,\chi_k^i \chi_{-k}^f\right]}$$

$$\exp\left[-\frac{1}{2}\frac{\pi^{\frac{n-1}{2}} m^{n-1}}{2(2\pi)^{n-1}} V_{n-1}\left\{\frac{4(imT)^{1-\frac{n}{2}}}{\sqrt{\pi}}\sum_{j\geq 1} j^{-\frac{n}{2}} K_{\frac{n}{2}}(2imTj) - \right.\right.$$
$$\left. -\Gamma\left(\frac{1-n}{2}\right) + \frac{(imT)(-1)^{\frac{n}{2}}}{\sqrt{\pi}(n/2)!}\left[\gamma - \log(m^2/2\mu^2) + \psi^{(0)}(1+n/2)\right]\right\}$$

Fields in de Sitter space

Field redefinition

$$\phi_{\rm new} \equiv a^{\frac{n-2}{2}} \phi_{\rm old}$$

The lagrangian now reads (remember, now $\phi \equiv \phi_{\rm new}$)

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m(z)^2}{2}\phi^2 - \frac{g(z)}{6}\phi^3 - \frac{\lambda(z)}{24}\phi^4 + \frac{2-n}{4l^2}\frac{d}{dz}\left(\frac{\dot{a}}{a}\phi^2\right)$$

where

$$m^2(z) \equiv \; m^2a^2 \pm \left(1-\frac{n}{2}\right)\frac{\ddot{a}}{al^2} \mp \left(\frac{n}{2}-2\right)\left(\frac{n}{2}-1\right)\frac{\dot{a}^2}{a^2l^2}$$

$$g(z) \equiv a^{3-\frac{n}{2}}\; g$$

$$\lambda(z) \equiv a^{4-n}\; \lambda$$

There is a new boundary term only in the de Sitter case

$$S_{dS}=\frac{1}{2}\int d^{n-1}x\, l dz\left(\frac{1}{l^2}\ (\partial_z\phi)^2-(\nabla\phi)^2-\frac{m^2\ l^2-\frac{n}{2}\left(\frac{n}{2}-1\right)}{l^2\ z^2}\phi^2-\frac{\lambda}{12}\frac{z^{n-4}}{l}\phi^4\right)+\int d^{n-1}x\ \frac{n-2}{4zl}\phi^2\Big|_{z}^{z}$$

$$S_{AdS}=\frac{1}{2}\int d^{n-1}x\, l dz\left(-\frac{1}{l^2}\ (\partial_z\phi)^2+\dot{\phi}^2-(\nabla\phi)^2+\frac{m^2\ l^2-\frac{n}{2}\left(\frac{n}{2}-1\right)}{l^2\ z^2}\phi^2-\frac{\lambda}{12}\frac{z^{n-4}}{l}\phi^4\right)$$

$$\gamma\omega_k=\frac{n-2}{z}\pi T$$

Conformal time-dependent interaction term

$$V\left(z,\phi\right)\equiv-L_I\left(z,\phi\right)\equiv\frac{m^2\ l^2\left(\pm1-z^2\right)\mp\frac{n}{2}\left(\frac{n}{2}-1\right)}{2\ l^2\ z^2}\phi^2+\frac{\lambda}{24}z^{n-4}\phi^4$$

Finite time Feynman boundary conditions

New boundary conditions in flat space owing to de sitter boundary terms after field redefinition

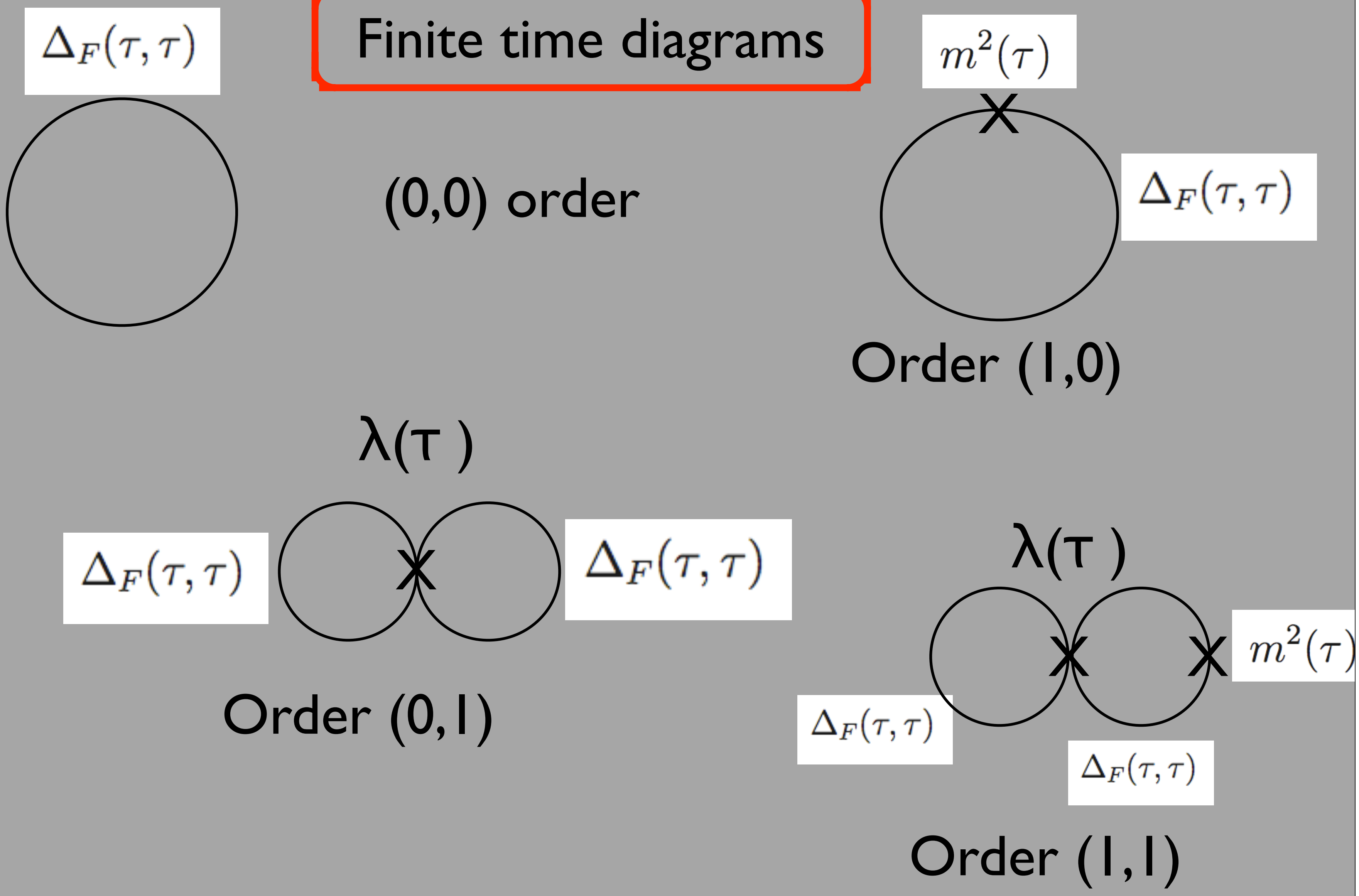
$$i\dot{\phi}_k^c(z_f) - \omega_k l \phi_k^c(z_f) = -i\gamma_f \omega_k l \phi_k^c(z_f)$$

$$i\dot{\phi}_k^c(z_i) + \omega_k l \phi_k^c(z_i) = -i\gamma_f \omega_k l \phi_k^c(z_i)$$

Schrodinger's functional: Diagrams with finite-time de Sitter propagators

$$K = \left. e^{i\int_{z_i}^{z_f} dz\ V\left(z, \frac{\delta}{i\delta J(z)}\right)}\ e^{\frac{i}{2}\int_{z_i}^{z_f} dz\ dz'\ J(z)\ \Delta_T(z,z')\ J(z')} \right|_{J=0} K_0[0]$$

Finite time diagrams



Asymptotic berhavior of free diagram (should include particle creation)

$$M_{0,0} \xrightarrow{Z\to\infty} \frac{V_{n-1}\Omega_{n-2}m^{n-3}}{2(2\pi)^{n-1}} \left\{ imlZ\, J\left(\frac{n-3}{2}, 0\right) + \frac{\gamma_i}{2} I_{00}(Z) \right\}$$

$$M_{0,0} \xrightarrow{Z\to 0} \frac{V_{n-1}\Omega_{n-2}m^{n-2}}{2(2\pi)^{n-1}} \left\{ ilZ\, J\left(\frac{n-3}{2}, 0\right) + \frac{\gamma_i}{2z_i} lZ^2\, J\left(\frac{n-3}{2}, -1\right) \right\}$$

$$J(a,b) = \int_1^\infty dx (x^2-1)^a\, x^b$$

$$I_{00}(Z) = \int_1^\infty dx (x^2-1)^{\frac{n-3}{2}} \frac{(e^{-2imlZx}-1)}{(\gamma_i - 2ix)x}$$

Mass insertion: transients

$$M_{1,0} \to -\left(\alpha + \frac{\beta}{z_i^2}\right) M_{00} + V_{n-1}\Omega_{n-2}\frac{m^{n-2}}{2(2\pi)^{n-1} z_i^3} ilZ^2 J\left(\frac{n-3}{2}, 0\right)$$

Mass insertion: asymptotics

$$M_{10} \to C$$

Self interaction:asymptotics

$$M_{0,1} \to \frac{i\lambda m^{2n-4}}{32(2\pi)^{2n-2}} V_{n-1}\Omega^2_{n-2} \left(J\left(\frac{n-3}{2}\right)^2 \frac{lZ^{n-3}}{n-3} + \left(\frac{\gamma_i}{ml} I_{01}(Z) - J\left(\frac{n-3}{2},0\right)^2\right) lZ^{n-4}\right)$$

Widths?

$$\Gamma(Z) \to \frac{\alpha m^{n-3}\gamma_i}{2(2\pi)^{n-1}Z} V_{n-1}\Omega_{n-2} Re\, I_{00}(Z) + \frac{\lambda m^{2n-5}\gamma_i}{16(2\pi)^{2n-2}} Z^{n-5} Im\, I_{01}(Z)$$

The integrals can be bound by Z-independent constants

Final Comments

Not fully conclusive results; positive indications in higher dimensions

Other vacua; other coordinates should be studied

Consistency with semiclassical equations of motion ?

This points towards inconsistency at freezing quantum-gravitational degrees of freedom

Full theory diff invariant: Need for a gauge invariant definition of vacuum decay