Design of a $\mu^+\mu^-$ Higgs Factory Lattice

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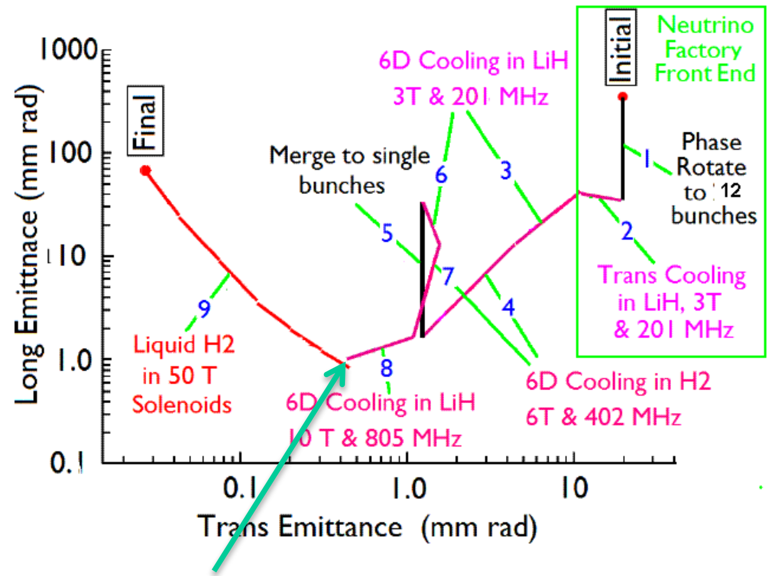
Neuffer's Scenario

The major advantage of a $\mu^+\mu^-$ Higgs Factory – the possibility of direct measurement of the Higgs boson width (Γ~3MeV FWHM expected) $\Rightarrow$ a very small beam energy spread is required, R~0.003%



Dave proposed to stop cooling here:

$\varepsilon_{\perp N}$ =0.3(π)mm·rad*, $\varepsilon_{||N}$ =1(π)mm·rad (**σ_s=5.6cm** with **$\sigma_p/p=3\cdot10^{-5}$**)

W/o final cooling the muons losses are reduced ~ by half:

$N_\mu=4\cdot10^{12}$ @ f_{rep}=15Hz for 4MW p-driver power ,

or we can afford ~ twice the reprate :

$N_\mu=2\cdot10^{12}$ @ f_{rep}=30Hz

*) The machine must be able to digest even higher emittances (at the price of larger β^*)

HF Lattice Design Issues

- Large $\varepsilon_{\perp N} \rightarrow$ small β^* to achieve the required luminosity $\rightarrow$ very large IR magnet apertures (up to ID=50cm). Are they feasible?
- Detector protection from backgrounds $\rightarrow$ it's desirable that longest & strongest IR quad was defocusing
- β^* variation in wide range (1.5cm – 10cm)
- Preservation of small σ_E in the presence of strong self-fields (Ipeak ~ 0.5-1kA !) $\rightarrow$ requirements on RF and momentum compaction
- Do we need chromaticity correction for σ_E/E as low as $3{\cdot}10^{-5}$? – There are effects which may require it
- Effect of fringe fields and multipole errors in large aperture IR magnets
- Beam-beam effect (both transverse and longitudinal) in long bunches

Chromaticity Correction?

Thanks to small σ_E/E~ 3·10^{-5} the chromaticity by itself is not a problem (though it may be such for larger σ_E/E needed for initial scan).

But there are other effects which require chromaticity correction, most notably the path length dependence on betatron amplitude (L. Emery, HEACC'92, Hamburg) which translates into additional energy spread*:

$$\frac{\Delta E}{E} \approx \frac{1}{\alpha_c R}(Q'_x I_x + Q'_y I_y) \rightarrow \left\langle \frac{\Delta E}{E} \right\rangle = \frac{2|Q'_\perp|\varepsilon_\perp}{\alpha_c R}, \quad \varepsilon_x = \langle I_x \rangle$$

$$A_x / \sigma = \sqrt{2I_x / \varepsilon_x}$$

With uncorrected $Q'_\perp$~ -100 and α_c=0.05 we would have

$$\left\langle \frac{\Delta E}{E} \right\rangle \sim 6 \cdot 10^{-5}$$

$\rightarrow$ we need chromaticity correction! $\rightarrow$ "three sextupoles" scheme developed for high-energy muon collider not to compromise the dynamic aperture.

In contrast to high-energy case we need sufficiently high α_c

Chromaticity correction is also needed from operational considerations.

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*) Look for detail in the support slide

Momentum Compaction Factor

To obtain small σ_E with high $\varepsilon_{||} \to$ high $\beta_{||}$ is required:

$$\sigma_E / E = \sqrt{\varepsilon_\| / \beta_\|} = 3\cdot 10^{-5} \;\to\; \beta_\| \approx 1880m,\;\; \beta_\| = \frac{\alpha_c C}{2\pi Q_s} = C\sqrt{\frac{\alpha_c E}{2\pi h V_{RF}\cos\varphi_s}}$$

$\to$ low V_{RF} and/or high momentum compaction α_c is required.

But V_{RF} should be high enough to minimize the effect of strong self-fields (~50kV for $Z_{||}/n$~0.1Ω in the GHz range*). For V_{RF} =100kV and f_{RF} =200MHz

$$\alpha_c = \frac{2\pi h e V_{RF}\cos\varphi_s}{E}\left(\frac{\beta_\|}{C}\right)^2 \approx 0.08\cos\varphi_s$$

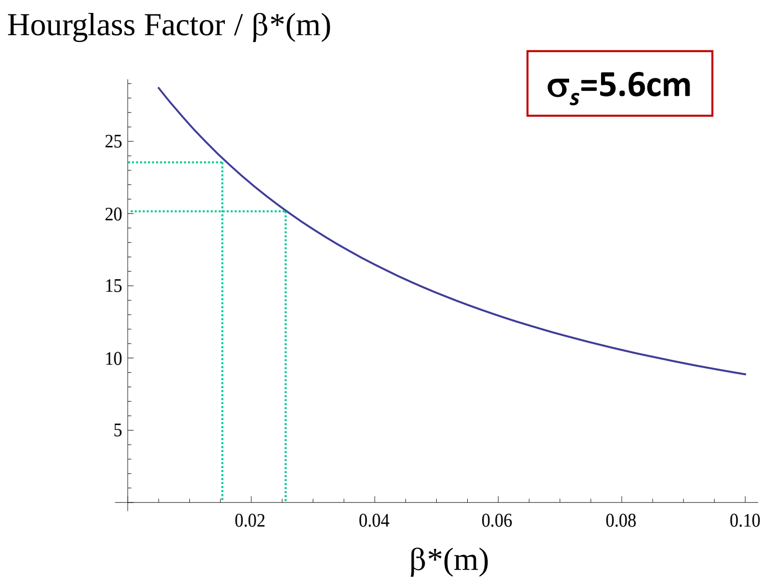
Another way to look at this is to use the Keil-Schnell criterion (+ Boussard conjecture)

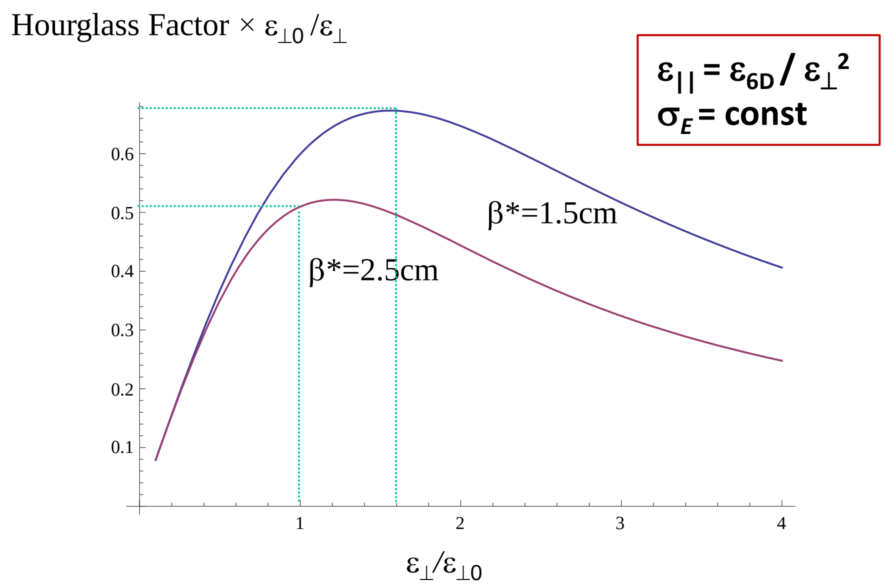
$$\left|\frac{Z_\|}{n}\right| \le \frac{2\pi E\,|\alpha_c|}{eI_{peak}}\left(\frac{\sigma_E}{E}\right)^2 \to |\alpha_c| \ge 0.13$$

Obviously we have to limit $Z_{||}/n$ to well below 0.1Ω and make α_c as large as possible (I set the goal at $\alpha_c > 0.05$)

*) Characteristic bunch frequency is $c/4\sigma_s$ ~1GHz whereas the rotational frequency is ~1MHz $\to$ n~10^3

Optimum β* for Given Emittances





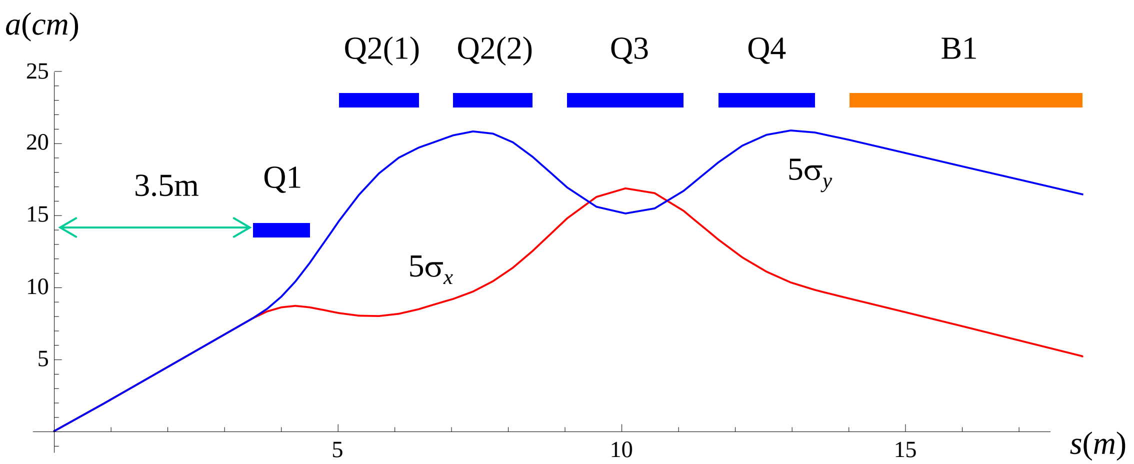
The hourglass factor takes away half of the luminosity gain from β* reduction: β*=1.5cm gives just 17% higher luminosity compared to β*=2.5cm (still can be worthwhile).

The gain could be higher if the emittances are redistributed so as to reduce the longitudinal one, but this would require even larger aperture of the FF quads.

β*=2.5cm is close to the optimum for $\varepsilon_{\perp N}$ =0.3(π)mm·rad, $\varepsilon_{||N}$ =1(π)mm·rad .

Quadruplet Final Focus

- For detector protection longest quad (Q2) is defocusing
- For dispersion matching the last quad (Q4) is also defocusing → quadruplet FF
- d*=3.5m is a guess, compare with ILC d*=2m



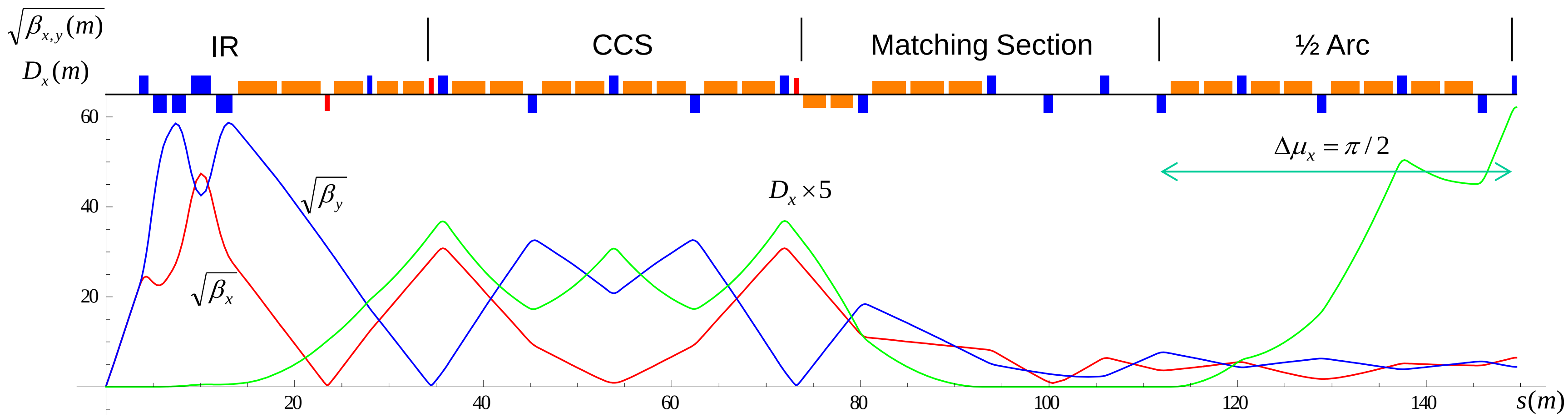
β*=2.5cm,
$\varepsilon_{\perp N}$ =0.3mm

	Q1	Q2	Q3	Q4
aperture (cm)	27	45	45	45
gradient (T/m)	74	-36	44	-25
dipole field (T)	0	2	0	2
length (m)	1.0	1.4	2.05	1.7

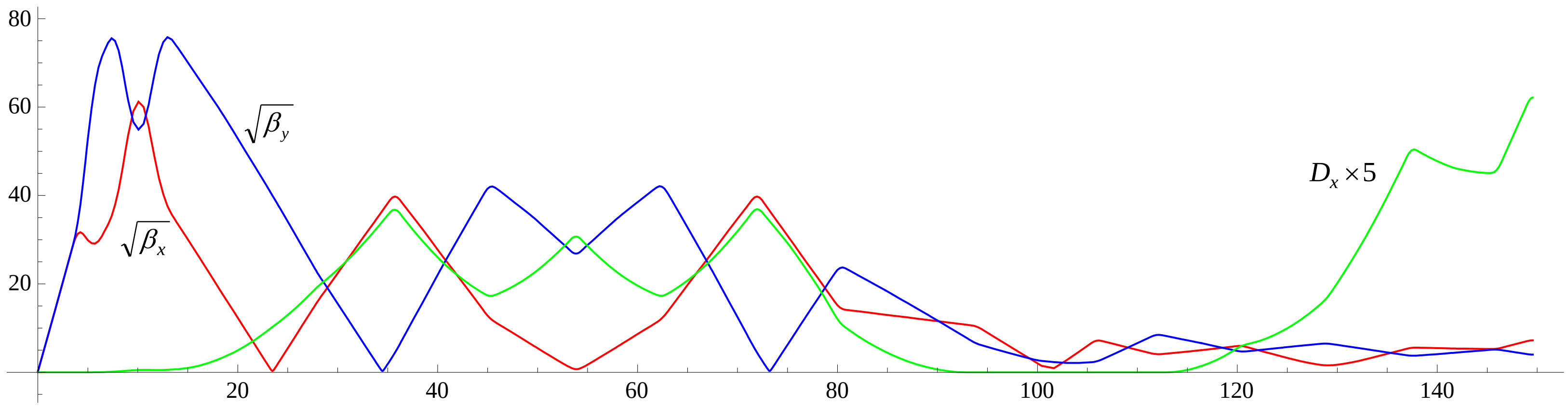
The pole tip field = 10T in all quads, B=8T in B1

Optics Functions (from IP to SP)

β*=2.5cm

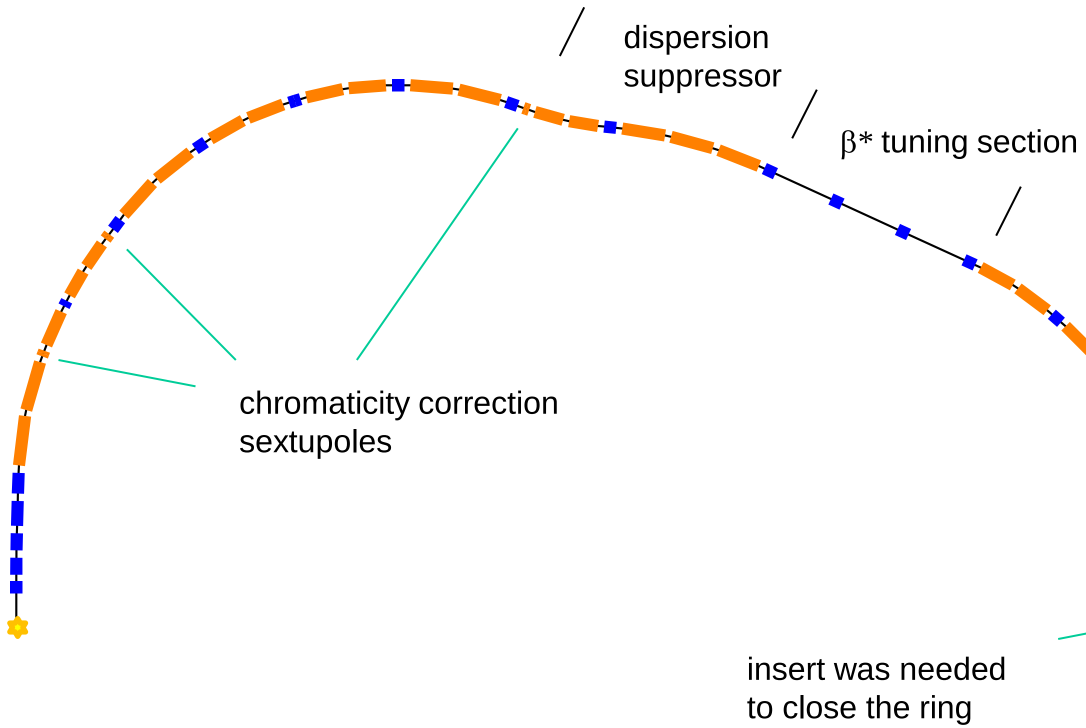


β*=1.5cm



β* can be increased up to 10 cm by changing quad gradients in the matching section

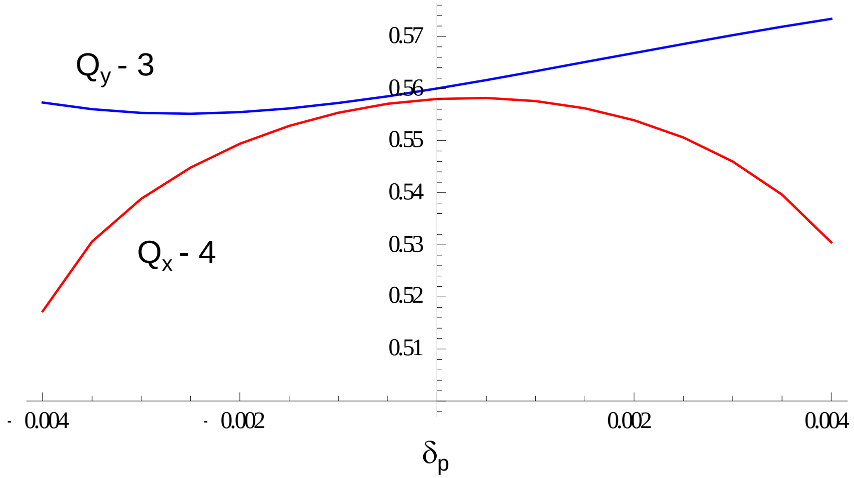
(Half) Ring Layout



Dispersion suppressor and β* tuning section noticeably increase the ring circumference, but they are probably indispensable

Momentum Acceptance

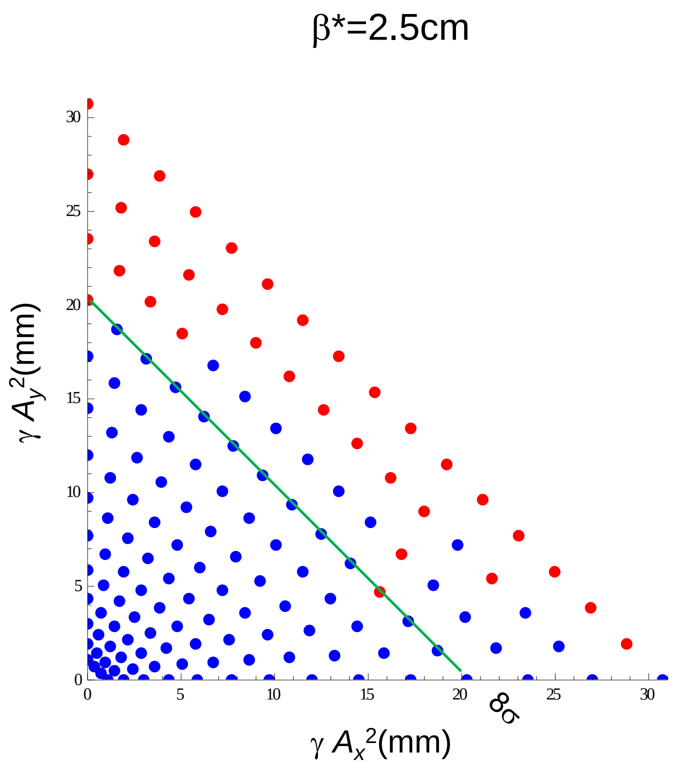
β*=1.5cm

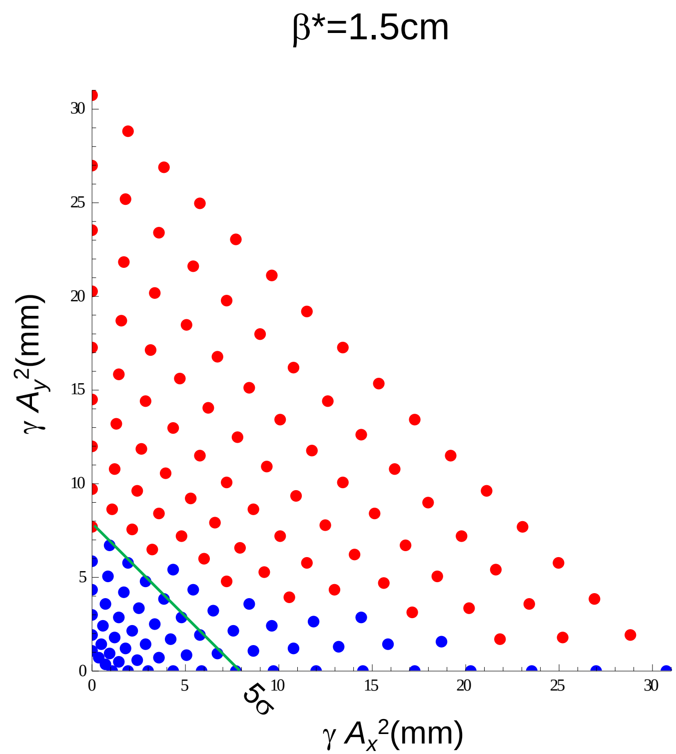


No attempt to correct nonlinear chromaticity has been made (should not be a problem).
Acceptance at β*=2.5cm exceeds ±0.5%

N.B. Chromaticity computed with MAD8 TWISS command is completely wrong!

Dynamic Aperture w/o Errors

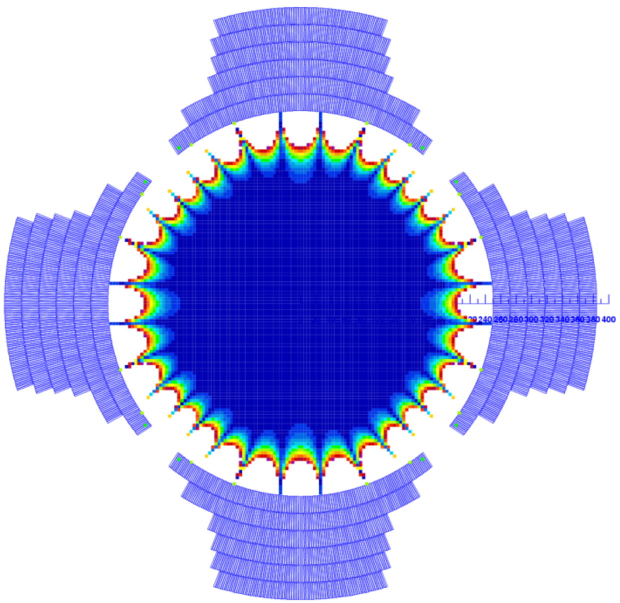




2048 turns Dynamic Aperture (DA) computed with MAD8 LIE4 method. Octupole correctors were used to correct vertical detuning with amplitude.
In both cases the DA significantly exceeds the physical aperture (5σ and 4σ respectively)

IR Quadrupole Design (V. Kashikhin)

Vadim K.:
"The pole tip field in 500mm quad at 44 T/m is 11.9 T and the quench field is 16.9 T"
– but I requested only 10T in a 450mm aperture, there is a huge margin!
For Q1 Vadim made design with 320mm aperture – again with a good margin.



name	b_n	name	k_{n-1}
b_2	10000	k1	+2.0859E-01
b_6	-0.1840	k5	-1.7970E-01
b_{10}	-0.5747	k9	-6.6227E+05
b_{14}	-0.9377	k13	-7.2352E+12
b_{18}	-10.4111	k17	-1.7903E+21
b_{22}	7.6648	k21	+7.3874E+28
b_{26}	-5.3676	k25	-6.1282E+36
b_{30}	3.0646	k29	+7.7823E+44

Cross section of 500mm quad coils and field quality

Systematic multipole errors, b_n corresponds to a 2n-pole, R_{ref}=225mm

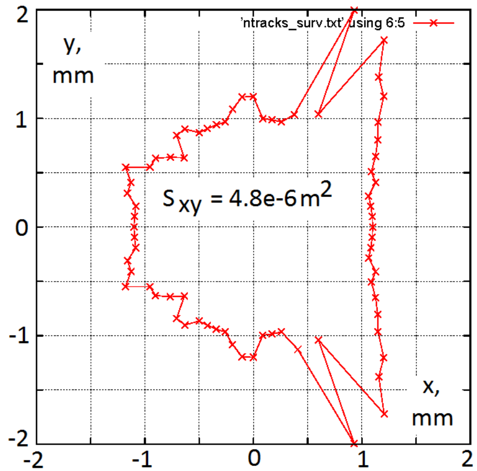
$$B_y + iB_x = B\big|_{r=R_{\rm ref}} \times 10^{-4} \sum_{n=1}^{\infty} b_n (x+iy)^{n-1} / R_{\rm ref}^{n-1}$$

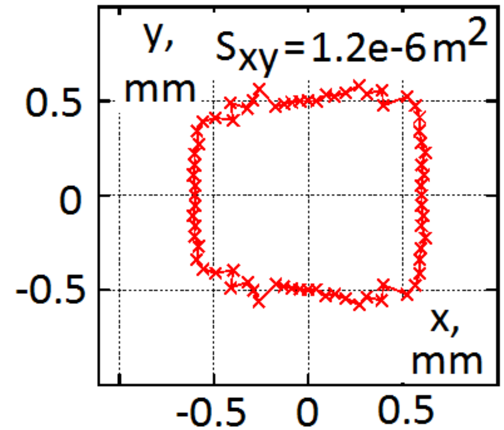
$$B_y + iB_x = B\rho \cdot \sum_{m=0}^{\infty} k_m^{\rm norm} (x+iy)^m / m!$$

Dynamic Aperture with IR Quad Multipole Errors (V. Kapin)

β*=2.5cm

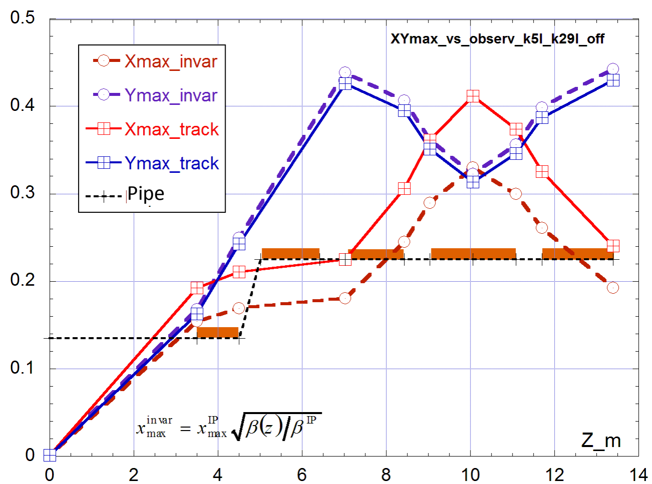
Errors OFF

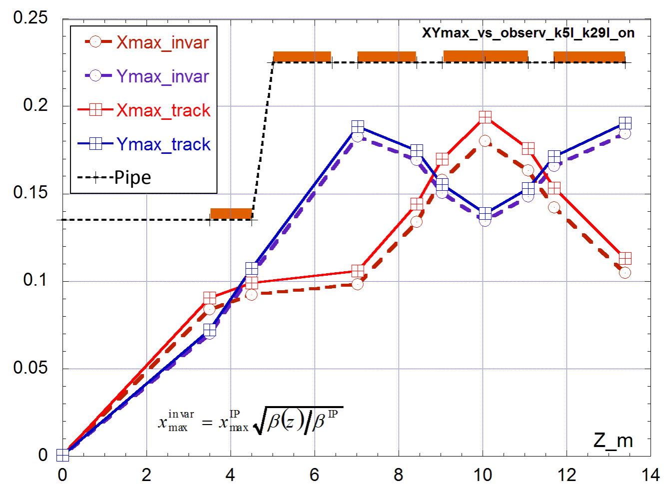




Errors ON

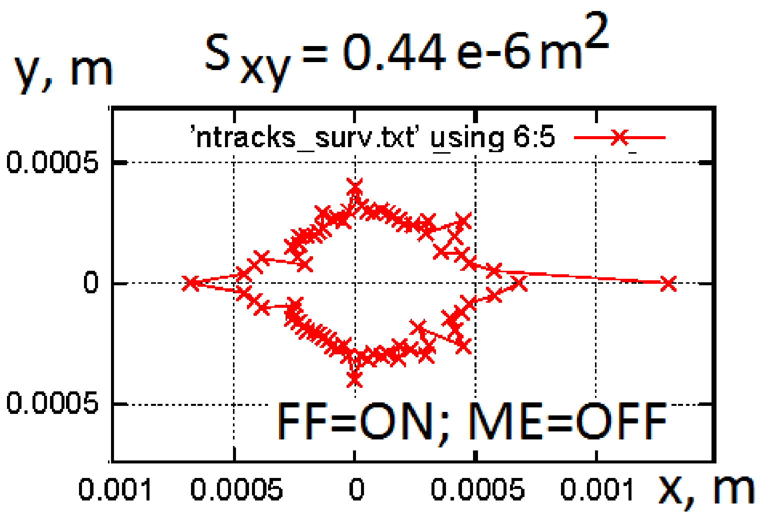
DA in the plane of initial coordinates *x, y* (at IP), Sxy = stable area

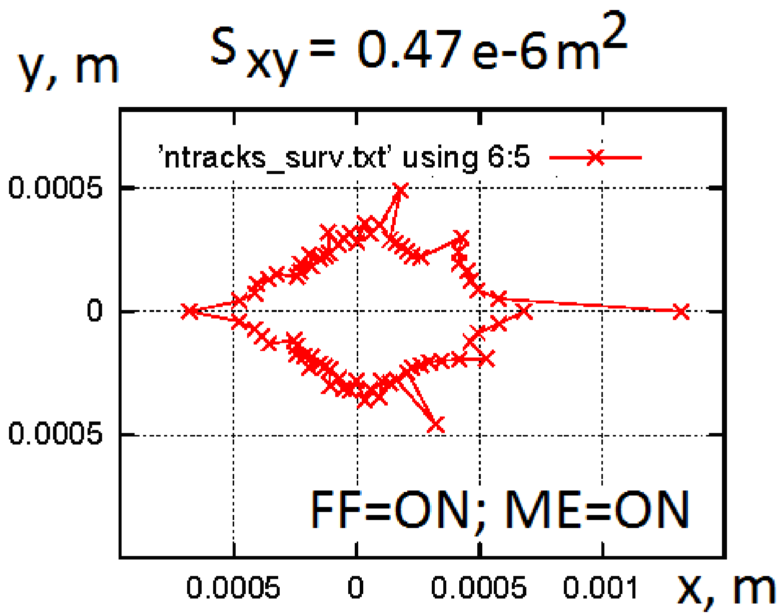




Stable beam envelopes obtained by tracking (solid lines) and via beta-functions (dashed lines)

Dynamic Aperture with IR Quad Fringe Fields





DA in the plane of initial coordinates *x, y* (at IP) with Fringe Fields (FF) on and the Multipole Errors (ME) off (left plot) and on (right plot).

MADX PTC block was used which computes Fringe Fields in the hard-edge approximation. The FF effect is much stronger than ME effect, it was traced to a large cross-detuning dQy/dEx.

Next steps:

- Effect of smooth Fringe Fields (described by Enge functions)

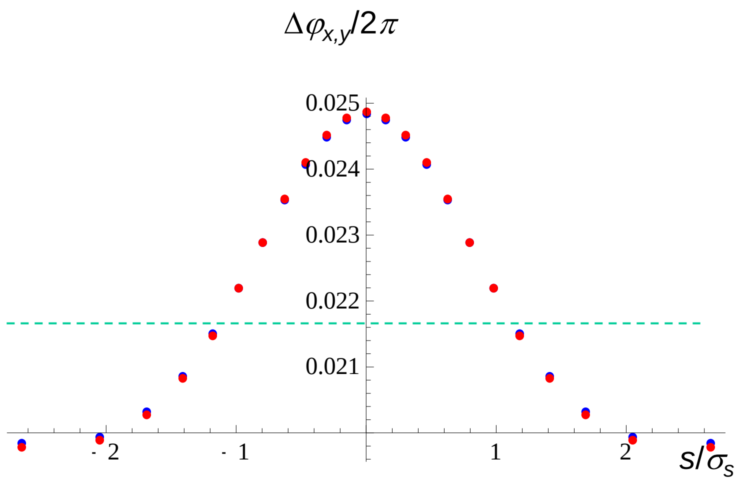
- Correction of the FF and ME effects

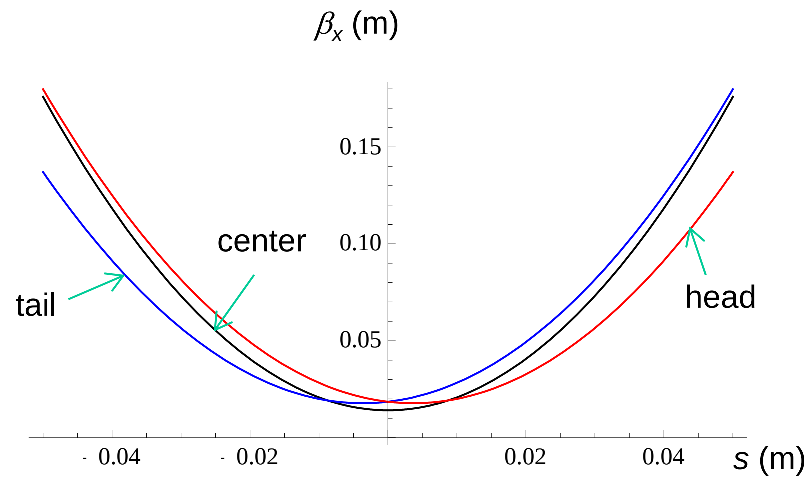
Strong-Strong Beam-Beam Effect in Long Bunches

Different parts of the bunch have different dynamic β-values at a given point → the beam-beam tuneshift can be increased/decreased.

Upgrade parameters assumed:

$$\varepsilon_{\perp N}=0.2(\pi)\text{mm}\cdot\text{rad},\ N_\mu=4\cdot 10^{12} \rightarrow \xi=0.0217,\ \ \beta^*=1.5\text{ cm},\ (\sigma_s=5.6\text{cm})$$





Left – tuneshift vs particle position in the bunch, right – dynamic β-values as seen by particle in head (red), center (black) and tail of the bunch (blue). Fractional tunes = 0.56

The transverse beam-beam effect is unlikely to cause any problem.

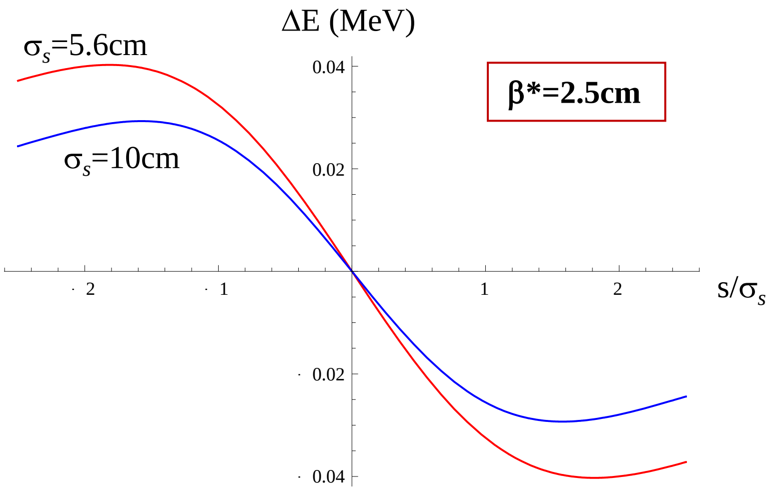
Longitudinal Beam-Beam Effect (Derbenev & Skrinsky, 1972)

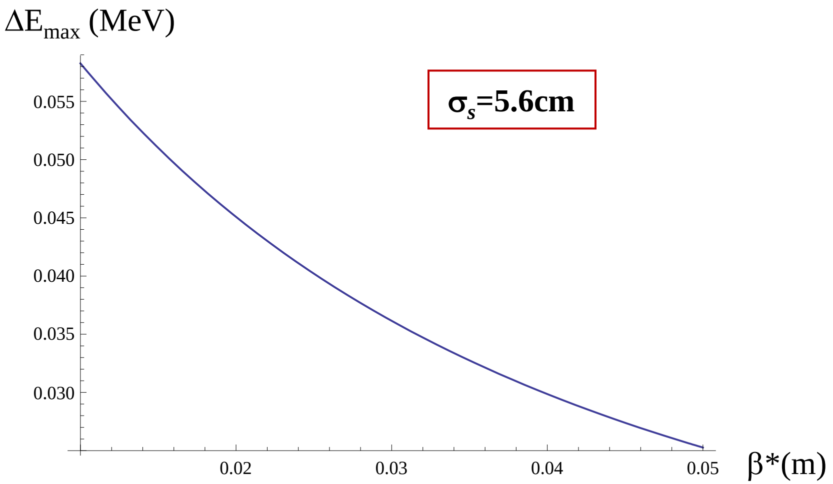
Collision with a thin slice of Ns particles leads to energy change

$$\Delta E = \frac{e^2 N_s}{2\beta_\perp} \frac{d\beta_\perp}{ds}\bigg|_{\text{collisionpoint}}, \quad \Delta E_{\max} = \frac{e^2 N_s}{2\beta^*} \sim 58\text{kV} \ \text{ for } \ N_s = 2\cdot 10^{12} \ \text{ and } \ \beta^* = 2.5cm$$

For $\alpha_c > 0$ the effect is defocusing (good), but it is strongly nonlinear (not so good).

The finite bunch length reduces it somewhat:





Effective gradient is ~0.7 MV/m for cited parameters, can exceed 2 MV/m for the upgrade.

Higher-frequency (500MHz) RF for compensation?

Parameters of the Present Design

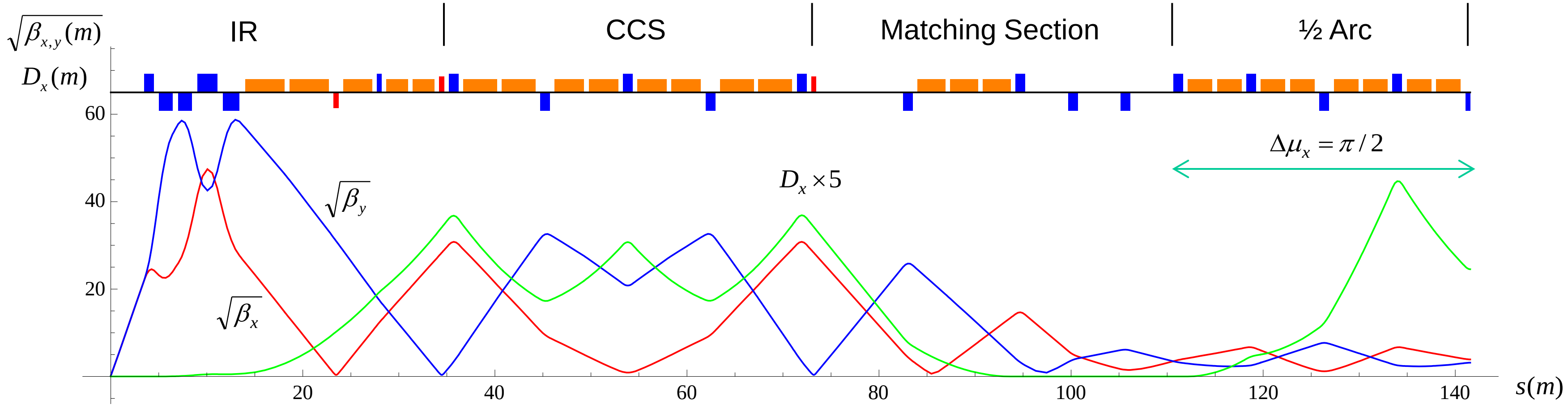
Parameter	Unit	Value
Circumference, C	m	299
β^*	cm	2.5 (1.5-10)
Momentum compaction, α_p	-	0.0793
Betatron tunes	-	4.56 / 3.56
Bare lattice chromaticity	-	-124 / -197
Synchrotron tune* (100kV, 200MHz)	-	0.002
Number of muons / bunch	10^{12}	2
Normalized emittance, $\varepsilon_{\perp N}$	π·mm·rad	0.3
Long. emittance, $\varepsilon_{\parallel N}$	π·mm	1.0
Beam energy spread	%	0.003
Bunch length, σ_s	cm	5.64
Beam-beam parameter	-	0.0072
Repetition rate	Hz	30
Average luminosity	10^{31}/cm^2/s	2.5

*) Without wake-fields and longitudinal beam-beam effect

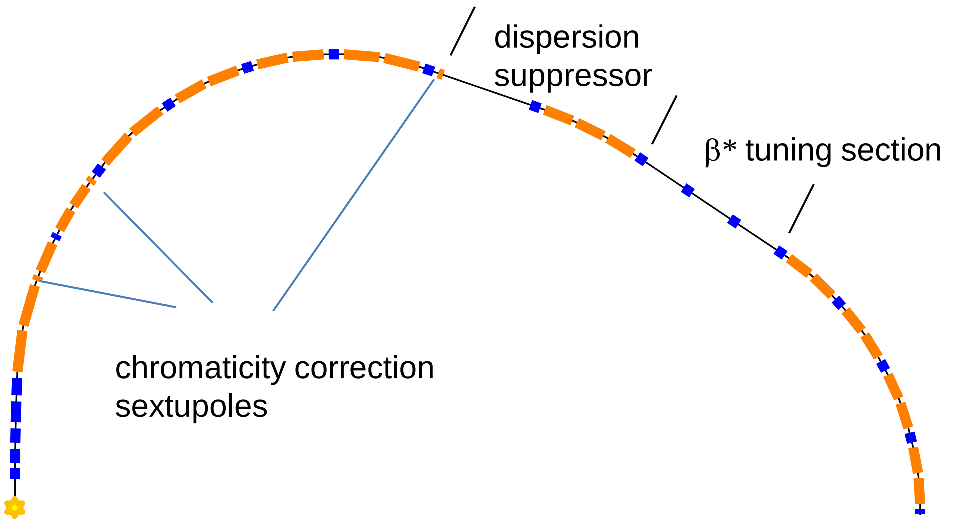
Summary & Outlook

- Unlike high-energy MC, the Higgs Factory needs highly non-isochronous ring (α_c~0.1) to preserve small energy spread
- Larger $\varepsilon_{\perp N}$ in D.Neuffer's proposal $\rightarrow$ smaller β^* to achieve the required luminosity $\rightarrow$ very large IR magnet apertures (up to 50cm) $\rightarrow$ gradients ~ twice lower than in the early designs
- A quadruplet Final Focus is proposed for better detector protection from backgrounds
- The proposed design is versatile enough to adapt to changes in the input parameters, β^* can be varied in a wide range (1.5cm – 10cm)
- Fringe fields are the major factor limiting the dynamic aperture, simulations with realistic profile are underway
- Longitudinal dynamics with wake-fields and beam-beam effect is another critical issue
- Lattice design: taking advantage of higher quad gradient available, halo extraction, separation at the symmetry point (if needed)

Higgs Factory Lattice Update



Optics functions in half ring for β*=2.5cm



Half ring layout

No reverse bends → circumference reduced to 283m (from 300m).

Difficulty in adjustment of the horizontal tune (now Qx=5.16, Qy=4.56).

Some problem with vertical dynamic aperture: 8σ → 6.5σ.

Higgs Factory Upgrade Parameters

Parameter	Unit	Baseline	Upgrade
Beam energy	GeV	63	63
Average luminosity	10^{31}/cm^2/s	1.7	8.0
Collision energy spread	MeV	3	4
Circumference, C	m	300	300
Number of IPs	-	1	1
β^*	cm	3.3	1.7
Number of muons / bunch	10^{12}	2	4
Number of bunches / beam	-	1	1
Beam energy spread	%	0.003	0.004
Normalized emittance, $\varepsilon_{\perp N}$	π·mm·rad	0.4	0.2
Longitudinal emittance, $\varepsilon_{\|\|N}$	π·mm	1.0	1.5
Bunch length, σ_s	cm	5.6	6.3
Beam size at IP, r.m.s.	mm	0.15	0.075
Beam size in IR quads, r.m.s.	cm	4	4
Beam-beam parameter	-	0.005	0.02
Repetition rate	Hz	30	15
Proton driver power	MW	4	4

Path Lengthening Effect

Hamiltonian with action-angle variables for the transverse motion and $z=s-v_0t$ and $\delta_p=(p-p_0)/p_0$ for longitudinal:

$$\mathcal{H} \approx \frac{1}{R}[Q_x(\delta_p)I_x + Q_y(\delta_p)I_y] - \eta_0 \frac{\delta_p^2}{2} - \frac{he}{2cp_0R}\sum_i V_{RF}\cos\varphi_s \cdot \delta(s-s_i)z^2$$

$$\eta_0 = \alpha_0 - \frac{1}{\gamma_0^2}$$

$$\mathcal{H}_\perp \approx \frac{1}{R}(Q_{x0}I_x + Q_{y0}I_y) + \frac{1}{R}(Q'_xI_x + Q'_yI_y)\delta_p + \frac{1}{2R}(Q''_xI_x + Q''_yI_y)\delta_p^2 + \dots$$

Equation of motion for $z=s-v_0t$:

$$\frac{dz}{ds} = \frac{d}{d\delta_p}\mathcal{H} = \frac{1}{R}(Q'_xI_x + Q'_yI_y) - [\eta_0 - \frac{1}{R}(Q''_xI_x + Q''_yI_y)]\delta_p + \dots$$

Looking for a stationary solution ($dz/ds=0$) we obtain betatron amplitude-dependent momentum shift $\Delta\delta_p$:

$$\Delta\delta_p \approx (Q'_xI_x + Q'_yI_y)/[\eta_0R - \cancel{(Q''_xI_x + Q''_yI_y)}]$$