Outlook Post-Higgs

Christopher T. Hill
Fermilab

UCLA Higgs Workshop March 22, 2013
A dynamical “Higgs” mechanism was supposed to explain the origin of electroweak mass.
A dynamical “Higgs” mechanism was supposed to explain the origin of electroweak mass.

We now know that, evidently, a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z.
A dynamical “Higgs” mechanism was supposed to explain the origin of electroweak mass.

We now know that, evidently, a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z.

The Higgs Boson does NOT explain the origin of the electroweak mass-scale.
A dynamical “Higgs” mechanism was supposed to explain the origin of electroweak mass.

We now know that, evidently, a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z.

The Higgs Boson does NOT explain the origin of the electroweak mass-scale.

i.e., what is the origin of the Higgs Boson mass itself?
A dynamical “Higgs” mechanism was supposed to explain the origin of electroweak mass.

We now know that, evidently, a fundamental Higgs Boson exists and explains the masses of quarks, leptons, W and Z.

The Higgs Boson does NOT explain the origin of the electroweak mass-scale.

i.e., what is the origin of the Higgs Boson mass itself?

This may present opportunities for a deeper understanding.
QCD beautifully explains the origin of strong mass.
The magnificent scale anomaly:

The scale current: \[ S_\mu = x^\nu T_{\mu\nu} \]

\[ \partial_\mu S^\mu = T^\mu_\mu = 0 \] in a scale invariant theory
The magnificent scale anomaly:

The scale current: \[ S_\mu = x^\nu T_{\mu \nu} \]

\[ \partial_\mu S^\mu = T_\mu^\mu = 0 \] in a scale invariant theory

The old puzzle: \[ T_{\mu \nu} = \text{Tr}(G_{\mu \rho}G_{\nu}^\rho) - \frac{1}{4}g_{\mu \nu} \text{Tr}(G_{\rho \sigma}G^{\rho \sigma}) \]

\[ \partial_\mu S^\mu = T_\mu^\mu = \text{Tr}(G_{\mu \nu}G^{\mu \nu}) - \frac{4}{4} \text{Tr}(G_{\mu \nu}G^{\mu \nu}) = 0 \] !???

QCD is scale invariant !!!???
Resolution: The Scale Anomaly

$$\partial_\mu S^\mu = \frac{\beta(g)}{g} \text{Tr} G_{\mu\nu} G^{\mu\nu} = O(\hbar)$$
Resolution: The Scale Anomaly

\[ \partial_\mu S^\mu = \frac{\beta(g)}{g} \text{Tr} \ G_\mu \nu G^{\mu \nu} = \mathcal{O}(\hbar) \]

Origin of Mass in QCD = Quantum Mechanics
QCD beautifully explains the origin of strong mass:

$$\Lambda_{MS} = 217^{+25}_{-23} \text{ MeV}$$

**Gell-Mann and Low:**

$$\frac{dg}{d \ln \mu} = \beta(g)$$

**Gross, Politzer and Wilczek:**

$$\alpha_s(k^2) \equiv \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{\beta_0 \ln(k^2/\Lambda^2)}$$

$$\beta(g) = \bar{\lambda} \beta_0 g^3 \quad \text{where} \quad \beta_0 = -\frac{1}{16\pi^2} \left( \frac{11}{3} N_c - \frac{2}{3} n_f \right)$$
Many theories were proposed to imitate QCD for the electroweak scale

(1) Technicolor
(2) Supersymmetric Technicolor
(3) Extended Technicolor
(4) Multiscale Technicolor
(5) Walking Extended Technicolor
(6) Topcolor Assisted Technicolor
(7) Top Seesaw
(8) Supersymmetric Walking Extended Technicolor
(9) Strong dynamics from extra-dimensions
(10)....
Many theories were proposed to imitate QCD for the electroweak scale

(1) Technicolor
(2) Supersymmetric Technicolor
(3) Extended Technicolor
(4) Multiscale Technicolor
(5) Walking Extended Technicolor
(6) Topcolor Assisted Technicolor
(7) Top Seesaw
(8) Supersymmetric Walking Extended Technicolor
(9) Strong dynamics from extra-dimensions
(10)...

Most of these models were killed on July 4th 2012
Susy is still alive?
Susy is still alive?

We discovered a Standard Model Higgs and Susy is still alive?
Why EDM’s are so powerful:

\[
\frac{e m_e}{\Lambda^2} \Psi \gamma_5 \sigma_{\mu\nu} F^{\mu\nu} \Psi
\]

\[
d_e = \frac{e m_e}{\Lambda^2} = 0.2 \times 10^{-16} \text{ (e-cm)} \times \frac{(m_e/\text{MeV})}{(\Lambda/\text{GeV})^2}
\]

Current limit: \( d_e < 10^{-27} \text{ e-cm} \)

\( \Lambda > 1.4 \times 10^5 \text{ GeV} \)
Why EDM’s are so powerful:

\[
\frac{e m_e}{\Lambda^2} \bar{\Psi} \gamma_5 \sigma_{\mu\nu} F^{\mu\nu} \Psi
\]

\[
1/(\Lambda)^2 = (\alpha \sin(\gamma) / 4\pi \sin^2 \theta) / (M_{\text{selectron}})^2
\]

\[
M_{\text{selectron}} > 7 \text{ TeV} \ (\sin(\gamma))^{1/2}
\]

Future limit: \( d_e < 10^{-29} \text{ e-cm} -- 10^{-32} \text{ e-cm} \)?
Susy is still alive?

Weak Scale SUSY was seriously wounded before the LHC turned on (e.g. EDM’s)
Susy is still alive?

Weak Scale SUSY was seriously wounded before the LHC turned on (e.g. EDM’s)

Natural Weak Scale SUSY e.g. MSSM, was killed by LHC with new, severe direct limits
A very heretical conjecture:

All mass scales in physics are intrinsically quantum mechanical and associated with scale anomalies. The $\hbar \to 0$ limit of nature is exactly scale invariant.
“Predictions” of the Conjecture:

We live in D=4!

\[ T_\mu = \text{Tr} \ G_{\mu \nu} G^{\mu \nu} - \frac{D}{4} \text{Tr} \ G_{\mu \nu} G^{\mu \nu} \]

Cosmological constant is zero in classical limit

QCD scale is generated in this way; Hierarchy is naturally generated

Testable in the Weak Interactions?

Weyl Gravity in D=4 is QCD-like:

\[ \frac{1}{h^2} \sqrt{-g} (R_{\mu \nu} R^{\mu \nu} - \frac{1}{3} R^2) \]

Is the Higgs technically natural?

On naturalness in the standard model.

Conjecture on the physical implications of the scale anomaly.
Christopher T. Hill (Fermilab) . hep-th/0510177
Mass is always associated with conformal symmetry breaking
QCD: Explicitly Broken Scale symmetry by perturbative anomaly

The Origin of the Nucleon Mass

(aka, the visible mass in the universe)
Spontaneously Broken Scale Symmetry?

Nambu-Goldstone Boson

“Dilaton”
Spontaneously Broken Scale Symmetry?

Nambu-Goldstone Boson

Is the Higgs boson a “Dilaton”?
Higgs Potential

\[-M^2 |\phi|^2 + \frac{\lambda}{2} |\phi|^4\]
$$\text{Scale Invariant Higgs Potential}$$

$$-M^2 |\phi|^2 + \frac{\lambda}{2} |\phi|^4$$
Scale Invariant Higgs Potential

\[-M^2 |\phi|^2 + \frac{\lambda}{2} |\phi|^4\]
As usual, the ground state has a minimum for:

\[ \langle H^i \rangle = \theta^i \quad \text{where} \quad \theta^i = (v, 0) \]

\[ v = M / \lambda^{1/2} \]
In the present case it pulls $H^i$ to the minimum VEV. But once we take the limit $\lambda \to 0$ the Lagrangian acquires a “shift symmetry,”

$$\delta H^i = \theta^i \epsilon \quad \rightarrow \quad \delta \partial_\mu H^i \partial^\mu H = 0$$
In the present case it pulls $H^i$ to the minimum VEV. But once we take the limit $\lambda \to 0$ the Lagrangian acquires a “shift symmetry,”

$$\delta H^i = \theta^i \epsilon \quad \rightarrow \quad \delta \partial_\mu H^{\dagger} \partial^\mu H = 0$$
The Noether current is:

\[ J_\mu = \frac{\delta \mathcal{L}_0}{\delta \partial_\mu \epsilon} = \theta^\dagger \partial_\mu H + H^\dagger \partial_\mu \theta \]

In the broken phase of the theory

\[ \frac{1}{2v} (\theta^\dagger H + H^\dagger \theta) = v + \frac{\dot{h}}{\sqrt{2}} \]

\[ J_\mu \rightarrow \sqrt{2}v \partial_\mu h \]

“dilatonic” current

Scale Invariant Higgs Potential

0 x $|\phi|^4$
This permits mass with hidden scale symmetry
This permits mass with hidden scale symmetry

To see this, consider the top quark mass term:

\[ g\bar{\psi}_L t_R H + h.c. \rightarrow m_{tt} \left( 1 + \frac{h}{\sqrt{2}v} \right) \]
This permits mass with hidden scale symmetry

To see this, consider the top quark mass term:

\[ g \bar{\psi}_L t_R H + h.c. \quad \Rightarrow \quad m_t \bar{t}t \left( 1 + \frac{h}{\sqrt{2}v} \right) \]

Under an infinitesimal scale transformation

\[ t(x) \Rightarrow (1 - \epsilon)^{3/2} t(x') \quad h(x) \Rightarrow (1 - \epsilon) h(x') \quad x_\mu \Rightarrow (1 + \epsilon) x'_\mu \]
This permits mass with hidden scale symmetry

To see this, consider the top quark mass term:

\[ g \bar{\psi}_L t_R H + h.c. \quad \Longrightarrow \quad m_t \tilde{t} t \left( 1 + \frac{h}{\sqrt{2}v} \right) \]

Under an infinitesimal scale transformation

\[ t(x) \Longrightarrow (1 - \epsilon)^{3/2} t(x') \quad h(x) \Longrightarrow (1 - \epsilon) h(x') \quad x_\mu \Longrightarrow (1 + \epsilon) x'_\mu \]

Hence the action transforms as:

\[ S_0 = \int d^4x \ m_t \tilde{t} t(x) \left( 1 + \frac{h(x)}{\sqrt{2}v} \right) \]

\[ \Longrightarrow \int d^4x \ (1 - \epsilon)^3 m_t \tilde{t} t(x') \left( 1 + \frac{h(x')(1 - \epsilon)}{\sqrt{2}v} \right) \]

\[ = \int d^4x' (1 + \epsilon)^4(1 - \epsilon)^3 m_t \tilde{t} t(x') \left( 1 + \frac{h(x')(1 - \epsilon)}{\sqrt{2}v} \right) \]

\[ = \int d^4x' \left( (1 + \epsilon)m_t \tilde{t} t(x') + m_t \tilde{t} t(x') \frac{h(x')}{\sqrt{2}v} \right) \]
However, with the dilaton we can compensate the rescaled mass term by a shift

\[ h(x') \Rightarrow h(x') - \sqrt{2}v\epsilon \]

we see that:

\[
\int d^4x' \left( (1 + \epsilon)m_t\bar{t}t(x') + m_t\bar{t}t(x') \frac{h(x')}{\sqrt{2}v} \right) \Rightarrow \int d^4x' \left( m_t\bar{t}t(x') + m_t\bar{t}t(x') \frac{h(x')}{\sqrt{2}v} \right) = S_0
\]

Hence, the top quark mass is ultimately invariant and the scale symmetry is broken spontaneously. The same conclusion applies to the masses of all fermions, and of the gauge fields, \( W \) and \( Z \). The Higgs self-interactions that involve nonzero \( \lambda \) would not be invariant under scale transformations with dilatonic shifts in \( h \).
The Higgs is massive, so it would be a pseudo-dilaton at mass-shell.
The Higgs is massive, so it would be a pseudo-dilaton at mass-shell.

Scale symmetry is broken by quantum loops, (i.e., by renormalization group running).
The Higgs is massive, 
so it would be a pseudo-dilaton at mass-shell 

Scale symmetry is broken by quantum loops, 
(ie by renormalization group running) 

Higgs quartic coupling \( \lambda \) runs to zero, but...
The Higgs is massive, so it would be a pseudo-dilaton at mass-shell.

Scale symmetry is broken by quantum loops, (ie by renormalization group running)

Higgs quartic coupling $\lambda$ runs to zero, but .... then negative at $M_{\text{GUT}} = 10^{10}$ to $10^{13}$ (very sensitive to $m_{\text{top}}$)
The Higgs is massive, so it would be a pseudo-dilaton at mass-shell.

Scale symmetry is broken by quantum loops, (ie by renormalization group running)

Higgs quartic coupling $\lambda$ runs to zero, but ....
then **negative** at

$M_{\text{GUT}} = 10^{10}$ to $10^{13}$ (very sensitive to $m_{\text{top}}$)

Standard Model Vacuum is Unstable!
Figure 3: Evolution of the Higgs coupling $\lambda(\mu)$ and its beta function, eq. (50), as a function of the renormalization scale, compared to the evolution of the effective coupling $\lambda_{\text{eff}}(h)$, defined in eq. (51), as a function of the field value. **Left:** curves plotted for the best-fit value of $M_t$. **Right:** curves plotted for the lower value of $M_t$ that corresponds to $\lambda(M_{\text{Pl}}) = 0$. 
Higgs mass and vacuum stability in the Standard Model at NNLO

Giuseppe Degrassi, Stefano Di Vita, Joan Elias-Miró, José R. Espinosa, Gian F. Giudice, Gino Isidori, Alessandro Strumia

Figure 4: The instability scale $\Lambda_I$ at which the SM potential becomes negative as a function of the Higgs mass (left) and of the top mass (right). The theoretical error is not shown and corresponds to a $\pm 1 \text{ GeV}$ uncertainty in $M_h$. 

1σ band in $M_t = 173.1 \pm 0.7 \text{ GeV}$
$\alpha_s(M_Z) = 0.1184 \pm 0.0007$

1σ bands in $M_t = 174 \text{ GeV}$
$\alpha_s(M_Z) = 0.1184 \pm 0.0007$
Figure 5: Regions of absolute stability, meta-stability and instability of the SM vacuum in the $M_t$–$M_h$ plane. **Right:** Zoom in the region of the preferred experimental range of $M_h$ and $M_t$ (the gray areas denote the allowed region at 1, 2, and 3σ). The three boundaries lines correspond to $\alpha_s(M_Z) = 0.1184 \pm 0.0007$, and the grading of the colors indicates the size of the theoretical error. The dotted contour-lines show the instability scale $\Delta$ in GeV assuming $\alpha_s(M_Z) = 0.1184$. 
The SM Higgs is Dilatonic at the $\lambda = 0$ scale

$M_{\text{GUT}} = 10^{10}$ to $10^{13}$ GeV

Higgs is pseudo-Dilatonic at the $m_{\text{Higgs}}$ scale
due to the $\lambda$ running (scale anomaly)

Dilatonic Higgs seems to imply a vacuum instability
Is there any hope left for dynamics?
Is there any hope left for dynamics? Coincidence?

\[ m_{Higgs}^2 \approx \frac{1}{2} m_{top}^2 \quad m_{top} \approx v_{weak} \]

\[ v_{weak} \approx 175 \text{ GeV} \]
Is there any hope left for dynamics? Coincidence?

\[ m^2_{Higgs} \approx \frac{1}{2} m^2_{top} \quad m_{top} \approx v_{weak} \]

\[ v_{weak} \approx 175 \text{ GeV} \]

\[ \mathcal{L} = \mathcal{L}_{\text{kinetic}} + g_t \bar{\psi}_L t_R H + h.c. - \frac{\lambda}{2} \left( H^\dagger H - v_{weak}^2 \right)^2 \]

\[ g_t \approx 1, \quad \lambda \approx \frac{1}{4} \]

Our main interest is in mechanisms that can relate the quantities \( m_H, m_t \) and \( v_{weak} \).
I find this intriguing:

Generalize dilatational c-number shift
Generalize dilatational c-number shift to a q-number shift: nonlinear “super”-symmetry:

“Super” Dilatation Symmetry of the Top-Higgs System

Generalize dilatational shift to a nonlinear “super”-symmetry:

\[
\mathcal{L}_K = \bar{\psi}_L i\partial \psi_L + \bar{t}_R i\partial t_R + \partial H^\dagger \partial H
\]

… is invariant under the following “SUSY dilatation:”

\[
\begin{align*}
\delta \psi_L^{ia} &= \theta_L^{ia} \epsilon_0 - i \frac{\partial H^i \theta_R^a}{\Lambda^2} \epsilon; \\
\delta t_R^a &= \theta_R^a \epsilon_0 - i \frac{\partial H^\dagger \theta_L^{ia}}{\Lambda^2} \epsilon; \\
\delta H^i &= \frac{\bar{\theta}_R \psi_L^{ia} + \bar{t}_R \theta_L^i}{\Lambda^2} \epsilon; \\
\delta H_i^\dagger &= \frac{\bar{\psi}_L \theta_R^a + \bar{\theta}_L t_R^a}{\Lambda^2} \epsilon.
\end{align*}
\]
A minimal “super” dilatation: shift only $t_R$ 

\[
\begin{align*}
\delta \psi^i_L &= -i \frac{\partial H^i \theta^a_R}{\Lambda^2} \epsilon; \\
\delta t^a_R &= \theta^a_R \epsilon_0; \\
\delta H^i &= \frac{\theta^a_R \psi^i_L}{\Lambda^2} \epsilon; \\
\delta H^i &= \psi^i_L \theta^a_R \epsilon. \\
\end{align*}
\]

Full Lagrangian of Top-Higgs System

\[
\mathcal{L}_H = \overline{\psi}_L i \not{\partial} \psi_L + i \overline{t}_R \not{\partial} t_R + \partial H^\dagger \partial H \\
+ g (\overline{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 \\
+ \frac{\kappa}{\Lambda^2} (\overline{\psi}_L t_R \overline{t}_R \psi_L) - \frac{\kappa}{\Lambda^2} (\overline{t}_R \gamma_\mu t_R) (H^\dagger i \not{\partial}^\mu H) + \mathcal{O} \left( \frac{1}{\Lambda^4} \right)
\]

Invariance under SD: \[\lambda = \frac{1}{2} g^2\]

\[m_H^2 = 2 \lambda v_{weak}^2 = m_t^2 \text{ in the broken phase.}\]
A UV Completion Theory invariant under super-dilatation

\[ \mathcal{L}_H = \bar{\psi}_L i \partial \psi_L + i t_R \partial t_R + \partial H^\dagger \partial H + g(\bar{\psi}_L t_R H + h.c.) - M_H^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 + \tilde{k} \bar{\psi}_L t_R \Phi - M^2 \Phi^\dagger \Phi + h.c. + \partial \Phi^\dagger \partial \Phi + \tilde{k} t_R H^\dagger \chi_L + \frac{\tilde{k}}{\Lambda} t_R \partial H^\dagger \chi_R - M \chi_L \chi_R + h.c. + \bar{\chi} \partial \chi \]

“predicts” Higgs boson recurrence(s)

and heavy Dirac fermion(s)

Mass scale \[ \Lambda \]

Few TeV

Recurrences of Higgs and, composite fermionic models
Otherwise:
Otherwise: The Desert
A Muon Collider Higgs Factory provides a Staged Pathway from the Intensity Frontier to the Energy frontier via a Unique Higgs Factory

Only a Low energy Muon Collider can directly measure full width of \( H(125) \)

Offers most precise measurement of Higgs mass

Offers most precise measurement of non-third generation Higgs-Yukawa coupling constant; can check running of cc.

\[ s_{\text{peak}} \times 10^{31} \times (1 \text{ SMyr}) = 10k \text{ Higgs} \]

Multi-Higgs \( H^0 \ A^0 \) accessible with → 500 GeV cms machine

Sensitivity begins at \( L = (\text{few}) \times 10^{29} \)
end